

Design of a bench hardware-in-the-loop system for the study of chatter in turning

Iker Mancisidor, Rafael Bárcena, *Member IEEE*, Jokin Munoa and Ainhoa Etxebarria, *Member IEEE*

Abstract—The violent vibrations due to regenerative effects occurring in machining processes, also known as chatter, are a classical problem that limits the productivity in machine tools. These effects motivate a reduction of the practical life of the tool and mechanical components of the machine, and prevent obtaining required surface quality. Therefore, the chatter has been a prevalent topic for academic and industrial research. On the other hand, experimental investigations on the active reduction of chatter may be difficult in a real machining environment, given the exigency of numerous machining tests and the associated problems of repeatability, due to the involvement of a large number of practical parameters. In this paper, a mechatronic Hardware-In-the-Loop (HIL) simulator for chatter in turning is presented. Such system reproduces experimentally, on a simple mechanical structure in the laboratory, *certain* regenerative chatter, described by stability lobes previously obtained from the real turning machine of interest. Then, by adding inertial actuators to the HIL simulator, the performance of different algorithms for the active control of *such* chatter may be *realistically* tested.

I. INTRODUCTION

THE presence of regenerative vibrations in machining processes has turned out to be a classic, work rate limiting, problem. Such self-excited vibrations in a machine are usually ominous since, on the one hand, they avoid the procurement of the required quality of surface finish and, on the other hand, reduce the life of the tool and the mechanical components of the machine. In addition, such regenerative effect is a complex phenomenon, affected by the cut attributes (spindle speed, depth of cut, ...) and the dynamic properties of the machine structure (stiffness, inertia, damping,...), described firstly by Tobias [1] and Tlustý [2]. Then, Merritt [3] introduced a feedback model for the chatter, describing it as a closed loop interaction between the structural dynamics and the cutting process. In this context,

Manuscript received October 29, 2012. This work was supported in part by the European Community under the DYNXPPTS Project (No. FP7-2010-NMP-ICT-FoF-260073).

I. Mancisidor is with the Dynamics and Control department, Ik4-Ideko, Elgoibar, 20870, Basque Country, Spain (e-mail: imancisidor@ideko.es).

R. Bárcena is with the Electronic Technology department, University of the Basque Country UPV/EHU, EUITI, Bilbao, 48012, Basque Country, Spain (corresponding author: phone: +34 94 601 43 01; fax: +34 94 601 43 00; e-mail: rafa.barcena@ehu.es).

J. Munoa is with the Dynamics and Control department, Ik4-Ideko, Elgoibar, 20870, Basque Country, Spain (e-mail: jmunoa@ideko.es).

A. Etxebarria is with the Electronic Technology department, University of the Basque Country UPV/EHU, EUITI, Bilbao, 48012, Basque Country, Spain (e-mail: ainhoa.etxebarria@ehu.es)

the stability lobe diagram establishes the limiting values for the machining parameters (depth of cut and spindle speed), in order to maintain a stable cut. Therefore, such diagram may be used as the main means to describe the chatter effect observed in the machining process under study.

In order to enhance stability limits of a particular machining operation, several techniques have been developed. They could be divided into two methodologies: passive control (see, e.g. [1] and [4]-[12]) and active control ([13]-[21]). Passive devices can be appropriate in many cases, but they present limitations in others where operation parameters can vary considerably. Active control can overcome these limitations by means of its adaptability to changing conditions. So, the focus of this work is on the active control of chatter. These kinds of controllers are based on actuators, normally equipped with a sensor to measure the vibration, that allow us to produce a compensating force.

Nonetheless, the factual experiments on active chatter stabilization strategies may be very problematic, due to the large number of uncertain machining parameters and unavoidable cutting tests. Furthermore, the chatter characterization process always forces the whole system to reach the limit of the stability and this may decrement the life of the machine tool. In this way, this work is devoted to the development of a mechatronic Hardware-In-the-Loop (HIL) simulator for chatter in turning. Such HIL simulator reproduces experimentally, on a simple mechanical structure-resistant and well suited for the application of active controllers-, a *previously defined* regenerative chatter, in order to allow practical investigations on different active control laws.

The paper is organized as follows. Section II outlines the feedback model for the regenerative chatter. The detailed description of the HIL chatter simulator system is presented in section III, including the experimental results. Such results exhibit a significant deviation from the expected, due to the delay of the actuator. Section IV purposes a procedure to model and correct such deviation, in order to obtain exactly the same behavior than the real machine under study. Finally, the conclusions appear in section V.

II. REGENERATIVE CHATTER IN TURNING

According to Altintas [22], machine tool chatter vibrations result from a self-excitation mechanism in the

generation of chip thickness during machining operations. Cutting forces excite one of the structural modes of the machine-tool-workpiece system and a wavy surface is left behind on the workpiece.

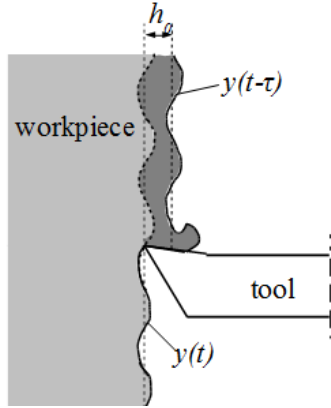


Fig. 1. Scheme of regenerative chatter vibrations in turning.

As showed in the figure 1, after a full rotation, the tool confront the mentioned waves and the resultant chip thickness $h(t)$ turn out to be time variant and dependant of the relative phase of the present $y(t)$ and previous $y(t-\tau)$ vibration displacements of the tool [22]

$$h(t) = h_0 - [y(t) - y(t - \tau)] \quad (1)$$

where h_0 is the constant feed of the tool into the workpiece and τ is the time of one full rotation. It is well known that the variable cutting force $F(t)$ may be considered proportional to the frontal area of the chip, that is, the product of the chip thickness $h(t)$ and the depth of the cut a_p

$$F(t) = K_f \cdot a_p \cdot h(t) \quad (2)$$

where K_f is the cutting coefficient of the process. Now, observing (1) and (2), the dynamic equation of motion is

$$m \cdot \ddot{y}(t) + c \cdot \dot{y}(t) + k \cdot y(t) = F_c(t) = K_f \cdot a_p \cdot [h_0 + y(t - \tau) - y(t)] \quad (3)$$

If the machine-tool transfer function between the force and the displacement is defined as $\Phi(s)$ in Laplace domain and making $y(t - \tau) = y(s)e^{-s\tau} = y_0(s)$, we have from (3)

$$\frac{h(s)}{h_0(s)} = \frac{1}{1 + K_f \cdot a_p \cdot \Phi(s) \cdot (1 - e^{-s\tau})} \quad (4)$$

being

$$\Phi(s) = \frac{y(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \quad (5)$$

From (4), a closed loop feedback diagram -figure 2- for regenerative chatter has been introduced by Merrit [3]. Such diagram may be used for analyzing the stability of the regenerative effect, from a control engineering perspective.

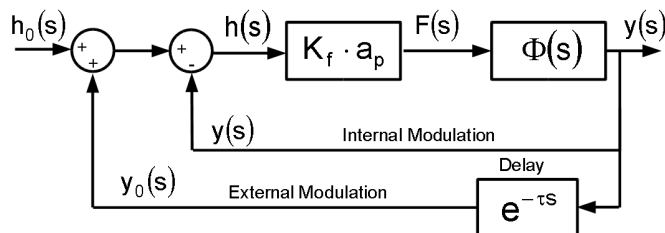


Fig. 2. Closed loop feedback Merrit's model for regenerative chatter.

The characteristic equation of the closed loop is then

$$1 + K_f \cdot a_p \cdot \lim \Phi(j\omega_c) (1 - e^{-j\omega_c \tau}) = 0 \quad (6)$$

By dividing equation (6) into the real and imaginary parts

$$\left\{ 1 + K_f \cdot a_p \cdot \lim \cdot \left[G \cdot (1 - \cos(\omega_c \tau)) - H \cdot \sin(\omega_c \tau) \right] \right\} + \quad (7)$$

$$+ j \left\{ K_f \cdot a_p \cdot \lim \cdot \left[G \cdot \sin(\omega_c \tau) + H \cdot (1 - \cos(\omega_c \tau)) \right] \right\} = 0$$

where $\Phi(j\omega_c) = G + jH$.

And, equaling both parts of equation (7) to zero, we have

$$1 + K_f \cdot a_p \cdot \lim \cdot \left[G \cdot (1 - \cos(\omega_c \tau)) - H \cdot \sin(\omega_c \tau) \right] = 0 \quad (8)$$

$$\frac{H}{G} = \frac{\sin(\omega_c \tau)}{\cos(\omega_c \tau) - 1} \quad (9)$$

Obtaining the limiting value for the depth of cut from (8) and using (9) to simplify the resulting expression, it follows

$$a_{p \lim} = \frac{-1}{2 \cdot K_f \cdot G(\omega_c)} \quad (10)$$

Expression (10) gives -see [3]- the limiting depth of cut as a function of the chatter frequency ω_c . Now, we have to describe the relationship between the chatter frequency and the spindle speed N . On the one hand, we define

$$\tan \psi = \frac{H}{G} \quad (11)$$

where Ψ is the phase of the system transfer function. In addition, we consider the trigonometric relations

$$\cos(\omega_c \tau) = \cos^2\left(\frac{\omega_c \tau}{2}\right) - \sin^2\left(\frac{\omega_c \tau}{2}\right) \quad (12)$$

$$\sin(\omega_c \tau) = 2 \sin\left(\frac{\omega_c \tau}{2}\right) \cdot \cos\left(\frac{\omega_c \tau}{2}\right)$$

By using (9) and (12) in (11), it follows

$$2\psi = \omega_c \tau - 3\pi \Rightarrow 2\pi \cdot f_c \cdot \tau = 2\psi + 3\pi \quad (13)$$

On the other hand, if ε is the phase difference between successive undulations on the workpiece surface and k is the number of complete waves during one period of revolution τ

$$\omega_c \cdot \tau = 2\pi f_c \cdot \tau = 2\pi \cdot k + \varepsilon \quad (14)$$

Note that, by using equations (13) and (14), we can obtain a relation between the system transfer function and ε

$$\varepsilon = 2\psi + 3\pi - 2\pi k \quad (15)$$

Finally, by obtaining τ from (14)

$$\tau = \frac{2\pi \cdot k + \varepsilon}{2\pi \cdot f_c} \quad (16)$$

and considering the spindle speed $N = 60/\tau$ in r.p.m., we have the relation between the chatter frequency and the spindle speed N - see [3]-

$$N = \frac{60}{\tau} = \frac{120\pi \cdot f_c}{2\pi \cdot k + \varepsilon} = \frac{120\pi \cdot f_c}{2\psi + 3\pi} \quad (17)$$

I. HARDWARE-IN-LOOP CHATTER DEMONSTRATOR

The regenerative feedback model of chatter presented in figure 2 yields the conceptual idea of the HIL simulator. A steel cantilever beam represents the dynamics for a turning machine $\Phi(s)$ and an actuator ("shaker") applies the cutting force $F(t)$ -expression (2)- to it. A sensor installed on the

beam provides the vibration displacement $y(t)$ and a fast processor, included on a programmable automation controller (PAC), calculates $F(t)$ in real-time. The whole scheme is presented in figure 3. It is inspired in the interesting work by Ganguli [23].

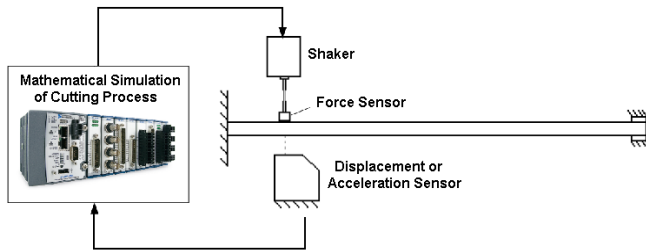


Fig. 3. HIL chatter demonstrator scheme to be implemented.

A. Mechanical system design.

It is well-known that the insertion of energy on flexible structures, near their natural frequencies, is very difficult for shakers, due to the deterioration of the ratio force/voltage (F/V) to such frequencies. This fact is very important in this project, given that the chatter frequencies appear normally in the vicinity of the resonant frequencies of the mechanical structure.

The deteriorated ratio F/V could be avoided by incrementing the stiffness -short cantilever-, but that makes the natural frequencies of the structure to increase and, if the frequencies are too high, the delay of the shaker could introduce important problems of phase differences between the signals of the closed loop.

Therefore, the cantilever has been designed sufficiently long to maintain the natural frequencies low enough and the damping has been increased, in order to conserve the ratio F/V, by fixing the loose end of the beam by means of two pieces of polyurethane (*Sylomer*). In addition, the exciting force is applied near the clamping point of the beam in order to augment the stiffness, without boosting the natural frequencies. The final design for the mechanical structure may be observed in figure 4. The dynamical behavior of the designed structure is described on figure 5 and table 1.

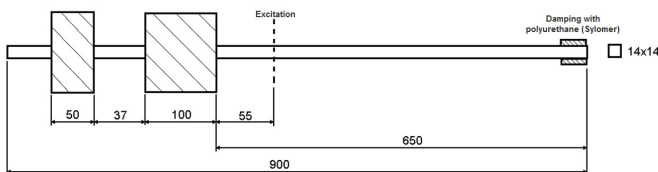


Fig. 4. Proposed mechanical structure design.

The characterization of the shaker applied to the structure is exposed in figure 6. It may be observed that the ratio F/V has been preserved and appears quasi-linearly frequency dependant (5.2 N/V in the vicinity of the first natural frequency).

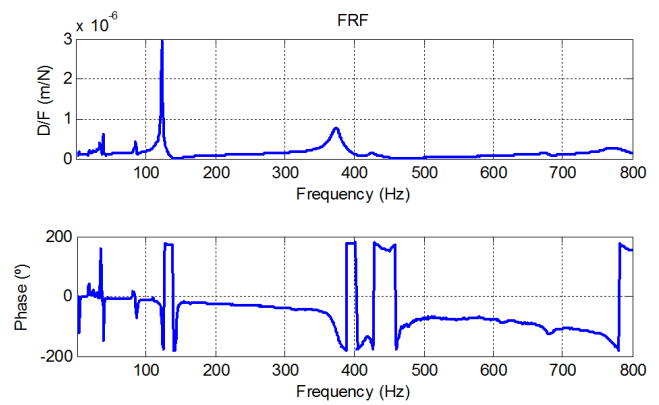


Fig. 5. Mechanical structure dynamical behavior, described by a frequency response function (FRF).

TABLE I
DYNAMICAL PARAMETERS OF THE STRUCTURE

	ω_n [Hz]	ξ [%]	k [N/m]
1	122,879	0,672	$3,0809 \cdot 10^7$
2	374,400	1,594	$3,4802 \cdot 10^7$

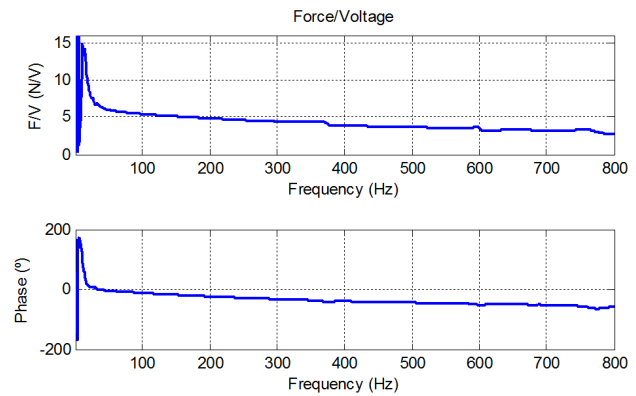


Fig. 6. F/V ratio and phase of the shaker versus frequency.

B. Practical implementation and equipment.

The equipment used for the implementation of the HIL chatter demonstrator is summarized in figure 7.

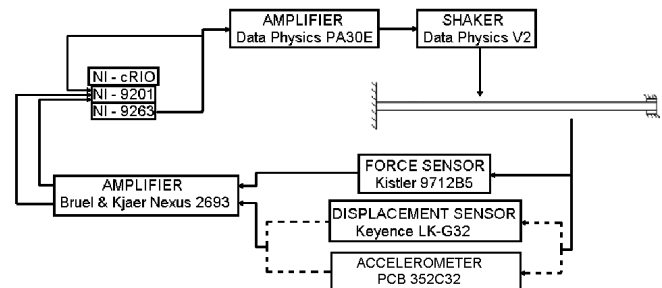


Fig. 7. HIL chatter demonstrator equipment.

As may be observed in figure 7, the *NI CompactRIO* has three input signal, although only the vibration (displacement sensor or accelerometer) signal is imperative to the correct operation of the HIL demonstrator. The other two signals (applied force sensor and computed force output) are collected for informative reasons.

Note 1: The vibration signal has been collected by two alternative sensors during the experimentation period. The results presented here have been obtained by using the laser displacement sensor LK-G32 for measuring the vibration displacement, given its best ratio signal/noise and the simpler computation associated to such signal (double integration is unnecessary). Anyway, the accelerometer 352C32 may be also interesting to such kind of applications, given its low-cost and dissemination.

Sampling period has been fixed to $T=50 \mu s.$, corresponding to less than a 1 degree resolution if the spindle speed in turning is supposed to be less than 3000 r.p.m.

Note 2: The computation of the chatter model has been implemented on the FPGA Virtex-5 LX 85 of the *NI CompactRIO 9022*, given the high timing requirements of this closed-loop in order to obtain realistic results, that is, the HIL demonstrator should behave very similar to the real turning machine.

The described HIL chatter demonstrator has been carried out in the laboratory and may be observed in figure 8. Once the beam vibrates as the tool of a turning machine suffering regenerative chatter, an inertial actuator may be attached to it in order to test the performance of different active control laws [24].

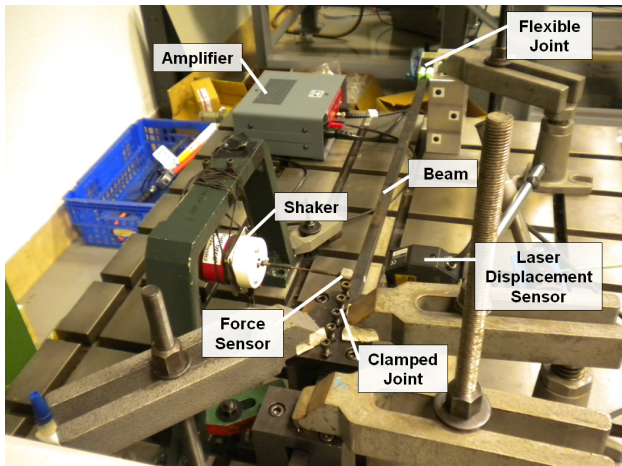


Fig. 8. HIL chatter demonstrator.

C. Experimental results.

The constant feed of the tool into the workpiece and the cutting coefficient of the metal have been considered constants ($h_0=1e^{-6}$ m. and $K_f=2500$ N/mm², respectively) in the implementation of the regenerative effect model. Then, for different values of the spindle speed N , the depth of cut a_p has been increased until the vibration becomes unstable (the modeled breaking of the chip saturates the vibration within some limits). In this way, the experimental stability lobes have been obtained, formed by discrete points, marked as crosses, and are presented in figure 9. By measuring the frequency of the vibration in such limiting points, figure 10 is configured.

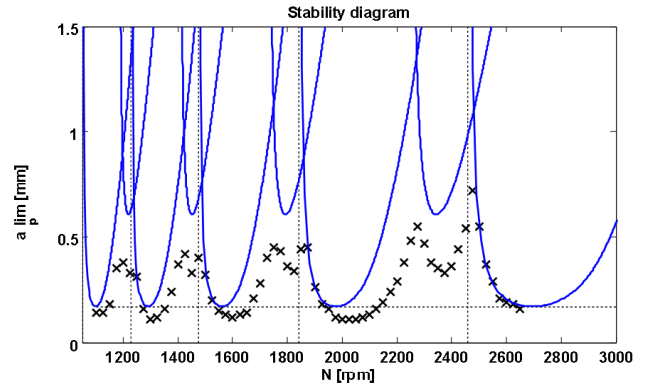


Fig. 9. Theoretical and experimental results: Critical depth of cut.

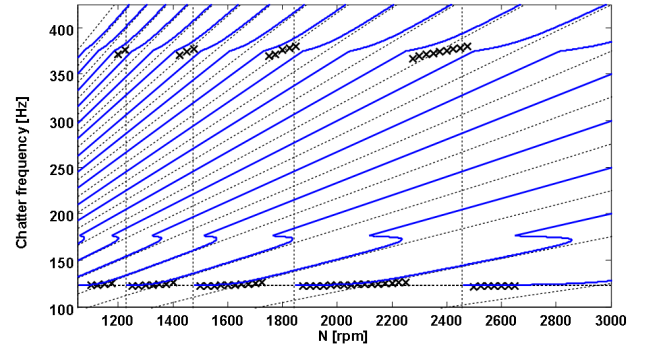


Fig. 10. Theoretical and experimental results: Chatter frequency.

Then, in order to compare experimental and theoretical results, the lobes for this structure have been obtained from expressions (10) and (17), by giving values to N and k . Such stability lobes are also depicted in blue in figures 9 and 10. A noticeable deviation is observed between experimental and theoretical results, partly preventing the desired behavior of the HIL demonstrator from occur.

II. ACTUATOR DELAY MODELLING AND COMPENSATION

A. Actuator delay modeling.

At this point, the delay of the actuator, described in the phase diagram of figure 6, is put forward as the main source of the discrepancies observed in figures 9 and 10. In order to confirm (or not) such hypothesis, the actuator delay τ_a has been got into the regenerative chatter model -see figure 11-.

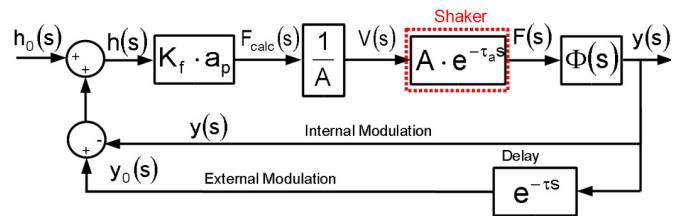


Fig. 11. Closed loop feedback model for regenerative chatter with the actuator delay included.

In this way, the transfer function describing such system is

$$\frac{h(s)}{h_0(s)} = \frac{1}{1 + K_f \cdot a_p \cdot \Phi(s) \cdot e^{-s\tau_a} \cdot (1 - e^{-s\tau})} \quad (18)$$

And operating from (18) in the same fashion as in section II from expression (4), we have

$$a_{p \text{ lim}} = \frac{-1}{2 \cdot K_f \cdot [G(\omega_c) \cos(\omega_c \tau_a) + H(\omega_c) \sin(\omega_c \tau_a)]} \quad (19)$$

that describes the limiting values for the depth of cut in order to maintain the stability of the system when the delay of the shaker is considered. Now, the stability lobes are again obtained and presented in red -see figures 12 and 13-

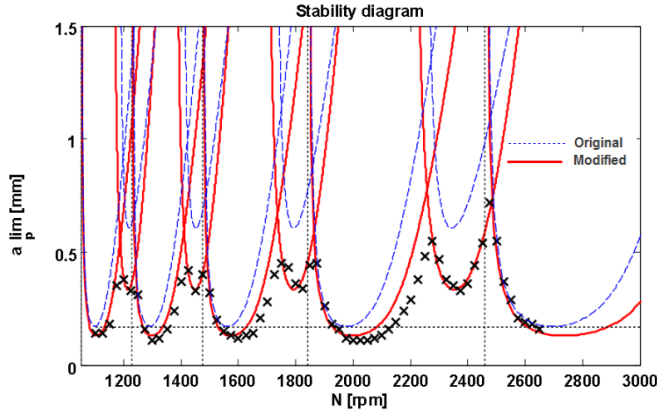


Fig. 12. Theoretical and experimental results with the actuator delay included: Critical depth of cut.

As may be observed, now the correspondence with the experimental results has been greatly enhanced -the remaining differences are probably due to errors in the factual delay measure τ_a - and the importance of the actuator delay has been confirmed.

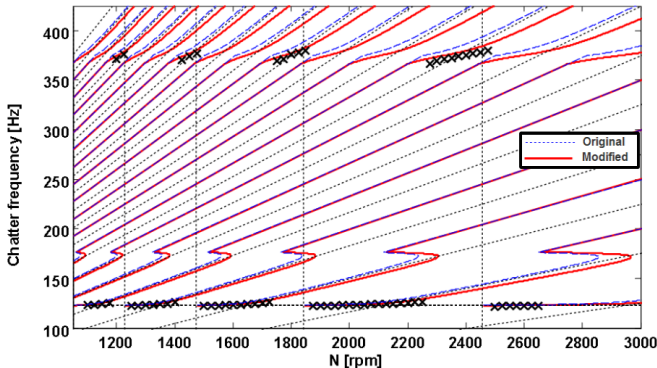


Fig. 13. Theoretical and experimental results with the actuator delay included: Chatter frequency.

B. Actuator delay compensation.

As it has been proved in the previous subsection, the delay of the actuator means an important drawback in order to reproduce *exactly* the behavior of the assumed feedback model of regenerative chatter.

At this point, it is useful to recall that the main goal is to reproduce, as exactly as possible, on the simple mechanical structure located at our lab, the behavior described by some stability diagrams -for example, the blue curves in figures 9 and 10-

Bearing in mind the previous facts, a procedure for the compensation of the actuator delay, via software, is presented. The flexibility of the HIL demonstrator, provided

by the system software, will be used to compensate the delay imposed by the hardware. Specifically, two parameters - the cutting coefficient of the process (K_f) and the period of revolution (τ)-, considered *a priori* as prefixed (for each workpiece-tool and value of the spindle speed), will be used as *fictitious* variables in order to compensate the delay of the actuator (and other components, if necessary). The mentioned procedure is divided into four sequential steps:

Step 1: Procurement of the feasible chatter frequencies for each N (rpm): First, from any given value of the spindle speed, a set of feasible values for the chatter frequency (f_c) is obtained from the blue f_c/N curves of figure 10 -the lobes we want to reproduce-, related with the number of full waves between cuts (k).

Step 2: Election of the actual chatter frequency for each N (rpm): By using the a_{plim}/N functions we want to implement -blue curves of figure 9-, the factual value of k is obtained (corresponding to the lowest stability lobe) and, by using such k , the exact f_c (accuracy is important) is elected from the set obtained in step 1.

Step 3: The compensating delay is obtained: By using the values of f_c and k obtained in the previous step, the compensating delay between successive passes of the tool is

$$\tau' = \frac{2\pi k + \varepsilon'(f_c)}{2\pi f_c} \quad (20)$$

where, evidently, now τ' do not match $60/N$, but will be the value introduced in the HIL in order to compensate the hardware delay. The compensating phase between subsequent undulations $\varepsilon'(f_c)$ is calculated, considering the delay of the shaker τ_a , by

$$\varepsilon'(f_c) = 2\Psi'(f_c) + 3\pi - 2\pi k \quad (21)$$

given that

$$\Psi' = \arctan \frac{H'}{G'} \quad \wedge \quad \Phi'(s) = \Phi(s)e^{-s\tau_a} = G'(\omega_c) + jH'(\omega_c) \quad (22)$$

The expressions presented above are obtained from the imaginary part of the stability condition of the model with the actuator delay included. Specifically, (20) is deduced on the analogy of (16), (21) on (15) and (22) on (11).

Note 3: The accuracy on the value τ_a is important, given the observed sensibility of the system to such value. Therefore, the function of the phase of the actuator related to the frequency of the signal, included in figure 6, is used to obtain τ_a , given that such frequency is known since step 2. In order to increase even more the accuracy, other known hardware delays (computational, sensors,...) may be added to the hardware delay.

Step 4: The compensating force “constant” of the process is obtained: Finally, by using the value of f_c obtained in the step 2, the compensating force “constant” K'_f for the studied process is calculated by equalizing equations (19) and (10)

in order to implement the stability lobes "without" the deviation due to the delay of the shaker:

$$a'_{p\lim} = \frac{-1}{2 \cdot K'_f \cdot [G(\omega_c) \cos(\omega_c \tau_a) + H(\omega_c) \sin(\omega_c \tau_a)]} = a_{p\lim} = \frac{-1}{2 \cdot K_f \cdot G(\omega_c)} \quad (23)$$

Resulting in:

$$K'_f = K_f \frac{G(\omega_c)}{[G(\omega_c) \cos(\omega_c \tau_a) + H(\omega_c) \sin(\omega_c \tau_a)]} \quad (24)$$

Remarks:

- The "compensating" items (20) and (24) must be recalculated for each value of the spindle speed. In this way, they turn out to be factual *variables* in the HIL software. Until now, the delay between passes and the force constant for the process have been considered as fixed parameters on the HIL chatter demonstrator.

- Evidently, when the delay of the shaker τ_a tends to 0, $\varepsilon'(f_c)$ tends to $\varepsilon(f_c)$ and τ' to $\tau=60/N$. In the same fashion K'_f will converge to K_f .

III. CONCLUSIONS

Since chatter reduction is a very specialized field with deep technological content, practical experimentation regarding this matter is especially relevant. Unfortunately, the factual experiments on active chatter reduction control laws used to be very problematical, because a large number of cutting tests are normally necessary and such tests require the whole system to become unstable several times. In this way, the life of the machine-tool may be considerably reduced. In addition, a large number of machining parameters are in practice rather uncertain and a monitored context for experimentation is very interesting, in order to improve the reliability of the obtained results.

In this paper, a complete hardware-in-the-loop (HIL) mechatronic simulator -chatter demonstrator- has been presented in depth. Such system allows us to reproduce, on a simple mechanical structure -specifically designed for the matter-, the regenerative chatter occurred on a real turning machine. On the one hand, the system grants the detailed comprehension of the regenerative effect. On the other hand, the experimentation on active chatter reduction controllers is considerably facilitated. The main difficulty presented during the design and implementation of such chatter demonstrator is related to the delay of the system actuator. Such drawback has been overcome by compensating -once modeled-, via software, the delays introduced by the hardware of the system.

REFERENCES

[1] S.A. Tobias and W. Fishwick, "Theory of regenerative machine tool chatter". *The Engineer*, vol. 205, 1958.
 [2] J. Tlustý and M. Poláček, "The stability of machine tools against self-excited vibrations in machining". *International Research in Production Engineering*, pp. 465-474, 1963.
 [3] H. E. Merrit, "Theory of self-excited machine-tool chatter-contribution to machine tool chatter research". *ASME Journal of Engineering for Industry*, vol. 87, no. 4, pp. 447-454, 1965.

[4] Y. Altintas and M. Weck, "Chatter Stability of Metal Cutting and Grinding". *CIRP Annals – manufacturing Technology*, vol. 53, no. 2, pp. 619-642, 2004.
 [5] J. Slavicek, "The Effect of Irregular Tooth Pitch Stability of Milling". *Proceedings of the 6th MTDR Conference*, pp. 15-22, 1965.
 [6] Z. Dombovari, Y. Altintas, and G. Stepan, "The effect of serration on mechanics and stability of milling cutters". *International Journal of Machine Tools & Manufacture*, vol. 50, pp. 511-520, 2010.
 [7] S. Smith and J. Tlustý. "Stabilizing chatter by automatic spindle speed regulation". *CIRP Annals*, vol. 41, no. 1, pp. 433-436, 1992.
 [8] I. Bediaga, J. Muñoa, J. Hernández, and L.N. López de Lacalle, "An automatic spindle speed selection strategy to obtain stability in high-speed milling", *International Journal of Machine Tools & Manufacture*, vol. 49, pp. 384-394, 2009.
 [9] T. Hoshi, N. Sakisaka, I. Moriyama, and M. Sato, "Study of practical application of actuating speed cutting for regenerative chatter control". *CIRP Annals*, vol. 25, pp. 175-179, 1977.
 [10] B. J. Sexton, J. S. Stone, "The stability of machining with continuously varying spindle speed". *Annals of the CIRP*, 27, pp.: 321-326, 1978.
 [11] M. Zatarain, I. Bediaga, J. Muñoa, and R. Lizarralde, "Stability of milling processes with continuous spindle speed variation: Analysis in the frequency domains, and experimental correlation". *CIRP Annals – Manufacturing Technology*, vol. 57, pp. 379-384, 2008.
 [12] N.D. Sims, "Vibration absorbers for chatter suppression: A new analytical tuning methodology", *Journal of Sound and Vibration*, vol. 301, pp. 592-607, 2007.
 [13] A. Cowley and A. Boyle, "Active dampers for machine tools". *CIRP Annals*, vol. 18, pp. 213-222, 1970.
 [14] S.G. Tewani, K.E. Rouch, and B.L. Walcott, "A Study of Cutting Process Stability of a Boring Bar with Active Dynamic Absorber". *International Journal of machine Tools & Manufacture*, vol. 35, no. 1, pp. 91-108, 1995.
 [15] J.R. Pratt, "Vibration Control for Chatter Suppression with Application to Boring Bars". *PhD Thesis, Virginia Polytechnic Institute and State University*. 1997.
 [16] C. Ehmann and R. Nordmann, "Low Cost Actuator for Active Damping of Large Machines", *IFAC Conference on Mechatronic Systems*, December 2002, Berkeley, California.
 [17] C. Ma, J. Ma, E. Shamoto, and T. Moriwaki, "Analysis of regenerative chatter suppression with adding the ultrasonic elliptical vibration on the cutting tool", *Precision Engineering*, vol. 35, no. 2, pp. 329-338, April 2011.
 [18] C. Nachtigal, "Design of a force feedback chatter control system". *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 94, no.1, pp. 5-10, 1972.
 [19] Y. S. Tarn, J. Y. Kao, and E. C. Lee. "Chatter suppression in turning operations with a tuned vibration absorber". *International Journal of Materials Processing Technology*, vol. 105, pp. 55-60, 2000.
 [20] A. Bilbao-Guillerna, A. Azpeitia, S. Luyckx, N. Loix, and J. Muñoa, "Low Frequency Chatter Suppression using an Inertial Actuator", *Proceedings of 9th International Conference on High Speed Machining 2012*, 7-8 March 2012, San Sebastian, Spain.
 [21] A. Harms, B. Denkena, and N. Lhermet. "Tool adaptor for active vibration control in turning operations". *Actuator 2004*, pp. 694-697, Bremen, Germany, 2004.
 [22] Y. Altintas, "Manufacturing Automation. Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design", *Cambridge University Press*, New York, 2000.
 [23] A. Ganguli. "Chatter reduction through active vibration damping". *PhD Thesis, Dept. Mechanical Eng., Université Libre de Bruxelles*. 2005.
 [24] J. Munoa, I. Mancisidor, N. Loix, L.G. Uriarte, R. Bárcena, M. Zatarain "Chatter suppression in ram type travelling column milling machines using a biaxial inertial actuator", *CIRP Annals – Manufacturing Technology*, 2013. (Approved for publication).