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# Easily Adaptable Model of Test Benches for Internal Combustion Engines

J. Blumenschein, P. Schrangl, T. E. Passenbrunner, H. Trogmann and L. del Re

*Abstract*— **Internal combustion engine test benches allow to simulate the operation of an engine under various conditions and are essential in engine development in the automotive industry. Models of such test benches are often used in the design and improvement of test bench controls and to simulate their behavior. Therefore, the paper summarizes a modeling procedure based on a simple model structure. In particular, the individual components of a test bench are described and their models are combined. A torque observer is used to compensate the lack of torque measurements. Finally, a model for a test bench equipped with a heavy duty engine is determined and validated with measurements.**

## I. Introduction

In a vehicle, the velocity and the rotational speed $\omega_C$ of the engine are the result of the engine torque $T_C$ and the load torque, this being the direct consequence of the road and vehicle conditions. The purpose of a dynamic test bench is the simulation of the operation of an internal combustion engine in a passenger car, truck or other vehicle without the need for this vehicle. To this end, both the engine speed $\omega_C$ and the shaft torque $T_S$ need to track the values the engine would experience in a vehicle. The load conditions have to be computed and enforced by a dynamometer connected to the engine crankshaft at a test bench. A test bench guarantees reproducible conditions in terms of temperature and air pressure and reduces the costs and time required for development and calibration.

In industrial practice the control task is usually solved by two separate control loops. For larger internal combustion engines, in most cases the accelerator pedal position $\alpha$ is directly related to the engine speed $\omega_C$ while the torque is applied by the dynamometer. However, the significance of experiments on a test bench is a direct consequence of the precision of the control system. Therefore, the subject has received attention in different ways [1], [2], many of these advanced control approaches are based on a model of the test bench.

The test bench model in [3] considers parameters such as the speed of the vehicle, aerodynamic forces or the slope of the road to compute the load. Its main drawback is the need to estimate those parameters. The torque provided by the internal combustion engine is estimated using the measured shaft torque and the moment of inertia of the engine. In [1] a fourth-order test bench model is introduced and subsequently used to design a robust control [4].

The modeling of the individual components of a test bench – internal combustion engine, asynchronous motor and connection shaft linking these two actuators – is well-covered in the literature. In [5] an extended Hammerstein system is used to model the internal combustion engine. The advantage of such an approach is that no a-priori information about the engine is needed since all model parameters are obtained from measurements. A completely different approach is described in [6]–[8] where thermodynamical models are used to simulate the dynamic behavior of the internal combustion engine. Controllers based on these models are difficult to tune due to the high complexity of the model.

Since the engine torque cannot be measured directly at the test bench, an observer is necessary. The observer approach presented in [9] introduces a high-gain observer. The major advantage of this method is its robustness against disturbances and parameter uncertainties. However, a drawback of high-gain approaches is the significant sensitivity to noise. In [10] an unknown input proportional-integral observer based on a linear model is introduced. The same approach is used in this paper in order to observe the internal combustion engine torque with reasonable accuracy.

Typically at test benches, asynchronous motors are used as dynamometer to load the engine. In [11] different data-based approaches to identify the dynamics of an asynchronous motor are introduced and compared. The comparison shows that an output error structure (see e.g. [12]) is a useful approach for the identification of a linear model of the asynchronous motor. In [13] the asynchronous motor and the frequency converter are modeled by means of a second-order transfer function for the entire operating region.

In order to model the connection shaft, an additional gear transmission is introduced in [14]. The shaft itself is modeled as a two-mass oscillator with a cubic characteristic of the spring coefficient, a Coulomb friction torque and a hysteresis. In [13] the shaft is also modeled as a two-mass oscillator using linear spring characteristics as well as viscous friction. All parameters are assumed to be known. The identification of the parameters of a connection shaft used in test benches is shown in [15], where an ARMAX structure is used to identify the eigenfrequency and the damping coefficient. These parameters depend on the shaft speed due to the slightly nonlinear operating point-dependent behavior of the test bench. The disadvantage of this approach is, that it is necessary to measure both torques, the asynchronous motor's as well as the internal combustion engine's.

J. Blumenschein and P. Schrangl have written their bachelor thesis at the Institute for Design and Control of Mechatronical Systems, Johannes Kepler University Linz, 4040 Linz, Austria

T. E. Passenbrunner, H. Trogmann and L. del Re are with the Institute for Design and Control of Mechatronical Systems, Johannes Kepler University Linz, 4040 Linz, Austria thomas.passenbrunner@jku.at

The main contribution of this paper is to provide a simple and sufficiently accurate model of the test bench system. Another topic covered in this paper is the estimation of the unknown torque of the internal combustion engine based on the knowledge of the mechanical system, which covers the connecting shaft between the internal combustion engine and the asynchronous motor. In particular, an observer is used to compensate for a lack of torque measurement when determining the model of the internal combustion engine.

The paper is organized as follows: Section II deals with internal combustion engine test benches and their modeling. Section III provides a comparison between the proposed model and the measurements recorded at the test bench. The paper is concluded with comments and an outlook in Section IV.

## II. Modeling of the Test Bench

The typical setup of an internal combustion engine test bench is shown in Fig. 1. The engine under test is connected via a flexible shaft to a second actuator, an electric dynamometer. The inputs of the test bench are the accelerator pedal position $\alpha$ and the desired torque $T_{A_{\text{set}}}$ of the dynamometer. The engine speed $\omega_C$, the dynamometer speed $\omega_A$ and the shaft torque $T_S$ can be measured, an estimate $\hat{T}_A$ of the dynamometer torque $T_A$ is calculated in the frequency converter. The estimation $\hat{T}_C$ calculated from the injected fuel amount by the engine control unit shows a slow dynamic behavior and therefore is not used in the following. The engine torque $T_C$ can instead be estimated from available measurements using the observer stated below.

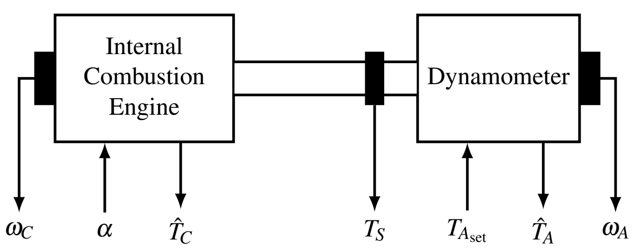


Fig. 1. Typical setup of an internal combustion engine test bench equipped with an asynchronous motor.

### A. Mechanical system

The mechanical part of the test bench can be modeled as a two-mass oscillator of the form

$$\begin{aligned} J_C\, \dot{\omega}_C &= T_C - T_S \\ J_A\, \dot{\omega}_A &= -T_A + T_S \\ \Delta\dot{\varphi} &= \omega_C - \omega_A \end{aligned} \tag{1}$$

with the shaft torque

$$T_S = c\, \Delta\varphi + d\, (\omega_C - \omega_A),$$

$\omega_C = \dot{\varphi}_C$, $\omega_A = \dot{\varphi}_A$ and the twist angle $\Delta\varphi = \varphi_C - \varphi_A$ of the connection shaft. $J_C$ and $J_A$ are the inertias of the internal combustion engine and of the dynamometer, respectively. The inertias of the connecting shaft and the measurement flange have already been included in these values. The constant parameter $c$ is the stiffness of the connecting shaft, $d$ denotes the damping parameter of the connecting shaft.

The parameter $J_A$ has been identified using a least squares algorithm to fit the second Equation of (1) with the estimated dynamometer torque $\hat{T}_A$, the measured shaft torque $T_S$ and the measured speed $\omega_A$ of the asynchronous motor. The derivative $\dot{\omega}_A$ has been determined using a high-pass filter.

Note that, in the following the dynamometer torque $T_A$ is calculated using the second Equation of (1) since the signal $\hat{T}_A$ is corrupted by disturbances in some recorded measurement data.

The parameters $J_C$ and $c$ have been obtained from data sheets provided by the manufacturers. An initial value of the damping $d$ has been determined by matching the poles of the transfer function $\omega_C(s)/T_A(s)$ of system (1), the variable $s$ denotes the Laplace variable, with the identified transfer function based on measurements recorded at the test bench.

Subsequently, the parameters $J_C$, $c$ and $d$ of the mechanical system have been optimized in a closed-loop simulation. A proportional-integral (PI) controller has been used to regulate the speed $\omega_C$ to the desired engine speed $\omega_{C_{\text{set}}}$ by the dynamometer torque $T_A$. Since the engine was unfired (accelerator pedal position $\alpha = 0\,\%$) during these measurements its friction torque $T_C$ has been obtained from the static engine map (see Fig. 7). The simulated engine speed $\omega_C$ has been matched with the measured one. The parameters $J_C$, $J_A$, $c$ and $d$ turned out to be speed-independent. Hence, the mechanical system is considered a linear time-invariant system. Note that the cooling water temperature is kept constant by a control loop.

### B. Internal combustion engine

The internal combustion engine is modeled using a Hammerstein system consisting of a static map and an operating point dependent linear dynamic system. A linear scheduling strategy is used to switch between the different dynamic systems. Fig. 2 shows the structure of the model of the internal combustion engine. The static map links both inputs – the accelerator pedal position $\alpha$ and the actual engine speed $\omega_C$ – with the output – the static torque $T_{C_{\text{stat}}}$. It has been recorded using the approach described in [16]. The same PI controller as in Section II-A has been used to keep the speed $\omega_C$ constant. The dynamics of the shaft have no influence on the measurements, since the engine has been operated in stationary operating points.

The dynamics of the internal combustion engine cannot be determined as easily as the static map since the estimation of the torque of the internal combustion engine is necessary. The estimation is based on the known mechanical system. The linear approach described in [10] has been used and will briefly be discussed in the following. The model is assumed to have the form

$$\begin{aligned} \dot{x}(t) &= A\, x(t) + B\, u(t) + E\, f(t) \\ y(t) &= C\, x(t) + D\, w(t), \end{aligned} \tag{2}$$

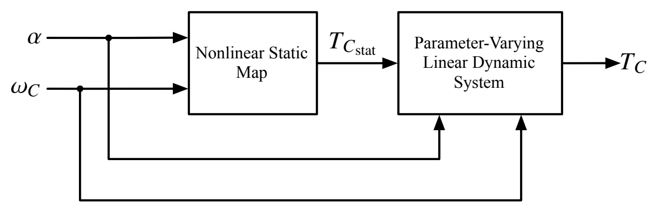


Fig. 2. Hammerstein model of the internal combustion engine.

where $x(t) \in \mathbb{R}^n$ represents the state of the system, $y(t) \in \mathbb{R}^m$ its measured output, $u(t) \in \mathbb{R}^p$ the known input, $f(t) \in \mathbb{R}^q$ the unknown input and $w(t) \in \mathbb{R}^r$ the measurement noise, which is assumed to be white noise.

The mechanical system (1) can be rewritten in form (2) with $x(t) = [\Delta\varphi(t)\ \ \omega_C(t)\ \ \omega_A(t)]^\top$, $u(t) = T_A(t)$, $f(t) = T_C(t)$ and the matrices

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{c}{J_C} & -\frac{d}{J_C} & \frac{d}{J_C} \\ \frac{c}{J_A} & \frac{d}{J_A} & -\frac{d}{J_A} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{J_A} \end{bmatrix},\ E = \begin{bmatrix} 0 \\ \frac{1}{J_C} \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 10^{-3} & 0 \\ 0 & 10^{-3} \end{bmatrix}.$$

The matrix $D$ has been chosen such that it represents the order of magnitude of the measurement noise. As a matter of fact, setting $D$ to zero would imply that no noise is present in the measured output signals. Since real measurement data is used, the measured noise, e. g. caused by combustion oscillations overlaying the signals, has to be taken into account.

The observer to estimate the state $x(t)$ and the unknown input $f(t)$ is of the form

$$\begin{aligned} \dot{\hat{x}}(t) &= A\,\hat{x}(t) + B\,u(t) + E\,\hat{f}(t) + K\,(y(t) - \hat{y}(t)) \\ \dot{\hat{f}}(t) &= L\,(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\,\hat{x}(t) \end{aligned} \tag{3}$$

with the estimated state $\hat{x}(t)$, the estimated unknown input $\hat{f}(t)$, the estimated output $\hat{y}(t)$, the proportional gain matrix $K$ and the integral gain matrix $L$. The system (2) and the observer (3) can be rewritten using the state error $\tilde{x} = (x - \hat{x})$, the estimated input error $\tilde{f} = (f - \hat{f})$, $\varphi(t) = [\tilde{x}\ \ \tilde{f}]^\top$ and $\varepsilon(t) = [w\ \ \dot{f}]^\top$ as

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{f}} \end{bmatrix} = \begin{bmatrix} A - K\,C & E \\ -L\,C & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{f} \end{bmatrix} + \begin{bmatrix} -K\,D & 0 \\ -L\,D & I \end{bmatrix} \begin{bmatrix} w \\ \dot{f} \end{bmatrix},$$

where $I$ is the identity matrix of proper dimension.

The Lyapunov function

$$V(t) = \varphi(t)^\top P \varphi(t), \tag{4}$$

with the positive definite matrix $P$ is chosen such that the estimation errors $\tilde{x}$ and $\tilde{f}$ tend towards zero, if $\dot{V}(t) < 0$. In order to take the error of the unknown input into account the inequality is modified as follows

$$\dot{V}(t) + \varphi(t)^\top\, Q_\varphi\, \varphi(t) + m\,\varepsilon(t)^\top\, Q_\varepsilon\, \varepsilon(t) < 0, \tag{5}$$

where $Q_\varphi$ and $Q_\varepsilon$ are user-defined positive definite matrices. Identity matrices of appropriate dimensions have been used as the components of $\varphi(t)$ and $\varepsilon(t)$ are in the same order of magnitude. The aim is to minimize the weighting coefficient $m > 0$, subject to the constraint (5) and $P = P^\top > 0$, which can be written as

$$\begin{aligned} &\min\ m \\ &\text{s. t.}\quad \dot{V}(t) + \varphi(t)^\top\, Q_\varphi\, \varphi(t) + m\,\varepsilon(t)^\top\, Q_\varepsilon\, \varepsilon(t) < 0 \\ &\qquad\ \ P > 0. \end{aligned} \tag{6}$$

The structure of (6) is a so-called linear matrix inequality (LMI). The matrices $K$ and $L$ of observer (3) can be computed by solving the LMI (6). The LMI solution has been computed using the solver SeDuMi for MATLAB, which is presented in [17]. The resulting observer (3) is a dynamic system which estimates the engine torque in operation on the test bench.

Using the obtained observer the dynamic part of the Hammerstein model in Fig. 2 can be identified using an output error model. First order linear dynamic systems turn out to be sufficient to model the engine dynamics. Additionally, it emerges that in different operating regions, different parameters for the dynamic system are necessary to describe the internal combustion engine adequately.

### C. Electric dynamometer

The electric dynamometer – an asynchronous motor – is driven by a frequency converter, the desired torque $T_{A_{\text{set}}}$ can be set. The model of the electric dynamometer covers both the frequency converter and the asynchronous motor itself. In order to model these components the model structure shown in Fig. 3 has been used.

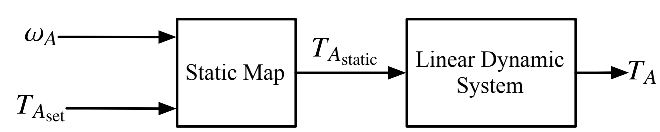


Fig. 3. Structure of the asynchronous motor model.

The proposed model structure uses a linear dynamic system for all operating points and a static map, which covers the nonlinearities of the signal path. The static map and the linear dynamic system were obtained from measurements, where an output error approach was used to identify the linear dynamics.

## III. VALIDATION

The models presented in Section II have been validated using measurement data recorded at a test bench. The test bench consists of three parts: a 200 kW four-cylinder in-line Diesel engine for construction machines, a 240 kW asynchronous motor used as dynamometer and a flexible connection shaft. Measurement data has been captured using a dSPACE system linked to SIMULINK. At the beginning of this Section the results of each test bench component model are shown whereas at the end of this Section the results of the overall test bench model will be presented.

### A. Mechanical system

The mechanical system is validated with the same data set as used for identification, but the transfer function $\omega_A(s)/T_A(s)$ is used to achieve results independent of the optimized transfer function $\omega_C(s)/T_A(s)$. The results are presented in Fig. 4.

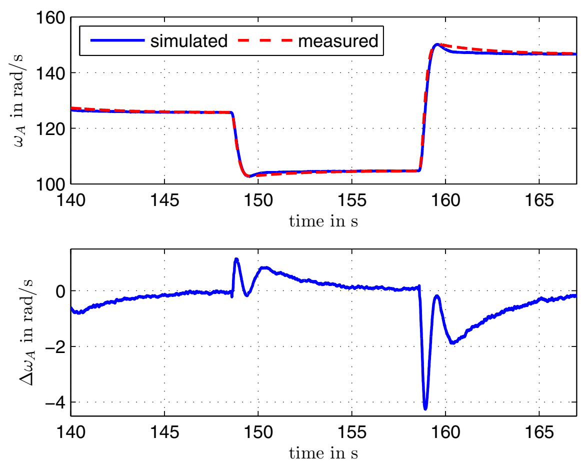


Fig. 4. Validation of the mechanical system. The upper part shows a comparison between the simulated and the measured dynamometer speed $\omega_A$ and the lower part displays the error between simulated and measured speed $\omega_A$. The correlation coefficient of the speed $\omega_A$ is 93 %.

Note that the motored internal combustion engine causes a friction torque, which has to be taken into account in the validation and can be estimated using the static map of the unfired engine. To ensure that the integrating term of the transfer function does not distort the speed estimation, the validation is also performed in closed-loop simulation as the parameter estimation in Section II-A. The correlation coefficient of the mechanical system is approximately 93 %.

### B. Internal combustion engine

First it is necessary to test the observer for the validation of the internal combustion engine. To verify the quality of the estimated engine torque $\hat{T}_C$, the measured dynamometer torque $T_A$ and the estimated engine torque $\hat{T}_C$ are used as inputs to the mechanical system model and the measured engine speed $\omega_C$ as well as the measured dynamometer speed $\omega_A$ are compared with the simulated ones ($\bar{\omega}_A$ and $\bar{\omega}_C$). The scheme in Fig. 5 shows the structure of the observer validation. The results are displayed in Fig. 6.

Since the observer provides accurate results, it is possible to validate the model of the internal combustion engine in the next step. The static engine map is shown in Fig. 7 and the engine dynamics in Fig. 8. The validation data is shown in

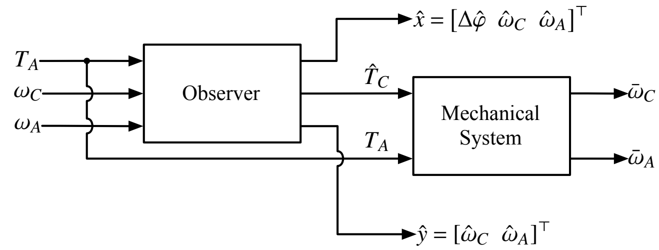


Fig. 5. Scheme for the validation of the observer for the engine torque $T_C$.

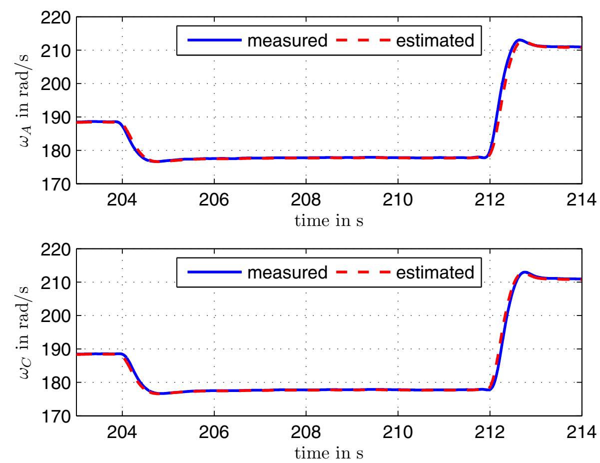


Fig. 6. Validation of the proposed torque observer. The estimated torque feeds the mechanical system model, the resulting speeds $\bar{\omega}_A$ and $\bar{\omega}_C$ are compared to the corresponding measured speeds $\omega_A$ and $\omega_C$.

Fig. 9. The correlation coefficient of the engine dynamics is approximately 90 %. The error at stationary operating points can be explained by the fact that the validation data and the measurement data for the static map of the internal combustion engine have been recorded on different days and therefore the static torque is affected by several factors, e. g. the actual air inlet temperature. The error in the transient estimation is caused by the linear interpolation of the parameters of the dynamic systems.

### C. Electric dynamometer

The model validation of the asynchronous motor model shows correlation coefficients between 88 % and 95 % when using application-typical test trajectories. An example of a validation trajectory is shown in Fig. 10.

### D. Overall test bench model

The validation of the overall test bench model – consisting of the internal combustion engine, the asynchronous motor and the mechanical system – is shown in Fig. 11 and Fig. 12. The torque $T_A$ is compared in Fig. 12 as it can be calculated directly from measurements at the test bench and – differently from the engine torque $T_C$ – does not need to be estimated by an observer. The errors of the dynamometer torque $T_A$ in the static points as well as in the dynamics can be explained as in Section III-B. Such errors also influence

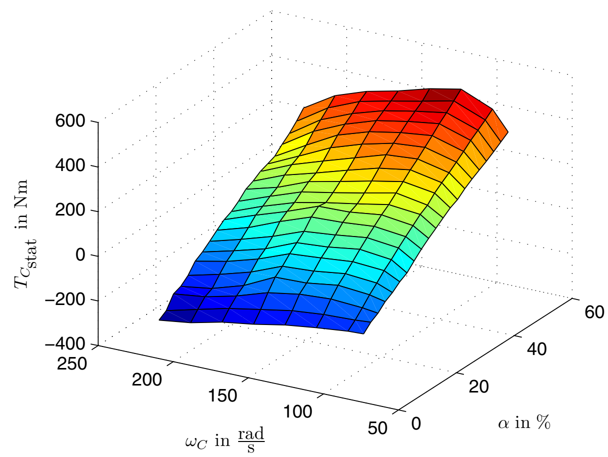


Fig. 7. Static torque map of the internal combustion engine. The actual torque configuration of the engine control unit is nearly linear at fixed speeds.

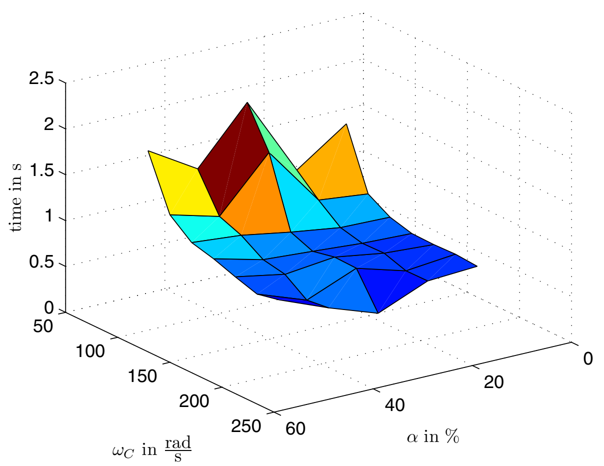


Fig. 8. Eigenfrequncies of the internal combustion engine. The minimum time constant is 0.4 s, the maximum time constant is 2.1 s.

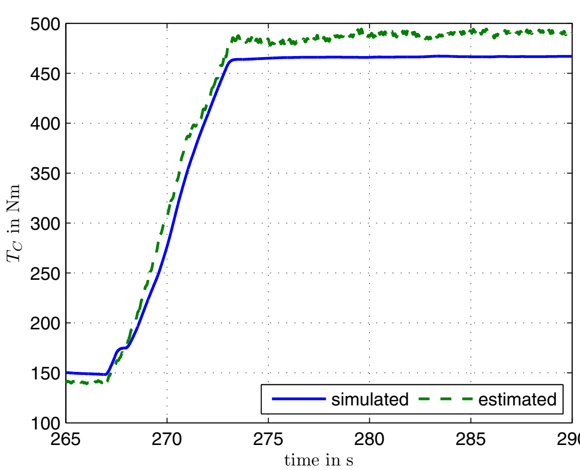


Fig. 9. Validation of the internal combustion engine. The Figure shows a comparison between the simulated and the estimated engine torque $T_C$. The estimated torque was provided by the observer using real measurement data from the test bench.

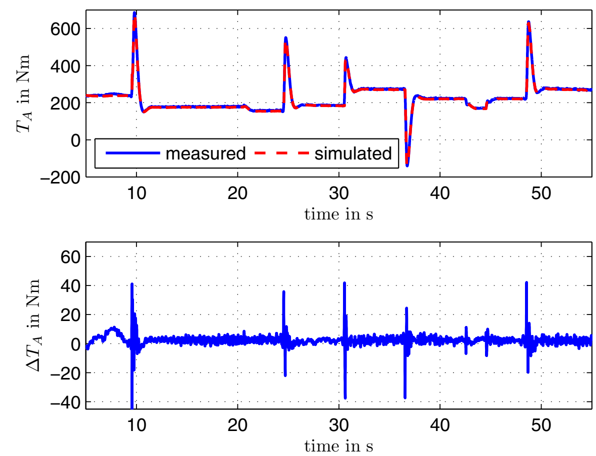


Fig. 10. Validation of the asynchronous motor model. The measured dynamometer torque $T_A$ is compared to the simulated torque. The correlation coefficient of this trajectory is 95 %. The error peaks in the lower part of the figure result from a slight time shift between the simulated and the measured torque $T_A$.

the speeds $\omega_C$ and $\omega_A$ as these are a direct result of both torques.

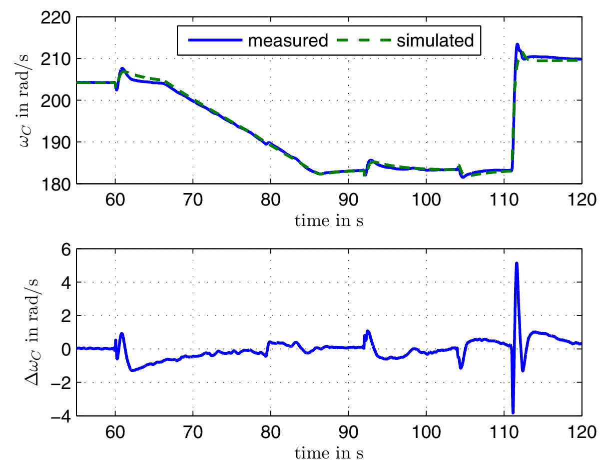


Fig. 11. Validation of the test bench model. The upper graph shows a comparison between the simulated and the measured engine speed $\omega_C$. The lower graph displays the error between simulated and measured speed $\omega_C$.

To control the test bench an observer for the torque $T_C$ is needed. To check the performance of the observer, particularly with regard to controlling, in simulation the output of the engine model, $T_C$, is used. The estimated torque provided by the observer and the simulated torque have been compared. Using a Butterworth-filter to reduce noise, the observer provides accurate results and is suitable for controlling the test bench.

## IV. Conclusion and Outlook

Using the models of the individual test bench components – internal combustion engine, asynchronous motor and mechanical system – a simple continuous-time simulation

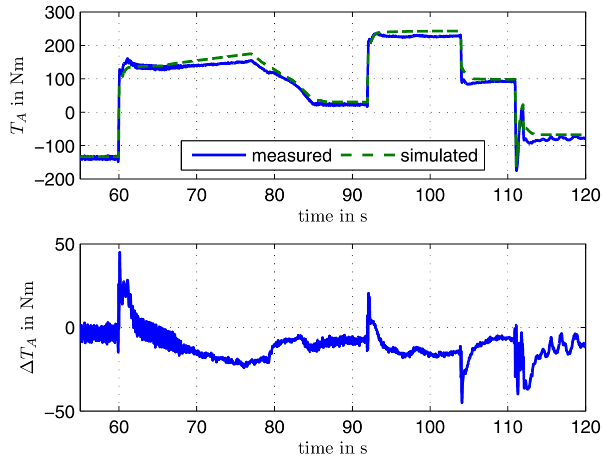


Fig. 12. Validation of the test bench model. The upper graph shows a comparison between the simulated and the measured dynamometer torque $T_A$. The lower graph displays the error between simulated and measured torque $T_A$.

model of a test bench has been determined. An outstanding consequence of this work is that an accurate reproduction of the behavior of the real system can be achieved using simple approaches like linear models, static maps and a linear scheduling strategy. The results of this paper can be used to design controllers for the test bench, in order to track given trajectories of engine speed and torque.

The torque observer used in this paper is simple and fast to compute by solving a LMI problem and can be used both to develop a test bench model and to control the test bench. The main advantage of this torque observer is that – as far as the engine is concerned – it only depends on a single engine parameter, the inertia of the engine. Since it is a common task to test different engines at a single test bench and therefore engines are frequently changed, this observer can be adapted rapidly to the engine. A drawback of the torque observer is that a linear time-invariant model of the mechanics is necessary. However, the observer can easily be extended to nonlinear systems by switching between operating point-dependent linear systems as described in [10].

The advantage of the model, with particular regard to a future control task, is the low model complexity. Another benefit is that all measurements and identification routines can easily be automated, when components are changed the identification of the test bench model is done rapidly and cost-effective.

Future steps based upon this work are the development of controllers based on the obtained simulation model.

## V. Acknowledgments

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