2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Scenario-Based MPC for Energy-Efficient Building Climate Control under Weather and Occupancy Uncertainty

Xiaojing Zhang, Georg Schildbach, David Sturzenegger, and Manfred Morari

Abstract— **Heating, ventilation and air conditioning (HVAC) systems regulate comfort levels in buildings, but also consume a large amount of energy, which makes them an attractive target for efficiency improvements. In this paper, a novel technique called Randomized Model Predictive Control (RMPC) is investigated to improve the control of existing HVAC systems. RMPC uses weather and occupancy predictions to minimize the building's energy consumption. It accounts for the prediction uncertainties by basing its control actions on a given number of sampled uncertainty scenarios. The main advantage of RMPC over existing methods is the absence of a probabilistic disturbance model. This makes the handling of uncertainties straightforward, even if they are non-Gaussian or non-additive. Moreover, the method of removing adverse samples after solving the initial control problem (RMPC-SR) can lead to a further improvement in the control performance, up to a saturation limit. Although theoretical bounds for choosing the sample sizes are available, our simulations show that only a fraction of these numbers is required for a good performance of RMPC and RMPC-SR. The performance of RMPC and RMPC-SR is investigated through extensive simulations on different models, based on empirically collected data. The results demonstrate that both techniques are attractive alternatives to other Model Predictive Control methods, because they show a higher energy saving potential, and are computationally tractable.**

I. INTRODUCTION

Buildings nowadays account for roughly 40% of total energy use in the world, as well as 76% and 73% of electricity consumption in Europe and the United States, respectively [1], [2]. Heating, ventilation and air conditioning (HVAC) systems regulate carbon dioxide levels, illuminance, and keep temperature and airflow within comfort ranges in buildings. However, they also consume approximately half of a building's energy usage, making them an attractive target for saving energy [3].

A. Review of Existing Strategies

Different attempts are being taken towards reducing energy consumption of HVAC systems. One direction deals with more energy-efficient HVAC components and better architectural design [4]. Another direction is aimed at improving the efficiency of existing, or slightly retrofitted, HVAC systems through better configurations and new control algorithms [5]–[8]. We chose to focus on the latter because buildings and HVAC equipment are often refurbished slowly. We investigated a new control method based on a basic HVAC system, present in many office buildings. Our aim is to reduce energy usage and costs while guaranteeing comfort for occupants. According to European building standards [9], the room temperature is allowed to occasionally leave the comfort bounds during extreme weather conditions. To deal with this, chance-constraint MPC has been proposed in [5].

The current practice in the building control industry is to use (non-predictive) Rule-Based Controllers (RBC) that perform control actions based on typically uncoordinated "if-then-else" rules. Recent work using Model Predictive Control (MPC) has shown that significant energy savings are possible if weather and occupancy predictions are incorporated into the controller [10]–[14]. In [5], a framework for Stochastic MPC (SMPC) considering weather uncertainty was presented. SMPC, however, is limited by two factors. First, SMPC does not consider the errors made when linearizing the bilinear system dynamics. Second, SMPC requires an additive Gaussian disturbance model which does not hold for the weather and occupants uncertainty, because these can be non-additive and are bounded.

B. Scenario-Based MPC

This project evaluates the energy saving potential of two novel techniques. The first is called Randomized MPC (RMPC) [15], [16], which is based on the scenario (also called sampling) approach [17]–[19]. Similar to SMPC, RMPC handles chance constraints but with the following two advantages: first, RMPC drops the necessity of the disturbance model being Gaussian. Instead, RMPC works with data of any distribution, both discrete and continuous. Second, RMPC is able to tackle the linearization error of the system dynamics. The second technique is RMPC with Sample Removal (RMPC-SR), which is an extension of RMPC, and aims to further improve the energy efficiency by a posteriori removal of adverse samples (sampling-and-discarding approach) [19], [20]. We compare two discarding methods and examine the effect of removing a high number of samples. RMPC and RMPC-SR are studied on two building types at two building sites. Based on those four scenarios, large-scale simulations have been carried out to assess the energy saving potential of RMPC and RMPC-SR, which are compared against SMPC.

We begin by formally introducing the uncertainty and building model in Section II. Section III describes the investigated control strategies, in particular RMPC and RMPC-SR. We present simulation results in Section IV, and conclude this paper with a discussion of the results in Section V.

X. Zhang, G. Schildbach, D. Sturzenegger, and M. Morari are with the Automatic Control Laboratory, Department of Electrical Engineering and Information Technology, Swiss Federal Institute of Technology Zurich (ETH Zurich), 8092 Zurich, Switzerland. {xiaozhan, schildbach, sturzenegger, morari}@control.ee.ethz.ch

II. Modeling

A building is subject to a number of external influences such as weather conditions and the presence of occupants and equipment (internal gains). We start by introducing the weather and internal gain uncertainty, before stating the building dynamics. The models used in this paper are taken from the OptiControl project[1]. Further details can be found in [5], [21], [22], and the references therein.

A. Weather Uncertainty Model

Weather uncertainty arises due to the discrepancy between the actual weather $v^{\mathrm{w}} \in \mathbb{R}^{n_{\mathrm{w}}}$ and its forecast $\bar{v}^{\mathrm{w}} \in \mathbb{R}^{n_{\mathrm{w}}}$. We consider three weather variables: outside air temperature, wetbulb temperature, and solar radiation. The (raw) weather forecast data and realizations were provided by MeteoSwiss[2] for the years 2006 and 2007. The forecast error at time t is then defined as $\tilde{v}^{\mathrm{w}}_t \triangleq v^{\mathrm{w}}_t - \bar{v}^{\mathrm{w}}_t$. Since the forecast is updated every 12 hours, it is desirable to exploit temporal correlation in the prediction error [5], [21]. To this end, we use the following autoregressive model to improve the forecast

$$\tilde{v}^{\mathrm{w}}_{t+i+1|t} = F^{\mathrm{w}} \tilde{v}^{\mathrm{w}}_{t+i|t} + \bar{w}^{\mathrm{w}}_{t+i|t} + w^{\mathrm{w}}_{t+i|t}, \tag{1}$$

where $F^{\mathrm{w}} \in \mathbb{R}^{n_{\mathrm{w}} \times n_{\mathrm{w}}}$ and $\bar{w}^{\mathrm{w}}_{t+i|t} \in \mathbb{R}^{n_{\mathrm{w}}}$ are the parameters of the uncertainty model. $w^{\mathrm{w}}_{t+i|t}$ are residuals and can be interpreted as the disturbance acting on the system. F^{w} and $\bar{w}^{\mathrm{w}}_{t+i|t}$ were identified based on 2006 data ("design data"), but model (1) was used to improve the prediction of the year 2007 ("simulation data"). If we define the estimated forecast error by $\hat{\tilde{v}}^{\mathrm{w}}_{t+i+1|t} \triangleq F^{\mathrm{w}} \hat{\tilde{v}}^{\mathrm{w}}_{t+i|t} + \bar{w}^{\mathrm{w}}_{t+i|t}$, then the (improved) weather prediction is

$$\hat{v}^{\mathrm{w}}_{t+i|t} \triangleq \bar{v}^{\mathrm{w}}_{t+i|t} + \hat{\tilde{v}}^{\mathrm{w}}_{t+i|t}. \tag{2}$$

Therefore, the weather disturbance acting on the building can be written as

$$v^{\mathrm{w}}_{t+i|t} = \hat{v}^{\mathrm{w}}_{t+i|t} + w^{\mathrm{w}}_{t+i|t}. \tag{3}$$

Hence, at each time step t, the measured forecast error $\tilde{v}^{\mathrm{w}}_{t-1|t}$ is taken to compute the predictions $\hat{v}^{\mathrm{w}}_{t|t}, \ldots, \hat{v}^{\mathrm{w}}_{t+N-1|t}$ by first iterating (1) and then applying (2).

B. Internal Gain Uncertainty

Internal gains (IG) are assumed to consist of two parts: the IG resulting from occupants and the IG due to equipment such as computers and other electric components. The simulation was carried out using real data collected from the Actelion building in Basel, Switzerland. More precisely, occupancy measurement data from eight single offices equipped with motion sensors was evaluated. The data from four rooms was collected in the so-called "design data" set, and the data from the remaining four rooms was put into the "simulation data" set. The mean of the "design data" set is used to compute a weekly schedule $\hat{v}^{\mathrm{IG}} \in \mathbb{R}^{n_{\mathrm{IG}}}$, which serves as the IG prediction. Hence, the IG disturbance acting on the building can be decomposed as

$$v^{\mathrm{IG}}_{t+i|t} = \hat{v}^{\mathrm{IG}}_{t+i|t} + \tilde{v}^{\mathrm{IG}}_{t+i|t}, \tag{4}$$

in analogy to the weather uncertainty (3). Here $\tilde{v}^{\mathrm{IG}}$ denotes the IG prediction error which, like w^{w}, can be interpreted as disturbances acting on the building.

C. Building Model

Many different models of varying complexity and purposes have been proposed in the literature. On one hand, complex and (strongly) nonlinear models that offer detailed representation are challenging for numerical simulations. On the other hand, simple linear models may fail to incorporate the essential underlying physics. As a reasonable trade-off between both cases, we use the 12th order bilinear model from [5], which incorporates the thermal and light dynamics of a single zone. This model was shown to capture the essential building dynamics with a temporal resolution of one hour, and is simple enough to be integrated in an MPC controller.

Let $\theta \in \mathbb{R}^{n_\theta}$ denote the state, $u \in \mathbb{R}^{n_u}$ the input and $v \triangleq (v^{\mathrm{w}}, v^{\mathrm{IG}}) \in \mathbb{R}^{n_{\mathrm{w}}+n_{\mathrm{IG}}}$ the external influences. Then the discrete-time room dynamics are given by

$$\theta_{t+1} = A\theta_t + \left(B_u + B_{\theta u}[\theta_t] + B_{vu}[v_t]\right) u_t + B_v v_t, \tag{5}$$

where A, B_u, B_v are matrices of appropriate sizes and $B_{\theta u}[\theta], B_{vu}[v]$ are defined as follows:

$$B_{\theta u}[\theta] \triangleq \begin{pmatrix} B_{\theta u,1}\theta & B_{\theta u,2}\theta & \cdots & B_{\theta u,m}\theta \end{pmatrix} \in \mathbb{R}^{n_\theta \times n_u},$$

where $B_{\theta u,i} \in \mathbb{R}^{n_\theta \times n_\theta}$. $B_{vu}[v]$ is defined accordingly. Note that the bilinearities appear between state-input and uncertainty-input, and represent heat fluxes that depend on the control input. Four actuators are modeled by the input u which corresponds to heating (radiator), cooling (evaporative cooling), lighting, and blind position. Further details can be found in [21].

III. Control Strategies

Given some initial state $\theta_{t|t}$, our objective is to reduce the consumption of Non-Renewable Primary Energy (NRPE) [5], which at time t is given by

$$c_t^T u_t \in \mathbb{R}, \tag{6}$$

where $c_t \in \mathbb{R}^{n_u}$ is the cost vector. Two types of constraints are enforced: input constraints and comfort level constraints, where the latter require the room temperature to lie within predefined bounds. Inspired by European building standards [9], the temperature constraints need not be satisfied at all times, but only below 70 Kh/a[3], which is formulated as chance constraints [5]. Hence, the constraints are

$$\begin{aligned} S_t u_t &\leq s_t, \\ \mathrm{P}[G_t\theta_t \leq g_t] &\geq 1 - \epsilon_t, \end{aligned} \tag{7}$$

where $S_t \in \mathbb{R}^{n_q \times n_u}, G_t \in \mathbb{R}^{n_r \times n_\theta}$, $\mathrm{P}[\cdot]$ denotes probability of an event and ϵ_t the violation probability. n_q and n_r denote the number of input and state constraints, respectively. Note that setbacks in the constraints are considered to save energy, i.e. the comfort level constraints are relaxed during non-working hours and weekends.

[1] http://www.opticontrol.ethz.ch
[2] http://www.meteoschweiz.ch

[3] Kelvin Hour per year

A. DMPC with Constraint Tightening

Deterministic MPC, also known as Certainty Equivalence MPC, assumes that the predictions are perfect and uses these predictions to make a decision. It is known that DMPC can perform acceptably when the uncertainties are small due to the feedback introduced by the receding horizon approach. However, it has also been observed that DMPC performs poorly in the presence of large uncertainties. In such cases, disturbances must be considered when designing the controller [23]. One way to do this is to provide a buffer zone and artificially keep the room temperature away from the bounds. Hence, we tighten the constraints by $\tau \in \mathbb{R}$, which is a tuning variable:

$$\begin{aligned} \min_{U_t} \quad & \sum_{i=0}^{N-1} c_{t+i|t}^T u_{t+i|t} \qquad (8) \\ \text{s.t.} \quad & \theta_{t+i+1|t} = A\theta_{t+i|t} + B[\hat{\theta}_t, \hat{v}_{t+i|t}]u_{t+i|t} + B_v \hat{v}_{t+i|t} \\ & S_{t+i} u_{t+i|t} \leq s_{t+i}, \quad G_{t+i}\theta_{t+i|t} \leq g_{t+i} - \tau \\ & \theta_{t|t} = \hat{\theta}_t, \qquad i = 0, \ldots, N-1, \end{aligned}$$

where $U_t \triangleq (u_{t|t}, \ldots, u_{t+N-1|t})$, $\hat{\theta}_t$ is the initial state (estimate), $B[\theta, v] \triangleq B_u + B_{\theta u}[\theta] + B_{vu}[v]$ and N is the prediction horizon. Note that the state-input bilinearity has been linearized around the initial state, and the disturbance-input bilinearity linearized around the external influence prediction. Constraint tightening is needed to counter the errors made when assuming the external influence v equals its prediction $\hat{v}$, because those errors cannot be compensated for by feedback only. In fact, feedback alone cannot guarantee a reasonable violation level in our problem setup.

B. SMPC with Additive Gaussian Noise

In contrast to DMPC, Stochastic MPC interprets the comfort constraints as chance constraints [24]–[27] because they do not have to be satisfied at all times [9]. The approach taken in this project is based on [5], but does not use the Affine Disturbance Feedback (ADF) technique [28], because no measurable performance degradation was observed in our simulations. The basic idea is to augment the dynamics (5) with the AR-model (1). SMPC ignores the uncertainty in the disturbance-input bilinearity and assumes that the uncertainty enters only in the additive term, i.e. w^{w} in (3) and $\tilde{v}^{\mathrm{IG}}$ in (4) are assumed to be Gaussian random variables. At each time step, SMPC then solves a chance-constraint optimization problem given an violation probability, which is a tuning variable itself. For further details the reader is referred to [21].

C. RMPC based on Scenario Approach

While DMPC and SMPC have deterministic reformulations, Randomized MPC is stochastic in nature as it is based on the scenario approach [17]–[19]. The scenario approach is an attractive method to handle random convex programs that are convex when the uncertainty is fixed. Its theory guarantees that if the number of samples M from the uncertainty set is large enough, then the solution of the scenario program is feasible with high confidence for the corresponding chance-constraint problem. Hence, the scenario approach can be used to approximate chance-constraint MPC problems [15]. In our case, the uncertainty set is the prediction error and we sample from $\tilde{v} \triangleq (w^{\mathrm{w}}, \tilde{v}^{\mathrm{IG}})$ constructed from the "design data" set. Hence, RMPC takes the following form:

$$\begin{aligned} \min_{U_t} \quad & \sum_{i=0}^{N-1} c_{t+i|t}^T u_{t+i|t} \qquad (9) \\ \text{s.t.} \quad & \theta_{t+i+1|t} = A\theta_{t+i|t} + B[\hat{\theta}_t, \hat{v}_{t+i|t} + \tilde{v}_{t+i|t}^{(j)}]u_{t+i|t} \\ & \qquad + B_v(\hat{v}_{t+i|t} + \tilde{v}_{t+i|t}^{(j)}) \\ & S_{t+i} u_{t+i|t} \leq s_{t+i}, \qquad G_{t+i}\theta_{t+i|t} \leq g_{t+i} \\ & \theta_{t|t} = \hat{\theta}_t, \qquad i = 0, \ldots, N-1, \quad j = 1, \ldots, M. \end{aligned}$$

Note that RMPC (9) is an LP and therefore can be solved efficiently even for large M. We have, however, observed that M can be chosen much smaller than the theoretical numbers indicated in [19], [29], [30], which is discussed later in Section V.

The major advantages of RMPC over SMPC are twofold: first, it is able to take the prediction error in the bilinearity into account. Second, RMPC directly samples from the uncertainty set, releasing the assumption of the uncertainty being Gaussian distributed. In addition, RMPC does not require artificial constraint tightening as DMPC does.

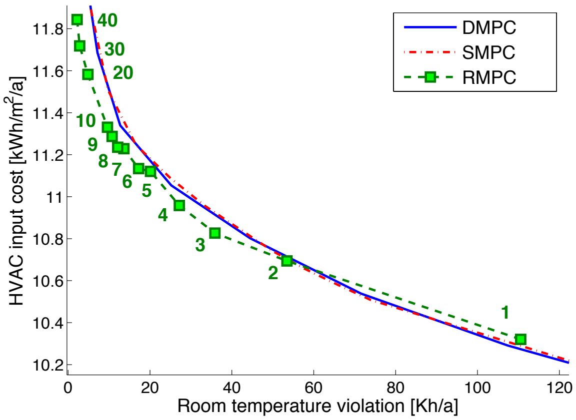
D. RMPC with Sample Removal

The goal of sample removal is to improve the objective value of (standard) randomized programming while guaranteeing a low violation probability of the nominal constraints. In RMPC-SR, a greater number of scenarios is sampled compared to RMPC, but some are discarded a posteriori to improve the cost function. It is shown in [19], [20] that if the numbers of sampled and discarded scenarios are well-chosen, then the solution of the sampling-and-discarding problem is still feasible with high confidence for the chance-constraint problem. Further details can be found in the references indicated above.

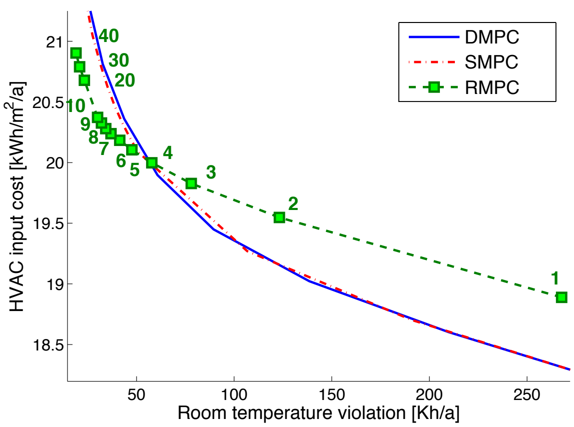
In the first part, we compare the performance of the greedy removal algorithm and the multiplier removal algorithm ([19], [20], [30]) for RMPC-SR when just one sample is removed. In the second part, we investigate further performance improvements when several samples are discarded by applying the removal algorithms sequentially multiple times.

IV. Simulation Results

This section presents simulation results of the controllers DMPC, SMPC, RMPC and RMPC-SR. The findings are discussed in the next section. Large scale simulations were run for two building types (passive house, PA, and Swiss average house, SA) at two locations (Marseille-Marignane in France, MSM, and Wien-Hohe-Warte in Austria, WHW), resulting in four cases in total. A detailed description of the location and building types can be found in [22]. We illustrate our findings based on two cases (*WHW PA* and *MSM SA*)

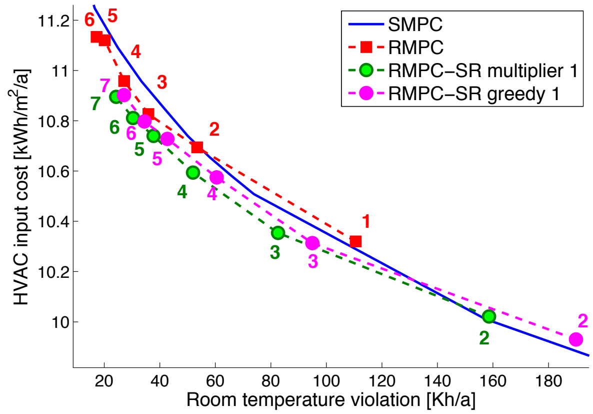


(a) *MSM PA*

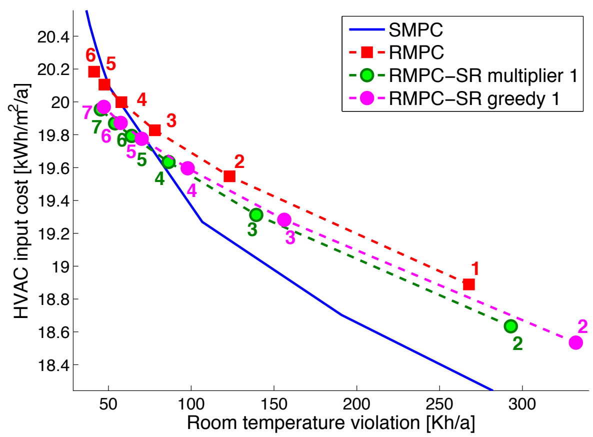


(b) *WHW SA*

Fig. 1. (a)–(b) Pareto frontier of DMPC, SMPC and RMPC. The numbers indicate M, the number of samples drawn in RMPC.



(a) *MSM PA*



(b) *WHW SA*

Fig. 2. (a)–(b) Pareto frontier of RMPC-SR. Labels of RMPC indicate the number of samples drawn, whereas labels of RMPC-SR enumerate the initial number of samples. Hence, a label 4 for "RMPC-SR multiplier/greedy 1" indicates that one sample was removed from a total of 4 samples.

which reflect the overall observations. The annual non-renewable Primary Energy (NRPE) usage of all automated HVAC systems and the total amount of thermal comfort violations over one year were evaluated. According to European standard, temperature violation below 70 Kh/a is tolerated [9]. The prediction horizon in MPC was set to the maximum of 60 hours allowed by the weather forecast data. Shorter horizons were observed to deteriorate the performance of the controller.

A. DMPC, SMPC and RMPC

Fig. 1 illustrates the obtained Pareto frontiers [21] of DMPC, SMPC and RMPC for *MSM PA* and *WHW SA*. The Pareto frontiers have been obtained by varying the constraint tightening factor τ, the open-loop violation probability and the number of samples M for DMPC, SMPC and RMPC, respectively. Since our particular interest lies with RMPC, the number of samples is indicated explicitly. Note that the RMPC Pareto lines are mean values of several runs, with variances less than 0.5%.

B. RMPC-SR: Removal Algorithms

Fig. 2 depicts simulation results of RMPC-SR with one sample removed. The Pareto frontiers of two removal algorithms (greedy and multiplier removal) are depicted. Since one sample is discarded, greedy removal is equivalent to optimal removal. For the purpose of comparison, (standard) RMPC and SMPC are depicted as well.

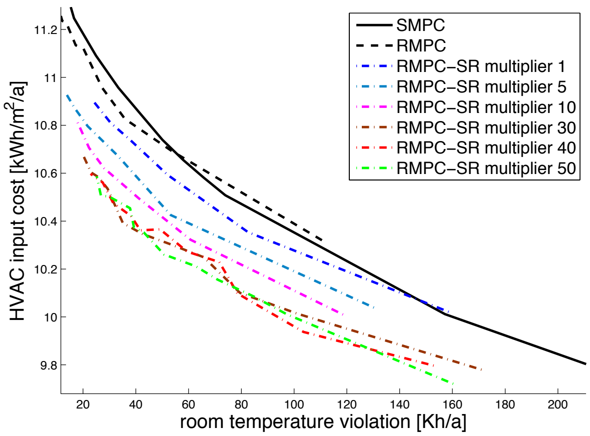
C. RMPC-SR: Multiple Sample Removal

The simulation results of RMPC-SR with multiple samples removed are depicted in Fig. 3. The removal of 50 samples requires 60 to 80 scenarios in both *MSM PA* and *WHW SA*.

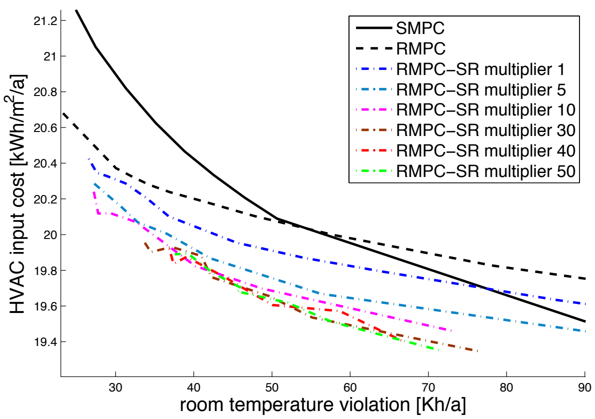
V. Discussion

In this section we discuss the results presented in Section IV. In particular, we focus on two questions:

A Performance of RMPC and RMPC-SR: How does RMPC perform compared to DMPC and SMPC? And what is the added value of RMPC-SR in terms of energy savings?



(a) *MSM PA*



(b) *WHW SA*

Fig. 3. (a)–(b) Pareto frontiers of RMPC-SR with up to 50 constraints removed. Saturation can be observed after removal of more than 30 samples in both cases. Generally, a higher number of removal leads to lower cost, while the comfort violation can be controlled with the number of samples drawn.

B Implementation of RMPC and RMPC-SR: We focus on the real-time solvability of the resulting optimization problems, and discuss the efficiency of the removal algorithms compared in RMPC-SR.

A. Performance of RMPC and RMPC-SR

1) DMPC and SMPC: From Fig. 1 we observe that DMPC and SMPC show very similar Pareto frontiers and are, for practical purposes, equivalent. One possible explanation lies in their conceptual similarity: Both linearize the disturbance-input bilinearity around the prediction $\hat{v}$, giving them the same dynamics, apart from the additive uncertainty modeled in SMPC. The effect of the additive uncertainty is an implicit constraint tightening which grows in the prediction horizon.

2) RMPC: It can be seen in Fig. 1 that RMPC outperforms DMPC and SMPC for violations below 70 Kh/a, i.e. in the area of interest. We believe that the better approximation of the disturbance-input bilinearity, which comes with the higher number of samples, is the explanation for the superior performance of RMPC. In contrast to DMPC and SMPC which linearize around one fixed operation point, RMPC explicitly considers the linearization error due to different scenarios. Our hypothesis is supported by our observation that the strongest uncertainty effect comes from the linearization process and not the additive uncertainty, and that the bilinearity uncertainty greatly affects the performance of the controller.

3) Removal Algorithms for RMPC-SR: Recall that greedy removal is optimal for discarding one sample. Fig. 2 well illustrates this fact as greedy removal leads to higher cost reduction (but also higher violations) compared to multiplier removal. Although multiplier removal does not outperform greedy removal in terms of cost for a fixed number of samples, its Pareto frontier lies below greedy's.

4) Performance of RMPC-SR: From Fig. 3 it can be observed that removing a higher number of samples considerably improves the objective function while comfort violations are kept low. The figures also empirically suggest the convergence of the Pareto frontiers in the number of removed samples. This may partly be due to the fact that we sampled from archived data, which is a finite set. In the extreme case, all possible samples would be considered, and the worst of them discarded, resulting in the "optimal" RMPC-SR. Since data is limited, simulations indicate that removing 30 samples is a good trade-off between performance improvement and computational tractability, as removing 40 and 50 samples does not significantly improve performance but comes at a higher cost.

B. Implementation of RMPC and RMPC-SR

1) Number of Samples: In all investigated cases it has been found that no more than four samples are required in RMPC to keep violation below 70 Kh/a, as exemplified in Fig. 1. Although it is difficult to relate a violation of 70 Kh/a to a percentage in theory, numerical simulations have shown that a violation of 70 Kh/a roughly corresponds to 5%. Therefore, a conservative estimate of the number of samples required to keep the expected violation probability below 5% is 4799 according to [20], [29], [30]. The discrepancy is due to two reasons. The first reason lies in the conservatism in the random convex programming approach. The second reason is the receding horizon approach, which generally results in closed-loop violation probabilities that are much lower than the open-loop violation probabilities. In practice, the number of samples can be treated as a tuning variable which determines the movement along the Pareto frontier. Similar phenomena regarding the number of samples can also be observed in RMPC-SR, where the conservatism is due to the same reasons as in RMPC.

2) Computational Effort: The computational efficiency of how fast the optimization problems can be solved is important for practical implementation. We have observed that one step in RMPC problems with fewer than 10 samples can be solved extremely fast using CPLEX [31] (less than 40 milliseconds on a standard desktop computer), and even problems with 80 samples are solved in around 400 milliseconds at each step. This is mainly due to the fact that the RMPC formulation is an LP, which can be solved very efficiently. In the case of RMPC-SR it has been found that even if a high number of samples is removed, the computational complexity is well below the typical sampling time in building control, which is in the range of 15 min to one hour. For example, removing 50 out of 80 samples can be done in less than 90 seconds.

Another issue that arises in RMPC-SR is the choice of the removal algorithm and their respective computational complexity. More precisely, the optimality of greedy removal for one sample removed entails higher computational cost: greedy removal requires the solution of M optimization problems, whereas multiplier removal only needs two optimization problems to be solved. Because the latter is computationally more attractive while delivering good performance, it should be preferred. This is especially true if a high number of samples is to be removed since greedy removal grows quadratically in the number of samples, whereas multiplier only grows linearly.

VI. CONCLUSION

This paper evaluates the energy saving potential of HVAC systems in buildings by using the RMPC and RMPC-SR approaches. Both methods use weather and internal gain predictions to compute the control input. RMPC was found to outperform the compared methods at low violations because it better accommodates for the linearization errors and deals with uncertainties that are non-additive and non-Gaussian. Moreover, higher energy savings are possible by applying RMPC-SR with multiple samples removed. Convergence of the Pareto frontiers in RMPC-SR suggests that no more than 30 samples need to be removed. Another benefit of RMPC and RMPC-SR are their computational tractability. Finally, both approaches are also attractive for the control practitioner because they are intuitive to use and easy to tune.

ACKNOWLEDGMENT

The authors thank Frauke Oldewurtel for fruitful discussions and Jenna Tomasevich for proofreading this paper.

REFERENCES

[1] J. Laustsen, “Energy efficiency requirements in building codes, energy efficiency policies for new buildings,” *International Energy Agency (IEA)*, 2008.

[2] J. McQuade, “A system approach to high performance buildings,” United Technologies Corporation, Tech. Rep., 2009.

[3] “Buildings energy data book,” *U.S. Department of Energy*, 2009, available: http://buildingsdatabook.eren.doe.gov/.

[4] S. Schiavon, K. H. Lee, F. Bauman, and T. Webster, “Influence of raised floor on zone design cooling load in commercial buildings,” *Energy and Buildings*, vol. 42, no. 8, pp. 1182–1191, 2010.

[5] F. Oldewurtel, A. Parisio, C. Jones, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari, “Use of model predictive control and weather forecasts for energy efficient building climate control,” *Energy and Buildings*, vol. 45, pp. 15–27, Feb. 2012.

[6] T. Nghiem and G. Pappas, “Receeding-horizon supervisory control of green buildings,” in *ACC*, 2011.

[7] V. Chandan and A. G. Alleyne, “Optimal control architecture selection for thermal control of buildings,” in *ACC*, 2011, pp. 2071–2076.

[8] A. Aswani, N. Master, J. Taneja, D. Culler, and C. Tomlin, “Reducing transient and steady state electricity consumption in HVAC using learning-based model predictive control,” *Proceedings of the IEEE*, vol. 100, no. 1, pp. 240–253, 2011.

[9] SIA Standard 382/1, “Lüftungs- und Klimaanlagen – Allgemeine Grundlagen und Anforderungen,” 2006.

[10] M. Gwerder and J. Tödtli, “Predictive control for integrated room automation,” in *Proc. of Clima – RHEVA World Congress 2005*, Lausanne, Switzerland, 2005.

[11] R. Andersen, B. Olesen, and J. Toftum, “Simulation of the effects of occupant behaviour on indoor climate and energy consumption,” in *Proc. of Clima – RHEVA World Congress 2007*, Helsinki, Finland.

[12] G. Henze, D. Kalz, S. Liu, and C. Felsmann, “Experimental analysis of model-based predictive optimal control for active and passive building thermal storage inventory,” *International Journal of HVAC and Research*, vol. 11, pp. 189–214, 2005.

[13] Y. Ma, F. Borrelli, B. Hencey, A. Packard, and S. Bortoff, “Model predictive control of thermal energy storage in building cooling systems,” in *Proc. of 48th IEEE CDC and 28th CCC*, Shanghai, China, 2009.

[14] F. Oldewurtel, D. Sturzenegger, and M. Morari, “Importance of occupancy information for building climate control,” *Applied Energy*, 2012.

[15] G. Schildbach, G. Calafiore, L. Fagiano, and M. Morari, “Randomized model predictive control for stochastic linear systems,” in *ACC*, 2012.

[16] M. Prandini, Garatti, S., and J. Lygeros, “A Randomized Approach to Stochastic Model Predictive Control,” in *IEEE Conference on Decision and Control*, Maui, Hawaii, USA, Dec. 2012.

[17] G. C. Calafiore and M. C. Campi, “Uncertain convex programs: Randomized solutions and confidence levels,” *Mathematical Programming, Series A*, vol. 102, pp. 25–46, 2005.

[18] M. C. Campi and S. Garatti, “The exact feasibility of randomized solutions of uncertain convex programs,” *SIAM Journal of Optimization*, vol. 19, pp. 1211–1230, 2008.

[19] G. C. Calafiore, “Random convex programs,” *SIAM Jornal of Optimization*, vol. 20, no. 6, pp. 3427–3464, 2010.

[20] M. C. Campi and S. Garatti, “A sampling-and-discarding approach to chance-constrained optimization: Feasibility and optimality,” *Journal of Optimization Theory and Applications*, vol. 148, pp. 257–280, 2011.

[21] F. Oldewurtel, “Stochastic model predictive control for building climate control,” Ph.D. dissertation, ETH Zurich, Aug. 2011.

[22] D. Gyalistras and M. Gwerder, “Use of weather and occupancy forecasts for optimal building climate control (opticontrol): Two years progress report - main report,” ETH Zurich, Tech. Rep., 2010.

[23] D. Mayne, M. Seron, and S. Raković, “Robust model predictive control of constrained linear systems with bounded disturbances,” *Automatica*, vol. 41, pp. 219–224, 2005.

[24] A. Prekopa, *Stochastic Programming*. Dordrecht, Netherlands: Kluwer Academic Publishers, 1995.

[25] P. Kall and J. Mayer, *Stochastic Linear Programming*. Kluwer Academic Publishers, 2005.

[26] J. Birge and F. Louveaux, *Introduction to Stochastic Programming*. New York: Springer, 1997.

[27] A. Shapiro, D. Dentcheva, and A. Ruszczyński, *Stochastic Programming, Modeling and Theory*. Philadelphia: SIAM, 2009.

[28] P. Goulart, E. Kerrigan, and J. Maciejowski, “Optimization over state feedback policies for robust control with constraints,” *Automatica*, vol. 42, pp. 522–533, 2006.

[29] G. C. Calafiore, “On the expected probability of constraint violation in sampled convex programs,” *Journal of Optimization Theory and Applications*, vol. 143, pp. 405–412, 2009.

[30] G. Schildbach, L. Fagiano, and M. Morari, “Randomized solutions to convex programs with multiple chance constraints,” *arXiv:1205.2190v2*, 2012. [Online]. Available: http://arxiv.org/abs/1205.2190v2

[31] IBM ILOG , “Cplex optimization studio,” [online] http://www-01.ibm.com/software/integration/optimization/cplex-optimization-studio/, last accessed Oct 26, 2012.