2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Digital stabilization of finite sampled nonlinear dynamics with delays: the unicycle example

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Abstract— **The paper illustrates through the example of a mobile robot how discretization makes easier the design of a predictor based stabilizer for nonlinear dynamics with delayed input admitting finite sampled equivalent models.**

I. INTRODUCTION

The study of systems with delays in continuous time invariably relies on an infinite dimensional problem settlement [1], [2], [3]. On the other side in discrete-time, as well known from the linear literature, the introduction of delays can be easily handled by considering suitable extensions of the dynamics [4]. As a consequence, in a digital context the computation of the controller can be strongly simplified if it is possible to re-state the problem on the sampled-data equivalent model *(direct-digital design)*. Such an approach has been recently proposed in [5], for continuous-time input-affine dynamics, admitting a delay free stabilizing feedback with known Lyapunov function and known delay τ satisfying $\tau = N\delta$ with $N \in Z^+$ and sampling period δ. The resulting sampled-data predictor based controller is composed by the digital Lyapunov matching control law, computed for the delay free system, implemented from the output of a discrete-time state predictor.

In this context , the most favorable case is represented by continuous-time dynamics admitting finite order sampled equivalent models: that is when the associated flow under piecewise constant control is described by a series expansion of finite order in the sampling period. Sampled equivalent models of finite order with respect to the sampling period δ are thus finitely computable. This occurs when the delay free dynamics exhibits a lower triangular form with polynomial non linearities. A typical example is represented by the well known strict feedforward forms for which different design procedures are available [6], [7], [8], [9], [10].

In this paper we show how the approach proposed in [5] can be profitably applied to dynamics which are finitely computable under feedback. This will be done making reference to the example of a wheeled mobile robot, which exhibits, through coordinates change and feedback, a chain form structure [11] resulting in a finitely computable discrete-time equivalent sampled model. This example represents a popular case study for which various continuous-time controllers have been proposed. We refer in the delay free case to the multi-rate digital control strategy introduced in [12], [13] and then we complete the scheme with a predictor for compensating input or measurement delays.

The paper is organized as follows: preliminary recalls on single and multi-rate samplings are set in section II; the discrete-time equivalent model of the wheeled mobile robot is computed in section III. Section IV deals with the design of a digital stabilizing control for the delay free model, and in Section IV the characterization of the predictor completes the design. In section VI some simulation results are discussed.

Notations Maps and vector fields are assumed smooth (i.e. infinitely differentiable - C^∞), vector fields are forward complete. The set $\mathscr{U}$ (resp. $\mathscr{U}_d$) of admissible inputs consists of all U-valued piecewise continuous (resp. piecewise constant) functions on R. $\mathrm{L}_f = \sum_{i=1}^n f_i(.)\frac{\partial}{\partial x_i}$, denotes the Lie derivative operator and $e^f := 1 + \sum_{i\geq 1}\frac{\mathrm{L}_f^i}{i!}$ the Lie series exponential operator, associated with a vector field f (1 is the identity operator on R^n); "(x)" or "$\big|_x$" denote the evaluation at a point x of a generic map. Given two vector fields on R^n, $ad_f g = [f,g] = [\mathrm{L}_f, \mathrm{L}_g] = \mathrm{L}_f{\circ}\mathrm{L}_g - \mathrm{L}_g{\circ}\mathrm{L}_f$ indicates their Lie bracket. Time dependency in the discrete-time domain of a function λ, is denoted $\lambda(k)$ or λ_k.

II. SINGLE-RATE AND MULTI-RATE SAMPLING

Consider the continuous-time input-affine dynamics

$$\dot{x}_c(t) = f(x_c(t)) + u(t)g(x_c(t)) \tag{1}$$

on R^n where x_c denotes the continuous-time state behavior. Assuming the control $u(.) \in \mathscr{U}_d$ constant over time intervals of length δ: $u(t) = u(k)$ for $t \in [k\delta, (k+1)\delta[, k \geq 0$, the *equivalent single-rate sampled-data dynamics* at times $t = k\delta$ takes the form

$$x_d(k+1) = F^\delta(x_d(k), u(k)) = e^{\delta(f+u(k)g)}x_d(k) \tag{2}$$

where x_d denotes the discrete-time state behavior. With $x_d(0) = x_c(0)$, $x_d(k)$ from (2) matches $x_c(t)$ from (1) at the sampling instants. When the series expansion in δ of the exponential form in (2) is of finite order we get a *finite order sampled model*. Otherwise, we can consider truncations of the series expansion so defining *approximate sampled models*. When the differential equation (1) is integrable under constant control, we get an *exact sampled model*.

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The work of V. Tanasa was supported in part by Romanian Ministry of Education, Research, Youth and Sport through UEFISCDI, under grant TE232/2010

Remark In practice, one computes $x_d(k+1)$ by applying to each state component the sampling rule $x_i(k+1) = x_i(k) + \sum_{j\geq 1} \frac{\delta^j}{j!} x_i^{(j)}(k)$ where $x_i^{(j)}(k)$ indicates the time derivative of order j of $x_i(t)$ computed at time $t = k\delta$ under constant control $u(k)$.

A m^{th}-order multi-rate sampling means, in the present context, that the control variables are actuated faster than the data is measured; $u(t)$ is thus assumed constant over intervals of length $\bar{\delta} = \frac{\delta}{m}$ so allowing m different control values on each interval of length δ; i.e

- $u_i(k)$ is active and constant for all $t \in [k\delta + (i-1)\bar{\delta},\ k\delta + i\bar{\delta}[$ where $1 \leq i \leq m$.

Generalizing (2), one describes the *equivalent multi-rate sampled-data dynamics* of order m by the difference equation

$$x_{MR}(k+1) = F^{\delta}_{MR}(x(k), u_1(k), \cdots, u_m(k)) \quad (3)$$

where

$$\begin{aligned} F^{\delta}_{MR}(x, u_1, \cdots, u_m) &= e^{\bar{\delta}(f+u_1 g)} \circ \cdots \circ e^{\bar{\delta}(f+u_m g)} x \\ &= e^{\mathscr{BCH}^m\left(\bar{\delta}(f+u_1 g), \cdots, \bar{\delta}(f+u_m g)\right)} x \end{aligned}$$

where $\mathscr{BCH}^m$ Baker-Campbell-Hausdorff exponent associated with the non commutative composition of m exponential operators $e^{\mathscr{BCH}^m(f_1,\ldots,f_m)} := e^{f_1} \circ \ldots \circ e^{f_m}$. Finite order, approximate and exact sampled models can be defined accordingly.

III. The unicycle example

A. The unicycle dynamics in the delay free case

Consider the kinematics equations of a wheeled vehicle

$$\begin{aligned} \dot{x}(t) &= v(t)\cos\theta(t) \\ \dot{y}(t) &= v(t)\sin\theta(t) \\ \dot{\theta}(t) &= \omega(t) \end{aligned} \quad (4)$$

where v denotes the forward velocity and ω the steering velocity. We recall that in (4), slipping is not admitted which corresponds to the non-holonomic constraint [14]

$$\dot{x}(t) sin\theta(t) - \dot{y}(t) cos\theta(t) = 0.$$

As well known [15], the coordinates change

$$\begin{aligned} x_1 &= x\cos\theta + y\sin\theta \\ x_2 &= x\sin\theta - y\cos\theta \\ x_3 &= \theta \end{aligned} \quad (5)$$

and the preliminary state feedback

$$u_1(t) = v(t) - x_2(t)\omega(t), \quad u_2(t) = \omega(t) \quad (6)$$

bring the dynamics (4) into the chained form

$$\begin{aligned} \dot{x}_1(t) &= u_1(t) \\ \dot{x}_2(t) &= u_2(t)x_1(t) \\ \dot{x}_3(t) &= u_2(t) \end{aligned} \quad (7)$$

The peculiarity of such a dynamics is to admit an exact and finite sampled equivalent dynamics of order 2 in δ. Considering the controls u_1 and u_2 constant over time intervals of length δ, one gets according to section II and over time intervals of length δ, the single rate exact sampled-data equivalent model of order 2 in δ

$$\begin{aligned} x_1(k+1) &= x_1(k) + \delta u_1(k) \\ x_2(k+1) &= x_2(k) + \delta u_2(k)x_1(k) + \frac{\delta^2}{2}u_2(k)u_1(k) \\ x_3(k+1) &= x_3(k) + \delta u_2(k) \end{aligned} \quad (8)$$

when setting

- $u_1(t) = u_1(k)$ for $t \in [k\delta, (k+1)\delta[$
- $u_2(t) = u_2(k)$ for $t \in [k\delta, (k+1)\delta[$.

B. The wheeled mobile robot dynamics with input delays

In presence of a communication delay τ in the inputs, assumed equal to $N\delta$, with N a positive integer, the kinematics equations (4) become

$$\begin{aligned} \dot{x}(t) &= v(t-\tau)\cos\theta(t) \\ \dot{y}(t) &= v(t-\tau)\sin\theta(t) \\ \dot{\theta}(t) &= \omega(t-\tau). \end{aligned} \quad (9)$$

Again, under the coordinates change (5) and the feedback

$$u_1(t-\tau) = v(t-\tau) - x_2(t)\omega(t-\tau), \quad u_2(t-\tau) = \omega(t-\tau) \quad (10)$$

one gets the input delayed chained form below

$$\begin{aligned} \dot{x}_1(t) &= u_1(t-\tau) \\ \dot{x}_2(t) &= u_2(t-\tau)x_1(t) \\ \dot{x}_3(t) &= u_2(t-\tau). \end{aligned} \quad (11)$$

IV. The delay free digital control design

In the delay free case, we first recall from [16] and [15] a digital procedure to steer the evolution of (4) to a given target position. More precisely, we look for a 2-rate digital controller achieving steering in one step of length δ (equivalently in 2 steps of length $\delta/2$), of any $x(t = k\delta)$ to a desired state x_f at time $t = (k+1)\delta$. With this in mind, assuming a multi-rate of order 2 on the control variable u_1 and a single rate on u_2 i.e.

- $u_1(t) = u_1(k)$ for $t \in [k\delta, (k+\frac{1}{2})\delta[$
- $u_2(t) = u_2(k)$ for $t \in [k\delta, (k+1)\delta[$
- $u_1(t) = u_3(k)$ for $t \in [(k+\frac{1}{2})\delta, (k+1)\delta[$

one computes through composition of the single rate sampled dynamics (8) over two time-intervals of length $\delta/2$, the double rate sampled equivalent dynamics

$$\begin{aligned} x_1(k+1) &= x_1(k) + \frac{\delta}{2}(u_1(k) + u_3(k)) \\ x_2(k+1) &= x_2(k) + \delta u_2(k)x_1(k) \\ &+ \frac{\delta^2}{8}u_2(k)(3u_1(k) + u_3(k)) \\ x_3(k+1) &= x_3(k) + \delta u_2(k). \end{aligned} \quad (12)$$

By construction, setting $u_1(k) = u_3(k)$ in (12), one recovers the single rate dynamics (8).

It is a matter of computations to verify that setting $x(k+1)=x_f=(x_{f1},x_{f2},x_{f3})^T$ in (12), one computes through simple map inversion the steering control

$$\begin{aligned} u_1(k) &= \frac{4}{\delta}\Big(\frac{x_{f2}-x_2(k)-x_1(k)(x_{f3}-x_3(k))}{x_{f3}-x_3(k)}\Big) \\ &- \frac{x_{f1}-x_1(k)}{\delta} \\ u_2(k) &= \frac{x_{f3}-x_3(k)}{\delta} \\ u_3(k) &= -\frac{4}{\delta}\Big(\frac{x_{f2}-x_2(k)-x_1(k)(x_{f3}-x_3(k))}{x_{f3}-x_3(k)}\Big) \\ &+ \frac{3(x_{f1}-x_1(k))}{\delta}. \end{aligned} \tag{13}$$

$u(k)$ described in (13) and denoted by $\gamma^\delta(x(k),x_f)$, defines the steering feedback from $x(k)$ to x_f in the delay free case. We note that it is not defined for $x_{f3}=x_3(k)$ which corresponds to the parallel parking problem [17]. In this case, one can impose an intermediary target point to achieve the parallel parking in two steps of length δ (see [16]).
Another steering solution which achieves stabilization in 3 steps has been proposed in [18], we perform some comparative simulations in section VI.

A. About the digital implementation

In terms of the velocities in dynamics (4), one gets from (13)

$$\begin{aligned} v(t) &= u_1+x_2(t)u_2 \quad \text{for} \quad t\in[k\delta,(k+\frac{1}{2})\delta[\\ v(t) &= u_3+x_2(t)u_2 \quad \text{for} \quad t\in[(k+\frac{1}{2})\delta,(k+1)\delta[\\ \omega(t) &= u_2 \quad \text{for} \quad t\in[k,(k+1)\delta[\end{aligned} \tag{14}$$

where the k dependency in (13) is omitted when no confusion is possible.
It results that ω remains constant over the sampling time intervals while v is piecewise continuous because of its dependence on $x_2(t)$. Such a controller is denoted by **MRCont** and could be ideally implemented from the continuous measurement of $x_2(t)$ and the sampled measures $(x_1(k),x_3(k))$.
An alternate predictor based solution can be implemented from the the sampled measures $(x_1(k),x_2(k),x_3(k))$ only by substituting in (14), $x_2(t)$ with

$$x_2(t) = x_2(k)+tu_2x_1(k)+\frac{t^2}{2}u_2u_1 \tag{15}$$

computed according to the integrated dynamics (8) for $t\in[k\delta,(k+\frac{1}{2})\delta[$ and with

$$x_2(t)= x_2(k+\tfrac{1}{2})+tu_2x_1(k+\tfrac{1}{2})+\tfrac{t^2}{2}u_2u_3 \tag{16}$$

for $t\in[(k+\frac{1}{2})\delta,(k+1)\delta[$ and

$$\begin{aligned} x_1(k+\tfrac{1}{2}) &= x_1(k)+\tfrac{\delta}{2}u_1(k) \\ x_2(k+\tfrac{1}{2}) &= x_2(k)+\tfrac{\delta}{2}u_2x_1(k)+\tfrac{\delta^2}{8}u_2u_1. \end{aligned} \tag{17}$$

In a strict digital context, if one looks for a piecewise constant control over the dynamics (4), a first approximate solution denoted by **MRHold** consists in the implementation of (14) maintaining $x_2(t)$ constant over time intervals of length $\delta/2$ so setting

$$v(k) = u_1+x_2(k)u_2 \quad \text{on} \quad [k\delta,(k+\frac{1}{2})\delta[\tag{18}$$

$$v(k) = u_3+x_2(k+\frac{1}{2})u_2 \quad \text{on} \quad [(k+\frac{1}{2})\delta,(k+1)\delta[\tag{19}$$

with $x_2(k+\frac{1}{2})$ computed according to (17).
Improved piecewise constant controllers can be computed assuming $x_2(t)$ in (14) constant again but equal to suitably computed values $\tilde{x}_2(k)$ and $\tilde{x}_2(k+\frac{1}{2})$

$$v(k) = u_1+\tilde{x}_2(k)u_2 \quad \text{on} \quad [k\delta,(k+\frac{1}{2})\delta[\tag{20}$$

$$v(k) = u_3+\tilde{x}_2(k+\frac{1}{2})u_2 \quad \text{on} \quad [(k+\frac{1}{2})\delta,(k+1)\delta[. \tag{21}$$

In the sequel we denote by **MRCorr** a controller of this form (20,21) where $\tilde{x}_2(k)$ and $\tilde{x}_2(k+\frac{1}{2})$) are respectively computed to annihilate the average errors

$$\frac{2}{\delta}\int_0^{\delta/2}(x_2(t)-\tilde{x}_2(k))dt = 0$$

and

$$\frac{2}{\delta}\int_{\delta/2}^{\delta}(x_2(t)-\tilde{x}_2(k+\frac{1}{2}))dt=0.$$

After easy computations, one gets from (15)

$$\begin{aligned} &\frac{2}{\delta}\int_0^{\delta/2}(x_2(k)-\tilde{x}_2(k)+tu_2x_1(k)+\frac{t^2}{2}u_2u_1)dt \\ &= (x_2(k)-\tilde{x}_2(k))+\frac{\delta}{4}u_2x_1(k)+\frac{\delta^2}{24}u_2u_1 \end{aligned}$$

and from (16)

$$\begin{aligned} &\frac{2}{\delta}\int_{\delta/2}^{\delta}(x_2(k+\frac{1}{2})+tu_2x_1(k+\frac{1}{2})+\frac{t^2}{2}u_2u_3-\tilde{x}_2(k+\frac{1}{2}))dt \\ &= \Big(x_2(k+\frac{1}{2})-\tilde{x}_2(k+\frac{1}{2})\Big)+\frac{3\delta}{4}u_2x_1(k+\frac{1}{2})+\frac{7\delta^2}{24}u_2u_3 \end{aligned}$$

which bring to

$$\tilde{x}_2(k) = x_2(k)+\frac{\delta}{4}u_2x_1(k)+\frac{\delta^2}{24}u_2u_1 \tag{22}$$

$$\tilde{x}_2(k+\frac{1}{2}) = x_2(k)+\frac{5\delta}{4}u_2x_1(k)+\frac{\delta^2}{2}u_2u_1+\frac{7\delta^2}{24}u_2u_3. \tag{23}$$

The **MRCorr** controller is so given by equations (20)-(21) with (22)-(23) and (u_1,u_2,u_3) as in (13).

V. The digital control with delay predictor

In presence of an input delay $\tau=N\delta$, the same approach can be performed on the input delayed chain form (11) so getting the solution denoted as **MRCont** described in the delay case by

$$\begin{aligned} v(t-N\delta) &= u_1(k-N)+x_2(t)u_2(k-N), t\in[k\delta,(k+\frac{1}{2})\delta[\\ v(t-N\delta) &= u_3(k-N)+x_2(t)u_2(k-N), t\in[(k+\frac{1}{2})\delta,(k+1)\delta[\\ \omega(k-N) &= u_2(k-N) \end{aligned}$$

with $u(k-N)=\gamma^\delta(x(k),x_f)$ and $\gamma^\delta(.)$ defined in (13). Such a controller can be rewritten in terms of the predicted state

$z(t) = x(t+N\delta)$ as $u(k) = \gamma^{\delta}(z(k), x_f)$. One recovers, with respect to the predicted state z, the delay free controller; i.e.

$$
\begin{aligned}
v(t) &= u_1(k) + z_2(t)u_2(k), t \in [k\delta, (k+\frac{1}{2})\delta[\\
v(t) &= u_3(k) + z_2(t)u_2(k), t \in [(k+\frac{1}{2})\delta, (k+1)\delta[\\
\omega(k) &= u_2(k)
\end{aligned} \tag{24}
$$

with according to (15) and (16)

$$
z_2(t) = z_2(k) + tu_2(k)z_1(k) + \frac{t^2}{2}u_2(k)u_1(k)
$$

for $t \in [k\delta, (k+\frac{1}{2})\delta[$ and

$$
z_2(t) = z_2(k+\tfrac{1}{2}) + tu_2(k)z_1(k+\tfrac{1}{2}) + \tfrac{t^2}{2}u_2(k)u_3(k)
$$

for $t \in [(k+\frac{1}{2})\delta, (k+1)\delta[$.

The predictor dynamics can be easily computed from the multi-rate sampled dynamics (12) and through N compositions to get

$$
\begin{aligned}
z_1(k) &= x_1(k) + \frac{\delta}{2}\sum_{i=1}^{N}\big(u_1(k-i) + u_3(k-i)\big), \\
z_2(k) &= x_2(k) + \delta x_1(k)\sum_{i=1}^{N} u_2(k-i) \\
&+ \frac{\delta^2}{8}\sum_{i=1}^{N} u_2(k-i)(3u_1(k-i) + u_3(k-i)) \\
&+ \frac{\delta^2}{2}\sum_{i=1}^{N-1} u_2(k-i)\sum_{j=1}^{N-i}(u_1(k-j) + u_3(k-j)) \\
z_3(k) &= x_3(k) + \delta\sum_{i=1}^{N} u_2(k-i)
\end{aligned} \tag{25}
$$

with suitable initial conditions $(z(0), u(-1), u(-N))$ so that $z(0) = x(N)$.

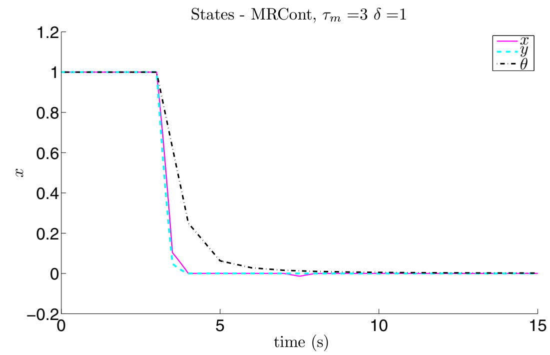
Accordingly, one defines the **MRHold** digital design when maintaining $z_2(t)$ constant over time intervals of length δ and the **MRCorr** design when making use of the values $\tilde{z}_2(k)$ and $\tilde{z}_2(k+1/2)$ computed according to (22) and (23) by substituting the x variable with the predicted z one.

VI. Simulation Results

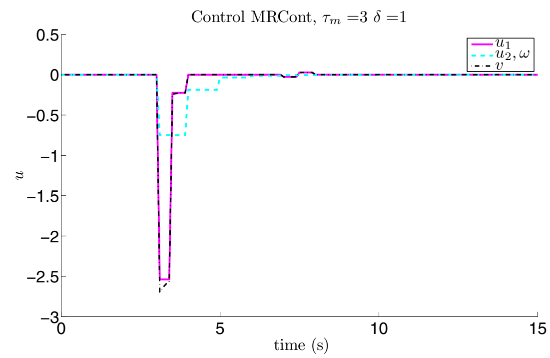
The simulations are performed according to two scenarios. The chosen initial condition is $(x = 1, y = 1, \theta = 1)$ and the target state is zero. The first scenario considers that v is piecewise continuous (no holding device on v). The simulation results in Figures 1-2 and 5a) respectively compare the performances of our controller **MRCont** defined by the equations (14) and **IKMK11** which corresponds to a stabilizing controller proposed in [18].

The second scenario considers that v is held constant. In this case two controllers are compared. The **MRHold** defined by equations (18)-(19) and the **MRCorr** defined by (20)-(21) with optimal values (22)-(23). Simulations results are illustrated in Figures 3-4 and 5b) respectively.

All these controllers are implemented with the multi-rate state predictor (25) for the multi-rate controllers and the single rate predictor (26) for the single rate controller **IKMK11** respectively.



(a) States evolution



(b) Control

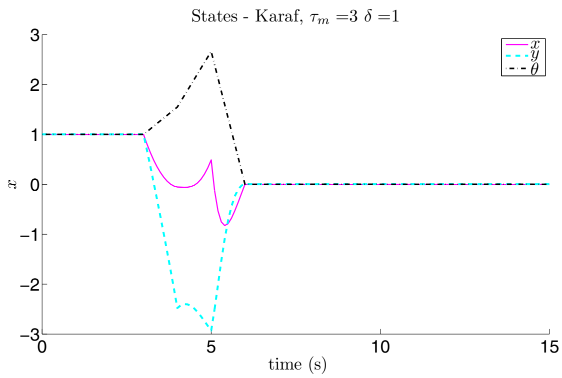
Fig. 1. Unicycle example -**MRCont**, $\delta = 1$s, $N = 3$

The N step single rate predictor can be easily computed from (8) by successive iterations, so getting:

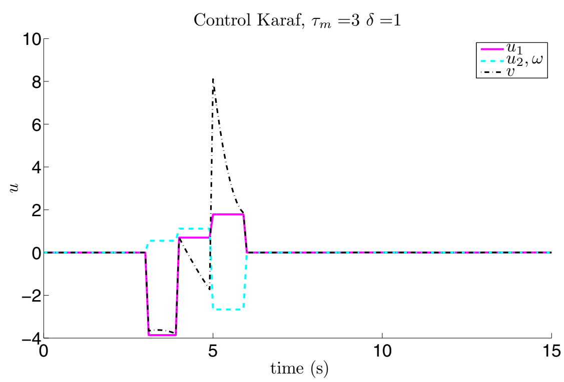
$$
\begin{aligned}
z_1(k) &= x_1(k) + \delta\sum_{i=1}^{N} u_1(k-i), \\
z_2(k) &= x_2(k) + \delta x_1(k)\sum_{i=1}^{N} u_2(k-i) \\
&+ \frac{\delta^2}{2}\sum_{i=1}^{N}\big(u_1(k-i)u_2(k-i) \\
&+ \delta^2\sum_{i=1}^{N-1} u_2(k-i)\sum_{j=1}^{N-i} u_1(k-j)\big) \\
z_3(k) &= x_3(k) + \delta\sum_{i=1}^{N} u_2(k-i)
\end{aligned} \tag{26}
$$

In the Figures 1-2 the control inputs and the state evolutions are depicted when considering that $v(t)$ is piecewise continuous. The simulation parameters are set as follows:

We can observe that the solution **MRCont** performs well compared with the **IKMK11** controller in terms of control amplitude and trajectories. The peaks that can be observed at time $t = 7$ seconds are due to numerical error when

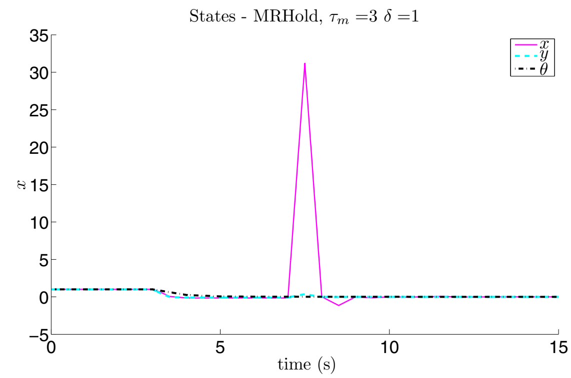


(a) States evolution

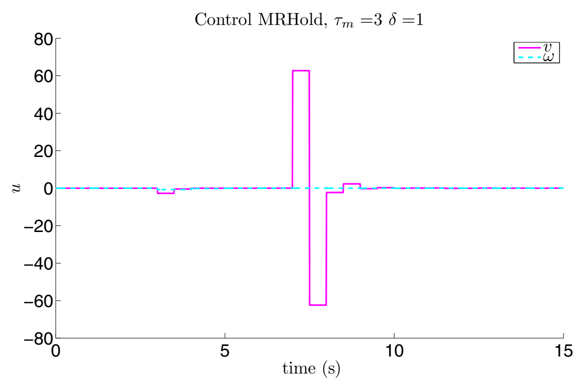


(b) Control

Fig. 2. Unicycle example -**IKMK11**, $\delta = 1$s, $N = 3$



(a) States evolution



(b) Control

Fig. 3. Unicycle example -**MRHold**, $\delta = 1$s, $N = 3$

δ(s)	N	t_f(s)	x_0	y_0	z_0
1	3	15	1	1	1

TABLE I

SIMULATION PARAMETERS

integrating the continuous-time dynamics (see Figure 1). This error is not present in the **IKMK11** controller because the method includes some suitable thresholds. We can also look to Figure 5a), where the phase portrait indicates that the **MRCont** has the minimal trajectory.

In the Figures 3-4 the control inputs and the state evolutions are depicted when considering that $v(t)$ is piecewise constant. The simulation parameters are given in Table I. The first solution, depicted in Figure 3 uses the same controller expression and predictor as **MRCont**, but it sets $v(t)$ constant. The second solution **MRCorr** adds a correction to the digital implementation of $v(t)$ constant and the performances are clear. The peaks on the system's response are still present due to numerical integration of the continuous-time system. This is also linked to the delay amplitude.

VII. Conclusions

We illustrate on the unicycle example in a comparative way the performances of a multi-rate control strategy in presence of input delays. This example is interesting because, thanks to suitable coordinates change and preliminary feedback, an exact point to point steering digital controller can be exactly computed in the delay free case and because, in presence of input delays, a multi-rate state predictor can be exactly computed too. This strategy can be applied to nonlinear dynamics admitting, through preliminary transformations (coordinates change plus state feedback), exact sampled-data equivalent dynamics. In this paper, we have considered the case of a delay multiple of the sampling period but this assumption could be relaxed and a solution proposed through higher order multi-rate devices for fractional delays.

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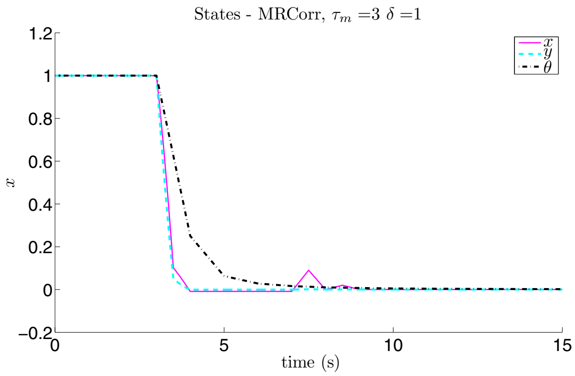
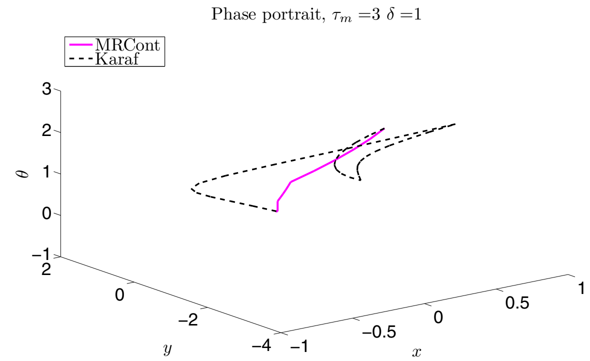
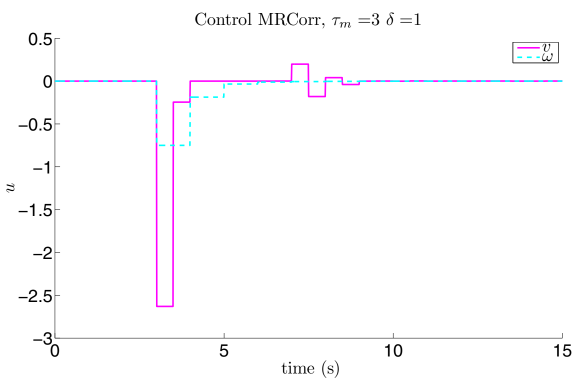


(a) States evolution

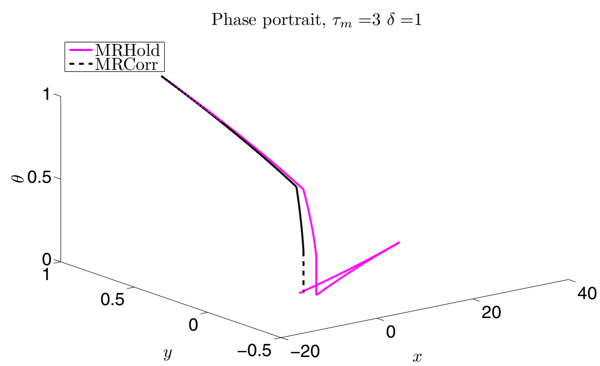


(a) Piecewise continuous controllers



(b) Control

Fig. 4. Unicycle example -**MRCorr**, $\delta = 1$s, $N = 3$



(b) Piecewise constant controllers

Fig. 5. Phase portrait, for $\delta = 1$s, $N = 3$

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