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A multiple model-based controller for NOx reduction in a Selective Catalytic Reduction system

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Abstract— **This paper deals with the design of a multiple model based controller for the nitrogen oxide emissions (NO_x), from vehicles using the selective catalyst as an aftertreatment system. The selective catalyst reduction (SCR) system is nonlinear, since the chemical reactions involved are highly depending on the operating points. Local linear models were used for identification. Local internal controllers are integrated into a global controller. A Bayesian approach is used to mix the local output of the controllers. A detailed simulator is used for the multiple model identification and testing the controller. For validation, experimental data based on a standard transient test developed for Euro VI testing are used in the simulator. Results obtained for this control approach are compared to one model controller.**

I. INTRODUCTION

The reduction of NO_x emissions is a worldwide problem. The harmful emissions contribute, among others, to the formation of acid rain and ground level ozone. In order to reduce these adverse effects, emission standards for a number of atmospheric pollutants, such as NO_x, have been established decades ago and are stringently updated. Recently, [1] introduced Euro VI emission standards which are in effect from January 2013. Euro VI emission standards propose a NO_x emission limit value of 0.4 g/kWh for heavy-duty engines.

Among the current technologies for NO_x reduction, selective catalytic reduction systems, SCR, is the more convenient one for heavy-duty applications. SCR commonly consist of a honeycomb monolith where the exhaust gas is combined with a dosified reduction agent, e.g. urea, causing several chemical reactions to take place. The SCR system presents high nonlinearities and is subject to a large range of disturbance variations. These variations are a product of the driving profile of the vehicle, i.e. speed and load, resulting in variations of gas flow, temperature and NO_x concentrations.

Several modelling approaches have been developed for SCR systems, they can be divided into principle-based models (see e.g. [2], [3]), and black-box models (see e.g. [4], [5]). The first group usually contains a detailed set of differential equations and requires specific tests for obtaining the model parameters, on the other hand, the second group usually involves less equations and parameters to be obtained, and also the use of several identification techniques. From the control point of view, it is attractive to obtain simpler models that can more easily be used in controller design.

An interesting analysis of the closed loop SCR control from a practical control point of view is illustrated in [6], where also three control strategies based on static maps are evaluated using a simulator. [7] presents a combination of feed-forward and feedback control structures for a SCR process. Software sensors are used for measurements of the nitric oxide concentration at the input of the catalytic converter and the flue gas flow rate. [8] presents a model-based feed-forward controller. The issue of the cross-sensitivity in NO_x sensors is analysed in [9]. Ammonia coverage ratio is estimated by the use of nonlinear observers in [10]. These works highlight the need of using advanced control strategies to meet the stringent requirements.

The multiple model approach for modelling and control of nonlinear processes, mainly chemical processes, have been studied previously ([11], [12], [13], [14]). The main idea is to represent the nonlinear system as a combination of the linear models at the different operating points. They can be combined by switching rules or by using weights. Linear models allows the use of linear control techniques, that simplifies the controller design task and brings benefits as a low computational load and an easy real implementation. One of the first questions that arises is how many linear models are required and also what are their respective operating ranges. There are some mathematical techniques that provide this information, however in some applications, the operating ranges can be defined based on the knowledge of the process or on simple experimental test. When multiple linear models are used, then the controller can be designed using well-known linear design control techniques. The controller can use the bank of models for tuning one controller or can use a bank of controller to produce the input signal.

The SCR has a number of features that makes appropriate the application of multiple model-based controllers: i) the system is nonlinear, ii) the system has few variables for the operating point characterization, iii) the system requires of controllers with low computational load.

The main contributions of this paper consist of identification and control design of an SCR system based on the multiple model approach. The multiple model-based controller is designed for NO_x reduction in the SCR system, that uses the injected urea as control signal. The operating regions of the system are characterized based on the load and the injected urea to the system such that the change in dynamics of the system is significant. ARX models are identified for each operating regions. Internal model controllers are designed for each local model. A Bayesian approach has been used to mix the local output of the controllers. The modelling and control approaches are done using a detailed simulator provided by Scania AB. Real signals for speed and load of the engine are used as inputs of the simulator for validation purposes.

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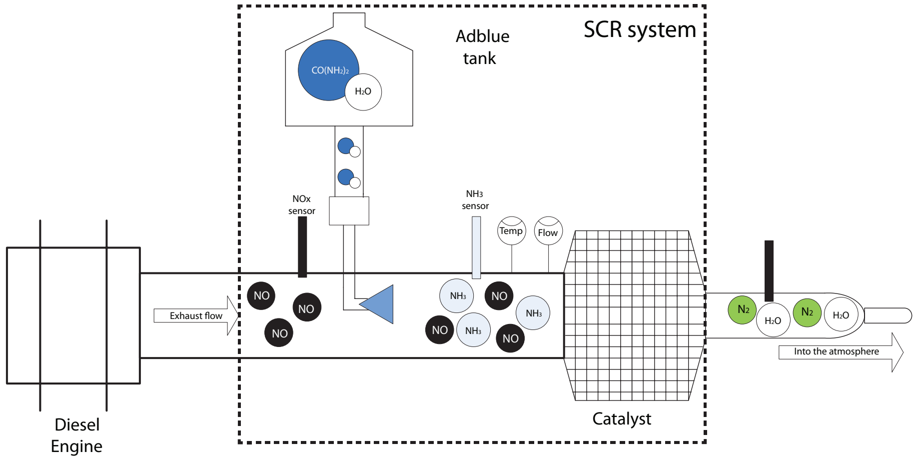

Fig. 1. SCR system.

The structure of the paper is as follows: Section II explains briefly the system, the simulator and the data used for validation. Section III describes the multiple model approach for identification, and the controller design is given in Section IV. Section V applies the multiple model approach and the controller to the SCR system. The experimental results are shown in Section VI and finally the conclusions are summarised in Section VII.

II. The process

A. Selective Catalytic Reduction system

For automotive applications, the selective catalytic reduction exhaust gas aftertreatment system commonly consists of a honeycomb monolith, where several chemical reactions take place. The system utilizes ammonia as a reduction agent. Ammonia is injected into the system using a dosification system for NO_x reduction.

Fig. 1 shows the main components of the system: the urea injection system, the catalyst and the sensors for exhaust flow, temperature and concentrations.

The SCR system consists of two main stages. Firstly, a reduction agent, in this case urea, is injected upstream through a nozzle and mixed with the exhaust flow at the input of the catalyst. Urea is contained in a harmless aqueous solution commercially named AdBlue, which consist of 32.5% urea. Urea is converted to ammonia as is shown in the following reaction

$$CO(NH_2)_2 + H_2O \rightarrow 2NH_3 + CO_2 \tag{1}$$

Secondly, the ammonia is partially adsorbed on the surface of the catalyst, where finally the dominant reactions occur, i.e. the ammonia reacts with the NO_x emitted by the engine (NO_x is composed primarily of NO with lesser amounts of NO_2) in order to get nitrogen gas (N_2) and water (H_2O) as final products:

- Standard SCR reaction

$$4NH_3 + 4NO + O_2 \rightarrow 4N_2 + 6H_2O \tag{2}$$

- Fast SCR reaction

$$4NH_3 + 2NO + 2NO_2 \rightarrow 4N_2 + 6H_2O \tag{3}$$

- NO_2 SCR reaction

$$8NH_3 + 6NO_2 \rightarrow 7N_2 + 12H_2O \tag{4}$$

The optimal operation of a SCR system requires taking into account the complex relationships governing the products formation. There are many factors affecting these formations. The amount of formed NH_3 depends on temperature and space velocity [15]. The homogeneous injected urea distribution improves the NO_x conversion. Some undesirable products can be formed depending on the temperature value (see [6]), NH_3 oxidation is produced for high temperatures, i.e. temperatures above 450°C, and ammonium nitrate (NH_4NO_3) and nitrous oxide (N_2O) are formed at respectively temperatures below 200°C and above 450°C. In automotive applications the temperature can vary significantly, from the cold start of the engine until hot operating conditions.

B. The simulator

The simulator layout is composed by static engine maps that provide the values of NO_x inlet concentration, $u_{1,r}$, inlet temperature, $u_{3,r}$ and exhaust flow, $u_{4,r}$ based on the input values of speed and load. These three signals together with the injected urea $u_{2,r}$, i.e. the control signal, are the inputs of a detailed kinetic SCR model. The output is the NO_x output concentration, y_r at the tailpipe, that is the controlled variable.

C. Validation data

The World Harmonized Transient Cycle (WHTC) [16] are used for Euro VI testing. For this reason, it was used for the identification and control validation in this paper. The WHTC is a transient test, as shown in Fig. 2 which specifies engine speed and load. It starts with a highly transient part, i.e. urban driving, and ends with higher load and less transience, i.e. highway driving.

III. Multiple model identification approach

A. Local models

A multiple model approach is the characterization of a system for the entire operating region by splitting it into

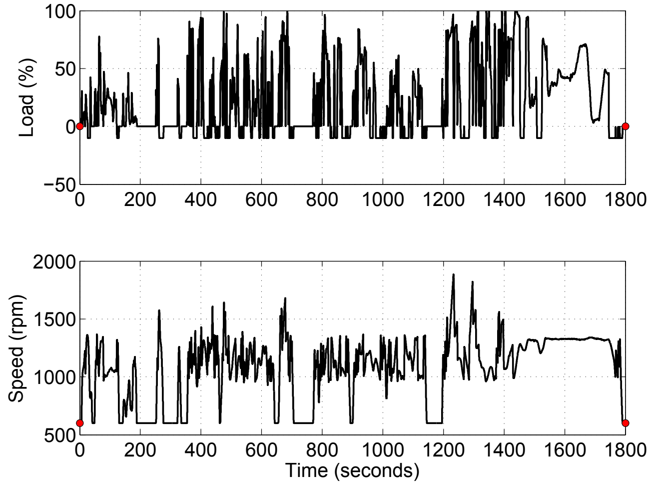

Fig. 2. Inputs of validation data.

small operating regions, where local linear models behaves very well.

In this paper, the multiple model approach is based on output error (OE) models, the local linear models have the general form for a general multiple-input single-output (MISO) system given in

$$y(t) = \sum_{l=1}^{n_u} \frac{B_l(z^{-1})}{F_l(z^{-1})} u_l(t) + v(t) \tag{5}$$

where $y(t)$, $u_l(t)$ and $v(t)$ are the system output, inputs and noise respectively, and n_u is the number of inputs for a MISO system. $B_l(z^{-1})$ and $F_l(z^{-1})$ are polynomials defined in the backward shift operator z^{-1}.

B. Model weight calculation

The weights provide information about how close the model outputs are to the real measured output. The use of weights for choosing the contribution of each model has advantages such as avoiding the switching between models, which something produce undesirable changes on the output of the system. The weights are calculated based on residuals of each local model. The residuals are given by (6):

$${}^i\varepsilon_k = y_k - {}^iy_{k|k}, \tag{6}$$

where ${}^iy_{k|k}$ is the output for i^{th} local model and y_k is the measured output of the system.

The model weight calculation is based on Bayes' probability theorem given by

$${}^ip_k = \frac{e^{(-\frac{1}{2}{}^i\varepsilon_k^T \Lambda {}^i\varepsilon_k)}\,{}^ip_{k-1}}{\sum_{j=1}^{n} e^{(-\frac{1}{2}{}^j\varepsilon_k^T \Lambda {}^j\varepsilon_k)}\,{}^jp_{k-1}}, \tag{7}$$

where ${}^ip_k$ represents the probability of the i^{th} model for representing the model at the k^{th} time step. Λ is a diagonal scaling matrix for the residuals, which is related to the covariances of each model and is considered as a tuning parameter.

The model weights are calculated using the normalised probabilities such that the sum of all weights is equal to 1. The model weights are calculated according to the following formula

$${}^iw_k = \begin{pmatrix} \frac{{}^ip_k}{\sum_{j=1}^{n} {}^jp_k} & \text{for} & {}^ip_k > \mu \\ 0 & \text{for} & {}^ip_k \leq \mu \end{pmatrix}. \tag{8}$$

It is clear that the probability calculation is of a recursive manner, hence it is possible that at any time for the probability to reach zero. Once that happens, the probability will never be able to adapt to changes and the value will remain zero. For this reason, the value μ is introduced in (8), which works as a lower limit for the probability. This way, that certain probability will not give rise to any weighting of the output or the controlled input but will contribute to future computation of the probability.

C. Global output

The global output of the system is given by

$$\hat{y}_k = \sum_{i=1}^{m} {}^iw_k \, {}^i\hat{y}_{k|k}, \tag{9}$$

where m is the number of local regions. Here, ${}^iw_k$ is the weight of the i^{th} model at time step k, and ${}^i\hat{y}_{k|k}$ is the predicted output from model i at step k.

IV. Multiple model-based controller

Internal Model Control, IMC, has become one of the most popular control design tool due to its disturbance rejection properties and the simple controller design procedure. In this paper, our focus is on using the IMC for control of the plant using the given models at each local region. The control structure for the proposed strategy is represented in Fig.3.

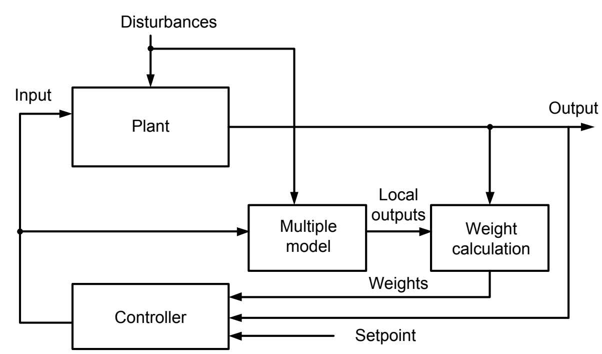

Fig. 3. Control scheme of the complete system.

A. Internal Model Control

Fig. 4 illustrates the IMC scheme which is directly based on the plant model, [17]. The advantages of the IMC structure is mainly the stability properties of the controller, which is solely dependent on the plant model. Also, the structure is very simple to design and tune. IMC can also be modified for non-minimum phase systems such that stable controllers are obtained. The IMC controller can also be transferred into a feedback system since it is mathematically equivalent to the feedback controller, [17]. For instance, the system can represent a PI controller if the model is a first order time-lag process and a PID controller if it is a second order process.

The general structure of the IMC, i.e. the transfer function from the reference input r to the error $e = r - y$, is given by

$$G_f = (I - G_q G)^{-1} G_q, \tag{10}$$

where G is the plant model and G_q is a tunable filter.

The IMC design procedure differs for different type of systems. Here a brief description of them are given as follows:

1) Nominal case: In the nominal case, the filter G_q can be chosen as the inverse of the process model

$$G_q = G^{-1} \tag{11}$$

leading to a controller reaching infinity. Even though this structure is not possible due to the fact that the controller reaches infinity it also might cause problems due to existence of more poles than zeros in G, however it can be used as a guideline for generation of G_q.

2) Stable minimum phase system: If the system is stable and minimum phase, and if G has more poles than zeros, the design process consists of tuning the controller using a low-pass filter. This is useful for reducing the gain for higher frequencies.

$$G_q = \left(\frac{1-\rho}{z-\rho}\right)^n G^{-1}. \tag{12}$$

Since the inverse of G is generally not physically possible, n must be chosen such that the degree of the denominator is less than or equal to the degree of the numerator and the resulting system becomes causal. Next, ρ is a tuning parameter that designs the bandwidth frequency of the closed loop system and defines the desired closed-loop behaviour.

3) Stable non-minimum phase: If the system is non-minimum phase, i.e. has zeros that are outside the unit circle, the above procedure cannot be directly used for control design because it will lead to unstable poles in the controller. In this case, there are several methods available for overcoming this problem. For instance, the unstable zero could either be omitted or be replaced by a stable version.

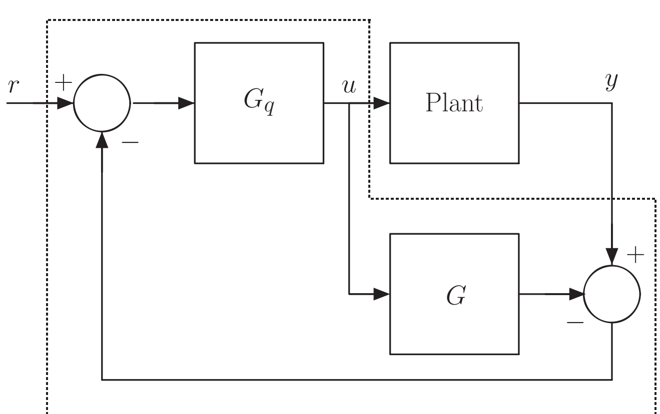

Fig. 4. Internal model control block diagram.

TABLE I
VALUES USED FOR DEFINITION OF REGIONS

Region	Load (%)	ANR
R1	-10@15	0@0.5
R2	-10@15	0.5@1
R3	15@40	0@0.5
R4	15@40	0.5@1
R5	40@100	0@1

In this paper, the model of region two is non-minimum phase, and to overcome the unstable controller it gives rise to, the unstable zero $(z + \beta)$ is replaced in the design of G_q by $(\beta z + 1)$, by using an all-pass decomposition. This way, the zero is reflected into the unit circle since $\frac{1}{\beta} < 1$.

B. Multiple model controller design for SCR

Using the IMC control design structure mentioned above, a controller is defined for each operation region based on the linear model obtained at that certain region. The controller output at each region is given by

$$^{i}u_{2,k} = \left(\frac{1-{}^{i}\rho}{z-{}^{i}\rho}\right)^n \frac{^{i}F(z^{-1})}{^{i}B_2(z^{-1})} e. \tag{13}$$

The output of each controller is then weighted using the same weights that were defined in (8) and the controller output is given by a weighted output of the local controllers.

$$u_{2,k} = \sum_{i=1}^{m} {}^{i}w_k {}^{i}u_{2,k} \tag{14}$$

V. SETUP

A. Operating regions

The temperature plays an important role in the reaction rate of the SCR. From the static engine maps in the Scania AB simulator, it can be noticed that it is mainly affected by the load. The main idea is to define the regions where the dynamics are fairly linear and no significant changes are notable. To be able to define these regions, different steps for the load and the injected urea were used as input signals to the system. For all regions, the speed is assumed to vary between 600 and 2300 rpm and does not affect the dynamics of the process significantly. From several step response tests for the urea, for different static combinations of load, it can be seen that the dynamic of the system changes significantly for low and high values of urea. This can be evaluated using the ammonia to NO_x ratio, ANR, which is defined as

$$\text{ANR} = \frac{u_1}{u_2} c, \tag{15}$$

where c is a constant defined by the molar mass of NH_3 and NO_2, this constant is approximately 2.7. Based on these experimental tests, the split of the operating region into 5 local regions is suggested in Table I.

B. Local models identification

The original data was normalised using the values shown in Table II. In this paper, pseudo random multilevel signals, with a sampling time of 500 ms are used for identification of the local models. The 5 local models were obtained using

TABLE II

VALUES USED FOR PREPROCESSING

Variable	Normalization value
y [ppm]	1825
u_1 [ppm]	1825
u_2 [g/h]	1349
u_3	3.63447 $\times 10^{-6}$
u_4 [kg/h]	1662

TABLE III

CONTROLLER TUNING PARAMETER FOR EACH REGION

Region	ρ
R1	0.9986
R2	0.995
R3	0.999
R4	0.9835
R5	0.990

the System Identification Toolbox. The order chosen for the local models was 2.

The input temperature was modified by a static nonlinear function which is based on the way temperature is involved in the chemical reaction rates in the first principle based model. This static nonlinear function was used in ([4], [5]) and it showed to be a beneficial choice. The modified input u_3 is given by $u_3(t) = e^{(-a/u_{3,r}(t))}$, where a is set as 10^4. The preprocessed variables are denoted by y, u_1, u_2, u_3 and u_4 respectively.

An additional local output ${}^6y_k$ was added and set equal to u_1, that means the NO_x output is the same as the NO_x input in case that there is no reduction. The tuning parameters for the multiple model approach are: ${}^ip(0) = \frac{1}{6}$ for $i = 1, \ldots, 6$, $\Lambda = 100I$, the recursion horizon is 4, and the lowest probability value μ is set to 0.01.

C. Local controllers design

Since total NO_x reduction is desired, the set point of the controller is set to zero. The same tuning parameters and initial values that were used for identification to compute the weights were also used for control. Whenever the reduction of the system was equal to zero, the injected urea was set to a value such that the ANR would be equal to one.

For all regions, the order of the filter in (13), n was set to 1. The parameters ${}^i\rho$ in (13) were set such that the desired rise time of the closed loop system could be obtained. Since the system is a chemical process and the reactions are slow at low temperatures, it is important to achieve rise time for each region in a way so that the urea is not injected too fast. For the regions where the temperature is lower, the rise time is set to a higher value. The chosen values of ${}^i\rho$ are shown in Table III. Note that in region 2, a modified controller is used to remove the unstable zero.

The local controllers are defined for the normalised models and therefore, the controller output is multiplied by the normalisation factor for NH_3 given in Table II, before injected into the SCR system.

VI. RESULTS

A. Identification

The identification of the models were performed using pseudo random multilevel signals for the load and the speed as inputs to the simulator. The validation was performed on the WHTC data. To measure the accuracy of the model, the model fit γ was used to compare the identified output with the output from the simulator. The model fit is calculated as

$$\gamma = \left(1 - \frac{||y_r - \hat{y}||}{||y_r - \bar{y}_r||}\right) \times 100, \qquad (16)$$

where $\bar{y}_r$ denotes the mean of the real output. The fit using WHTC data is 77.6%. Fig. 5 shows the validation of the multiple model approach for the 30 min of WHTC data. Notice that each subplot represents a window of 10 min.

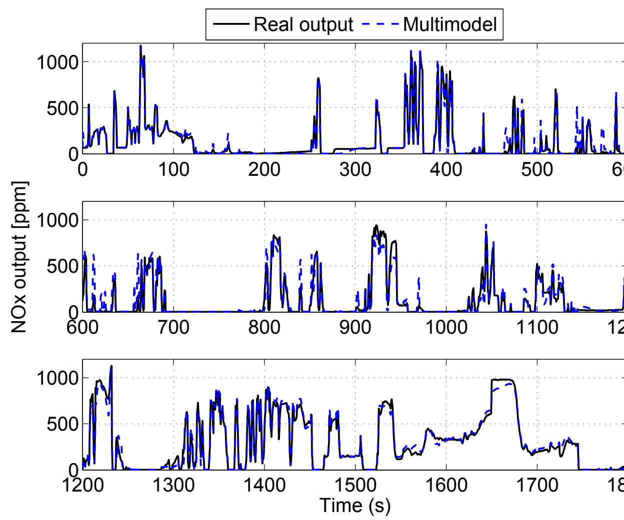

Fig. 5. Validation of the multiple model based identification.

B. Control

The control algorithm described in this paper was implemented in the simulator provided by Scania and applied on two sets of data. First different steps were performed to validate the controller performance, at this point, the multiple model based controller was compared with a controller based on a single linear model. The weight computation is together with the step performance of the controller illustrated in Fig 6. Second, the controller was validated using the WHTC data. The obtained results are shown in Fig 7. In the beginning of the WHTC, almost no NO_x is noticable, this is due to the cold start of the SCR system. It takes time for the system to reach a temperatures above 200°C, where the NO_x is minimal or nonexistent. This is due to the low reaction rate of NH_3 with the NO_x

VII. CONCLUSIONS AND FUTURE WORK

A multiple model based identification algorithm using local linear models is proposed for identification of a selective catalytic reduction system. The output of the models are weighted using Bayes' probability theorem. The multiple

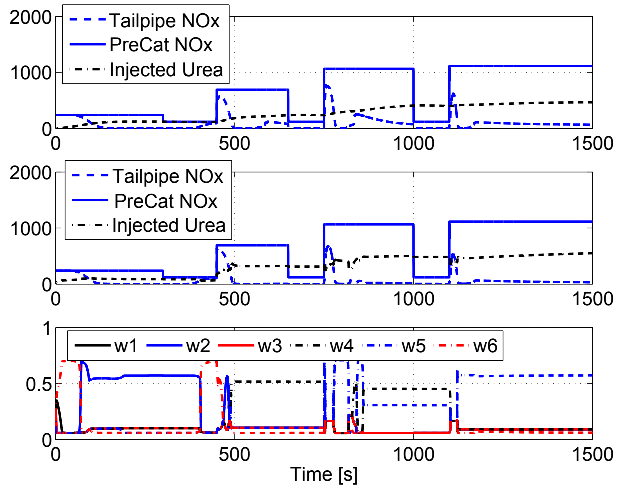

Fig. 6. Validation of the controller performance for different step levels together with the computed weights for the different models compared to a controller based on one overall model.

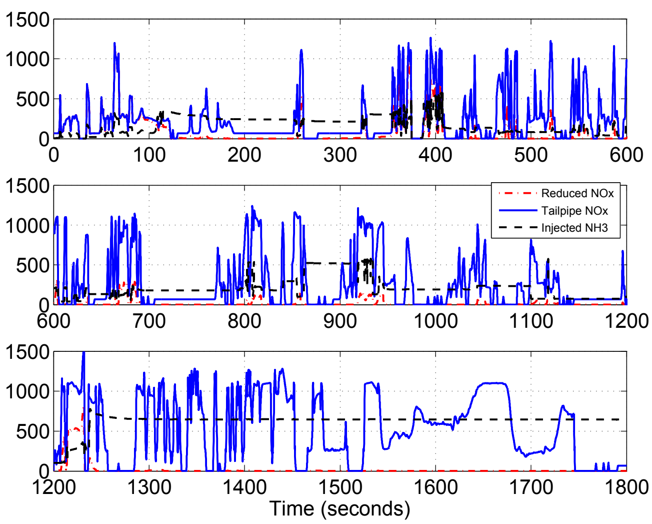

Fig. 7. Validation of the controller for the WHTC data.

model gives a high accuracy of the process with a model fit of above 77 % and describes the system well for all regions.

The proposed multiple model is used for local control design using internal model control structure. The local IMC is implemented for the process and the control output is weighted using the same technique as for identification. The controller performs well in NO_x reduction and for all regions and satisfy the given controller requirements.

The proposed scheme provides significant benefits for the SCR process, as for example: a practical and feasible identification procedure, and also, a low computational load controller that provides stability and good performance facing the faster variations of the speed and load.

In the future, the proposed controller could be compared to nonlinear controllers for validation purposes. The implementation of the multi-model identification algorithm and the controllers for the real system is also of interest for the future work.

VIII. Acknowledgements

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