

Hybrid Controllers for Mode-Observability of Switching Linear Systems: Existence and genericity

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Abstract—This paper presents recent developments in the study of non-autonomous switching linear systems. For such systems, we address the issue of how to systematically design linear controllers allowing the active process mode to be observable from closed-loop data. The results are stated formally by introducing the notions of *feedback distinguishability* and *discerning controllers*. Both existence and genericity problems are discussed. It is finally shown how a given family of discerning controllers can be implemented as a single hybrid controller which preserves the discerning capability of the original controllers.

I. INTRODUCTION

Recent years have seen intensive research into dynamic systems that are characterised by the interaction of both continuous and discrete dynamics, commonly referred to as hybrid systems [1]. Switching linear systems represent a special class of hybrid systems, namely those systems composed of several linear subsystems (*modes*) and a switching signal that specifies the active subsystem at each instant of time. Switching systems can be classified in many ways: an important distinction is between *autonomous* and *non-autonomous* systems. This paper deals with the latter case, and, in particular, focuses on the feedback configuration of Figure 1. The process to be controlled is assumed to be modeled by a SISO switching linear system. Then, a high-level unit called the supervisor drives a bank of candidate controllers (typically linear time-invariant) and, by monitoring the process input/output data, determines the control signal to be applied at each instant of time.

Feedback configurations of this type are found in a very large number of control applications, and arise when dealing with systems which naturally exhibit multiple operating conditions, as well as when dealing with systems subjects to failures, where different modes are introduced to model possible process faults —[1]-[3] are an excellent source for the interested reader. To cope with these situations, a viable solution is then to design a family of controllers so that, in each configuration, the process performs satisfactorily when controlled by at least one of the candidate controllers.

One of the key issues in re-configurable control is the fact that it is not always possible to stipulate that the process switching signal is available for measurement (in real-time or with delay). In this case, in order to select the control signal

properly, the current process mode must be estimated using the available data. A natural question therefore arises about the possibility of achieving *exact* process mode identification. In the relevant literature, this problem is usually referred to as the *mode-observability* problem [4]-[7]. Most of the control architectures of the type described above base controller selection on the idea of *certainty equivalence*, which amounts to applying at each instant of time the controller designed for the model that best fits the available data [3], [8]. Mode-observability has therefore clear implications on certainty-equivalence-based control, since it addresses the question of whether or not a certain process mode can be identified from the measured data.

While several results on mode-observability have been reported for autonomous systems, the question of mode-observability for non-autonomous systems is still largely unexplored and, in fact, more subtle. Specifically, the question of mode-observability for non-autonomous systems has been studied mainly with the purpose of delineating the theoretical limits of what may be achieved when the control action is allowed to be *arbitrary* [6], [7], [9], [10]. Standard notions of *controlled-discernibility* and *discerning controls*, in fact, aim at establishing the conditions that the process must obey for the existence of *any* control action that solves the mode-observability problem (including open-loop strategies), as well as the constraints that *any* control action must obey for allowing process mode-observability. This analysis is therefore mainly oriented toward establishing connections between input selection and mode-observability, in close analogy with the problem of input selection for parameter identification [11]. When objectives other than mode-observability are present, the situation is, however, substantially different. It may be neither desirable, nor even possible, to use control signals which ensure mode-observability but are otherwise arbitrary. As an example, a “probing” signal which is injected into the plant as an additive perturbation input, superimposed to the control variable, may destroy desired regulation properties. Also, it is not clear whether (and, in the affirmative case, how) it is possible to achieve mode-observability with relatively simple controllers, such as linear time-invariant controllers. Hence, it is not clear whether it is possible to achieve mode-observability within simple (but relevant) feedback control architectures such as the one of Figure 1.

Motivated by these facts, the aim of this paper is to address the issue of mode-observability for feedback control architectures, considering the case where the control action is given by: either a single linear time-invariant controller; or

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a multi-controller of the type shown in Figure 1. The paper is organized into three parts. In Section II, we introduce a notion of mode-observability under feedback (which we call *feedback-distinguishability*), which is related to the open-loop conditions that the process modes must obey for the existence of a linear time-invariant controller of a given order that allows process mode identification. Also, we introduce a notion of *discerning controller*, which is related to the condition that a linear time-invariant controller of a given order must obey in order to solve the mode-observability problem given a family of feedback-distinguishable process modes. In Section III, we address both the questions of *existence* and *genericity* of discerning controllers. We first derive necessary and sufficient condition for feedback-distinguishability. Then, we prove that, under feedback-distinguishability, a discerning controller always exists generically for any given controller order, where by “generically” we mean that the controller turns out to be discerning for almost all choices of its parameters. A geometric characterization of all discerning controllers is also given, which allows a simple interpretation of mode-observability in terms of location of closed-loop poles. Finally, Section IV shows how a given family of discerning controllers can be implemented, rather than in the multi-controller configuration of Figure 1, as a single *hybrid* controller which preserves the discerning capability of the original controllers. For an application of the results to adaptive stabilization of switching linear systems, see [12], [13], [14].

II. PROBLEM DEFINITION

Consider a process described by the switching linear system

$$\begin{cases} \dot{x} = A_\rho x + b_\rho u \\ y = c_\rho x \end{cases} \quad (1)$$

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}$ is the input, $y \in \mathbb{R}$ is the output and $\rho : \mathbb{R}_+ \mapsto \mathcal{N} := \{1, 2, \dots, N\}$ is the switching signal, *i.e.* the signal (right continuous) which identifies the index of the active system at each instant of time, assumed to be *unknown*. A_i , b_i , and c_i , $i \in \mathcal{N}$, are constant matrices of appropriate dimensions. In the sequel, we shall denote by \mathcal{P}_i the linear time-invariant (LTI) system with state-space realization $\{A_i, b_i, c_i\}$.

The scenario under consideration is as follows: suppose that it is desired to design a family of LTI controllers

$$\mathcal{C} \triangleq \{C_j, j = 1, 2, \dots, M\} \quad (2)$$

supervised by a high-level unit whose task is to monitor the process input/output data, and possibly switch between the candidate controllers. Such a control architecture can be represented as in Figure 1, where, at each instant of time, the control signal applied to the process is $u = u_\sigma$, where $\sigma : \mathbb{R}_+ \mapsto \mathcal{M} := \{1, 2, \dots, M\}$. Notice that M can be different from N . In particular, $M < N$ accounts for the case where a single controller takes care of multiple process modes. An extreme case is when $M = 1$. In this

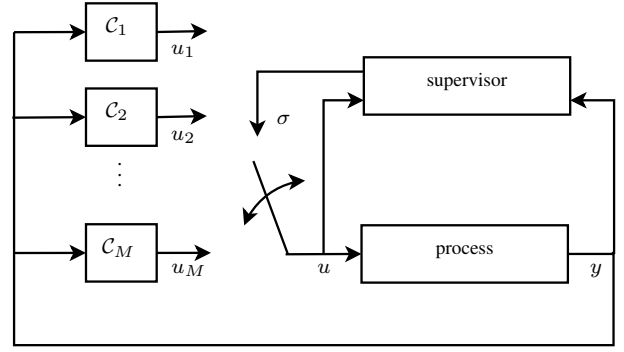


Fig. 1. Feedback loop with a multi-controller.

situation, no controller switching takes place. This case is of relevance in fault-tolerant control schemes where a single (robust) stabilizing controller is designed for one nominal process mode plus $N - 1$ faulty modes, and the supervisor simply acts as a fault detection/isolation device.

A. Feedback-distinguishability and discerning controllers

We now introduce some definitions relevant for the discussion to follow. Consider a family \mathcal{C} of LTI controllers and let $\{F_j, g_j, h_j, k_j\}$ be a state-space realization of controller C_j . According to Figure 1, the dynamics of the j -th candidate controller can be expressed as

$$\begin{cases} \dot{q}_j = F_j q_j + g_j y \\ u_j = h_j q_j + k_j y \end{cases} \quad (3)$$

where $q \in \mathbb{R}^{n_{qj}}$; F_j , g_j , h_j , and k_j are constant matrices of appropriate dimensions. By the notation $\{F_j, g_j, h_j, k_j\}$, it is implicitly assumed that the j -th element of \mathcal{C} is a dynamic system (*i.e.* a system with memory). For clarity of exposition, we shall suppose that this is the situation. The case of purely algebraic controllers is addressed in the Appendix.

Define

$$\Phi_{i/j} \triangleq \begin{bmatrix} A_i + b_i k_j c_i & b_i h_j \\ g_j c_i & F_j \end{bmatrix}, \quad \Delta_{i/j} \triangleq \begin{bmatrix} k_j c_i & h_j \\ c_i & 0 \end{bmatrix}$$

where $i \in \mathcal{N}$ and $j \in \mathcal{M}$. Given a pair (t_0, t) of non-negative reals with $t > t_0$ and a vector $w_0 \in \mathbb{R}^{n_x + n_{qj}}$, define

$$z_{i/j}(t, t_0, w_0) \triangleq \Delta_{i/j} e^{\Phi_{i/j}(t-t_0)} w_0 \quad (4)$$

Definition 1: Consider a process as in (1). Two process modes $i, \ell \in \mathcal{N}$ are said to be *feedback-distinguishable* if there exists a controller C_j of the form (3) such that ¹

$$z_{i/j}(\cdot, t_0, w_0) - z_{\ell/j}(\cdot, t_0, w'_0) \neq 0 \quad \text{a.e. on } \mathcal{I} \quad (5)$$

for all $\mathcal{I} := [t_0, t_0 + T)$ with $T > 0$ and all pairs (w_0, w'_0) with either $w_0 \neq 0$ or $w'_0 \neq 0$. \square

Definition 2: A controller C_j of the form (3) satisfying condition (5) is said to be (i, ℓ) -discerning. Furthermore, C_j

¹By “a.e.” we mean “almost everywhere”, *i.e.* everywhere except on a set of Lebesgue measure zero.

is said to be *globally discerning* if it satisfies condition (5) for all pairs (i, ℓ) of process modes with $i \neq \ell$. \square

One easily recognizes that if $\rho(t) = i$ for all $t \in \mathcal{I}$, and $\sigma(t) = j$ for all $t \in \mathcal{I}$, then

$$z_{i/j} \left(t, t_0, \begin{bmatrix} x(t_0) \\ q_j(t_0) \end{bmatrix} \right) = \begin{bmatrix} u(t) \\ y(t) \end{bmatrix} \quad (6)$$

for all $t \in \mathcal{I}$. Thus, the rationale behind Definition 1 is that if two process modes $i, \ell \in \mathcal{N}$ are feedback-distinguishable and \mathcal{C}_j is (i, ℓ) -discerning, then i and ℓ cannot give rise to the same observation data when \mathcal{C}_j is in the feedback loop. In turns, this opens up the possibility of identifying the active mode of the process from the measured data. This is the reason by which in the literature on switching linear systems distinguishability is also referred to as *mode-observability*, so as to indicate the possibility of reconstructing ρ by observing u and y [4], [7]. It is worth noting that the case where $w_0 = 0$ and $w'_0 = 0$ is explicitly excluded from Definition 1. This is because linear systems are never distinguishable when no motion occurs. Thus, we single out such a non-interesting case by considering non-zero trajectories only.

Definition 2 is (somehow) dual to Definition 1 as it addresses the question of mode-observability from the point of view of the controller. It is convenient to consider both these standpoints since they complement each other. Specifically, Definition 1 is related to the open-loop conditions that a given pair (i, ℓ) of process modes must obey for the *existence* of any (i, ℓ) -discerning controller; Definition 2 is instead related to the conditions that a controller \mathcal{C}_j must obey in order to satisfy condition (5) given any pair (i, ℓ) of feedback-distinguishable process modes.

III. MAIN RESULTS

In this section, we provide necessary and sufficient conditions for any pair (i, ℓ) of process modes to be feedback-distinguishable. We also provide necessary and sufficient conditions for an LTI controller \mathcal{C}_j to be (i, ℓ) -discerning given any pair (i, ℓ) of feedback-distinguishable process modes. We finally show that, under feedback-distinguishability, \mathcal{C}_j turns out to be (i, ℓ) -discerning for almost all choices of the controller parameters.

As a first step toward this end, some notation is needed. Let $\mathbb{R}[s]$ denote the ring of polynomials with reals coefficients. Given $a(s), b(s) \in \mathbb{R}[s]$ we denote by $\gcd\{a(s), b(s)\}$ the greatest common divisor of $a(s)$ and $b(s)$. To avoid heavy notation, given a signal u and a polynomial $a(s)$, by the notation $a(s)u$ we mean the action the differential operator polynomial $a(s)|_{s=d/dt}$ on u [15].

Consider the realization $\{A_i, b_i, c_i\}$ of the i -th process mode, and let

$$\mathcal{P}_i(s) \triangleq \frac{Q_i(s)}{U_i(s)}, \quad \det U_i(s) = \det(sI - A_i) \quad (7)$$

be the corresponding non-reduced (*i.e.* before possible pole-zero cancellations) transfer function. Likewise, consider the

realization $\{F_j, g_j, h_j, k_j\}$ of the controller \mathcal{C}_j , and let

$$\mathcal{C}_j(s) \triangleq \frac{S_j(s)}{R_j(s)}, \quad \det R_j(s) = \det(sI - F_j) \quad (8)$$

be the corresponding (non-reduced) transfer function.

Finally, define

$$\Theta_{i,\ell/j}(s) \triangleq \begin{bmatrix} -Q_i(s) & U_i(s) \\ -Q_\ell(s) & U_\ell(s) \\ -R_j(s) & S_j(s) \end{bmatrix}. \quad (9)$$

Next two results provide necessary and sufficient conditions for a given controller \mathcal{C}_j to be (i, ℓ) -discerning given any pair (i, ℓ) of process modes. We consider both the cases where $\{F_j, g_j, h_j, k_j\}$ is non-minimal ($\mathcal{C}_j(s)$ is reducible) and minimal ($\mathcal{C}_j(s)$ is irreducible). The reason for considering also non-minimal controller realizations is because this is what is required when the multi-controller of Figure 1 is implemented as a single hybrid dynamical system (Section IV).

Theorem 1: (Non-minimal controller realizations) Consider two different process modes $i, \ell \in \mathcal{N}$ with realizations $\{A_i, b_i, c_i\}$ and $\{A_\ell, b_\ell, c_\ell\}$, respectively. Also consider an n_{q_j} -dimensional controller \mathcal{C}_j of the form (3) and suppose that $\{F_j, g_j, h_j, k_j\}$ is non-minimal. Then \mathcal{C}_j is (i, ℓ) -discerning if and only if the following three conditions hold:

- i) (c_i, A_i) and (c_ℓ, A_ℓ) are observable;
- ii) (h_j, F_j) is observable;
- iii) $\text{rank } \Theta_{i,\ell/j}(s) = 2$ for all $s \in \mathbb{C}$.

For minimal controller realizations, a simplified variant of Theorem 1 is as follows.

Let

$$\varphi_{i/j}(s) \triangleq \det(sI - \Phi_{i/j}) = \det \begin{bmatrix} -Q_i(s) & U_i(s) \\ -R_j(s) & S_j(s) \end{bmatrix} \quad (10)$$

be the characteristic polynomial of $\Phi_{i/j}$.

Theorem 2: (Minimal controller realizations) Consider two different process modes $i, \ell \in \mathcal{N}$ with realizations $\{A_i, b_i, c_i\}$ and $\{A_\ell, b_\ell, c_\ell\}$, respectively. Also consider an n_{q_j} -dimensional controller \mathcal{C}_j of the form (3) and suppose that $\{F_j, g_j, h_j, k_j\}$ is minimal. Then \mathcal{C}_j is (i, ℓ) -discerning if and only if the following two conditions hold:

- i) (c_i, A_i) and (c_ℓ, A_ℓ) are observable;
- iv) $\gcd\{\varphi_{i/j}(s), \varphi_{\ell/j}(s)\} = 1$.

Remark 1: One sees from Theorem 1 and 2 that observability of (c_i, A_i) , (c_ℓ, A_ℓ) and (h_j, F_j) is a necessary prerequisite for the modes i and ℓ to be feedback-distinguishable as well as for \mathcal{C}_j to be (i, ℓ) -discerning. This means in particular that only not controllable realizations $\{F_j, g_j, h_j, k_j\}$ are allowed in order for \mathcal{C}_j to be (i, ℓ) -discerning. The importance of observable controller realizations for feedback-distinguishability is not surprising, and arises as a natural generalization of condition i) for distinguishability of autonomous linear systems [4]. \square

A. Existence and genericity of discerning controllers

We now exploit both Theorem 1 and 2 to analyze existence and genericity of discerning controllers. As a first step, we address the existence part, *i.e.* necessary and sufficient conditions for feedback-distinguishability.

Let

$$\text{adj}(sI - A_i) b_i =: [q_{i1}(s) \quad q_{i2}(s) \quad \cdots \quad q_{in_x}(s)]^\top$$

Theorem 3: Consider two different process modes $i, \ell \in \mathcal{N}$ with realizations $\{A_i, b_i, c_i\}$ and $\{A_\ell, b_\ell, c_\ell\}$, respectively. They are feedback-distinguishable if and only if the following three conditions hold:

- i) (c_i, A_i) and (c_ℓ, A_ℓ) are observable;
- v) $Q_i(s) U_\ell(s) \neq Q_\ell(s) U_i(s)$;
- vi) $\gcd_k \{U_i(s), U_\ell(s), q_{ik}(s), q_{\ell k}(s)\} = 1$.

Remark 2: Notice that condition v) amounts to saying that the (reduced) transfer functions of the i -th and ℓ -th process mode are different. As for vi), recall that a realization $\{A_i, b_i, \cdot, \cdot\}$ of a single-input process is controllable if and only if $\gcd_k \{U_i(s), q_{ik}(s)\} = 1$. Therefore, condition vi) amounts to saying that the i -th and ℓ -th process mode do not have common uncontrollable dynamics. \square

Establishing that a certain discerning controller exists when conditions i), v) and vi) hold would be of modest practical interest, since such a controller might be unable to satisfy desired control objectives (*e.g.* stabilize the process for a number of process configurations). It turns out that one can actually prove something more than simple existence. In fact, it is possible to show that, under i), v) and vi), a controller of a given (but arbitrary) order is discerning for almost all values of its parameters. The basic idea is as follows. Consider an n_{q_j} -dimensional controller \mathcal{C}_j of the form (3) with realization $\{F_j, g_j, h_j, k_j\}$. Let $\mathcal{C}_j(s) = S_j(s)/R_j(s)$ denote its corresponding (non-reduced) transfer function, where

$$S_j(s) = \sum_{n=0}^{n_{q_j}} \alpha_{jn} s^n \quad (11)$$

and

$$R_j(s) = s^{n_{q_j}} + \sum_{n=0}^{n_{q_j}-1} \beta_{jn} s^n \quad (12)$$

with $\det R_j(s) = \det (sI - F_j)$.

Definition 3: A subset Θ of a topological space is said to be generic when the following two conditions hold: for any $\theta \in \Theta$ there exists a neighborhood of θ contained in Θ ; for any $\theta \ni \Theta$ every neighborhood of θ contains an element of Θ . Here, by “neighborhood of θ ” we mean an open set \mathcal{V} containing θ . \square

Notice now that the coefficients of $R_j(s)$ and $S_j(s)$ are polynomial functions of (depend with continuity on)

the entries of $\{F_j, g_j, h_j, k_j\}$. Thus, in order to prove that \mathcal{C}_j is discerning for generic $\{F_j, g_j, h_j, k_j\}$ -entries, one can show that under i), v) and vi) the set of coefficients $\{\alpha_{j0}, \dots, \alpha_{jn_{q_j}}, \beta_{j0}, \dots, \beta_{j(n_{q_j}-1)}\}$ which satisfy either conditions ii) and iii) of Theorem 1 or condition iv) of Theorem 2 is a generic subset of $\mathbb{R}^{2n_{q_j}+1}$. The theorem which follows shows precisely this fact. It shows in particular that for all choices of $\{\alpha_{j0}, \dots, \alpha_{jn_{q_j}}, \beta_{j0}, \dots, \beta_{j(n_{q_j}-1)}\}$ except for a finite union of linear varieties in $\mathbb{R}^{2n_{q_j}+1}$, the following property holds:

$$\gcd\{\varphi_{i/j}(s), \varphi_{\ell/j}(s)\} = 1 \quad (13)$$

(condition iv) of Theorem 2). This basically means that an n_{q_j} -dimensional controller with realization $\{F_j, g_j, h_j, k_j\}$ is generically minimal because (13) implies

$$\gcd\{R_j(s), S_j(s)\} = 1 \quad (14)$$

and, hence, that the resulting controller is generically discerning because of (13), (14) and Theorem 2.

Theorem 4: Consider two different plant modes $i, \ell \in \mathcal{N}$ with realizations $\{A_i, b_i, c_i\}$ and $\{A_\ell, b_\ell, c_\ell\}$, respectively. Suppose that conditions i), v) and vi) of Theorem 3 hold. Consider an n_{q_j} -dimensional controller \mathcal{C}_j of the form (3). Then, the set of entries of $\{F_j, g_j, h_j, k_j\}$ for which the controller is (i, ℓ) -discerning is generic. In particular, the subset of coefficients $\{\alpha_{j0}, \dots, \alpha_{jn_{q_j}}, \beta_{j0}, \dots, \beta_{j(n_{q_j}-1)}\}$ for which neither conditions ii) and iii) of Theorem 1 nor condition iv) of Theorem 2 hold is at most a finite union of linear varieties in $\mathbb{R}^{2n_{q_j}+1}$.

Remark 3: If all the pairs of process modes are feedback-distinguishable, then a *globally* discerning controller exists generically. Existence and genericity simply follow from the fact that the number of process modes is finite. \square

IV. HYBRID REALIZATION OF DISCERNING CONTROLLERS

Suppose that a family \mathcal{C} of controller is available and that each element of \mathcal{C} is globally discerning. The control architecture of Figure 1 therefore ensures that at each instant of time it is possible to reconstruct the active mode of the process from the measured data. Such a control architecture, however, is known to exhibit some deficiencies. Implementing each controller as a separate dynamical system raises the question of stability of the *latent* controllers, *i.e.* the controllers which are not in charge of the plant. It is then convenient to implement the multi-controller as a single controller \mathcal{C}_σ with switched parameters (Figure 2),

$$\begin{cases} \dot{q} = \mathfrak{F}_\sigma q + \mathfrak{g}_\sigma y \\ u = \mathfrak{h}_\sigma q + \mathfrak{k}_\sigma y \end{cases} \quad (15)$$

Such a controller is of a *hybrid* type and relies on the idea of “*state sharing*”, as originally introduced in [3].

Let conditions i), v) and vi) hold. Assume that L purely algebraic controllers are available,

$$u_j = k_j y \quad (16)$$

each of them globally discerning (*i.e.* satisfying condition vii) of Theorem 5 for all pairs (i, ℓ) of different process modes). Further assume that $M - L$ dynamic controllers are available, each of them globally discerning (*i.e.* satisfying either condition iii) of Theorem 1 or condition iv) of Theorem 2 for all pairs (i, ℓ) of different process modes). For each of these controllers, let $C_j(s) = S_j(s)/R_j(s) =: \tilde{S}_j(s)/R_j(s) + k_j$, $\deg \tilde{S}_j(s) < \deg R_j(s)$, $R_j(s)$ monic, *i.e.*

$$R_j(s) u_j = \tilde{S}_j(s) y + k_j R_j(s) y \quad (17)$$

Define

$$n_q \triangleq \max_j \deg R_j(s) \quad (18)$$

Thus, we wish to implement this family of controllers as a hybrid system of the form (15) in such a way that each realization $\{\mathfrak{F}_j, \mathfrak{g}_j, \mathfrak{h}_j, \mathfrak{k}_j\}$ has dimension n_q , yields a globally discerning controller and satisfies $\mathfrak{h}_j(sI - \mathfrak{F}_j)^{-1} \mathfrak{g}_j + \mathfrak{k}_j = k_j$ for the purely algebraic controllers and

$$\mathfrak{h}_j(sI - \mathfrak{F}_j)^{-1} \mathfrak{g}_j + \mathfrak{k}_j = \tilde{S}_j(s)/R_j(s) + k_j \quad (19)$$

for the dynamic controllers after cancellation of common poles and zeros in $\mathfrak{h}_j(sI - \mathfrak{F}_j)^{-1} \mathfrak{g}_j + \mathfrak{k}_j$. If the M controllers are not of the same order, then $\{\mathfrak{F}_j, \mathfrak{g}_j, \mathfrak{h}_j, \mathfrak{k}_j\}$ will necessarily be non-minimal. By Theorem 1, $\{\mathfrak{F}_j, \mathfrak{g}_j, \mathfrak{h}_j, \mathfrak{k}_j\}$ must be observable. The idea is then to search for observable, possibly uncontrollable, realizations which satisfy condition iii) of Theorem 1. A simple way for achieving this consists in letting

$$\{\mathfrak{F}_j, \mathfrak{g}_j, \mathfrak{h}_j, \mathfrak{k}_j\} := \{\mathfrak{F}_0 + \alpha_j \mathfrak{h}_0, \beta_j \mathfrak{h}_0, k_j\} \quad (20)$$

where $(\mathfrak{h}_0, \mathfrak{F}_0)$ is a n_q -dimensional parameter-independent observable pair, whereas α_j and β_j are free assignable parameters. We can assume without loss of generality that $(\mathfrak{h}_0, \mathfrak{F}_0)$ has the observer form

$$\mathfrak{F}_0 = \begin{bmatrix} -f_{n_q-1} & 1 & 0 & \cdots & 0 \\ -f_{n_q-2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -f_1 & 0 & 0 & \cdots & 1 \\ -f_0 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad (21)$$

$$\mathfrak{h}_0 = [1 \ 0 \ 0 \ \cdots \ 0] \quad (22)$$

Pick a monic polynomial $\phi_j(s)^2$ with $\deg \phi_j(s) = n_q$ for the purely algebraic controllers and $\deg \phi_j(s) = n_q - \deg R_j(s)$ for the dynamic controllers, and satisfying $\phi_j(s_0) \neq 0$ for all $s_0 \in \mathbb{C}$ such that

$$\begin{bmatrix} -Q_i(s_0) & U_i(s_0) \\ -Q_\ell(s_0) & U_\ell(s_0) \end{bmatrix} \quad (23)$$

has rank 1. Notice that (23) may have rank 1 only at a finite number of values $s_0 \in \mathbb{C}$, otherwise we would contradict v).

²Unstable roots in $\phi_j(s)$ as well as unstable pole-zero cancellations in the transfer functions $C_j(s)$ do not compromise mode-observability. Obviously, they are however necessary for $(\mathfrak{F}_j, \mathfrak{g}_j)$ to be stabilizable, *i.e.* for ensuring boundedness of the state q in (15) when u and y are such.

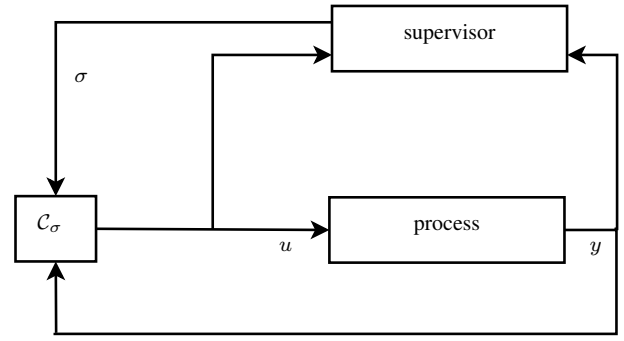


Fig. 2. Feedback loop with a hybrid controller.

Then, the parameters α_j and β_j are defined as the unique solutions of

$$\omega(s) - \mathfrak{h}_0 \text{adj}(sI - \mathfrak{F}_0) \alpha_j = R_j(s) \phi_j(s) \quad (24)$$

$$\mathfrak{h}_0 \text{adj}(sI - \mathfrak{F}_0) \beta_j = \tilde{S}_j(s) \phi_j(s) \quad (25)$$

where $\omega(s) \triangleq \det(sI - \mathfrak{F}_0)$. Existence and uniqueness of α_j and β_j follows simply by noting that

$$\mathfrak{h}_0 \text{adj}(sI - \mathfrak{F}_0) = [s^{n_q-1} \ \cdots \ s \ 1]$$

and that

$$\deg R_j(s) \phi_j(s) - \omega(s) < n_q, \quad \deg \tilde{S}_j(s) \phi_j(s) < n_q$$

It is simple to see that the realization $\{\mathfrak{F}_j, \mathfrak{g}_j, \mathfrak{h}_j, \mathfrak{k}_j\}$ is such that $\mathfrak{h}_j(sI - \mathfrak{F}_j)^{-1} \mathfrak{g}_j + \mathfrak{k}_j = k_j \phi_j(s) / \phi_j(s)$ for the purely algebraic controllers and

$$\mathfrak{h}_j(sI - \mathfrak{F}_j)^{-1} \mathfrak{g}_j + \mathfrak{k}_j = \frac{\tilde{S}_j(s) \phi_j(s)}{R_j(s) \phi_j(s)} + k_j \quad (26)$$

for the dynamic controllers. Also, it satisfies the conditions ii) and iii) of Theorem 1. Specifically, observability of $(\mathfrak{h}_j, \mathfrak{F}_j)$ follows directly from observability of $(\mathfrak{h}_0, \mathfrak{F}_0)$. Moreover, the polynomial matrix $\Theta_{i,\ell/j}(s)$ becomes

$$\Theta_{i,\ell/j}(s) = \begin{bmatrix} -Q_i(s) & U_i(s) \\ -Q_\ell(s) & U_\ell(s) \\ -\phi_j(s) & k_j \phi_j(s) \end{bmatrix} \quad (27)$$

for the purely algebraic controllers, and

$$\Theta_{i,\ell/j}(s) = \begin{bmatrix} -Q_i(s) & U_i(s) \\ -Q_\ell(s) & U_\ell(s) \\ -R_j(s) \phi_j(s) & S_j(s) \phi_j(s) \end{bmatrix} \quad (28)$$

for the dynamic controllers. Thus, in both cases $\Theta_{i,\ell/j}(s)$ has rank 2. In fact, $\text{rank } \Theta_{i,\ell/j}(s) = 2$ when $\phi_j(s) = 0$ since (23) has rank 2 for all s_0 such that $\phi_j(s_0) = 0$; in addition, $\text{rank } \Theta_{i,\ell/j}(s) = 2$ when $\phi_j(s) \neq 0$ because the original controllers are globally discerning by hypothesis.

V. CONCLUSIONS

In this paper we addressed the issue of mode-observability for non-autonomous switching linear systems. We introduced a notion of mode-observability under feedback (called *feedback-distinguishability*), related to the open-loop conditions that the process modes must obey for the existence of a linear time-invariant controller of a given order that allows process mode identification. We also introduced a notion of *discerning controller*, related to the condition that a linear time-invariant controller of a given order must obey in order to solve the mode-observability problem given a family of feedback-distinguishable process modes. We addressed both *existence* and *genericity* problems, and proved that, under feedback-distinguishability, a discerning controller always exists generically for any given controller order. It was finally shown that a given family of discerning controllers can always be implemented as a single *hybrid* dynamical system which preserves the discerning capability of the original controllers.

APPENDIX. PURELY ALGEBRAIC CONTROLLERS

Suppose that one or more candidate controllers are purely algebraic, *i.e.*

$$u_j = k_j y \quad (29)$$

The reason for considering such a case separately, is simply that for purely algebraic controllers there is no question of controller observability whatsoever. Definitions 1 and 2 must be interpreted with respect to the vector-valued sequence $z(\cdot, t_0, w_0)$ obtained by replacing the pair $(\Phi_{i/j}, \Delta_{i/j})$ with the pair $(\tilde{\Phi}_{i/j}, \tilde{\Delta}_{i/j})$,

$$\tilde{\Phi}_{i/j} \triangleq A_i + b_i k_j c_i, \quad \tilde{\Delta}_{i/j} \triangleq \begin{bmatrix} k_j c_i \\ c_i \end{bmatrix}$$

which means $n_{q_j} = 0$. With this in mind, it is simple to verify that the analysis of dynamics controllers carries over to the case of purely algebraic controllers with no extra difficulties. Specifically, by defining

$$\tilde{\varphi}_{i/j}(s) \triangleq \det(sI - \tilde{\Phi}_{i/j}) = \det \begin{bmatrix} -Q_i(s) & U_i(s) \\ -1 & k_j \end{bmatrix} \quad (30)$$

the following results holds.

Theorem 5: (Purely algebraic controllers) Consider two different plant modes $i, \ell \in \mathcal{N}$ with realizations $\{A_i, b_i, c_i\}$ and $\{A_\ell, b_\ell, c_\ell\}$, respectively. Also consider a controller \mathcal{C}_j of the form (29). Then \mathcal{C}_j is (i, ℓ) -discerning if and only if the following two conditions hold:

- i) (c_i, A_i) and (c_ℓ, A_ℓ) are observable;
- vii) $\gcd\{\tilde{\varphi}_{i/j}(s), \tilde{\varphi}_{\ell/j}(s)\} = 1$. □

Also genericity follows along the same lines as the ones for dynamics controllers. It can be obtained from the proof of Theorem 4 by letting $n_{q_j} = 0$. The conclusion is that a purely algebraic controller as in (29) turns out to be discerning for all k_j but a finite union of linear varieties in \mathbb{R} , *i.e.* for all k_j but a finite number of values.

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