

Limits of Controller Performance in the Heave Disturbance Attenuation Problem*

Ulf Jakob F. Aarsnes¹, Ole Morten Aamo¹, Espen Hauge¹ and Alexey Pavlov²

Abstract—The periodic heave movement of a floating drilling rig is transferred to an elastic pipe suspended in a drilling well. The pipe oscillates up and down in the well with the period of the heave motion pushing the mud in the well and exciting pressure fluctuations in the mud in the pipe. These dynamics are described by six coupled first-order linear partial differential equations of hyperbolic type. By using the Laplace transform, the system is solved in the frequency domain and the resulting irrational transfer functions, from the periodic heave disturbance and the controlled choke flow to the distributed pressure in the annulus around the pipe, are found for different boundary conditions. We show that the response can be improved by choosing an impedance matched boundary condition which can be achieved without the use of active control. We also derive the response when a theoretical 'perfect feed-forward control law', which yields full attenuation of the effect of the disturbance at a single point in the well-annulus, is applied. The limitations of this 'perfect point control' when considering an extended section of the well is shown.

I. INTRODUCTION

A. Problem Description

In drilling operations performed in the oil and gas industry a fluid called mud is pumped down through the drill string and flows through the drill bit in the bottom of the well. If the pressure in the mud at the bottom of the well is too low the well can collapse trapping the drill string, and if the pressure exceeds a certain threshold it can fracture the well. Hence, it is important to control the mud pressure in the well. In Managed Pressure Drilling (MPD) operations this is achieved by sealing the well and releasing mud from the well through a control choke. A back pressure pump allows the pressure to be controlled even when the main pump is stopped. Thus, the pressure in the bottom of the well can be regulated to a desired setpoint. This approach has proven successful when drilling from stationary platforms and results on MPD control can be found in papers such as [1]. MPD from floating drilling rigs, however, still face significant challenges due to the wave induced vertical motion of the floating drilling rig (known as heave).

During normal drilling the heave motion of the drilling rig is decoupled from the drill string by compensation techniques. However, when the drill string is to be extended by a drill string connection it is rigidly connected to the floater. It will then act as a piston in the well creating pressure oscillations which may exceed the upper or lower pressure thresholds one wishes to enforce. It is therefore desirable to use active control of the topside choke to compensate for the pressure changes due to the heave motion. In this scenario, the main pump is disconnected and there is no flow between the annulus and the drill string (the drill bit is equipped with

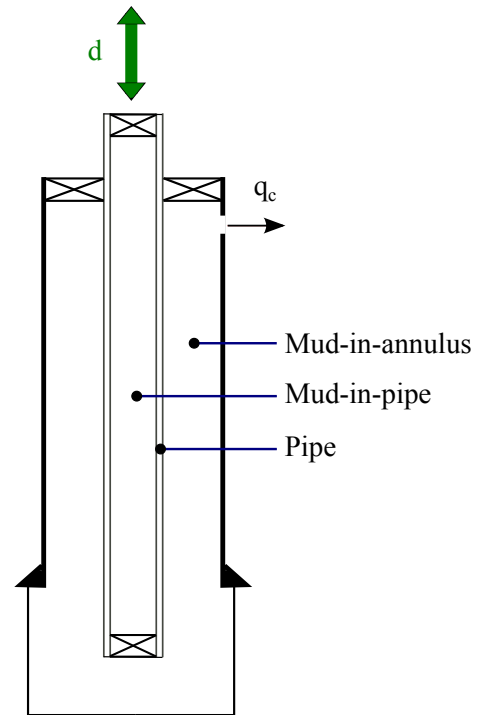


Fig. 1. Well schematic.

a one-way valve which prevents back flow from the annulus into the drill string.)

B. Existing Models

The pressure dynamics of the well can be described by a hyperbolic distributed parameter system. To obtain a model that can be simulated in the time domain a discretization must be made. In the original approach to this problem [2] a simple hydraulic model, developed in [3] using a single control volume, was used for controller design. However, in full scale testing, the controller was unable to successfully compensate for the heave disturbance. In [4], it was shown that more control volumes are needed to accurately describe the dynamics in the well, with increasing numbers of control volumes for longer wells. This was further investigated in [5], which also showed the significance of the resonance frequencies of the mud in the annulus. A controller design based on models with several control volumes is presented in [6],[7], and controller design based directly on the distributed model is presented in [8], [9], [10]. All these models only consider the dynamics in the annulus.

Heave induced dynamic loads on deepwater landing

strings is considered in [11], [12], [13]. The significance of the mud column dynamics in the pipe in some scenarios is confirmed in [14] and in experiments [15]. In these models the dynamics are considered in the frequency domain with a focus on the possible occurrence of resonance.

A time domain model to predict the surge and swab pressures in pipe tripping operations is given in [16], [17] (which is a significant extension of the seminal paper [18]), and this model is confirmed in tests in [19]. This model considers not only the dynamics in the annulus but the elasticity of the pipe and dynamics of the mud in the pipe as well. But, in the cases considered for this model, the transient pressure surge is largely determined by the initial acceleration of the pipe before converging to the predicted steady-state response. Unlike this excitation, caused by sudden acceleration of the pipe, the main frequency component of the heave of a drilling vessel is mainly a low frequency persistent excitation. The model in [16] is also too complex to be used for controller design.

Another model to predict the dynamic loading on a pipe suspended in a well is given in [20], [21]. The mud column in the annulus is included in the mathematical derivation but the discussion is based on the assumption that the fluid is incompressible which makes the model unsuitable for the control problem considered in this paper.

C. Contribution

In this paper we present a distributed parameter model of the coupled pipe-annulus dynamics and an exact solution in the frequency domain is given. This model shows how the coupling between the dynamics of the mud column in the annulus, the elastic pipe and the mud in the pipe all give significant contributions to the bottom-hole pressure. The solution is used to show the limitations of attenuating the down-hole pressure fluctuations over a long section in the well using by considering the response when a 'perfect point control' is applied. We also show how the effect of resonances in the well can be removed by using impedance matching and that improved disturbance attenuation is achieved by combining impedance matching with active control.

II. WAVE SPECTRUM AND FLOATER RESPONSE AMPLITUDE OPERATOR

The range of frequencies for which the heave disturbance, in the scenario described in the introduction, can be limited by various circumstances. To show this, we will in this section perform a short case study for the Aker H-6e Semisubmersible.

The Aker H-6e Semisubmersible is designed for drilling operations in the North Sea and can drill and complete wells of up to 10.000 meters in water depths ranging from 70 to 3.000 meters. It is a sixth generation semisubmersible and has one of the best heave response characteristics available. It is capable of performing connections in sea states up to 8 Bft on the Beaufort Scale [22], which in the North Sea can correspond to 7 meters significant wave height with a spectral peak period of 12 seconds. The wave energy spectrum, $S(\omega)$,

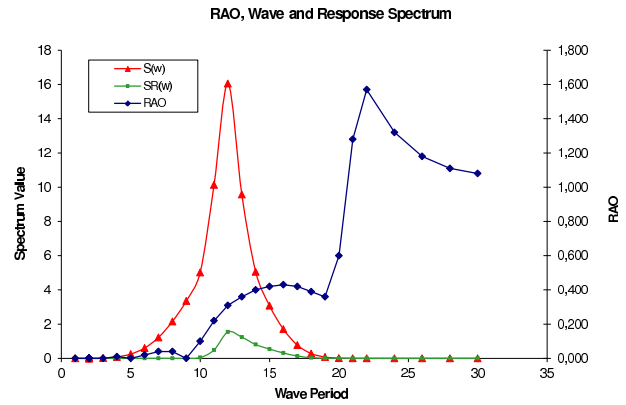


Fig. 2. Response Amplitude Operator (RAO), Wave and reponse energy spectrum of the Aker H-6e Semisubmersible. Data and figure provided by courtesy of Prof. Jan Vidar Aarsnes.

of this sea state is shown in Fig. 2. The energy spectrum of the resulting heave motion (called response spectrum, $SR(\omega)$) is given by $SR(\omega) = RAO(\omega)^2 S(\omega)$. For the Aker H-6e in the given sea state, this results in a significant response amplitude of 0.89 meters, and a zero crossing period of 12.57 seconds.

For sea states with a spectral peak period shorter than 12 seconds, the response amplitude of the Aker H-6e semisubmersible becomes increasingly small. Longer periods typically correspond to worse sea states, during which the Aker H-6e is prevented from drilling due to the limitations of the riser slip joint. Hence, in this case, the considered heave disturbance is limited to frequencies around 0.50 [rad/s] corresponding to the zero crossing period of 12.57 seconds.

III. MATHEMATICAL DERIVATION

Nomenclature:

- c Speed of sound
- E Youngs modulus of pipe
- M_{BHA} Mass of bottom hole assembly
- p pressure, distributed parameter
- s Laplace transform variable
- v velocity, distributed parameter
- β Effective bulk modulus
- ρ Density of drilling mud
- K BHA damping constant
- k Linear viscous friction coefficient

Subscripts:

- a Property relating to annulus
- p Property relating to pipe
- i Property relating to mud in pipe

Superscripts:

- L Boundary at $x = L$
- 0 Boundary at $x = 0$
- $'$ Differentiation w.r.t. x

A. Solution of Coupled System Dynamics in the Frequency Domain

The following system of linear Partial Differential Equations (PDE) is derived by the first principle relations of continuity and momentum balance. All velocity and pressure variables are functions of position, x , and time, t . However, the Laplace transform w.r.t. time has been applied so that the PDE system is transformed into a system of Ordinary Differential Equations (ODEs) with x as the independent variable and the Laplace variable s as a parameter. The system describes velocity and pressure dynamics of the mud in annulus, mud in pipe and the pipe itself, and is given by the following set of ODEs

Mud in Annulus:

$$p_a s + \beta_a \frac{\partial v_a}{\partial x} = 0, \quad (1a)$$

$$v_a s + \frac{\partial p_a}{\partial x} \frac{1}{\rho_a} + k_a v_a - k_d v_p = 0. \quad (1b)$$

Pipe:

$$p_p s + E \frac{\partial v_p}{\partial x} = 0, \quad (2a)$$

$$v_p s + \frac{\partial p_p}{\partial x} \frac{1}{\rho_p} + k_p (v_p - v_a) + k_i (v_p - v_i) = 0. \quad (2b)$$

Mud in Pipe:

$$p_i s + \beta_i \frac{\partial v_i}{\partial x} = 0, \quad (3a)$$

$$v_i s + \frac{\partial p_i}{\partial x} \frac{1}{\rho_a} + k_i (v_i - v_p) = 0. \quad (3b)$$

Here k_a, k_p, k_i, k_d are coefficients accounting for the viscous drag of the mud. Equations (1a,2a,3a) are derived based on continuity and compressibility, and (1b,2b,3b) are derived from momentum balances. Notice that each continuity/momentum balance pair can be combined into one second-order hyperbolic PDE, to obtain the form

$$v_j s^2 - \frac{\partial^2 v_j}{\partial x^2} c_j^2 + v_j \bar{k}_j s = f_j, \quad j = a, i, p \quad (4)$$

$$c_{a,i} = \sqrt{\frac{\beta_{a,i}}{\rho_a}}, c_p = \sqrt{\frac{E}{\rho_p}}, \quad (5)$$

which is the one dimensional wave equation with linear damping and a source term, f , due to the coupling with the other equations. Linear damping in a fluid transmission line is an acceptable approximation over a limited frequency range when using a friction coefficient fitted to the actual response [23], [24]. This approximation should be valid since the disturbance is limited to a short frequency range, as shown in Section II.

Letting the $'$ superscript denote differentiation w.r.t. x , the linear system of ODEs (1a)-(3b) can be written as

$$v_a' = -\frac{1}{\beta_a} s p_a \quad (6a)$$

$$p_a' = -\rho_a (s + k_a) v_a + \rho_a k_d v_p \quad (6b)$$

$$v_p' = -\frac{s}{E} p_p \quad (6c)$$

$$p_p' = -v_p \rho_p (s + k_p + k_i) + k_p \rho_p v_a + k_i \rho_p v_i \quad (6d)$$

$$v_i' = -\frac{1}{\beta_i} s p_i \quad (6e)$$

$$p_i' = -\rho_a (s + k_i) v_i + \rho_a k_i v_p. \quad (6f)$$

Arranging the distributed state variables in the vector \mathbf{y} , we have

$$\mathbf{y}' = \mathbf{A} \mathbf{y}, \quad (7)$$

where \mathbf{A} and \mathbf{y} are given in (8).

1) *General Solution:* The solution of Eq. (7) is given by

$$\mathbf{y}(x) = e^{\mathbf{A}x} \mathbf{C} \quad (9)$$

where the six constants of integration, c_1, \dots, c_6 , arranged in the column vector \mathbf{C} , must be determined by specifying six boundary conditions. The row vectors, $\bar{\mathbf{y}}_i$, $i = 1, \dots, 6$, of the matrix exponential, i.e.

$$\begin{bmatrix} \bar{\mathbf{y}}_1(x) \\ \vdots \\ \bar{\mathbf{y}}_6(x) \end{bmatrix} = e^{\mathbf{A}x}, \quad (10)$$

gives the basis of the solution.

B. Boundary Conditions

In the following we will use the superscripts $0, L$ to denote the $x = 0$ and $x = L$ boundaries of the distributed parameters, respectively, where the bottom-hole is at $x = 0$, and the top of the well at $x = L$. Six boundary conditions must be specified for the system to be fully determined. For a closed annulus they are

$$A_a v_a^L = q_{out}, \quad \text{Flow out of top of annulus} \quad (11a)$$

$$v_p^L = s d, \quad \text{Topside pipe-end movement} \quad (11b)$$

$$v_a^0 \frac{A_a}{A_p + A_i} = -v_p^0, \quad \text{Bottomhole annulus flow} \quad (11c)$$

$$\text{Eq. (12)}, \quad \text{Bottomhole momentum balance} \quad (11d)$$

$$v_i^0 = v_p^0, \quad \text{Pipe is closed at the bottom} \quad (11e)$$

$$v_i^L = v_p^L, \quad \text{Pipe is closed at the top} \quad (11f)$$

Here d is the position of the top of the pipe, which is the exogenous heave disturbance. During connections the pressure in the mud-in-pipe is significantly lower than in the annulus, and the drill bit is fitted with a non return valve, hence there is no flow through the bit and the mud in the bottom of the pipe moves with the same velocity as the pipe-end (11e).

The bottom-hole momentum balance is derived as follows. Assuming that the volume of the well below the bottom hole

$$\mathbf{A} = \begin{bmatrix} 0 & -\frac{1}{\beta_a}s & 0 & 0 & 0 & 0 \\ -\rho_a(s+k_a) & 0 & \rho_a k_d & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{s}{E} & 0 & 0 \\ \rho_p k_p & 0 & -\rho_p(s+k_p+k_i) & 0 & \rho_p k_i & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\beta_i}s \\ 0 & 0 & \rho_a k_i & 0 & -\rho_a(s+k_i) & 0 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} v_a \\ p_a \\ v_p \\ p_p \\ v_i \\ p_i \end{bmatrix} \quad (8)$$

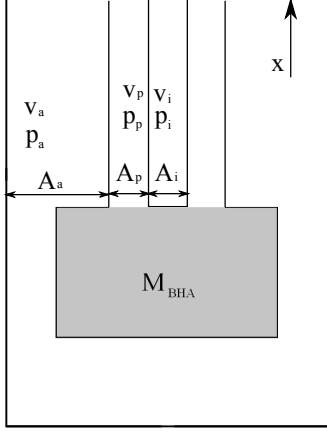


Fig. 3. Schematic showing bottom-hole boundary conditions

assembly (BHA) is small (see Fig. 3) we can model the damping due to flow being squeezed by the BHA as a piston-cylinder type dashpot with the damping constant K . Details on how to derive this damping constant can be found in [25].

The balance of forces acting on the BHA is given by

$$v_p^0 s M_{BHA} = -K v_p^0 + p_a^0 (A_p + A_i) - p_p^0 A_p - p_i^0 A_i. \quad (12)$$

Having specified the boundary conditions by a set of six linearly independent equations, the states in these equations are substituted by their basis vectors, given by (10), and put in a matrix, \mathbf{D} , while the exogenous input is put in the vector \mathbf{b}_j $j = 1, 2$. The constant of integration, \mathbf{C}_j $j = 1, 2$ corresponding to the two input vectors, is found by solving

$$\mathbf{D}\mathbf{C}_j = \mathbf{b}_j, \quad j = 1, 2. \quad (13)$$

\mathbf{D} and \mathbf{b} for the boundary conditions specified in Eq. (11a)-(11f) are given by (14).

where \mathbf{D} and \mathbf{b}_j are given by (14). \mathbf{b}_1 and \mathbf{b}_2 give the transfer function from d and q_{out} , respectively.

C. Procedure

The solution is developed based on a frequency domain approach where the Laplace variable is evaluated along the imaginary axis $s = j\omega$. At each frequency the solution is found by the following procedure:

- 1) Compute the matrix exponential given by Eq. (10) to find the basis of the states.
- 2) Find the constants of integration by solving (13) for \mathbf{C} , where \mathbf{D} and \mathbf{b} are given by the boundary conditions.
- 3) The solution is then given by (9).

IV. CONTROL PROBLEM

When drilling a well, the sections already drilled are fitted with casings and cemented, protecting the surrounding formation against pressure changes in the well-annulus. However, there is a limit to how many times this casing procedure can be performed, since the diameter of the well decreases each time. Hence, we want to attenuate the pressure oscillations over as long a section of the well as possible, so that the number of new casings is kept at a minimum.

Consider the well of Table I, and assume that the section containing the last 500 meters of the well (i.e. $x = 0$ to $x = 500$) has not been cemented. The control problem is to minimize the amplitude of the pressure fluctuations over this section of the well. In managed pressure drilling (MPD) the annulus is sealed topside and flow in and out of the annulus at the top is controlled with a choke and a back-pressure pump. The sum of these two flows are here denoted by the manipulated variable q_{out} .

A. Resonances and Impedance Matching

As the pipe moves down towards the bottom of the well, dragging along drilling mud in the annulus, the mud in the bottom of the well is compressed, creating an increase in pressure. This increase in pressure pushes the mud back upwards in the well, initiating a small flow which carries the overpressure upwards in the annulus in a pressure wave. When the pressure reaches the top of the annulus it will, in the case of a sealed annulus, reflect and start travelling back down. If the time it takes the pressure wave to travel back and forth in the annulus coincides with the period of the pipe movement, the amplitude of the pressure oscillations will increase and we get a resonance in the annulus. If, instead, the top-end of the annulus is a restriction which allows for the amount of mud which creates the overpressure to leave the annulus, there is no reflection and hence there will be no resonance. The same consideration is true for the mud in the pipe. This is achieved when the load impedance (the pressure to flow ratio in the frequency domain at the boundary) is matched to the characteristic line impedance (the pressure to flow ratio in the frequency domain in the transmission line) [26]. Hence this method is called impedance matching. Note that impedance matching does not require active control but can be achieved by designing the topside boundary condition in an appropriate way.

Impedance matching in the annulus is achieved by chang-

$$\mathbf{D} = \begin{bmatrix} \bar{y}_1(L) \\ \bar{y}_3(L)/s \\ \bar{y}_1(0)A_a + \bar{y}_3(0) \\ -s\bar{y}_3(0)M_{BHA} - K\bar{y}_3(0) + \bar{y}_2(0)(A_p + A_i) - \bar{y}_4(0)A_p - \bar{y}_6(0)A_i \\ \bar{y}_5(0) - \bar{y}_3(0) \\ \bar{y}_5(L) - \bar{y}_3(0) \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

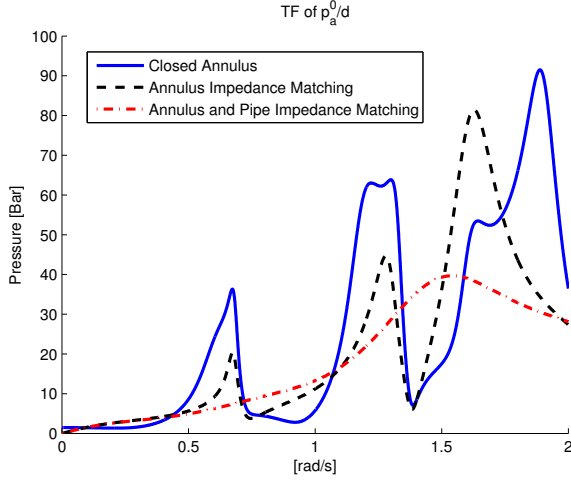


Fig. 4. Deviation of bottom-hole (i.e. $x = 0$) annulus pressure from hydrostatic.

ing the boundary condition in Eq. (11a) with

$$A_a v_a^L = \frac{1}{\sqrt{\rho_a \beta_a}} A_a p_a^L. \quad (15)$$

Similarly, impedance matching can be achieved on the mud in the pipe by changing the boundary condition in Eq. (11f) with

$$A_i v_i^L = \frac{1}{\sqrt{\rho_a \beta_i}} A_i p_i^L. \quad (16)$$

A comparison of the frequency response of the system, with these boundary condition, to the one where the annulus is kept closed can be seen in Fig. 4. Most notably, the resonance at $0.67[\text{rad/s}]$ is significantly reduced by the impedance matching in the annulus and completely removed by combining it with impedance matching in the pipe.

B. Perfect Point Control at $x = x_c$

Since the considered model is linear, we can superposition the effect of the optimal control input with the effect of the heave disturbance. With both the heave disturbance and the controlled inflow as inputs, the pressure in the annulus at $x = x_c$ can be written as

$$p_a(x_c) = h_1(x_c)d + h_2(x_c)q_{out}, \quad (17)$$

where h_1 and h_2 are the irrational transfer functions to the annulus pressure from the disturbance, d , and the controlled input, q_{out} , respectively. h_1 and h_2 are derived by following the procedure in Section III-C and using b_1, b_2 in 13 respectively. Using the boundary control action available through

q_{out} the best possible performance that can be achieved is when the control action completely cancels the effect of the heave disturbance at one point, i.e. when

$$q_{out} = -\frac{h_1(x_c)}{h_2(x_c)}d. \quad (18)$$

Finding a causal and stable approximation of this control law is beyond the scope of this paper. Here we are interested in the in the limitation of the boundary control approach. With this feed-forward control law, the distributed transfer function becomes

$$p_a(x) = \left(h_1(x) - h_2(x) \frac{h_1(x_c)}{h_2(x_c)} \right) d. \quad (19)$$

The result of using this feed-forward control law for $x_c = 250$ can be seen in Fig. 5. Although the disturbance is effectively attenuated over the uncemented section of the well for most frequencies in the excited frequency interval ($0.4 - 0.9[\text{rad/s}]$), there is a problem with the high amplitudes at the frequency $0.67[\text{rad/s}]$. This is due to the resonance in the mud-in-pipe which we saw for this frequency in Fig. 4. To avoid this, the perfect point control can be combined with the impedance matching of the mud-in-pipe discussed in Section IV-A. The result of this combined control approach can be seen in Fig. 6, where the resonance occurring at $0.67 [\text{rad/s}]$ is effectively removed.

Using the transfer function given by Eq. (19) and the H-6e's response spectrum considered in Section II we can also derive properties of the pressure response at $x = 0$. The properties given in Table I is derived based on a Rayleigh distribution of the individual wave heights. A typical connection operation during drilling takes 10 minutes and for some wells the drilling window may be as narrow as ± 2.5 bars [27]. Such stringent constraints would necessitate the use of sophisticated control algorithms, while for other wells a sufficient improvement is achieved by setting an appropriate passive boundary condition such as impedance matching.

REFERENCES

- [1] J. M. Godhavn and A. Pavlov, "Drilling seeking automatic control solutions," in *Proc. 18th IFAC World Congr.*, Milano, Italy, aug 2011, pp. 10 842–10 850.
- [2] A. Pavlov, G. O. Kaasa, and L. Imsland, "Experimental Disturbance Rejection on a Full-Scale Drilling Rig," in *Proc. 8th IFAC Symp. on Nonlinear Control Syst.*, Bologna, Italy, sept 2010.
- [3] G. Kaasa and O. Stamnes, "Intelligent Estimation of Downhole Pressure Using Simplified Hydraulic Model," in *Proc. IADC/SPE Managed Pressure Drilling and Underbalanced Operations Conf. & Exhibition*, Denver, Colorado, April 2011.

TABLE I
BOTTOM-HOLE PRESSURE RESPONSE PROPERTIES.

Property	Closed annulus	Imp.Mat. annulus	FF-Control & closed pipe	FF-Control & Imp.Mat. pipe
Response zero crossing	11.48 [s]	12.18 [s]	11.78 [s]	12.02 [s]
Significant response amplitude	18.48 [bar]	10.51 [bar]	1.40 [bar]	1.13 [bar]
Most prob. max amp., 10 min.	25.99 [bar]	14.68 [bar]	1.96 [bar]	1.59 [bar]
Most prob. max amp., 1 hour	31.33 [bar]	17.74 [bar]	2.36 [bar]	1.91 [bar]

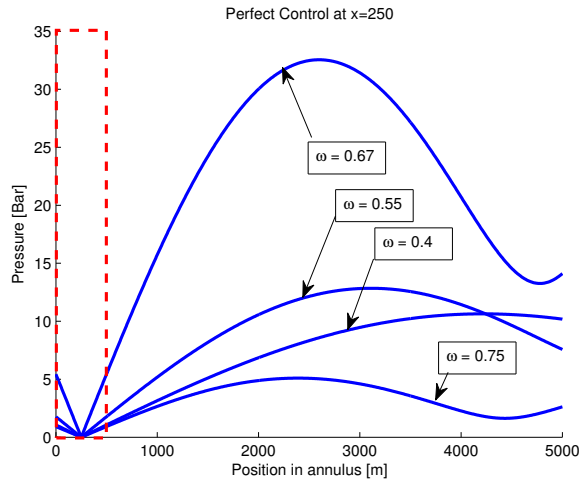


Fig. 5. Distributed pressure with perfect control at $x = 250m$.

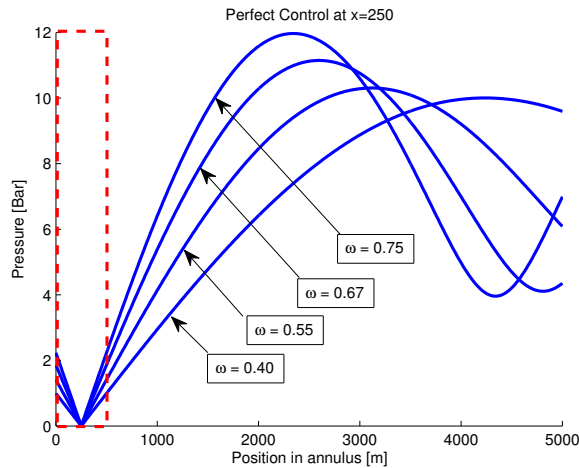


Fig. 6. Distributed pressure with perfect control at $x = 250m$, and impedance matching of mud-in-pipe.

[4] I. S. Landet, H. Mahdianfar, U. J. F. Aarsnes, A. Pavlov, and O. M. Aamo, "Modeling for MPD Operations With Experimental Validation," in *Proc. IADC/SPE Drilling Conf. and Exhibition*, San Diego, California, mar 2012.

[5] U. J. F. Aarsnes, O. M. Aamo, and A. Pavlov, "Quantifying Error Introduced by Finite Order Discretization of a Hydraulic Well Model," in *Proc. Australian Control Conf.*, Sydney, Australia, nov 2012.

[6] I. Landet, A. Pavlov, O. M. Aamo, and H. Mahdianfar, "Control of heave-induced pressure fluctuations in managed pressure drilling," in *Proc. American Control Conf.*, Montreal, Canada, jun 2012, pp. 2270–2275.

[7] I. Landet, A. Pavlov, and O. M. Aamo, "Modeling and control of heave-induced pressure fluctuations in managed pressure drilling," *IEEE Transactions on Control Systems Technology*, to be published.

[8] H. Mahdianfar, O. M. Aamo, and A. Pavlov, "Attenuation of heave-induced pressure oscillations in offshore drilling systems," in *Proc. American Control Conf.*, Montreal, Canada, jun 2012, pp. 4915–4920.

[9] —, "Suppression of Heave-Induced Pressure Fluctuations in MPD," in *Proc. Workshop on Automatic Control in Offshore Oil and Gas Production*, Trondheim, Norway, may–jun 2012, pp. 239–244.

[10] O. M. Aamo, "Disturbance rejection in 2 x 2 linear hyperbolic systems," *IEEE Transactions on Automatic Control*, to be published.

[11] J. Chung and A. Whitney, "Dynamic Vertical Stretching Oscillation of an 18,000-ft Ocean Mining Pipe," in *Proc. 13th Annu. Offshore Technology Conf.*, Texas, Houston, may 1981.

[12] J. Niedzwecki and S. Thampi, "Heave response of long riserless drill strings," *Ocean Eng.*, vol. 15, no. 5, pp. 457–469, 1988.

[13] V. S. Tikhonov and A. I. Safronov, "Effect of fluid column dynamics on longitudinal vibrations of an ultra deepwater riser covered by an orifice plug," *Ocean Eng.*, vol. 29, no. 1, pp. 99–112, Jan. 2002.

[14] J. Miller and R. Young, "Influence of Mud Column Dynamics on Top Tension of Suspended Deepwater Drilling Risers," in *Proc. 17th Annu. Offshore Technology Conf.*, Texas, Houston, may 1985.

[15] Y. Namba, S. Uto, and T. Nimura, "Experiments for Modeling Internal Fluid Effect on Hung-off Rigid Riser Under Axial Motion," *Int. J. of Offshore and Polar Eng.*, vol. 20, no. 1, pp. 7–14, 2010.

[16] R. F. Mitchell, "Dynamic surge/swab pressure predictions," *SPE drilling engineering*, pp. 325–333, September 1988.

[17] R. Mitchell, "Surge Pressures in Low-Clearance Liners," in *Proc. IADC/SPE Drilling Conf.*, Dallas, Texas, Mar. 2004.

[18] J. A. Burkhardt, "Wellbore Pressure Surges Produced by Pipe Movement," *Journal of Petroleum Technology*, vol. Volume 13, no. 6, pp. 595–605, 1961.

[19] R. R. Wagner, A. S. Halal, and M. A. Goodman, "Surge field tests highlight dynamic fluid response," in *Proc. SPE/IADC Drilling Conf.*, Amsterdam, Netherlands, feb 1993.

[20] N. Zheng, J. Baker, and S. Everage, "Further Considerations of Heave-Induced Dynamic Loading on Deepwater Landing Strings," in *Proc. SPE/IADC Drilling Conf.*, Amsterdam, Netherlands, feb 2005.

[21] S. Everage, N. Zheng, and S. Ellis, "Evaluation of heave-induced dynamic loading on deepwater landing strings," *SPE Drilling & Completion*, December 2005.

[22] N. Haug, K. B. Engebretsen, P. Lindstad, and M. Nygård, "Extreme Deepwater Drilling Semisubmersible for Harsh Environments," in *Deep Offshore Technology Int. Conf. and Exhibition*, Perth, Australia, dec 2008.

[23] J. Stecki and D. Davis, "Fluid transmission lines—distributed parameter models part 1: A review of the state of the art," *Proc. Institution of Mech. Engineers, Part A: J. of Power and Energy*, vol. 200, no. 4, pp. 215–228, 1986.

[24] J. S. Stecki and D. C. Davis, "Fluid transmission lines—distributed parameter models Part 2 : comparison of models," *Proc. Institution of Mech. Engineers, Part A: J. of Power and Energy*, vol. 200, no. 4, pp. 229–236, 1986.

[25] S. S. Rao and F. F. Yap, *Mechanical vibrations*, 2nd ed. Addison-Wesley Publishing Company, 1995, pp. 25–29.

[26] O. Egeland and J. T. Gravdahl, *Modeling and simulation for automatic control*. Marine Cybernetics AS, 2002, ch. 4.5.

[27] J. M. Godhavn, "Control requirements for high-end automatic MPD operations," in *SPE/IADC Drilling Conf. and Exhibition*, Amsterdam, Netherlands, mar 2009, pp. 17–19.