

A Robust Control of Contact Force of Pantograph-Catenary for The High-Speed Train

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Abstract

In this paper, a simplified model to three degree of freedom (3-DOF) of the pantograph-catenary (PAC) system is represented. A robust control of contact force between the pantograph and the catenary for the high-speed trains using the fuzzy sliding mode controller and the additional compensator is presented. A PID outer loop in the control law is used then the gains of the sliding term and PID term are tuned on-line by a fuzzy system. The result of the simulation is fit for the actual phenomenon. Thus, the method of this paper is valid.

1. INTRODUCTION

One of the main problems in high-speed train transportation systems is the control of the contact force between the catenary and the pantograph collector end. The force exerted by the pantograph on the contact wire oscillates a lot, such oscillations can originate electric arcs that damage the mechanical structure and reduce the system performance [1].

The contact wire is an elastic system which can be characterized from a static perspective by its stiffness and from a dynamic one by its frequency. In addition, by design, it contains non-linearities that make the modeling difficult.

The pantograph is an articulated system, also non-linear, which has its own dynamics and can be modeled more or less finely. Studies on the interac-

tion between the pantograph and the catenary [2, 3] have shown that the variation of the contact force is primarily caused by the variation of the elasticity along the reach, and the waves propagation in the catenary's contact wire (console and return arm).

When the pantograph slides under the catenary, the change in stiffness causes a periodic excitation which leads to the vibration of the pantograph and to fluctuations in the contact force. Moreover, as the pantograph head slides along the contact wire of the catenary, it causes an increase in wave's propagation along the contact wire. This wave's propagation affects the contact force and also the pantograph movement. To provide answers on the dynamic behavior of the system, an improved model; more general compared to the one proposed in [4, 5, 6], was designed.

The task of regulating the contact force to a pre-specified constant value of about 100N in the presence of model uncertainties and external disturbances is suggested. Different approaches have been followed in order to cope with this problem, such as, for instance, Optimal control strategy, [7, 8, 9, 10], H control method [11], fuzzy-sliding mode control [12, 13], Variable Structure Control (VSC) with Sliding Modes (SM) [4, 14, 15].

A Sliding Mode Fuzzy Controller (*SMFC*) inherits the robustness property of Sliding Mode Control and Interpolation property of Fuzzy Logic Control, so that the non-linear switching curve can be approximated and the robustness can be maintained. Sliding mode control is known for its robustness to the external disturbance and system modeling error [16]. In Sliding Mode Fuzzy Controller each fuzzy rule output function is exactly a sliding mode controller, the slope of the sliding mode controller in each rule is determined by the approximate slope of the nonlinear switching

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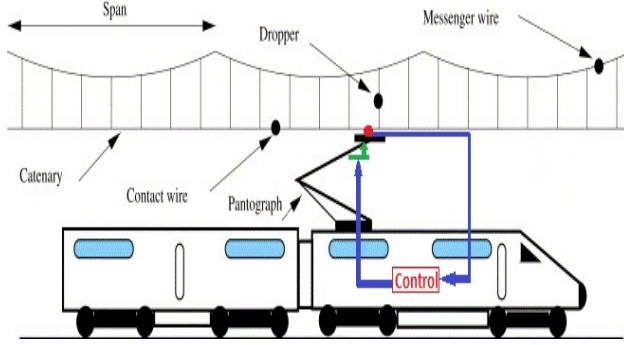


Figure 1. Principle of active control

curve in that partition of the phase plane where the rule covers [16].

In this study, a robust control of the contact force between the pantograph and the catenary, for the high-speed trains, using the fuzzy sliding mode controller and the additional compensator is presented. We have used a PID outer loop in the control law, then the gains of the sliding term and the PID term are tuned online by a fuzzy system. This control law has the advantage of controlling the pantograph without having to use a state observer, which is complicated to calculate for our system and difficult to implement (you must solve the differential equation of Riccati), because the parameters of the synthesized control are based on fuzzy logic, so they do not depend on the state of the system.

This paper is organized as following: In section II, the mathematical 3-DOF time-variant model of the pantograph-catenary system is represented. In section III, we present the controller design. Section IV presents the simulation results.

2. THE MATHEMATICAL MODEL OF SYSTEM PAC

In this section, we introduce the mathematical models for the pantograph and the catenary. In detail, the pantograph model used for the initial control system design is the tree-mass model shown in figure3. This model of the type CX pantograph is shown below. The upper mass (m_1) represents, the pantograph head, the mass in the middle (m_2)

and mass lower (m_3) represents, respectively, upper arm and lower arm of the pantograph. The linear time varying (LTV) model of the pantograph, like the one shown here, has the ability to describe the response of the pantograph in a wide frequency range.

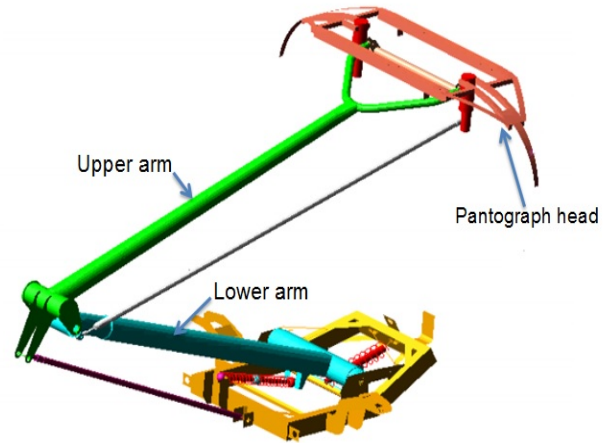


Figure 2. Mechanism of CX Pantograph

The equations governing this system (movement of the pantograph) are:

$$\begin{cases} m_c(t)\ddot{x}_c + b_c(t)\dot{x}_c + k_c(t)x_c \\ + m_1\ddot{x}_1 + b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0 \\ m_2\ddot{x}_2 + b_2(\dot{x}_2 - \dot{x}_3) + k_2(x_2 - x_3) \\ + b_1(\dot{x}_2 - \dot{x}_1) + k_1(x_2 - x_1) = 0 \\ m_3\ddot{x}_3 + b_3\dot{x}_3 + k_3x_3 + b_2(\dot{x}_3 - \dot{x}_2) \\ + k_2(x_3 - x_2) = F_u \end{cases} \quad (1)$$

During normal operation, the pantograph is in contact with the catenary, which implies $x_1 \equiv x_c$ and therefore the weight of the upper block is $m_3 + m_c$; where m_c is the weight of the catenary. Therefore, we obtain the following state equations:

$$\begin{cases} \dot{x} = A(t)x + Bu + Dw(t) \\ y = Cx \end{cases} \quad (2)$$

Where $u \in R^1$ is the input, $y \in R^1$ is the output, $x \in R^{1 \times n}$ is the state vector where $x =$

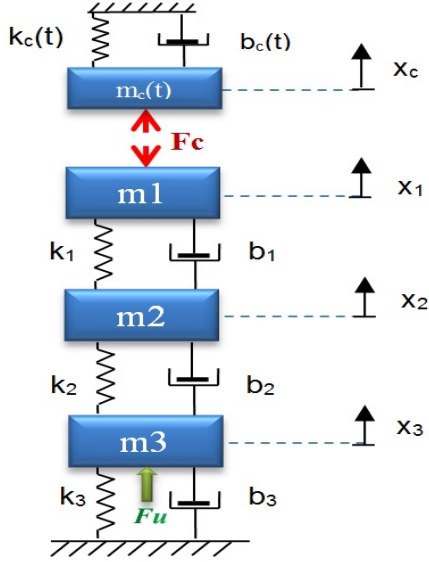


Figure 3. The simplified linear model

$[x_1 \ x_2 \ x_3 \ \dot{x}_1 \ \dot{x}_2 \ \dot{x}_3]^T$, $A \in R^{n \times n}$ is the nominal plant coefficient matrix, $B \in R^{n \times 1}$ is the nominal input coefficient vector, $C \in R^{1 \times n}$ is the output coefficient vector, and w represents the system uncertainty.

$$A(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\mu_{21} & -\mu_{22} & \frac{k_2}{m_1} & \frac{b_2}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & -\mu_{43} & -\mu_{44} & \frac{b_1}{m_2} & \frac{b_1}{m_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{k_2}{m_3} & \frac{b_2}{m_3} & -\mu_{65} & -\mu_{66} \end{bmatrix} \quad (3)$$

Where:

$$\mu_{21} = \frac{k_1 + k_c}{m_1 + m_c}, \quad \mu_{22} = \frac{b_1 + b_c}{m_1 + m_c}, \quad \mu_{43} = \frac{k_1 + k_2}{m_2}$$

$$\mu_{44} = \frac{b_1 + b_2}{m_2}, \quad \mu_{65} = \frac{k_3 + k_2}{m_3}, \quad \mu_{66} = \frac{b_3 + b_2}{m_3}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{m_3} \end{bmatrix}^T \quad (4)$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{m_3} \end{bmatrix}^T \quad (5)$$

$$C = [-H_1 \quad -H_2 \quad H_3 \quad H_4 \quad 0 \quad 0] \quad (6)$$

Where:

$$H_1 = \frac{m_1 * k_c + (2 * m_1 + m_c) * k_1}{m_c + m_1},$$

$$H_2 = \frac{m_1 * b_c + (2 * m_1 + m_c) * b_1}{m_c + m_1}$$

$$H_3 = \frac{(2 * m_1 + m_c) * k_1}{m_c + m_1}, \quad H_4 = \frac{(2 * m_1 + m_c) * k_1}{m_c + m_1}$$

The catenary is modeled using the system mass-spring and damper.

A more accurate model of the catenary modeling takes into account the variation of the mass and the damper along its scope. The catenary is modeled through the system mass, spring and damper. The mechanical properties of the catenary; mass m_c , spring stiffness K_c and the damping constant b_c , exhibit periodic characteristics throughout each section. Thus we will consider Fourier series expansion for the characteristic parameters of the catenary, taking into account the first, second and third harmonic [17].

$$m_c(t) = m_{c0} + \sum_{i=1}^3 m_{ci} \cos\left(\frac{2i\pi}{L} x(t)\right)$$

$$b_c(t) = b_{c0} + \sum_{i=1}^3 b_{ci} \cos\left(\frac{2i\pi}{L} x(t)\right) \quad (7)$$

$$k_c(t) = k_{c0} + \sum_{i=1}^3 k_{ci} \cos\left(\frac{2i\pi}{L} x(t)\right)$$

Where $x = x(t)$ the actual distance of the train from a tower, L is the span length. If the train speed is constant, $x(t) = V \times t$; V is train speed and t the time taken to travel that distance.

Parameter	Notation	Value
Catenary mass parameters	m_{c0}	195kg
	m_{c1}	100kg
	m_{c2}	20kg
	m_{c3}	5kg
Catenary damping parameters	b_{c0}	24Nm ⁻¹ s
	b_{c1}	240Nm ⁻¹ s
	b_{c2}	50Nm ⁻¹ s
	b_{c3}	12Nm ⁻¹ s
Catenary stiffness parameters	k_{c0}	7000Nm ⁻¹
	k_{c1}	3360Nm ⁻¹
	k_{c2}	650Nm ⁻¹
	k_{c3}	160Nm ⁻¹
Span Length	L	65.52m

Table 1. Baseline Catenary Parameters[5]

3. Active Control Strategy

- **Fuzzy PID Sliding mode controller**

Consider the output error $e = y(t) - F_c$; where $F_c = 100N$ is the desired constant value for the contact force. The sliding function is defined as:

$$S = \dot{e} + \lambda_1 e + \lambda_2 \int_0^t e dt \quad (8)$$

Where \dot{e} is the first derivative of error and λ_1, λ_2 are a strictly positive real constant.

The control law is defining as:

$$U = k_d S + k_s \text{sgn}(S) \quad (9)$$

Where:

$$k_d = N_d k_{fuzz}, k = N k_{fuzz}$$

Tracking loop for out PID and k_d, k are gains to ensure for stability and $\text{sgn}(S)$ is the sign function defined as:

$$\text{sgn}(S) = \begin{cases} 1 & \text{if } S > 0 \\ -1 & \text{if } S < 0 \end{cases} \quad (10)$$

The designed fuzzy logic controller has two inputs variables and the output variable. The inputs are sliding surface S and the change of the sliding surface $DS = \frac{S(t) - S(t-1)}{\Delta t}$ and the output is the fuzzy gain k_{fuzz} .

The fuzzy controller consists of three stages: fuzzyfication, inference engine and defuzzyfication. Both fuzzyfication and inference system were tuned [18, 19, 16]. The reasoning used for the inference engine is that of Takagi-Sugeno and for defuzzyfication, we use the method center of gravity to calculate the setpoint change. The membership functions of inputs variables are defined with triangular-shaped functions, as shown in figure4. The universe of discourse of each input is normalized over the interval $[-1 \ 1]$.

The ruleset has been built in an simple way, the idea is to build a table diagonally symmetric zero.

The constructed command tries to evolve the system as quickly as possible to the diagonal of zeros keeping it in this area until equilibrium point (0, 0). This type of rules is very simple; it provides a good control and a good robustness with respect to the parametric variations.

This table contains 25 decision rules composed by pairs situations/action of the form:

When S is (ZE) and DS is (PS), so $k_{fuzz} = 0.5$.

S \ DS	NB	NS	ZE	PS	PB
NB	-2	-1.4	-0.9	-0.5	0
NS	-1.4	-0.9	-0.5	0	0.5
ZE	-0.9	-0.5	0	0.5	0.9
PS	-0.5	0	0.5	0.9	1.4
PB	0	0.5	0.9	1.4	2

Table 2. Fuzzy rule base

Where: NB =Negative big; NS : Negative Small; ZE : Zero; PS : Positive Small; PB : Positive Big.

We took membership functions symmetrical, evenly distributed over the universe of discourse with a 50% overlap, they are symbolized by:

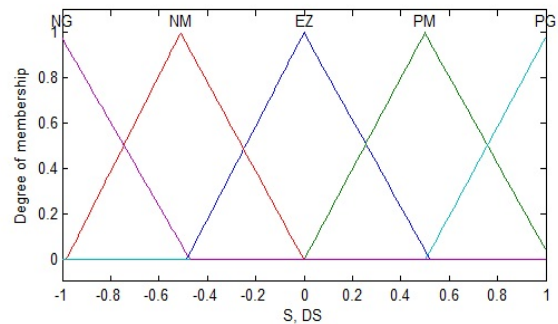


Figure 4. Membership functions for S and DS normalized inputs

The simulink model of the of Fuzzy PID Sliding controller is shown in figure5.

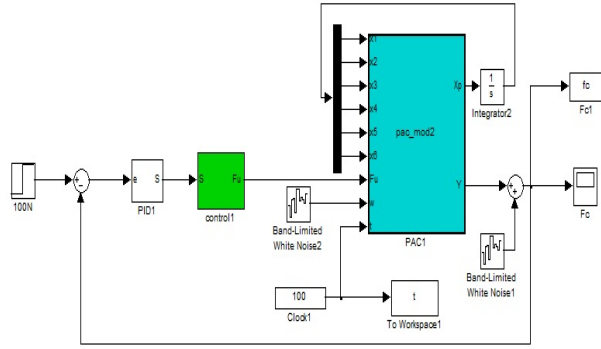


Figure 5. Simulink block diagram of Fuzzy PID Sliding controller

4. SIMULATION RESULTS

Simulation of pantograph-catenary system allows to know the evolution of the contact force influenced by the fluctuations caused by the dynamics of the pantograph and contact wire excitement of the catenary.

This section presents the simulation output of a pantograph traveling under a catenary at high-speed. The parameters for the baseline pantograph are summarized in table 3. The velocity of the train is $V = 350$ km/hr.

Parameter	Notation	Value
Head parameters	m_1	8.5kg
	b_1	$5\text{Nm}^{-1}\text{s}$
	k_1	6045Nm^{-1}
Upper frame parameters	m_2	4.63kg
	b_2	$5400\text{Nm}^{-1}\text{s}$
	k_2	50Nm^{-1}
Catenary stiffness parameters	m_3	4.8kg
	b_3	$32\text{Nm}^{-1}\text{s}$
	k_3	1Nm^{-1}

Table 3. Baseline Pantograph Parameters[20]

Variations in the positions of the pantograph and the catenary's masses allow getting the evolution of the contact force. Note that x_1 is the point of contact between the contact wire of the catenary and the pantograph, x_2 and x_3 are the states representing variation in the vertical positions of the masses m_2 and m_3 . These states reflect the vertical

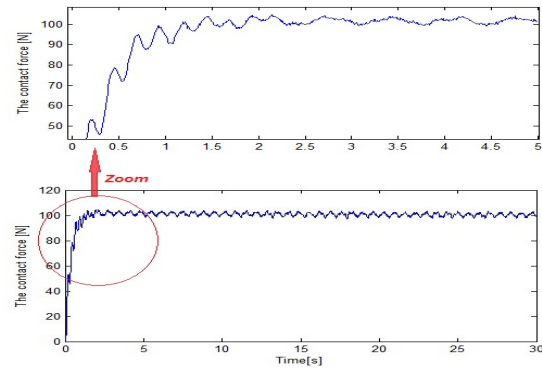


Figure 6. Contact force for the active Pantograph

displacements of the pantograph-catenary. Figure 7 shows the evolution of vertical positions variation during time.

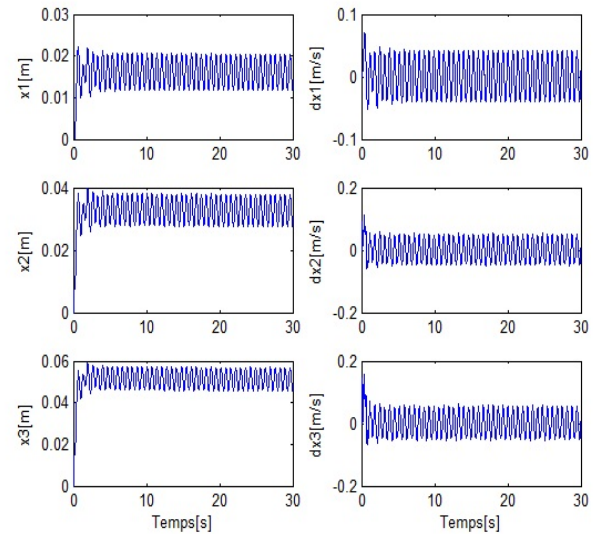


Figure 7. Simulation of displacement and velocities of the masses of the model

The evolution of x_1 indicates that the contact between the pantograph and the catenary is permanent. The observed oscillations show the vertical movements of the pantograph's floors. These oscillations are influenced by the parameters of the pantograph which depend on the speed of the train; therefore they simulate the fluctuations that can be observed in a real simulation of pantograph-catenary.

5. CONCLUSION

In this study, a simplified mathematical model to three degree of freedom (3-DOF) of pantograph is presented. The problem of regulating the contact force of pantograph-catenary is exposed; a solution to this problem based on the fuzzy logic and sliding modes with a PID compensator is presented. The additional compensator relaying on the sliding mode theory is used to improve the dynamical characteristics of the drive system. The simulations are applied to linear time-varying and uncertain of pantograph-catenary and a speed of 350 Km/h. Simulation results confirm the good performance and robustness to uncertainties and external disturbances of the proposed control.

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