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Incident parameter estimation

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Abstract— **The paper proposes a sampled data based estimation methodology to reconstruct local incident parameter of the macroscopic Incident Traffic Flow (ITF) models. The key idea in ITF models is to dynamically relax the traffic mean speed to the traffic equilibrium one based of an time and space varying incident term. First, the analysis of incident corrupted traffic flow models, described as an inhomogenuous nonlinear Partial Differential Equation (PDE), is presented in continuous time. Second, space and time discretization techniques are applied to derive traffic management oriented ITF models. Online parameter estimation is suggested to capture the severity of incident throughout the proposed parameter, i.e. to estimate the incident parameter. Numerical example is carried out to show the viability of macroscopic incident parameter estimation technique using data obtained from a high-fidelity microscopic simulation.**

Index Terms— **Traffic flow models, incident modeling, accident, parameter estimation, hyperbolic conservation laws, PDE, discretization**

I. INTRODUCTION

For traffic management and control purposes, macroscopic concepts offers alternative modeling frameworks in terms of dynamically changing averaged valued quantities such as traffic density, speed, and volume. Nowadays, advance traffic control algorithms are mainly derived on the basis of these models, e.g. freeway traffic flow control [14], [7]. Considering the degree of freedom involved in the conservation law, macroscopic traffic flow models can be divided in two main classes. First order models [10] guarantee vehicle conservation and have successfully been used for control/estimation purposes, e.g. [17]. Second order traffic flow models describe the dynamic evolution of the conserved density and speed/volume quantities, [15]. A discretized version of [15], called METANET [4], has been widely applied/extended for alternative traffic model parameter estimation/control oriented studies e.g. [21], [12], [11]. [6] pointed out the fact that models of [15] violate the anisotropy principle (stating that traffic flow is mostly influenced by what is happening ahead of it and not behind) as well as could generate negative vehicle speed. To overcome this drawback, [3] proposed to define the traffic pressure term as a Lagrangian quantity. By using the model obtained in [3], [13] discretized the continuous time conservation law and concluded on its accuracy of capturing low traffic flow phenomenon. Within this model framework, [8] suggested a locally optimal ramp metering solution.

As a traffic incident related extension, [18] modeled incidents as partial lane blockage. Hence, a modified merging term has been added to the METANET model in order to account lane number reduction. [20] used Extended Kalman Filtering to estimate abrupt changes in core model parameters of the equilibrium speed due to incident. By means of online monitoring these parameters, it was possible to capture the effect of incidents on real sites. As about this work, a preliminary idea communicated in [5] on dynamic relaxation terms in hyperbolic conservation laws for incident modeling, leads to introducing Incident Traffic Flow (ITF) models. Compared to [5], this note provides with a structured analysis of the most important properties of this continuous time ITF model and its discretization as well as validation of the model in capturing the incident effect with microscopic simulation. The ITF model structure can capture traffic incidents throughout the dynamism built into the relaxation term, the term which describes how the average driver tend to relax to traffic equilibrium speed. These dynamics result in an artificial density component perceived by the average driver. It is influenced by the severity of the congestion due to the incident. The paper presents a systematic analysis of the key properties of the augmented hyperbolic conservation law, thereof the ITF model (e.g. eigenvalue structure, shock wave propagation). The obtained inhomogenous continuous time PDE is discretized by means of Godunov scheme with the help of a two-step procedure. Nonlinear discrete time difference equations are then derived to characterize the possibly incident traffic flow in a way segment-wise. Procedure has been presented in the paper to reconstruct the incident parameter onine. The suggested macroscopic ITF model has beed validated through incident free and effected data obtained from microscopic traffic simulator, PTV VISSIM [1] in a case study.

The paper is organized as follows. After the introduction, Section II presents a novel way of describing the effect of the incident under the form of a nonlinear and inhomogenous continuous time PDE. In Section III both eigenvalues and eigenvectors of the extended system are analyzed. Shock wave propagation is analyzed under the presence of incident in the same Section. Godunov discretization combined with splitting method is applied in Section IV in order to discretize the obtained inhomogenous continuous time PDE. in section V online parameter estimation is presented in order to estimate incident parameter. This section is followed by a numerical case study with active incident term where the goal is to reconstruct the incident parameter in ITF

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model with data yielded from microscopic simulation. It is illustrated in the case study that ITF model along with its modified pressure term is capable in capturing the effect of incident. Further research questions are concluded in Section VI.

II. Continuous time incident effected model description

This Section introduces a novel way of modeling incidents in a macroscopic traffic model framework. As [15] introduced, second order traffic flow models can be written under the form of a continuous time PDE as:

$$\rho_t + (\rho v)_x = 0 \tag{1}$$

$$v_t + v v_x = \frac{V_e(\rho) - v}{\tau} - \frac{P_x(\rho)}{\rho}, \tag{2}$$

where ρ and v are density and average speed variables, respectively. Subscripts $\cdot_t, \cdot_x$ and $(\cdot)_t$, $(\cdot)_x$ accounts for partial derivatives with respect to time t and to space x respectively. The first term in the right hand side of eq. (2) is the relaxation term expressing the tendency of drivers to reach the traffic equilibrium speed $V_e(\rho)$. The second component in eq. (2) is the anticipation term which represents the drivers' anticipation on the traffic situation in front of them. $P(\rho)$ stays for the pressure term in analogy to gas dynamics [19] which is a nondecreasing function of density ρ. Since the traffic pressure term is a quality moving with traffic, [3] proposed to replace $P_x(\rho)$ by its material derivative as $D_t P(\rho) = P_t(\rho) + v P_x(\rho)$ with D_t standing for the material derivative. Substituting the modified pressure term back to (2) [15], the Aw-Rascle (AR) model [3] can be given by:

$$v_t + (v - \rho P'(\rho)) v_x = \frac{V_e(\rho) - v}{\tau}. \tag{3}$$

where prime denotes the partial derivation with respect to density. We can verify the eigenvalues of the corresponding hyperbolic system and find:

$$\lambda_1 = v - \rho P'(\rho) \leq \lambda_2 = v, \tag{4}$$

It is easy to see that the violation of anisotropy principle is obviated in (4). Zhang in [23] has developed a similar concept by virtue of traffic sound speed. Defining Pressure term as:

$$P(\rho) = V_{max} - V_e(\rho) \tag{5}$$

will create equivalence in between the two models [13], [9]. In the following, we will use the framework [3], [23] and we introduce a generic and novel method in order to capture the incident effect in macroscopic view. It worth to mention that by incidents we mean any type of off-nominal (abnormal) traffic conditions that might abrupt traffic flow dynamics.

Incident Traffic Flow (ITF) model

Inspired by [2], [5], the proposed model is based on defining a fictitious density term ρ^*. By definition, this density is the one that drivers anticipate in the upstream of the incident and can be given by:

$$\rho^* = \rho + \alpha D_t \rho = \rho + \alpha(\rho_t + v\rho_x), \tag{6}$$

where α is a model parameter needs to be identified. Nonzero values of α indicates the presence of changing driving conditions. The larger the magnitude of α, the larger the severity of incidents' effect. Note that the parameter α might depend both on time and space. Consequently, the speed to which drivers relax depends on ρ^* rather than on ρ. With ρ^*, the intention is to emphasize the effect of speed relaxation whenever (space or time related) changes in ρ are significant, i.e. to include the material derivative of density. Re-defining the relaxation term, the incident effected AR model can be written as:

$$v_t + (v - \rho P'(\rho)) v_x = \frac{1}{\tau}(V_e(\rho^*) - v). \tag{7}$$

By using (1) and Taylor expansion of $V_e(\rho^*)$ (while neglecting higher order terms such as higher order density derivatives) eq. (7) can be writen as:

$$\rho_t + (\rho v)_x = 0 \tag{8}$$

$$v_t + (v - \rho P'_{new}(\rho)) v_x = \frac{1}{\tau}(V_e(\rho) - v) \tag{9}$$

Equivalently we can give the following formal definition.

Definition 1: Incident Traffic Flow model can be given by,

$$\rho_t + (\rho v)_x = 0$$

$$(v + P_{new}(\rho))_t + v(v + P_{new}(\rho))_x = \frac{\rho}{\tau}(V_e(\rho) - v). \tag{10}$$

where, $P_{new}(\rho)$ is an increasing function of ρ and is defined as:

$$\begin{aligned} P_{new}(\rho) &= P(\rho) - \frac{\alpha}{\tau} V_e(\rho) \\ &= V_{max} - (1 + \frac{\alpha}{\tau}) V_e(\rho) \end{aligned} \tag{11}$$

Where the second equality is valid due to (5). This extended AR model, presented in eqs. (1)-(10), is a continuous time model an will be called (ITF) model. The new traffic pressure term is influenced by α throughout the dynamism built into the relaxation term. Comparing (5) and (11), the new pressure term can be regarded as an original one but with modified equilibrium speed. This expression is consistent with the idea of adaptive fundamental diagram presented in [20] for modeling the traffic incidents while ITF model describes a generic incident parametrization. Aforementioned modification in ITF model, however, leads to changes in the analytic properties of the original continuous time PDE [3]. The subsequent section therefore gives a detailed and structural analysis of the ITF model.

III. Structural Properties of the Model

Several important analytic properties are investigated in this Section. First, we prove the ITF model remains hyperbolic and investgate the controversial anisotropy property of the model. Comprehensive discussion on the change in the speed of shockwave in the ITF model is presented after calculating Riemann invariants which are crucial in solving Riemann Problems (RP) and performing Godunov discretization

A. Eigenvalues and anisotropy property

For eigenvalue structure analysis, the source-free PDE obtained from eqs. (8)-(9) is considered. Define $L = (\rho, v)$ and rewrite the equations into a generic form as,

$$L_t + A(L)L_x = 0 \tag{12}$$

where

$$A(L) = \begin{pmatrix} v & \rho \\ 0 & v - \rho P'_{new}(\rho) \end{pmatrix}. \tag{13}$$

This system has two eigenvalues, such as

$$\lambda_1 = v - \rho P'_{new}(\rho) \leq \lambda_2 = v. \tag{14}$$

As a conclusion, except for vacuum, the aforementioned system is strictly hyperbolic. The corresponding eigenvectors of the systems can be found as:

$$r_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; r_2 = \begin{pmatrix} 1 \\ -P'_{new}(\rho) \end{pmatrix}. \tag{15}$$

By simple manipulations it can be shown that λ_2 is linearly degenerate and therefore the corresponding waves are contact discontinuities, i.e. waves separating two states with similar speeds but different densities. As for the first eigenvalue, assuming the convexity of the function $\rho P'_{new}(\rho)$, [3], λ_1 is genuinely nonlinear and the waves associated to this eigenvalue are either shock waves (corresponding to braking) or rarefaction waves (corresponding to acceleration). Based on the aforementioned results and considering (14), the anisotropy properties of the system can be concluded, i.e. the speed of the wave propagation can not be higher than the velocity v. Riemann invariants corresponding to λ_1 and λ_2 can be given as,

$$w_1 = v + P_{new}(\rho), \; w_2 = v. \tag{16}$$

Riemann invariants are essential in solving initial condition problem, i.e. the Riemann Problem (RP), see Section IV .

B. Conserved variables and speed of the shock waves

To use the Godunov scheme for discretization and also to calculate the speed of the shockwave, the hyperbolic system has to be transformed into conservation form. Recall eqs. (1)-(10) and multiply eq. (1) by $v + P_{new}(\rho)$, the eq. (10) with ρ. After summing them up, the so-called conservation form of the hyperbolic system can be generated by

$$\rho_t + (\rho v)_x = 0 \tag{17}$$

$$(\rho(v + P_{new}(\rho)))_t + (\rho v(v + P_{new}(\rho)))_x = \frac{\rho}{\tau}(V_e(\rho) - v). \tag{18}$$

Given the pressure term in (11), the conserved variables can be defined by,

$$U = \begin{pmatrix} \rho \\ \rho(v - (1 + \frac{\alpha}{\tau})V_e) \end{pmatrix} = \begin{pmatrix} \rho \\ m \end{pmatrix}, \tag{19}$$

where the variable m denotes the incident corrupted traffic volume. Using the new notations of (17)-(18), we can write:

$$U_t + F_x(U) = G(U), \tag{20}$$

$$F(U) = \begin{pmatrix} m + \rho(1 + \frac{\alpha}{\tau})V_e(\rho) \\ \frac{m^2}{\rho} + m(1 + \frac{\alpha}{\tau})V_e(\rho) \end{pmatrix},$$

$$G(U) = \begin{pmatrix} 0 \\ -\frac{m}{\tau} - \frac{\alpha\rho}{\tau^2}V_e(\rho) \end{pmatrix}.$$

$F(U)$ can also be written in the following form:

$$F(U) = \begin{pmatrix} q \\ q(v - (1 + \frac{\alpha}{\tau})V_e(\rho)) \end{pmatrix},$$

where $q = \rho v$.

The new traffic incident term changes the propagation speed of the waves as well. Consider two states $L_l = (\rho_l, v_l)$ and $L_r = (\rho_r, v_r)$ $(\rho_r > \rho_l)$ which are separated by a shock wave, [22]. Since the relaxation term is finite, its effect in the transient phase (short time) is negligible compare to the infinite space derivative of ρ and v at the given shock location. For simplicity lets transform the problem into a new moving frame in which the speed of the shock in the new frame is zero. In the new frame, all of the states in both sides except for the speed are the same. Since in the incident we assume that the shock moving backwards, in new frame we have the following equality for speed:

$$\hat{v}^{new}_{r(l)} = S^{new} + v_{r(l)} \tag{21}$$

$$\hat{v}_{r(l)} = S + v_{r(l)} \tag{22}$$

where $\hat{v}^{new}_{r(l)}$ is speed of the right (left) hand side of the shock in the new coordinate of ITF model while $\hat{v}_{r(l)}$ are the corresponding speed from the AR model. S^{new} and S are obviously defined in the original frame and correspond to the speed of shock wave in the ITF model and AR model respectively. Having the shock waves' speed in new coordinate as zero and using the Rankine-Hugoniot condition, we obtain:

$$\hat{v}^{new}_r = \rho_l \frac{P^l_{new} - P^r_{new}}{\rho_l - \rho_r} \tag{23}$$

$$\hat{v}_r = \rho_l \frac{P^l - P^r}{\rho_l - \rho_r} \tag{24}$$

Considering the equations above as well as (11), (21) and (22) we can write:

$$S + \frac{\alpha}{\tau}\rho_l(V_e(\rho_r) - V_e(\rho_l)) = S^{new} \tag{25}$$

noticing that $V_e(\rho_r) - V_e(\rho_l) < 0$ it is straightforward to have the following inequality for negative α:

$$S_{new} > S. \tag{26}$$

In other words, this means that the speed of the shock wave generated by the ITF model is larger than the one by the AR model. The reason is that in the upstream of an incident, drivers who see the (effect of the) incident in front of them anticipate it and therefore feel unsafe about the velocity they have. This is the reason why drivers might decrease their speed accordingly.

IV. Discretization

In this Section, a discretization technique for the proposed ITF models is presented. Since the corresponding PDE is a non homogeneous one, splitting method suggests a two step procedure to first discretize a homogeneous PDE which is followed by the discretization of an ODE defined to take the source term into account [19].

Consider the initial value problem which is a non homogeneous PDE as:

$$\begin{cases} U_t + F_x(U) = G(U), \\ U(x,t^n) = U^n \text{ : initial condition} \end{cases} \tag{27}$$

In order to find the solution U^{n+1} from the initial value U^n, splitting method suggests to separate the source term from the original PDE by first solving a PDE without the source term:

$$\begin{cases} U_t + F_x(U) = 0, \\ U(x,t^n) = U^n \text{ : initial condition} \end{cases} \Rightarrow \overline{U}^{n+1} \tag{28}$$

followed by separately solving the ODE term as:

$$\begin{cases} \dfrac{d}{dt}U = G(U), \\ \\ \overline{U}^{n+1} \text{ : initial condition} \end{cases} \Rightarrow U^{n+1} \tag{29}$$

In this method, the solution of the initial value problem (28), which is a homogenous PDE, will be used as the initial condition for the ODE (29). Initialization is performed in a separate way in two consecutive steps. In the following sections, each of these two steps for discretizing the ITF model (18) with proper initial value is explained in more details.

A. Discretizing the homogeneous PDE

Recall a homogenous hyperbolic conservation law by:

$$U_t + F_x(U) = 0. \tag{30}$$

Godunov scheme can be used to define the computational cells for time and space discretization. Assume a uniform discretization grid on the domain $[x_L, x_R]$, where the discrete points are denoted by $x_j = x_L + (j + \frac{1}{2})\Delta x$ for $j = 0, 1, .., N$ with N standing for the number of subsegments. Since $\Delta x = \dfrac{x_L - x_R}{N+1}$, at the boundaries, we can write:

$$x_{j-1/2} = x_j - \frac{\Delta x}{2} = x_L + j\Delta x \tag{31}$$

for $j = 0, ...N+1$. Considering uniform discretization in time with step size Δt, the average value of $U(x,t)$ in cell i at the fixed time $t = t^n = n\Delta t$ is defined as:

$$U_i^n = \frac{1}{\Delta x}\int_{x_{j-1/2}}^{x_{j+1/2}} U(x,t^n)dx, \tag{32}$$

the set of cell average in (32) defines a piece-wise constant distribution of the solution at time t^n. Godunov scheme is based on the following conservation law:

$$U_i^{n+1} = U_i^n + \frac{\Delta t}{\Delta x}[F_{i-1/2} - F_{i+1/2}], \tag{33}$$

where the intercell fluxes $F_{i-1/2}$ and $F_{i+1/2}$ become:

$$\begin{cases} F_{i-1/2} = F(U_{i-1/2}(0)) \\ F_{i+1/2} = F(U_{i+1/2}(0)), \end{cases} \tag{34}$$

and $U_{i+1/2}(0)$ refers to the exact solution $U_{i+1/2}(x/t)$ of the Riemann Problem $RP(U_i^n, U_{i+1}^n)$ evaluated at $x/t = 0$; the solution is evaluated along the intercell boundary, which coincide with the t-axis in the local frame of the Riemann solution. Considering the ITF model in (18) but with no relaxation term, the resulting homogenous PDE can be discretized based on (33) using (19) and (20) to define the conserved variable U and the flux F respectively, while $F_{i-1/2}$ is calculated by solving a Riemann problem in each boundary point. As a result, the conserved variable will be updated for the next time step as the following equation:

$$\overline{\rho}_j^{n+1} = \rho_j^n + \frac{\Delta t}{\Delta x}(q_{j-1}^n - q_j^n) \tag{35}$$

$$\overline{m}_j^{n+1} = m_j^n + \frac{\Delta t}{\Delta x}(z_{j-1}^n - z_j^n), \tag{36}$$

where

$$z = q(v - (1 + \frac{\alpha}{\tau})V_e(\rho)). \tag{37}$$

In this note, the equilibrium speed-density relationship is chosen as below where the first equation is valid if $0 \leq \rho \leq \rho_{crit}$ while the second one is for $\rho_{crit} \leq \rho \leq \rho_{max}$.

$$V_e(\rho) = \begin{cases} V_{max} - \dfrac{\rho}{\rho_{cr}}(V_{max} - V_{cr}) \\ \dfrac{1}{\rho}(W_{max}(\rho_{max} - \rho_{cr}) + \beta(\rho_{max} - \rho)^2 \end{cases}$$

$$\beta = \frac{Q_{max}}{(\rho_{max} - \rho_{cr})^2} - \frac{W_{max}}{(\rho_{max} - \rho_{cr})} \tag{38}$$

In which ρ_{max}, V_{max} and ρ_{max} are critical density, free speed and jam density respectively. The corresponding Riemann solution can be obtained by modifying the solution in [13].

B. Discretizing the ODE

The final discretized solution can be obtained by solving the following ODE with initial value $\overline{U}^{t+1}$ which is nothing but the solution of (35) and

$$\begin{cases} \dfrac{d\rho}{dt} = 0 \\ \dfrac{dm}{dt} = -\dfrac{m}{\tau} - \dfrac{\alpha\rho}{\tau^2} V_e(\rho) \\ \text{Initial data : } [\overline{\rho}^{n+1}, \overline{m}^{n+1}]^T \end{cases} \quad (39)$$

By applying the trapezoidal method the solution of equation (39) can be formulated as follows:

$$\rho_j^{n+1} = \overline{\rho}_j^{n+1} \quad (40)$$

$$m_j^{n+1} = a_f \overline{m}_j^{n+1} - \frac{2\alpha_j \Delta t \overline{\rho}_j^{n+1}}{\tau(2\tau+\Delta t)} V_e(\overline{\rho}_j^{n+1}), \quad (41)$$

where

$$a_f = \left(\frac{1 - \dfrac{\Delta t}{2\tau}}{1 + \dfrac{\Delta t}{2\tau}} \right). \quad (42)$$

Note, α_j indicates the fact that the incident parameter can be space dependent; each segment can have different α.

V. INCIDENT PARAMETER ESTIMATION

A. Online Parameter Identification

The purpose of this section is to seek a method for the estimation of incident parameter α_j while other model parameters such as V_{max}, ρ_{max} and τ are considered to be known. For the sake of simplicity we assume if incident is happening in segment $j+1$, in all segments except for j, α will be zero. As it is stated in section IV, at each time step two Riemann problems need to be solved in order to calculate $U_{i-1/2}(0)$ and $U_{i+1/2}(0)$ for obtaining corresponding flux $F(U_{i-1/2}(0))$ and $F(U_{i+1/2}(0))$. The solution of each Riemann Problem is depend on the initial values and α itself and consequently have different structure at each time which makes the α identification a complex task. In this note, The online identification has been used in order to estimate α at each time step. In this sense, at each time step n an optimization problem with predefined cost function needs to be solved in order to have an estimation of α in step n which is called $\hat{\alpha}^n$. This estimation is then will be used in the ITF model to generate the corresponding state. The selected objective function in this work is:

$$J^n = (\hat{\rho}_j^n - \rho_j^n) + \eta(\hat{v}_j^n - v_j^n) \quad (43)$$

$\hat{\rho}$ and $\hat{v}$ are the fitted density and speed while η is simply a weight in the cost function. Consequently, the optimization problem can be formulated as:

$$\min_{\hat{\alpha}_j^n} \quad J^n$$
$$\text{subjected to (35), (36)} \quad (44)$$

That is a specific case of nonlinear moving horizon parameter estimation problem [16].

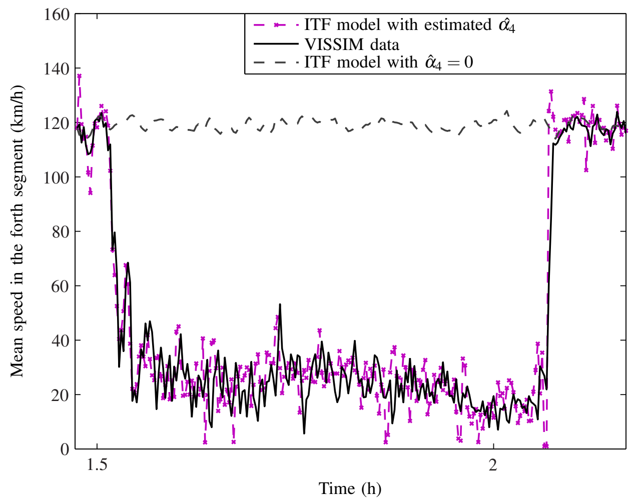

Fig. 1. Simulated speed in segment 4 using the ITF model with identified α and results with $\alpha = 0$ (AR model) which is compared with VISSIM data in solid line

B. Case study

The purpose of this case study is to evaluate the effect of α in incident modeling using data obtained from the traffic simulator VISSIM [1]. Therefore the validation of macroscopic incident estimation is given throughout the basis of microscopic simulation. For simplicity, the system of PDE describing the ITF model has been chosen as homogenous. Two data set was generated for describing the traffic behavior in a hypothesis motorway with 7 segments, each except the last one has length of 500 m. All segments have three lanes except for the last section which is 2 lane segment with length of 100 meter. 3 hours of traffic behavior in this motorway has been simulated using VISSIM, to generate 2 data set. The first data set is the incident-free case and has been used offline to identify all the model parameters except for α (α is kept zero in this phase). The setup in VISSIM has been chosen in a way that after 1.5 hours, an incident happens in the beginning of the fifth segment and lasts for half an hour. The second data set then has been collected from this incident corrupted case in order to be used for online α identification. This case study is mainly performed to demonstrate the effect of α in capturing the changes in traffic behavior due to incident and not for estate estimation. Therefore the optimal place of sensors is not discussed here and has been chosen as it is stated in section V.

The parameter identification results on the first data set can be seen in Table I. These identified parameters then have been chosen as nominal ones to model and identify α. Fig. (1) and (2) demonstrate the simulation results of speed and

TABLE I
PARAMETER IDENTIFICATION RESULT

ρ_{max}	V_{max}	ρ_{cr}	V_{cr}	W_{max}	τ
295.2	119.1	57.1	82.78	19.234	2.34

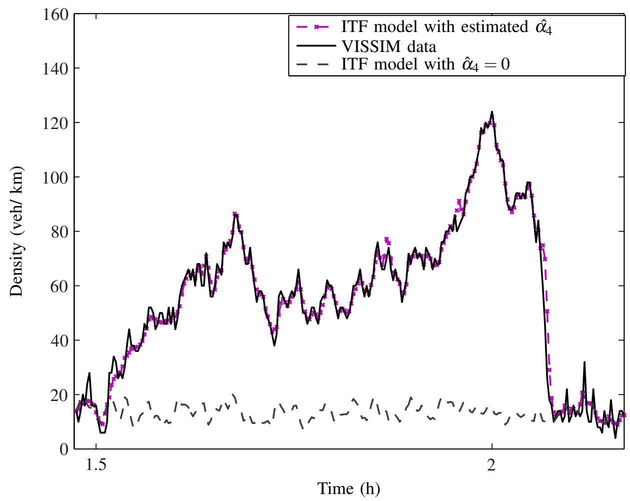

Fig. 2. Simulated density in segment 4 using the ITF model with identified α and with $\alpha = 0$ which is compared with VISSIM data in solid line

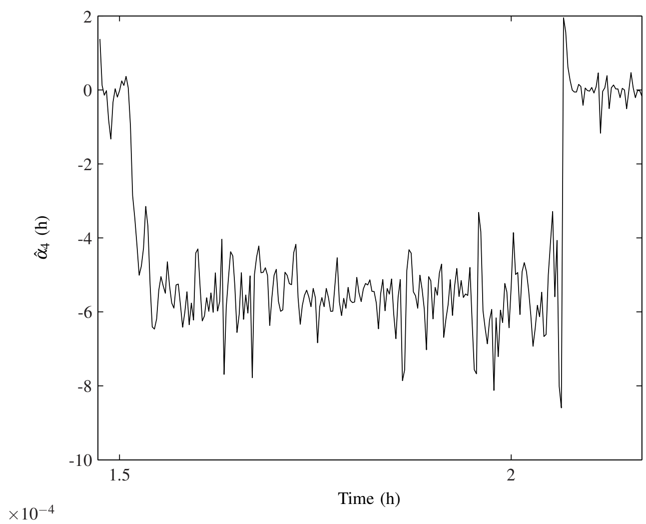

Fig. 3. Estimated α in segment 4

density of the forth segment of the motorway using the ITF model compared with data generated with VISSIM. As it can be seen from the figures, while original ITF model with $\alpha = 0$ is unable to capture the incident effect, results from the ITF model with the identified α is promising. The estimated α is depicted in Fig. (3). As it is illustrated from the figure, at time $t = 1.5h$ where incident has happened, α will take nonzero value and it gets zero again when the incident is obviated at around time $t = 2h$.

VI. CONCLUSION

Incident modeling in macroscopic traffic flow concepts has been studied in the paper. Dynamic relaxation technique applied for incident description has been analyzed within the framework of continuous time PDE. With Godunov discretization, a time discrete model has been developed. Validation by means of online α estimation has been carried out by using microscopic traffic simulator. Further works include incident traffic flow model validation by means of real measurements and development of incident tolerant traffic control algorithm.

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