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Model invalidation for repeated ℓ_1-bounded linear time-varying uncertainty models

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Abstract— **Conditions are derived for the existence of a repeated linear time-varying (LTV) ℓ_1-bounded perturbation and some ℓ_∞-bounded disturbance signals that are able to describe experimental data. The uncertainty structure is motivated by formation control problems where complex dynamic systems with similar dynamics are moving together and safety can be measured in terms of the relative distances. It is shown that by solving linear programming problems a disturbance-perturbation trade-off curve useful for unfalsified robust performance computation can be determined. The invalidation results are demonstrated on the simulation example of a homogeneous vehicle platoon.**

I. Introduction

The existing robust control design and analysis methods are able to deal with unmodelled dynamics, which is always present in model based control, for both structured and unstructured forms, see e.g. [1], [2], [3] for linear time-invariant (LTI) systems and [4], [5], [6] for linear parameter-varying (LPV) systems. A nominal model and an uncertainty structure embedding the behaviour of the real system are prerequisites for robustness analysis. Model (in)validation in the deterministic setting is an important tool to fill the gap between (possibly stochastic) system identification and robust control. A special class of uncertainty structures, i.e. uncertain repeated time-varying dynamics, may arise, for example, in formation control problems where each member of the fleet have similar dynamics sharing a synchronized time-varying nature due to the common task they perform. The similar but not necessarily the same unmodelled dynamics can be described by a repeated uncertainty structure and an additive disturbance for each of the members. One important question when analyzing formation control is whether a collision might occur when the allowed inputs are acting on the controlled system. A natural choice for the performance is the peak values of errors in the prescribed relative distances between the members. An adequate measure for the uncertainty is the induced peak-to-peak norm. Robust performance analysis subject to repeated uncertainties provide much less conservative performance results than subject to general uncertainty structures. The latter involves the former as a special case and does not take into consideration the similarities and synchronism between the members. This paper contributes on this field by developing an invalidation method for repeated LTV ℓ_1-bounded uncertainties.

Some worst-case identification methods deliver model sets directly applicable in robust control, see e.g. [7], [8], [9] and references therein. However, these methods require a priori information on the system which is not always available or might be too conservative [10, pages 287-290]. In these cases data-based uncertainty modelling allows to examine the achievable performance whenever the model is not invalidated [11], [12], [13]. In general, the goal of model invalidation is to check whether the uncertain model equipped with the assumed uncertainty bounds are able to describe experimental data, see e.g. [14], [15], [16], [17], [18], [19]. Unfortunately, none of these methods can be directly applied for the purpose of nonconservative formation analysis.

In this paper the model validation concept is extended to the case of repeated ℓ_1-bounded LTV perturbations. The methods are elaborated for only repeated scalar perturbations in order to simplify the notations, but the results can be generalized to the multi-variable case without any theoretical difficulty. Conditions for model consistency are derived and the calculation of a disturbance-perturbation trade-off curve is carried out by means of linear programming (LP).

After providing the basic notations applied in the paper, the problem is formulated in Section III for which the solution is presented in Section IV. An example originating from a vehicle platoon problem demonstrates the method in Section V. The achievements are summarized in Section VI.

II. Notations

The following notations are mainly from [20]. Let $\mathbb{N}$ denote the set of nonnegative integers, $\mathbb{R}$ denote the set of real numbers. Let $\mathbb{R}^n$ denote the space of n dimensional real vectors. The ith element of vector $f \in \mathbb{R}^n$ is referred as f_i. The ith row of an n-by-m matrix M is denoted by $(M)_i$, its jth column by $(M)^j$ and its ijth element by M_{ij}. The indexes may refer to elements of hyper-matrices or hyper-vectors, but this will be clear from the context. Let ℓ_∞ denote the normed space of bounded sequences of real numbers

$$\ell_\infty = \{f : \mathbb{N} \mapsto \mathbb{R},\ \|f\|_\infty := \sup_{k\in\mathbb{N}} |f(k)| < \infty\}$$

and ℓ_1 denote the normed space of absolutely summable sequences

$$\ell_1 = \{f : \mathbb{N} \mapsto \mathbb{R},\ \|f\|_1 := \sum_{k=0}^{\infty} |f(k)| < \infty\}$$

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ℓ_∞^n denote the vector of signals in ℓ_∞ and ℓ_1^{nm} the n-by-m matrices whose elements belong to ℓ_1. The corresponding vector and matrix norms are defined, respectively, by

$$\|f\|_\infty := \max_{i\in\{1..n\}} \|f_i\|_\infty$$
$$\|f\|_1 := \max_{i\in\{1..n\}} \sum_{j=1}^{m} \|f_{ij}\|_1$$

The ℓ_1 norm is also defined for real matrices $M \in \mathbb{R}^{nm}$ by

$$\|M\|_1 := \max_{i\in\{1..n\}} \sum_{j=1}^{m} |f_{ij}|$$

Let $\mathcal{L}_{LTV}^{nm}$ denote the set of all causal linear operators mapping ℓ_∞^m to ℓ_∞^n. A map $G \in \mathcal{L}_{LTV}^{nm}$ is said to be ℓ_∞-stable if it is bounded, i.e. there exists $\alpha > 0$ such that $\|Gu\|_\infty \le \alpha\|u\|_\infty$ for all $u \in \ell_\infty^m$. Each $G \in \mathcal{L}_{LTV}^{nm}$ can be completely characterized by its lower triangular pulse response matrix in the form

$$T_G := \begin{bmatrix} G_{00} & & 0 \\ G_{10} & G_{11} & \\ \vdots & \vdots & \ddots \end{bmatrix}$$

where $G_{ij} \in \mathbb{R}^{nm}$. The induced operator norm of G is equal to $\sup_i \|[G_{i0}, ..., G_{ii}]\|_1$. If $u \in \ell_\infty^m$, then $y = Gu \in \ell_\infty^n$ where $y(k) = \sum_{j=0}^{k} G_{kj}u(j) \in \mathbb{R}^n$.

Let π_k denote the truncation operator mapping the infinite sequence $[u(0), u(1), ..., u(k), u(k+1), u(k+2), \ldots]$ to $[u(0), u(1), ..., u(k), 0, 0, ...]$.

It will be useful, furthermore, to define the following operator that maps $u \in \mathbb{R}^m$ into a block-diagonal matrix $\Gamma_u \in \mathbb{R}^{m,(m+m^2)/2}$

$$\Gamma_u := \begin{bmatrix} u_0 & 0 & 0 & \cdots & & & 0 \\ 0 & u_0 & u_1 & & & & \vdots \\ \vdots & & & \ddots & & & 0 \\ 0 & 0 & \cdots & 0 & u_0 & \cdots & u_m \end{bmatrix}$$

Let Φ_G^k denote the operator that maps a pulse response matrix into the hyper vector

$$\Phi_G^k := \left[\begin{array}{c|cc|c|ccc} G_{00}^T & G_{10}^T & G_{11}^T & \cdots & G_{k0}^T & \cdots & G_{kk}^T \end{array}\right]^T$$

where ^{T} denotes transpose. The latter two notations enable us the reformulation of the operation $y = \pi_k(Gu)$ with $u \in \ell_\infty$, $G \in \mathcal{L}_{LTV}$ to $y = \Gamma_u \Phi_G^k$. The n-length vector with all elements being one is denoted by 1_n.

III. Problem setup

Suppose that input-output measurements $\mathcal{E}_i := \{y_i(k), u_i(k)\}_{k=0}^N$, $y_i \in \ell_\infty^{n_y}$ and $u_i \in \ell_\infty^{n_u}$, are available from a set of discrete time LTV systems $P_1, ..., P_L$ which generated the data in the form

$$y_i = P_i \begin{bmatrix} u_i \\ \nu_i \end{bmatrix}, \qquad i = 1, ..., L \tag{1}$$

where ν_i is some unmeasured disturbance. It is assumed that the experiments start from rest (zero initial conditions). Consider the following set of models

$$y_i = G_i u_i + \Delta u_i + \beta W_i d_i, \qquad i = 1, ..., L \tag{2}$$

where G_is are nominal models, practically of class LTI or LPV. The nominal model errors defined by $e_i := y_i - G_i u_i$ are characterized by the uncertainty model

$$e_i = \Delta u_i + \beta W_i d_i, \qquad i = 1, ..., L \tag{3}$$

where $\Delta \in \mathcal{L}_{LTV}^{n_y n_u}$ is the unmodelled dynamics which is common for all systems. The discrepancies between the uncertainties and the effects of disturbances ν_i are captured by disturbance model $\beta W_i d_i$, where W_i is a pre-specified transfer function characterizing the frequency dependence in the disturbance model and $0 < \beta \in \mathbb{R}$ is a trade-off parameter influencing the error distribution between disturbances and neglected dynamics.

The following problems are posed.

Problem 1 (Model invalidation): Given sequences $\mathcal{E}_i := \{y_i(k), u_i(k)\}_{k=0}^N$, $i = 1, ..., L$ and constants α and β, when does there exist a stable, causal operator $\Delta \in \mathcal{L}_{LTV}^{n_y n_u}$ with $\|\Delta\|_1 \le \alpha$ and disturbances $d_i \in \ell_\infty^{ny}$, $i = 1, ..., L$ with $\|d_i\|_\infty \le 1$ such that equation (3) is satisfied?
If there exist such Δ and d_is, the model is said to be consistent with the data.

Problem 2 (Model unfalsification): Given a constant β, what is the smallest $\alpha \ge 0$ with which the model is consistent?

Remark 1: Concerning the usability of this validation approach and the trade-off curve defined by function $\alpha(\beta)$, a question regarding stability may arise immediately. Certainly, any model error can be described purely by disturbances (large β, $\Delta = 0$), however, placing this model in an analysis framework, where the uncertain plant model and a controller is in feedback interconnection, nothing can be said about robust stability[1], since bounded disturbances do not cause stability problems. Information about neglected dynamics is required for determining robust stability. The resolution of this problem is similar to that presented in [12] for the $\mathcal{H}_\infty$ framework, which is based on the fact that disturbances influence the performance of the controlled system. Assume that model consistency with respect to $\mathcal{E}_i$ as defined above implies the consistency of the closed-loop model[2] with respect to $\mathcal{E}_i$ and other closed-loop signals (e.g. performance signals, controller inputs and outputs). In this case, since robust performance implies robust stability [20], the induced norm of the uncertain closed-loop model (calculated measure of the robust performance) is an upper-bound of the measure of the true robust performance *as long as the model is*

[1]Robust stability and performance is defined, e.g., in [20]. Roughly speaking, the uncertain feedback-configuration is robustly stable if it is stable for all allowable perturbations (Δ) in the model set. Robust performance can be tested by a robust stability problem where the performance outputs (e.g. tracking errors) are fed back through a fictive uncertainty block to the performance inputs (e.g. disturbances).

[2]The closed-loop model consists of the the nominal model, neglected dynamics and controller feedback configuration.

consistent with data $\mathcal{E}_i$ from closed-loop experiments. Then consistency implies robust stability. Although this statement is rather trivial, it follows that we do not need to care about the distribution of the model error between perturbations and disturbances, when robust performance is in question. It also follows that if one want to use the obtained uncertainty model for the purpose of controller synthesis then one can obtain destabilizing controllers, therefore, an iterative controller design-modelling scheme is advised, see e.g. [11], [6]. An application of the trade-off curve also follows from the above statement. When calculating robust performance based on the consistent uncertainty models, one have to select the $(\beta, \alpha(\beta))$ pair that allows the best robust performance calculation. This will be the smallest upper-bound on the true performance and is called unfalsified performance.

IV. Solution by linear programming

A. The non-repeated case

With non-repeated perturbation the uncertainty model

$$e_i = \Delta_i u_i + \beta W_i d_i, \qquad i = 1, ..., L \tag{4}$$

reduces the original invalidation problem to independent problems. Necessary and sufficient conditions are provided in [20][Lemma 4] which is copied here in order to highlight the differences between the two problems.

Lemma 1: Let $u = \{u(k)\}_{k=0}^{\infty} \in \ell_{\infty}^{m}$ and $y = \{y(k)\}_{k=0}^{\infty} \in \ell_{\infty}^{r}$. There exists $\Delta \in \mathcal{L}_{LTV}^{rm}$ with $\|\Delta\|_1 \leq 1$ such that $y = \Delta u$ if and only if

$$\|\pi_k y\|_{\infty} \leq \|\pi_k u\|_{\infty}, \qquad \forall k \geq 0, \tag{5}$$

where π_k denotes the truncation operator mapping the infinite sequence u to $[u(0), u(1), ..., u(k), 0, ...]$.
For uncertainty model (4) [15] provides necessary and sufficient conditions for finite length data.

Lemma 2 (Theorem 5.11/4 in [15]): Let $u = \{u(k)\}_{k=0}^{N}$, $u(k) \in \mathbb{R}^m$ and $e = \{e(k)\}_{k=0}^{N}$, $e(k) \in \mathbb{R}^r$. Uncertainty model (4) is not invalidated by the observed input-output data if and only if the following convex feasibility problem is solvable: Does there exist $d = \{d(k)\}_{k=0}^{N}$, $d(k) \in \mathbb{R}^r$, $\|d\|_{\infty} \leq 1$ such that

$$\|\pi_k(y - \beta W d)\|_{\infty} \leq \|\pi_k u\|_{\infty}, \qquad \text{for } k = 0, ..., N. \tag{6}$$

It will be seen in the next section that invalidation of repeated uncertainty characterizes convex problems of much larger complexity.

B. Conditions for consistency

The proof of sufficiency of Lemma 1 is based on the construction of the impulse response matrix T_{Δ} satisfying (5) that solves $y = T_{\Delta} u$. The elements of T_{Δ} depend on the data u and y. This concept cannot be generalized to the case when the conditions must hold for multiple pairs of input-output data. Instead, the following lemma can be stated. The solution is presented for the single input single output case in order to keep the notations transparent.

Lemma 3: Let $u = \{u(k)\}_{k=0}^{N-1}$, $u(k) \in \mathbb{R}$ and $e = \{e(k)\}_{k=0}^{N-1}$, $e(k) \in \mathbb{R}$. There exists $\Delta \in \mathcal{L}_{LTV}$ with $\|\Delta\|_1 \leq 1$ such that $e = \Delta u$ if and only if there exists vectors $x^+ \in \mathbb{R}^{(N+N^2)/2}$ and $x^- \in \mathbb{R}^{(N+N^2)/2}$ with $x^+ \geq 0$ and $x^- \geq 0$ that satisfy

$$\Gamma_u(x^+ - x^-) = e \tag{7}$$

$$|\Gamma_{1_N}(x^+ + x^-)| \leq 1_N \tag{8}$$

where the absolute value and the inequality are meant elementwise.

Proof: Equality constraint (7) is equivalent to $e = \Delta u$ by defining $\Phi_{\Delta}^{N} = (x^+ - x^-)$ and (8) is equivalent to $\|\Delta\|_1 \leq 1$, since the ith element of vector $\Gamma_{1_N}(x^+ + x^-)$ is nothing but $(\Gamma_{1_N}(x^+ + x^-))_i = \sum_{j=0}^{i} |\Delta_{ij}|$, where Δ_{ij} is the ij entry of the impulse response matrix T_{Δ}.

□

The generalization for uncertainty model (3) can be summarized by the following theorem.

Theorem 1 (Model invalidation): Let β and $u_i = \{u_i(k)\}_{k=0}^{N-1}$, $u_i(k) \in \mathbb{R}$ and $e_i = \{e_i(k)\}_{k=0}^{N-1}$, $e_i(k) \in \mathbb{R}$, $i = 1, ..., L$, i.e. data from L plants are given. There exists a common perturbation $\Delta \in \mathcal{L}_{LTV}$ with $\|\Delta\|_1 \leq 1$ and disturbances $d_i = \{d_i(k)\}_{k=0}^{N-1}$, $d_i(k) \in \mathbb{R}$ with $\|d_i\|_{\infty} \leq 1$, $i = 1, ..., L$ such that (3) holds if and only if there exists vectors $x^+ \in \mathbb{R}^{(N+N^2)/2}$ and $x^- \in \mathbb{R}^{(N+N^2)/2}$ with $x^+ \geq 0$ and $x^- \geq 0$ that satisfy

$$\left|\beta^{-1} T_W^{-1}\left(e_i - \Gamma_{u_i}(x^+ - x^-)\right)\right| \leq 1_N, \; i = 1, ..., L \tag{9}$$

$$|\Gamma_{1_N}(x^+ + x^-)| \leq 1_N \tag{10}$$

where the absolute value and the inequality are meant elementwise.

Proof: The necessity is trivial. If the conditions hold then $d_i := \beta^{-1} T_W^{-1}\left(e_i - \Gamma_{u_i}(x^+ - x^-)\right)$ solves (3).

□

C. Computation of uncertainty trade-off curve

The uncertainty trade-off curve as defined in [16] is a simple parametrization of the consistent models and can be applied in the computation of unfalsified performance. More general parametrization is also possible, for example, by parameterizing weighting functions, but it heavily increases the computation time. Practical frequency-domain methods exist for the $\mathcal{H}_{\infty}$ unfalsification problems, see e.g. [12].

In the following theorem the linear programming reformulation of Theorem 1 is provided that allows the pointwise computation of trade-off curve $\alpha(\beta)$.

Theorem 2 (Model unfalsification): Let β and $u_i = \{u_i(k)\}_{k=0}^{N-1}$, $u_i(k) \in \mathbb{R}$ and $e_i = \{e_i(k)\}_{k=0}^{N-1}$, $e_i(k) \in \mathbb{R}$, $i = 1, ..., L$, i.e. data from L plants are given. The smallest α that allows a consistent model such that $\|\Delta\|_1 \leq \alpha$ is given by the solution of the following linear programming problem

$$\min_{\alpha, x^+, x^-} \alpha \text{ subject to } Ax \leq b, \;\; Bx \leq 0 \tag{11}$$

where $x = [\alpha \;\; x^{+T} \;\; x^{-T}]^T$ and

$$A = \begin{bmatrix} 0 & \beta^{-1}T_W^{-1}\Gamma_{u_1} & -\beta^{-1}T_W^{-1}\Gamma_{u_1} \\ \vdots & \vdots & \vdots \\ 0 & \beta^{-1}T_W^{-1}\Gamma_{u_L} & -\beta^{-1}T_W^{-1}\Gamma_{u_L} \\ 0 & -\beta^{-1}T_W^{-1}\Gamma_{u_1} & \beta^{-1}T_W^{-1}\Gamma_{u_1} \\ \vdots & \vdots & \vdots \\ 0 & -\beta^{-1}T_W^{-1}\Gamma_{u_L} & \beta^{-1}T_W^{-1}\Gamma_{u_L} \end{bmatrix}$$

$$b = \begin{bmatrix} \beta^{-1}T_W^{-1}e_1 + 1_N \\ \vdots \\ \beta^{-1}T_W^{-1}e_L + 1_N \\ -\beta^{-1}T_W^{-1}e_1 + 1_N \\ \vdots \\ -\beta^{-1}T_W^{-1}e_L + 1_N \end{bmatrix}$$

$$B = \begin{bmatrix} -1_N & -\Gamma_{1_N} & -\Gamma_{1_N} \\ -1_N & \Gamma_{1_N} & \Gamma_{1_N} \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}$$

where I denotes the $\frac{N+N^2}{2}$-by-$\frac{N+N^2}{2}$ unit matrix.

Proof: Consistency condition $Ax \le b$ is a reformulation of the inequalities in (9). Norm condition $\|\Delta\|_1 \le \alpha$ is equivalent to $|\Gamma_{1_N}(x^+ + x^-)| \le \alpha 1_N$, which, together with the positiveness conditions for x^+ and x^- are guarantied if and only if $Bx \le 0$.

It can be seen that the size of the matrices and the length of the parameter vector increase with $\mathcal{O}(N^2)$, however, the matrices are sparse and contain only $\mathcal{O}(N)$ different elements. Problem specific sparse solvers can help in memory problems. The following example demonstrates the applicability and the computational burden of Theorem 2 by using CLP in MPT Toolbox for Matlab [21].

V. EXAMPLE

A. Data generating system

Consider the following LTI discrete-time nominal model

$$G_n(q) = \frac{0.6192}{q - 0.4159} \qquad (12)$$

where q is the forward shift operator and the sampling time is $T_s = 0.5s$. The unmodelled LTV dynamics $\Delta \in \mathcal{L}_{LTV}$ with time-varying state-space matrices $\{a_\Delta(k), b_\Delta(k), c_\Delta(k), d_\Delta(k)\}$ is generated with the help of a stable 8-dimensional random LTI system denoted by S with state-space matrices $\{a_S, b_S, c_S, d_S\}$ followed by an element-wise 20% random perturbation in each time step

$$\begin{bmatrix} a_\Delta(k) & b_\Delta(k) \\ c_\Delta(k) & d_\Delta(k) \end{bmatrix}_{ij} = (1 + 0.2r_{ij}(k)) \begin{bmatrix} a_S & b_S \\ c_S & d_S \end{bmatrix}_{ij} \qquad (13)$$

where $r_{ij}(k) \in [-1, 1]$ is a uniform distributed random number. Finally, Δ is normalized by a static gain so that $\|\Delta\|_1 = 1$. The magnitude plot of S is shown in Figure 1. The true plant is denoted by $G = G_n + \Delta$. Its impulse and step responses can be compared with those of the nominal model in Figures 2 and 3, respectively. In Figure 4 the responses to an input which is worst-case with respect to Δ are presented. At time $t_{wc} = 35.5s$ the maximal gain of Δ is achieved. The data generating systems are formulated by

$$y_i = Gu_i + Wd_i, \qquad i = 1, ..., L \qquad (14)$$

where $W(q) = 0.04\frac{q-0.6561}{q-0.9656}$.

B. Experiment data

Five experiments are taken, each of length $N = 80$. In four of them, the plant is driven by uniformly distributed random inputs, $u_i(k) \in [-2, 2]$, $d_i(k) \in [-1, 1]$, $i = 1, ..., 4$, $k = 1, ..., N$. The fifth experiment is constructed by the worst-case input u_5 which is plotted in Figure 4 and a worst-case disturbance d_5 maximizing the output of Wd_5 at time t_{wc}. If this experiment is included into the data set then the solution of the model unfalsification problem must recover $\alpha = 1$ when $\beta = 1$ is set.

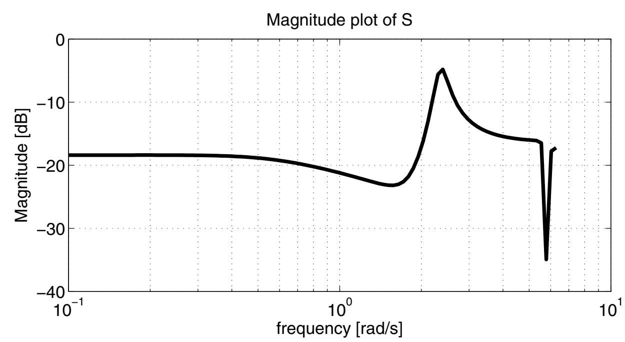

Fig. 1. Magnitude plot of $S(e^{j\omega})$

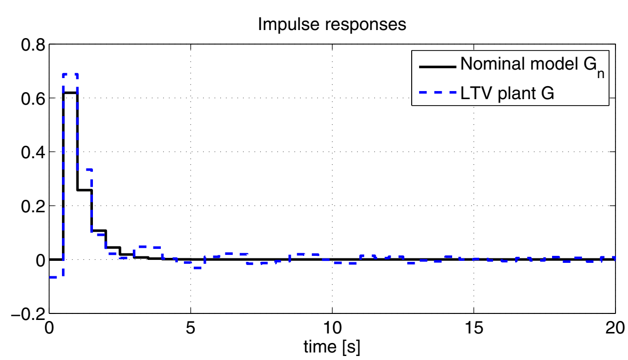

Fig. 2. Impulse responses of nominal model G_n and uncertain system G

C. Model unfalsification

Uncertainty model (3) is considered with $W_i = 1$, $i = 1, ..., 5$ and Theorem 2 is applied for a set of constants $\beta = 0.1, 0.2, ..., 3$. The following experimental data sets are tested

1) Random inputs $\mathcal{E}_1 := \{u_i, y_i\}_{i=1}^4$
2) Worst-case input $\mathcal{E}_2 := \{u_5, y_5\}$

The trade-off curves are shown in Figure 5. Since $\beta = 0.4$ is a good approximation of disturbance weighting function

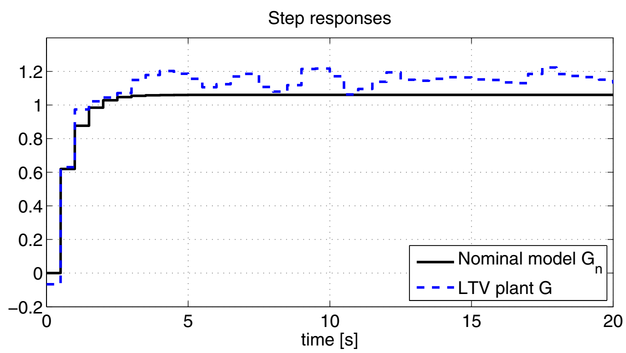

Fig. 3. Step responses of nominal model G_n and uncertain system G

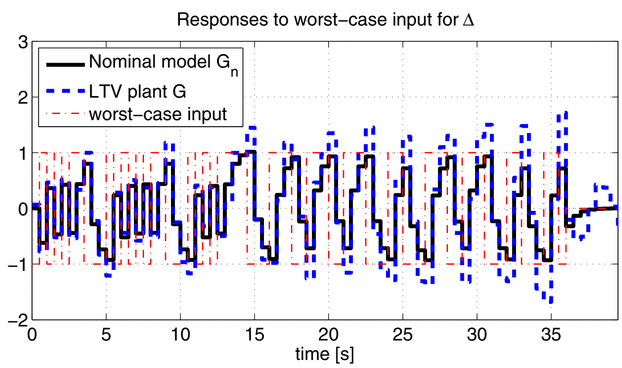

Fig. 4. Responses of nominal model G_n and uncertain system G to an input which is the worst-case with respect to Δ. The maximal gain is achieved at $t_{wc} = 35.5s$.

$W(q)$ at low frequencies, the $\alpha(\beta)$ value at $\beta = 0.4$ can be compared with the true perturbation. In case of $\mathcal{E}_2$, $\alpha(0.4)$ approximately equals to the true bound $\|\Delta\|_1 = 1$. The uniformly distributed random inputs are far from the worst-case in the sense that much smaller (less than 20%) perturbations can reproduce the plant output.

Concerning computational complexity, the LP problem in Theorem 2 for one constant β involves 6481 variables and 7281 inequality constraints and takes 1.6s on a computer with Intel Core 2 Duo CPU, 3GHz, 2GB of RAM and CLP solver of MPT Toolbox [21].

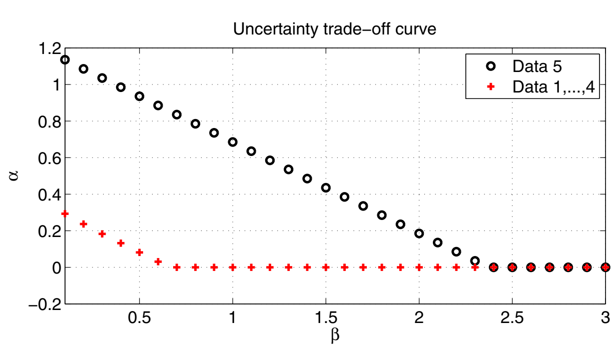

Fig. 5. Trade-off curve $\alpha(\beta)$

D. Discussion

In this discussion we follow the contexture of Remark 1. When the goal is to compute unfalsified robust performance, one has to collect experimental data which is close to that occurring in the closed-loop. It is unrealistic that a closed-loop controller produces inputs similar to that worst-case one in Figure 4. The goal of unfalsification is not to reproduce the true bounds of the uncertainties. In the closed-loop only subsets of the entire signal spaces can excite the perturbations. It is sufficient to prepare the model for these real-life excitations. For this reason, it is worth calculating the unfalsified performance, even if proper information is available about the size and structure of uncertainties and disturbances.

VI. Conclusions

Model invalidation and unfalsification are examined for systems with additive repeated LTV perturbations and disturbances. The conditions for invalidation are convex and characterized by linear equality and inequality constraints. The search for the minimal bound on the repeated perturbation leads to a linear programming problem of $\mathcal{O}(N^2)$ variables and $\mathcal{O}(N)$ constraints where N is the number of data involved. The applicability of the proposed algorithm is demonstrated on a simulation example.

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