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Integrated Fault Detection in Multiple Switched Systems

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Abstract— **This paper addresses a new strategy for constructing switched systems. This approach will be used to set an integrated fault detection filter (FDF). The basic idea behind this approach is to switch multiple sub-models with weighting factors that reflect the effect of each local sub-model. The main contributions of this paper are summarized as follows: (1) set a multiple switching technique for describing the system dynamic behavior. (2) integrate residual generation with evaluation and threshold setting. (3) enhance the fault detectability for the switched systems by utilizing available information provided by each local sub-model. (4) relax the stability condition for the switched observers in terms of switched Lyapunov functions (SLF). The proposed approach will be illustrated by lateral vehicle dynamics as an example.**

I. INTRODUCTION

Model-based fault detection (FD) technique has been investigated in the last two decades. Nowadays, it is accepted as a powerful tool to solve the fault diagnoses problems in technical processes, [5], [7], [8], [9] and references therein. The theory of the model-based FD is well-established for linear time invariant (LTI) systems, see e.g. [3], [5], [7], [9]. In comparison, FD solutions in the nonlinear systems are limited to some special kinds of nonlinearities, as in [14] and [15]. Due to the restricted FD solutions for nonlinear systems, it can be represented as a group of LTI sub-models. In this case, the LTI model-based FD techniques can be used, as in *Multiple Models, T-S Fuzzy System* and *Switched Systems*.

FD issues for linear discrete-time switched systems will be investigated in this paper for the following reasons:

- Switched systems show a promise and convince ability in modeling and representing the high complex systems in sub-models. Since these sub-models are normally linear and simple.
- Reduce system uncertainties. Define high complex systems in terms of sub-models is considered as an effective and powerful tool to handle the systems with large uncertainties.
- A lot of work has been done in designing a robust control law for switched systems. However, to the best of the author's knowledge, FD problems have not been intensively investigated for switched systems, and it is still an open issue.

In the last few years, some methods of FD have been applied for switched systems, which are reported in [4], [16], [17]. In these papers, the standard norm-based evaluation method has been used to evaluate the switched residual signal. The threshold has been set a constant overall the switched system. In [1] and [2] we have studied the robust FD problem for the switched systems with adaptive threshold setting taking into account the behavior of the switched systems. In this paper, we will continue the study on FD the switched systems and propose a new scheme for representing switched systems for the following objectives:

- Get a more accurate model for system dynamic behavior using multiple switched systems technique, keeping in the same time low computation effort.
- Integrate the residual generation with the evaluation and threshold computation in order to improve the fault detectability.
- Achieve a high FD performance for switched systems. Since, all the available information for each sub-model should be taken into account in the FD design.
- Relax the switched observers stability by SLF. Since, find one Lyapunov function for numbers of sub-models is difficult, or even it does not exist.

This paper is organized as follows. After the introduction in Section I, preliminaries and the basic idea are presented in Section II. In Section III, the problem to be addressed is formulated. The solution is given in Section IV. In Section V the proposed scheme is illustrated by an example.

Notations: The notations used throughout this paper are generally standard. X^T is the transpose of the matrix X. 0 is a zero matrix of appropriate dimension. I represent the identity matrix and $\mathscr{L}_2$-norm is defined as $||x_k||_2 = \sqrt{\sum_{k=0}^{\infty} x_k^T x_k}$.

II. PRELIMINARIES AND BASIC IDEA

Consider the following linear discrete-time switched systems:

$$x_{k+1} = A_{\sigma_k} x_k + B_{\sigma_k} u_k + E_{d,\sigma_k} d_k + E_{f\sigma_k} f_k \quad (1)$$

$$y_k = C_{\sigma_k} x_k + D_{\sigma_k} u_k + F_{d,\sigma_k} d_k + F_{f\sigma_k} f_k \quad (2)$$

where, $x \in \mathscr{R}^n$ is the system state vector, $y \in \mathscr{R}^m$ is the measurement output vector and $u \in \mathscr{R}^p$ is the input vector. $d \in \mathscr{R}^{k_d}$ represents the disturbance vector and $f \in \mathscr{R}^{k_f}$ is the vector of the faults to be detected.

Switching signal σ_k can be defined to be time-dependent, state-dependent, input/output dependent or parameter dependent. ***In this study, it is assumed that, the switching signal is unknown in a priory but its value is real-time available***. The switching rule σ_k

takes values in the finite set $i = \{1,2,...,N\}$, i.e. $\sigma_k \in i$. The Index i is related to the sub-models in the switched systems, where: $A_i \in \{A_1, A_2, ..., A_N\}$, $B_i \in \{B_1, B_2, ..., B_N\}$, $C_i \in \{C_1, C_2, ..., C_N\}$, and $D_i \in \{D_1, D_2, ..., D_N\}$. These matrices are known, and it is in appropriate dimensions.
In the switched systems, each linear sub-model may be affected by different disturbances and faults. Therefore, the disturbance and fault matrices are defined as follows: $E_{d,i} \in \{E_{d1}, E_{d2}, ..., E_{dN}\}$, $E_{f,i} \in \{E_{f1}, E_{f2}, ..., E_{fN}\}$, $F_{d,i} \in \{F_{d1}, F_{d2}, ..., F_{dN}\}$ and $F_{f,i} \in \{F_{f1}, F_{f2}, ..., F_{fN}\}$.

In this work, it is assumed that all sub-models are stable. The global stability of the switched systems can be ensured by SLF, [6], [13] and references therein.
Generally speaking, a model-based FD system consists of residual generation and residual evaluation with threshold setting, the followings will highlight the FD design procedure.

For the purpose of residual generator consider the following switched model-based FDF:

$$\hat{x}_{k+1} = A_i \hat{x}_k + B_i u_k + L_i(y_k - \hat{y}_k) \quad (3)$$
$$\hat{y}_k = C_i \hat{x}_k + D_i u_k \quad (4)$$
$$r_{i,k} = K_i(y_k - \hat{y}_k) \quad (5)$$

where $\hat{x}_k \in \mathscr{R}^n$ is the estimate of the state vector x_k, $\hat{y}_k \in \mathscr{R}^m$ is the estimate of the output vector y_k. $L \in \mathscr{R}^{n\times m}$ is the observer gain and $L_i \in \{L_1, L_2, ..., L_N\}$, $r_{i,k} \in \mathscr{R}^m$ is the residual vector for each sub-model, and $K \in \mathscr{R}^{m\times m}$, is the post filter matrix, and $K_i \in \{K_1, K_2, ..., K_N\}$.

Define the estimation error as follows $e_k = x_k - \hat{x}_k$. Then the dynamics of the residual generator (3)-(5) can be written as:

$$\begin{aligned} e_{k+1} &= [A_i - L_iC_i]e_k + [E_{d,i} - L_iF_{d,i}]d_k + [E_{f,i} - L_iF_{f,i}]f_k \\ &= \bar{A}_i e_k + \bar{E}_i d_k + \bar{E}_{f,i} f_k \quad (6) \\ r_{i,k} &= K_i[C_i e_k + F_{d,i} d_k + F_{f,i} f_k] \\ &= \bar{C}_i e_k + \bar{F}_{d,i} d_k + \bar{F}_{f,i} f_k \quad (7) \end{aligned}$$

where, $\bar{A}_i = A_i - L_iC_i$, $\bar{E}_{d,i} = E_{d,i} - L_iF_{d,i}$, $\bar{E}_{f,i} = E_{f,i} - L_iF_{f,i}$, $\bar{C}_i = K_iC_i$, $\bar{F}_{d,i} = K_iF_{d,i}$, $\bar{F}_{f,i} = K_iF_{f,i}$.

In this paper, the $\mathscr{L}_2$-norm is considered for residual evaluation, which is defined as follows:

$$\|r_{i,k}\|_{2,R} = \left(\sum_{k=0}^{R-1} r_i(k+l)^T r_i(k+l) \right)^{1/2}$$

where R is the size of the evaluation window.

Normally, the threshold is set as the maximum influence of unknown inputs on the residual signal in fault-free case, i.e.

$$J_{th} = \sup_{d,f=0} \|r_k\|_{2,R}$$

We can notice that, the norm-based threshold computation in the sense of $J_{th} = \sup_{d,f=0} \|r_k\|_{2,R}$ would cover all possible changes in the residual signal caused by unknown inputs. However, in switched systems, the plant is constructed by a set of sub-models. Therefore, using the knowledge of the possible maximum influence of disturbances on the overall sub-models will lead to a conservative threshold, which affect the fault detectability. To this end, the threshold can be set as follows:

$$J_{th,i} = \sup_{d,f=0} \|r_{i,k}\|_{2,R}$$

It is the essential idea behind this study that the switched logics will not only be used for residual generation but also for evaluation process. In this way, the available knowledge of each local sub-models will be integrated into residual evaluation and threshold computation schemes.

The decision logics for switched systems can be defined as follows:

$$\|r_{i,k}\|_{2,R} > J_{th,i} \Longrightarrow \text{fault} - \text{alarm}$$

$$\|r_{i,k}\|_{2,R} \le J_{th,i} \Longrightarrow \text{fault} - \text{free}$$

The followings are the basic idea of the multiple sub-models and the integrated FD design.

A. Basic Idea - switching multiple sub-models

The complex or nonlinear plant can be represented by a number of linear sub-models with switch signal. The transition between these sub-models is controlled by switching signal, where in each time-instant, there is only one active sub-model.
In this subsection, a new scheme for modeling switched systems is developed. Based on this scheme, a significant improvement on fault detectability can be achieved. The essential idea of this scheme is that, the switching signal will activate more than one sub-model to represent the dynamic of the operating point. Fig. (1) demonstrates the basic of this idea by considering the sub-models as references. Therefore, to model the actual operating point precisely, more than one reference sub-model can be used. The reference sub-models

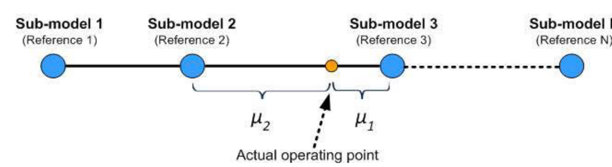

Fig. 1: Basic idea of multiple sub-models

are defined by the switching signal $\sigma(\zeta(k))$[1], $\sigma \in \{i = 1,2,...,N\}$, where N represents the number of sub-models, and $\zeta(k)$ is known value. The number of sub-models which can be used to represent the active point is trade-off between the intended accuracy and the computation cost. As shown in Fig. (1), the simplest case can be expressed as follows: If the operating point is near the reference sub-model, then the switching signal will activate the nearest two sub-models. A

[1]For simplicity $\sigma(\zeta(k))$ will represented by i and $\sigma(\zeta(k+1))$ by j.

weighting factor (μ_q) will be used to represent the weight of each sub-model as follows:

$$\mu_q \triangleq \{\mu_1, \mu_2\}, \qquad \sum_{q=1}^{2} \mu_q = 1 \tag{8}$$

$$\mu_2 = \left| \frac{\zeta_x - \zeta_{\sigma_1}}{\zeta_{\sigma_2} - \zeta_{\sigma_1}} \right|, \quad \mu_1 = \left| \frac{\zeta_x - \zeta_{\sigma_2}}{\zeta_{\sigma_2} - \zeta_{\sigma_1}} \right| \quad OR \quad \mu_1 = 1 - \mu_2$$

Based on these interpretations, the switched systems given in (1)-(2) is reformulated as follows:

$$x_{k+1} = \sum_{q=1}^{h} \mu_q \{A_q^i x_k + B_q^i u_k + E_{d,q}^i d_{q,k} + E_{f,q}^i f_k\} \tag{9}$$

$$y_k = \sum_{q=1}^{h} \mu_q \{C_q^i x_k + D_q^i u_k + F_{d,q}^i d_{q,k} + F_{f,q}^i f_k\} \tag{10}$$

where h is the number of weighted sub-models.
Fig. (2) shows the schematic modeling of the weighted switched systems.
The advantages of this extension scheme can be summarized into:

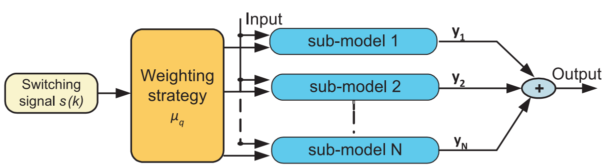

Fig. 2: Schematic of weighted switched system

- In comparison with the traditional switched systems, more than one sub-model will be activated. Therefore, a more accurate result can be obtained for representing the new operating point.
- In multiple model or T-S fuzzy schemes, all the sub-models should run in parallel. But in this scheme, there is a reduction of computational effort by running only selected sub-models based on the complexity of the process.
- The weighting in this scheme is constructed based on the closest sub-models, which is more significant than weighted all the sub-models.
- This scheme also played a role in reduce of the number of sub-models by presenting some operating points by weighting strategy.

B. Integrated FD design

Consider that in the fault-free case,

$$\sup_{f=0,d} \|r_i\|_2 \leq \gamma_i \sup \|d\|_2 \leq \gamma_i \delta_{d_i} := J_{th,i}, \quad \forall d \tag{11}$$

where δ_{d_i} is the $\mathscr{L}-2$ disturbance bound for each sub-model, and γ_i is an induced norm defined by:

$$\gamma_i = \frac{\sup_{f=0,d} \|r_i\|_2}{\sup \|d\|_2} \tag{12}$$

Consider the residual generator (6)-(7). The threshold can be defined as:

$$J_{th,i} := \gamma \sup \|d\|_2 \geq \sup_{f=0,d} \|r_i\|_2 \tag{13}$$

where $\gamma = \max\{\gamma_i | i = 1, 2, ..., N\}$.
Now we redefine the residual signal as follows:

$$\bar{r}_{i,k} = r_{i,k}/\gamma \tag{14}$$

and the threshold will be constant overall the switched systems

$$\bar{J}_{th,i} = \sup \|d\|_2, \quad \forall i = \{1, 2, ..., N\} \tag{15}$$

This definition of threshold handles the worst case of disturbances, that mean the existing differences between the local residuals generated by local sub-models have not been taken into account. This may lead to a significant reduction in fault detectability. Therefore, it motivated us to consider the performance index for each local sub-model in the evaluation and threshold setting as follows:

$$\tilde{r}_{i,k} = \omega_i K_i (y_k - \hat{y}_k) \tag{16}$$

where $\omega_i = 1/\gamma_i$, and the threshold will be adaptive based on the active sub-model

$$\tilde{J}_{th,i} = \delta_{d_i}$$

which will lead to weight each local residual generator differently instead of constantly.
Considering

$$\omega_i \geq \frac{1}{\gamma}, \qquad \forall i = \{1, 2, ..., N\}$$

means that, the FD system considering the local performance index $\omega_i \geq \gamma^{-1}$ with the threshold $\tilde{J}_{th,i}$ will enhance the fault detectability with respect to $\bar{r}_{i,k} = r_{i,k}/\gamma$.
The previous explanation can be summarized in the following remarks:

- If γ is considered as the maximum influence of disturbance on r_k, i.e. the worst case, then this value should cover the whole working point of the switched systems. In this case, the residual generator and the dynamics of the residual generator are given as follows:

$$\hat{x}_{k+1} = A_i \hat{x}_k + B_i u_k + L_i (y_k - \hat{y}_k) \tag{17}$$

$$\hat{y}_k = C \hat{x}_k + D_i u_k \tag{18}$$

$$\bar{r}_{i,k} = \omega K_i (y_k - \hat{y}_k) \tag{19}$$

where, $\omega = \gamma^{-1} = \min\{\omega_i | i = 1, ..., N\}$.
Based on it the dynamic of the FDF is given as follows

$$e_{k+1} = \bar{A}_i e_k + \bar{E}_{d,i} d_k + \bar{E}_{f,i} f_k \tag{20}$$

$$\bar{r}_{i,k} = \omega (\bar{C}_i e_k + \bar{F}_{d,i} d_k + \bar{F}_{f,i} f_k) \tag{21}$$

which lead to the following constant threshold,

$$\bar{J}_{th,i} = \sup_i \|d\|_2 = \delta_d \tag{22}$$

where δ_d is the maximum disturbance bound overall the sub-models.

- If the local performance of each sub-model is considered, then the residual generator and its dynamics are as follows, respectively:

$$\hat{x}_{k+1} = A_i\hat{x}_k + B_i u_k + L_i(y_k - \hat{y}_k) \quad (23)$$

$$\hat{y}_k = C\hat{x}_k + D_i u_k \quad (24)$$

$$\tilde{r}_{i,k} = \omega_i K_i(y_k - \hat{y}_k) \quad (25)$$

where, $\omega_i = 1/\gamma_i, \qquad \forall i = \{1,2,...,N\}$

$$e_{k+1} = \bar{A}_i e_k + \bar{E}_{d,i} d_k + \bar{E}_{f,i} f_k \quad (26)$$

$$\tilde{r}_{i,k} = \omega_i(\bar{C}_i e_k + \bar{F}_{d,i} d_k + \bar{F}_{f,i} f_k) \quad (27)$$

As a result, threshold adapted to the change of the active sub-model as follows,

$$\tilde{J}_{th,i} = \sup_i \|d\|_2 = \delta_{d_i}$$

- It will be concluded from Eq. (27) that, the local effect of each sub-models can be transferred to the residual evaluation and threshold setting.
- By using the weighting factor ω_i, the local residual signal will be weighted individually based on the influence of d on r_i. i.e. the local residual signal will be weekly weighted if the influence of d is high, and it will be strongly weighted if the influence of d is low.

III. Problem formulation

Based on this weighting criterion and the integrated FD design, the FDF given in (3)-(5) is rewritten as follows:

$$\hat{x}_{k+1} = \sum_{q=1}^{h} \mu_q\{A_q^i\hat{x}_k + B_q^i u_k + L_q^i(y_k - \hat{y}_k)\} \quad (28)$$

$$\hat{y}_k = \sum_{q=1}^{h} \mu_q\{C_q^i\hat{x}_k + D_q^i u_k\} \quad (29)$$

$$\tilde{r}_{i,k} = \sum_{q=1}^{h} \omega_i\mu_q K_q^i(y_k - \hat{y}_k) \quad (30)$$

Define the dynamics error $e_k = x_k - \hat{x}_k$. Then the dynamics of the residual generator (28)-(30) can be written as:

$$e_{k+1} = \sum_{q=1}^{h}\sum_{p=1}^{h} \mu_q\mu_p(\bar{A}_{qp}^i e_k + \bar{E}_{d,qp}^i d_k + \bar{E}_{f,qp}^i f_k) \quad (31)$$

$$\tilde{r}_{i,k} = \sum_{q=1}^{h}\sum_{p=1}^{h} \omega_i\mu_q\mu_p(\bar{C}_{qp}^i e_k + \bar{F}_{d,qp}^i d_k + \bar{F}_{f,qp}^i f_k) \quad (32)$$

where, $\bar{A}_{qp}^i = A_q^i - L_q^i C_p^i$, $\bar{E}_{d,qp}^i = E_{d,q}^i - L_q^i F_{d,p}^i$, $\bar{E}_{f,qp}^i = E_{f,q}^i - L_q^i F_{f,p}^i$, $\bar{C}_{qp}^i = K_q^i C_p^i$, $\bar{F}_{d,qp}^i = K_q^i F_{d,p}^i$, $\bar{F}_{f,qp}^i = K_q^i F_{f,q}^i$.

The problem is formulated in terms of weighting factors, and multiple switched systems as follows.

Problem I:
Given a stable FD system (31)-(32) with $e(0) = 0$, find a solution for

$$\max \omega_i, \quad i = \{1,2,...,N\}$$

$$\text{subject to } \tilde{J}_{th,i} = \delta_{d_i} > \sup_{f=0,d} \|\tilde{r}_{i,k}\|_{2,R}$$

In this problem $\omega_i = 1/\gamma_i$, which represents the effect of each sub-model, and it is integrated in the residual generation as ω_i.

IV. FD system design for switched systems

Given residual generator (28)-(30), whose dynamics is governed by (31)-(32), we will find the threshold $\tilde{J}_{th,i}$ such that $\forall d,\ f = 0,\ \|\tilde{r}_{i,k}\|_{2,R} < \tilde{J}_{th,i}$.
The solution is giving by the following theorem.

Theorem 1: *Given system (31)-(32), with $f = 0$, $e(0) = 0$ and constants $\omega_i = 1/\gamma_i > 0$, assume that $\sigma_k = i$, $\sigma_{k+1} = j$, $i = \{1,2,...,N\}$, then*

$$\tilde{J}_{th,i} = \delta_{d_i} > \|\tilde{r}_{i,k}\|_{2,R} \quad (33)$$

if there exist $P_i > 0$ and $P_j > 0$ such that:

$$\begin{bmatrix} -P_j & \Theta_{12}(qq) & \Theta_{13}(qq) \\ * & -P_i + \Theta_{22}(qq) & \Theta_{23}(qq) \\ * & * & \Theta_{33}(qq) - I \end{bmatrix} < 0 \quad (34)$$

for $i = \{1,...,N\}$
$\Theta_{12}(qq) = P_j(A_q^i - L_q^i C_q^i)$, $\Theta_{13}(qq) = P_j(E_{dq}^i - L_q^i F_{dq}^i)$
$\Theta_{22}(qq) = \omega_i^2(C_q^{i,T} K_q^{i,T} K_q^i C_q^i)$, $\Theta_{23}(qq) = \omega_i^2(C_q^{i,T} K_q^{i,T} K_q^i F_{dq}^i)$
$\Theta_{33}(qq) = \omega_i^2(F_{dq}^{i,T} K_q^{i,T} K_q^i F_{dq}^i)$.
and

$$\begin{bmatrix} -P_j & \Theta_{12}(qp) & \Theta_{13}(qp) \\ * & -4P_i + \Theta_{22}(qp) & \Theta_{23}(qp) \\ * & * & \Theta_{33}(qp) - 4I \end{bmatrix} < 0 \quad (35)$$

$\forall (i,j)$.

$$\begin{aligned}
\Theta_{12}(qp) &= P_j(A_q^i - L_q^i C_p^i + A_p^i - L_p^i C_q^i), \\
\Theta_{13}(qp) &= P_j(E_{dq}^i - L_q^i F_{dp}^i + E_{dp}^i - L_p^i F_{dq}^i), \\
\Theta_{22}(qp) &= \omega_i^2(C_p^{i,T} K_q^{i,T} K_p^i C_p^i + C_p^{i,T} K_q^{i,T} K_p^i C_q^i + C_q^{i,T} K_p^{i,T} K_q^i C_p^i \\
&+ C_q^{i,T} K_p^{i,T} K_p^i C_q^i), \\
\Theta_{23}(qp) &= \omega_i^2(C_p^{i,T} K_q^{i,T} K_q^i F_{dp}^i + C_q^{i,T} K_p^{i,T} K_q^i F_{dp}^i \\
&+ C_p^{i,T} K_q^{i,T} K_p^i F_{dq}^i + C_q^{i,T} K_p^{i,T} K_p^i F_{dq}^i), \\
\Theta_{33}(qp) &= \omega_i^2(F_{dp}^{i,T} K_q^{i,T} K_q^i F_{dp}^i + F_{dp}^{i,T} K_q^{i,T} K_p^i F_{dq}^i \\
&+ F_{dq}^{i,T} K_p^{i,T} K_q^i F_{dp}^i + F_{dq}^{i,T} K_p^{i,T} K_p^i F_{dq}^i).
\end{aligned}$$

To this end, the threshold $\tilde{J}_{th,i}$ can be set to:

$$\tilde{J}_{th,i} = \delta_{d_i} \quad (36)$$

Proof: the dynamics of the residual generator in fault-free case, i.e. $f = 0$ is given by:

$$e_{k+1} = \sum_{q=1}^{h}\sum_{p=1}^{h} \mu_q\mu_p(\bar{A}_{qp}^i e_k + \bar{E}_{d,qp}^i d_k) \quad (37)$$

$$\tilde{r}_{i,k} = \sum_{q=1}^{h}\sum_{p=1}^{h} \omega_i\mu_q\mu_p(\bar{C}_{qp}^i e_k + \bar{F}_{d,qp}^i d_k) \quad (38)$$

Define the following SLF,

$$V(e_k) = e_k^T P_{\sigma(k)} e_k \qquad \sigma(k) = i, \ \ i \in \{1,2,...,N\} \quad (39)$$

where $P_\sigma \in \{P_1, P_2, ..., P_N\}$ is a set of positive definite matrices, $V(e_k) > 0$, $V(0) = 0$, $V(e_k)$ is monotonic decreasing

function and $\Delta V(e_k) < 0$.
The difference equation of the switched Lyapunov function is given by:

$$\begin{aligned}\Delta V(e_k) &= e_{k+1}^T P_{\sigma(k+1)} e_{k+1} - e_k^T P_{\sigma(k)} e_k \\ &= \sum_{q=1}^{h}\sum_{p=1}^{h}\sum_{s=1}^{h}\sum_{t=1}^{h} \mu_q\mu_p\mu_s\mu_t \big(e_k^T \bar{A}_{qp}^{iT} P_j \bar{A}_{st}^i e_k \\ &+ e_k^T \bar{A}_{qp}^{iT} P_j \bar{E}_{d,st}^i d_k + d_k^T \bar{E}_{d,qp}^{iT} P_j \bar{A}_{st}^i e_k \\ &+ d_k^T \bar{E}_{d,qp}^{iT} P_j \bar{E}_{d,st}^i d_k^- e_k^T P_i e_k\big)\end{aligned}$$

The stability of system (37) and (38) is ensured if $\Delta V(e_k)$ is less than zero. $\Delta V(e_k)$ can be combined with the induced norm which reflects the effect of disturbances d_k on residual signal $r_i(k)$; i.e. $\{ \ ||r_{i,k}||_2 - \gamma_i ||d_k||_2 < 0 \ \}$ as follows:

$$\Delta V(e_k) + \tilde{r}_{i,k}^T \tilde{r}_{i,k} - d_k^T d_k < 0$$

where $\tilde{r}_{i,k} = r_{i,k}/\gamma_i$. This lead to:

$$\begin{aligned}&\sum_{q=1}^{h}\sum_{p=1}^{h}\sum_{s=1}^{h}\sum_{t=1}^{h} \mu_q\mu_p\mu_s\mu_t \big[e_k^T \bar{A}_{qp}^{iT} P_j \bar{A}_{st}^i e_k + e_k^T \bar{A}_{qp}^{iT} P_j \bar{E}_{d,st}^i d_k \\ &+ d_k^T \bar{E}_{d,qp}^{iT} P_j \bar{A}_{st}^i e_k + d_k^T \bar{E}_{d,qp}^{iT} P_j \bar{E}_{d,st}^i d_k + \omega_i^2 \big(e_k^T \bar{C}_{qp}^{iT} \bar{C}_{st}^i e_k \\ &+ e_k^T \bar{C}_{qp}^{iT} \bar{F}_{d,st}^i d_k + d_k^T \bar{F}_{d,qp}^{iT} \bar{C}_{st}^i e_k + d_k^T \bar{F}_{d,qp}^{iT} \bar{F}_{d,st}^i d_k\big) \\ &- e_k^T P_i e_k - d_k^T d_k\big] < 0 \qquad (40)\end{aligned}$$

The previous inequality can be written as:

$$\begin{aligned}\sum_{q=1}^{h} \mu_q^2 \quad [\quad & e_k^T \bar{A}_{qq}^{iT} P_j \bar{A}_{qq}^i e_k + \omega_i^2 (e_k^T \bar{C}_{qq}^{iT} \bar{C}_{qq}^i e_k) \\ + \quad & e_k^T \bar{A}_{qq}^{iT} P_j \bar{E}_{d,qq}^i d_k + \omega_i^2 (e_k^T \bar{C}_{qq}^{iT} \bar{F}_{d,qq}^i d_k) \\ + \quad & d_k^T \bar{E}_{d,qq}^{iT} P_j \bar{A}_{qq}^i e_k + \omega_i^2 (d_k^T \bar{F}_{d,qq}^{iT} \bar{C}_{qq}^i e_k) \\ + \quad & d_k^T \bar{E}_{d,qq}^{iT} P_j \bar{E}_{d,qq}^i d_k + \omega_i^2 (d_k^T \bar{F}_{d,qq}^{iT} \bar{F}_{d,qq}^i d_k) \\ - \quad & e_k^T P_i e_k - d_k^T d_k] \\ +\frac{1}{4}\sum_{q=1}^{h}\sum_{p=1}^{h} \mu_q\mu_p \quad [\quad & e_k^T (\bar{A}_{qp}^{iT} + \bar{A}_{pq}^{iT}) P_j (\bar{A}_{qp}^i + \bar{A}_{pq}^i) e_k \\ + \quad & \omega_i^2 e_k^T (\bar{C}_{qp}^{iT} + C_{pq}^{iT})(\bar{C}_{qp}^i + \bar{C}_{qp}^i) e_k \\ + \quad & e_k^T (\bar{A}_{qp}^{iT} + \bar{A}_{pq}^{iT}) P_j (\bar{E}_{d,qp}^i + \bar{E}_{d,pq}^i) d_k \\ + \quad & \omega_i^2 e_k^T (\bar{C}_{qp}^{iT} + \bar{C}_{pq}^{iT})(\bar{F}_{d,qp}^i + \bar{F}_{d,pq}^i) d_k \\ + \quad & d_k^T (\bar{E}_{d,qp}^{iT} + \bar{E}_{d,pq}^{iT}) P_j (\bar{A}_{qp}^i + \bar{A}_{pq}^i) e_k \\ + \quad & \omega_i^2 d_k^T (\bar{F}_{d,qp}^{iT} + \bar{F}_{d,pq}^{iT})(\bar{C}_{qp}^i + \bar{C}_{pq}^i) e_k \\ + \quad & d_k^T (\bar{E}_{d,qp}^{iT} + \bar{E}_{d,pq}^{iT}) P_j (\bar{E}_{d,pq}^i + \bar{E}_{d,pq}^i) d_k \\ + \quad & \omega_i^2 d_k^T (\bar{F}_{d,qp}^{iT} + \bar{F}_{d,pq}^{iT})(\bar{F}_{d,qq}^i + \bar{F}_{d,pq}^i) d_k \\ - \quad & 4(e_k^T P_i e_k + d_k^T d_k)] < 0 \qquad (41)\end{aligned}$$

Inequality (41) is negative if each sum is negative definite. Rewrite the previous inequality into two quadratic form and after some mathematical manipulation using Schur complements lemma, it will lead to (34) and (35) respectively.
To this end, the proof is completed. □

The following remarks discuss the improvements which can be achieved by applying the preceding strategy of the integrated FD system.

Remark 1: Matrix inequalities (34) and (35) are nonlinear matrix inequalities (NMI)'s. It can be transformed to LMI's by using the following definitions: $P_i L_i = X_i$ and K_i is set as a pre-chosen matrix. ω_i^2 is set to W_i. After solving the LMI's the observer gains and ω_i are computed respectively, as follows: $L_i = (P_i)^{-1} X_i$ and $\omega_i = (W_i)^{1/2}$.

Remark 2: In the residual signal given in (19), ω should hold for all the sub-models. Based on it a constant threshold is defined in (22) as worst-case handling.
It is clear that: $\gamma^{-1} = \omega = \min\{\omega_i | i = 1, ..., N\}$, but due to the different local sub-model behavior, it would be possible that there exist some ω_i which ensures $\omega_i \geq \omega$.
This point is important to have the following: If we have the following two structures of residual generators

$$e_{k+1} = \bar{A}_i e_k + \bar{E}_{f,i} f_k \qquad (42)$$

$$\bar{r}_{i,k} = \omega(\bar{C}_i e_k + \bar{F}_{f,i} f_k) \qquad (43)$$

$$\tilde{r}_{i,k} = \omega_i(\bar{C}_i e_k + \bar{F}_{f,i} f_k) \qquad (44)$$

Then the influence of the possible faults on $\tilde{r}_{f_i}(k)$ is enhanced in comparison with $\bar{r}_{f_i,k}$, which can be explained as follows:

$$\tilde{r}_{f_i,k} = \eta \bar{r}_{f_i,k}, \quad \eta = \frac{\omega_i}{\omega} \geq 1$$

Remark 3: If the disturbance which affects all the sub-models is constant, then the threshold in (36) will be constant overall the local sub-models. However, this problem can adopt a new strategy for threshold setting by considering different bound values of disturbance for each sub-model, as follows: $\delta_{d_i} \in \{\delta_{d_1}, \delta_{d_2}, ..., \delta_{d_N}\}$. In this way, an adaptive threshold is set as (36).

V. Example

Three sub-models of the one-track model of lateral vehicle dynamics are defined at different velocities, [7]: 30km/h, 50km/h and 70km/h as follows:
Sub-model 1 ($v_k = 30km/h$): Based on the following cornering stiffness values, $C_{\alpha V} = 303302$ and $C_{\alpha H} = 115407$, the discrete-time system matrices are:

$$A_1 = \begin{bmatrix} 0.53 & -0.03 \\ -0.92 & 0.47 \end{bmatrix}, B_1 = \begin{bmatrix} 0.30 \\ 2.29 \end{bmatrix}, E_{d1} = \begin{bmatrix} 0.015 \\ -0.012 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -286.2 & -14.1 \\ 0 & 1 \end{bmatrix}, D_1 = \begin{bmatrix} 207.3151 \\ 0 \end{bmatrix}, F_{d1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Sub-model 2 ($v_k = 50km/h$): Based on the following cornering stiffness values, $C_{\alpha V} = 106066$ and $C_{\alpha H} = 98089$, the discrete-time system matrices are:

$$A_2 = \begin{bmatrix} 0.81 & -0.01 \\ 0.19 & 0.77 \end{bmatrix}, B_2 = \begin{bmatrix} 0.08 \\ 1.16 \end{bmatrix}, E_{d2} = \begin{bmatrix} -0.02 \\ -0.002 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -139.5 & 1.1 \\ 0 & 1 \end{bmatrix}, D_2 = \begin{bmatrix} 72.5 \\ 0 \end{bmatrix}, F_{d2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Sub-model 3 ($v_k = 70km/h$): Based on the following cornering stiffness values, $C_{\alpha V} = 73266$ and $C_{\alpha H} = 77442$, the discrete-time system matrices are:

$$A_3 = \begin{bmatrix} 0.89 & -0.017 \\ 0.28 & 0.87 \end{bmatrix}, B_3 = \begin{bmatrix} 0.04 \\ 0.85 \end{bmatrix}, E_{d3} = \begin{bmatrix} -0.02 \\ -0.002 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} -103.0 & 1.01 \\ 0 & 1 \end{bmatrix}, D_3 = \begin{bmatrix} 50.1 \\ 0 \end{bmatrix}, F_{d3} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

A. Model Validation

The efficiency of the switching models can be seen in this sub-section. Due to the size limitation, the results of lateral acceleration sensor (a_y) will be shown in this paper. The three sub-models of the one-track model show a very good performance in presenting the lateral dynamics as in Fig. (3).

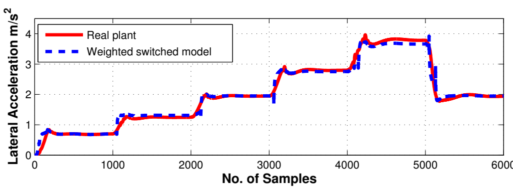

Fig. 3: Lateral acceleration (a_y) validation.

B. Residual evaluation with threshold setting

Fig. (4) shows two sensor faults of different size. Fig. (5) shows that the standard evaluation with constant threshold setting could not detect the small faults. But according to the proposed method, the small faults can be detected easily as shown in Fig. (6).

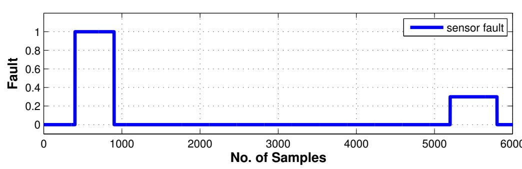

Fig. 4: Lateral acceleration sensor faults.

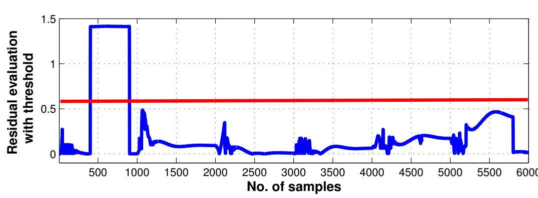

Fig. 5: a_y residual evaluation with constant threshold.

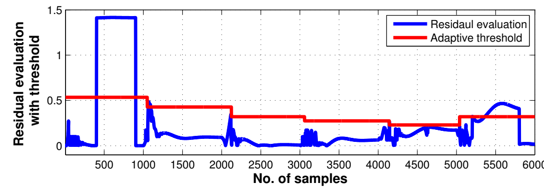

Fig. 6: a_y residual evaluation with adaptive threshold.

VI. Conclusion

In this paper, a new scheme for evaluating the residual signal has been proposed to design an integrated FD system for multiple switched systems. The challenge in this study is summarized into utilizing the available information for each local sub-model. This information is used to integrate the residual signals with the evaluation and threshold computation in FD system. As a result, the local residual signals are weighted individually, instead of constantly. The proposed design schemes can achieve higher fault detectability than the standard one. And by that, it will improve the model-based FD approach. The stability of the switched FDF observers has been guaranteed in terms of SLF. Furthermore, this chapter disclosed that, using the information provided from each local sub-model may lead to construct an adaptive threshold. The optimization problems have been formulated and solved using the LMI techniques with SLF.

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