2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Formation Control via Quasi-Time Optimal Protocol*

Yury Morozov[1]

Abstract— **The problem of formation control is considered. The dynamics of an agent are described as a Lagrangian system without/with internal dynamic of controler. A quasi-time-optimal decentralized control law is proposed for synchronization of networked agents. Construction of the formation is to be completed in finite time using signum control protocols. During the motion, the formation follows a leader, real or virtual. Every agent determines relative position, velocity and acceleration of its neighbors, links between which are determinated by a time-stable directed tree. Theoretical results are illustrated by numerical examples.**

I. INTRODUCTION

The design of distributed communication and control protocols is an important issue in the construction of networked systems. One typical protocol is the neighbor-based linear consensus protocol [11], [17]. This protocol achieves consensus over an infinite-time horizon with exponential convergence. The finite-time consensus problem was defined in [12]. Also the finite-time consensus tracking control problem was studied in [1], [15]. Chen and Lewis [6] proposed binary control protocols for synchronization of networked Lagrangian systems with/without tracking in a finite time, but these control protocols are not optimal in time or energy and cannot be used for synchronization of networked third-order systems. In this, paper we propose control protocol for formation of networked Lagrangian systems with tracking in a finite time, and each system has an internal controller (single integrator). If dynamics are neglekted, the control protokol is constracted for networked triple integrators.

Recall that Boltyanskiy [2] synthesized time-optimal control law for the second-order integrator in 1969, but Lyapunov function was developed for this case only in 2011 by Polyakov [4]. Matyuhin [5] upgraded this control law. In [1] used modification of Boltyanskiy's control law for mechanical systems.

Lee and Markus [8] designed a time-optimal control for servomechanisms without disturbances and unmodeled dynamics. In [7] a proximate time-optimal control for a third-order servomechanisms was proposed. That control is exactly time-optimal control for a triple integrator plant if control parameters are selected in a special way. Unfortunately, that control is not a finite-time control. The authors showed only that all trajectories lead to the attraction domain (an ellipsoid around the origin) in a finite time. In [18], the implicit Lyapunov function method is developed for construction of a finite-time control for some dynamic systems.

[1]Yu. Morozov, V.A. Trapeznikov Institute of Control Sciences RAS, Moscow, Russia ToT1983@inbox.ru

In this paper, our aim is to provide a quasi-time-optimal control protocols for groups of dynamic systems, such as cars, manipulator robots, third-order servomechanisms, etc.

The format of the paper is as follows. In Section II, we present a quasi-time-optimal control for synchronization of a networked Lagrangian. In Section III, we consider a perturbed triple integrator and define quasi-time-optimal control law for this system. A finite-time tracking control algorithm for a group of third-order integrators is given in Section IV. In section V, examples are presented to illustrate the proposed strategy. Finally, conclusions are summarized in Section VI.

II. PROBLEM STATEMENT

Consider a system consisting of $n+1$ agents, where the agent with the number $n+1$ acts as the leader and the other agents indexed by $\overline{1,n}$, are referred to as the followers. The dynamics of each agent can be described as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + D_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = u_i^l + u_i + \xi_i, \quad (1)$$

where $q_i \in \mathcal{R}^m$ are generalized configuration coordinates, $M_i(q_i) \in \mathcal{R}^{m\times m}$, $M_i(q_i) \succ 0$ is the inertia matrix, symbol $\succ$ denotes positive-definite matrix. $C_i(q_i,\dot{q}_i) \in \mathcal{R}^{m\times m}$ is the Coriolis/centrifugal matrix, $D_i(q_i,\dot{q}_i) \in \mathcal{R}^{m\times m}$ represents the damping force, $g_i(q_i) \in \mathcal{R}^m$ is the vector of gravitational torques (forces), $\xi_i \in \mathcal{R}^m$, $\|\xi_i\| \le \bar{\xi}_i < \infty$ represents input disturbances and system uncertainties, $u_i \in \mathcal{R}^m$ denotes the cooperative control term to be determined, and $v_i \in \mathcal{R}^m$ denotes the local feedback control term given as

$$u_i^l = C_i(q_i,\dot{q}_i)\dot{q}_i + D_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i). \quad (2)$$

The goal of this paper is to determine control inputs $u_i(t)$ for agents $i=\overline{1,n}$ to create a finite-time formation. The latter is defined as follow.

Definition 1: A group of agents is said to a finite-time formation if all agents (leader and followers) are attain the same velocity vector, the desired distances between the agents are stabilized in finite time, and no collisions among them occur.

The topology relationships between the leader and the followers is described by a directed graph $\tilde{\mathcal{G}}(\tilde{\mathcal{V}},\tilde{\mathcal{E}},\tilde{\mathcal{A}}) = \mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A}) \cup \mathcal{G}(\mathcal{V}_b,\mathcal{E}_b,\mathcal{B})$. Nodes v_i of the directed graph $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ are followers and the link between agent i and agent j is represented by the edge (v_i,v_j), link between follower i and the leader is the edge (v_i,v_{n+1}). Here $\tilde{\mathcal{V}} = \{v_1,,v_n,v_{n+1}\}$ is the set of nodes in $\tilde{\mathcal{G}}(\tilde{\mathcal{V}},\tilde{\mathcal{E}},\tilde{\mathcal{A}})$, $\tilde{\mathcal{E}} = \mathcal{E} \cup \mathcal{E}_b \in \mathcal{V}\times\mathcal{V} \cup \mathcal{V}_b \times \mathcal{V}$ is the set of edges and $\tilde{\mathcal{A}} = [\tilde{a}_{ij}]_{ij=1}^{n+1}$ is the adjacency matrix: $\tilde{a}_{ij} = 1$ if $(v_i,v_j) \in \tilde{\mathcal{E}}$

and $\tilde{a}_{ij} = 0$ otherwise. We set $a_{ij} = \tilde{a}_{ij}$, $i = \overline{1,n}$, $j = \overline{1,n}$; $b_i = \tilde{a}_{i,n+1}$, $i = 0, \ldots, n$.

Substituting v_i^l into (1), we obtain the equations:

$$\dot{q}_i = p_i,\ M_i(q_i)\dot{p}_i = u_i + \xi_i,\ i = \overline{1,n}. \quad (3)$$

Assumption 1: Suppose that the dynamics of the leader agent are described as

$$\dot{q}_0 = p_0,\ M_i(q_0)\dot{p}_0 = u_0(q_0, p_0),\ \|u_0\| < \bar{u}_0 \quad (4)$$

and there exits at least one follower i that is connected to the leader, i.e. for at least one i, $b_i > 0$. The Leader has no information about followers, i.e. $\tilde{a}_{n+1,j} = 0$, $j = \overline{1,n}$.

Assumption 2: The position of the leader q_0, its velocity $\dot{q}_0$ and control resource $\bar{u}_0$ are available to its neighbors.

Assumption 3: Let matrix $\mathcal{D} = [\Delta_{ij}]_{ij=1}^{nm}$ denote desired deviations between agents $i = \overline{1,n}$ and the leader. This matrix may vary in the course of time, and, hence, different formations can be formed.

Problem 1: (Construction of finite-time formation) Let $\tilde{\mathcal{G}}(\tilde{\mathcal{V}}, \tilde{\mathcal{E}}, \tilde{\mathcal{A}})$ be a fixed connected graph and $\mathcal{D}$ be a fixed matrix. Determine distributed control laws $u_i(t)$ for all agent i so that, for any initial state, there exists a finite time t^* such that $q_i(t) = q_0(t) - \Delta_i$, $\dot{q}_i(t) = \dot{q}_0(t)$ for all i and $t \geq t^*$.

To solve Problem 1, we apply the pinning control (see, e.g., [16]) $u_i(q_{\mathcal{N}_i}, p_{\mathcal{N}_i}, q_0, p_0, \bar{u}_0)$, where $\mathcal{N}_i$ is the set of neighbors of agent $i = \overline{1,n}$

$$\mathcal{N}_i = \{j : a_{ij} > 0\}. \quad (5)$$

Let us choose the control input as

$$u_i = -M(q_i)\alpha_i\Big(\sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sign}(S_{ij}) + b_i\mathrm{sign}(S_{i0})\Big), \quad (6)$$

where $\alpha_i = \bar{u}_i/l_i$, $l_i = b_i + \sum_{j\in\mathcal{N}_i} a_{ij}$, $S_{ij} = (p_i - p_j) + \beta(q_i + \Delta_i - q_j - \Delta_j)^{[0.5]}$, $\Delta_0 = 0$, $\beta < 2(\bar{\alpha} - \bar{C})$, $\bar{C} = M^*\Xi + \bar{u}_0$, $\bar{\alpha} = \min_{i=1,n}\{\alpha_i l_i\}$, $\bar{u}_i$ is a maximum possible acceleration of the agents.

We have the following finite-time tracking result.

Theorem 1: Consider the leader-follower system (4) and (1) closed by the cooperative control input (6). Let the communication graph $\tilde{\mathcal{G}}(\tilde{\mathcal{V}}, \tilde{\mathcal{E}}, \tilde{\mathcal{A}})$ be a directed tree. Then, agents construct a finite-time formation with the deviations $\mathcal{D}$ if the matrix $M(q_i)$ satisfies the inequality

$$\alpha_i > M^*\Xi + \bar{u}_0, \quad (7)$$

where $M^* = \max_{i=1,n} \|M^{-1}(q_i)\|$.

Proof of Theorem 1: Let $\mathcal{I}_1$ denote the set of nodes that receive information directly from the target system. Let $i \in \mathcal{I}_1$. Thus, the cooperative control input u_i becomes

$$u_i = -M(q_i)\alpha_i b_i \mathrm{sign}(S_{i0}), \quad (8)$$

Let us define the error vectors as

$$z_i^q = q_i + \Delta_i - q_0, z_i^p = p_i - p_0,\ i \in \mathcal{I}_1, \quad (9)$$

Substituting (8) into (31), we have

$$\dddot{z}_i^q = -\alpha_i\mathrm{sign}(S_{i0}) + M(q_i)^{-1}\xi_i - u_0(q_0, p_0). \quad (10)$$

Like in [1], we choose the Lyapunov in the term function

$$V_i(z_i^q, z_i^p) = \begin{cases} w_i^2/4 & , \quad S_i \neq 0 \\ 0 & , \quad S_{i0} = 0. \end{cases} \quad (11)$$

Here,

$$\begin{aligned} & w_i = \chi_i\sqrt{|\varphi_i|} - (z_i^p)^2/\tilde{\gamma}_i, \\ & \tilde{\gamma}_i = -\bar{u}\mathrm{sign}(S_{i0}) + \gamma_i,\ \gamma_i = (M^*\Xi + \bar{u}_0)\mathrm{sign}(S_{i0}), \\ & \varphi_i = z_i^q - (z_i^p)^2/\tilde{\gamma}_i, \chi_i = -\frac{\mathrm{sign}(\varphi_i)}{\tilde{\gamma}_i\sqrt{|1/\beta^2 - \mathrm{sign}(\phi_i)/2\tilde{\gamma}_i|}}. \end{aligned} \quad (12)$$

By Theorem 3 in [4], the systems in $\mathcal{I}_1$ achieve goal, with the convergence time being bounded by $t_0^* = t_r^0 + t_s^0$, where $t_r^0 = \max_{i\in\mathcal{I}_1}(t_r^0)_i$, $(t_r^0)_i = 2\sqrt{V_i(z_i^q(0), z_i^p(0))}$, $t_s^0 = \max_{i\in\mathcal{I}_1}(t_s^0)_i$, $(t_s^0)_i = 2\beta^{-1}\sqrt{z_i^q(0) + z_i^p(0)(t_r^0)_i + \delta_i^t}$, $\delta_i^t = ((t_r^0)_i)^2(M^*\Xi + \bar{u}_0 - \alpha_i)\mathrm{sign}(S_{i0}(z_i^q(0), z_i^p(0)))$. Let $\mathcal{I}_2$ denote the set of nodes that receive information directly from the nodes in $\mathcal{I}_1$. Let $k \in \mathcal{I}_2$. Since accelerations of all agents bounded and finite-time tracking control (8) is global, we have $q_0 = q_i$, $p_0 = p_i$, $i \in \mathcal{I}_1$ after $t > t_0^*$ and

$$u_k = -\bar{u}\mathrm{sign}(S_k), S_k = z_k^p + \beta(z_k^q)^{[0.5]}, k \in \mathcal{I}_2. \quad (13)$$

Repeating the above procedure, we prove the theorem.

Remark 1: The finite-time tracking control (8) is time-optimal control for all agents $i \in \mathcal{I}_1$, if noise ξ_i is the worst, i.e. $\xi_i = \bar{\xi}_i\mathrm{sign}(S_{i0})$. If $\|\xi_i\| < \bar{\xi}_i$, the finite-time tracking control (8) is quasi-time-optimal control for all agents $i \in \mathcal{I}_1$.

III. Quasi time-optimal control of the perturbed triple integrator

In this section, we consider the perturbed triple integrator

$$\dot{q} = p,\ \dot{p} = a,\ \dot{a} = f(q, p, a, t) + g(q, p, a, t)u, \quad (14)$$

where $f(\cdot)$ and $g(\cdot)$ are unknown terms satisfying the following inequalities

$$\begin{array}{lll} \forall(q, p, a, t) & \in \mathcal{R}^3 \times \mathcal{R}_{\geq 0}: & \\ & |f(q, p, a, t)| & \leq F(q, p, a, t), \\ 0 < G_1(q, p, a, t) \leq & |g(q, p, a, t)| & \leq G_2(q, p, a, t), \end{array} \quad (15)$$

$F(q, p, a, t)$, $G_1(q, p, a, t)$ and $G_2(q, p, a, t)$ known functions. The aim is to derive a robust terminal (i.e. finite-time) stabilizer for system (14) based on the idea, similar to that in [7], i.e. that of using the switching surface of quasi time-optimal control as a stable sliding surface for the implementation of a sliding-mode controller.

Theorem 1 Let the switching surface be defined as $S(q, p, a) = q + h(p, a) = 0$ and U be a positive arbitrary constant, where

$$h(p, a) = \frac{a^3}{3U^2} + u_2\Big[\frac{1}{\sqrt{(U)}}\Big(u_2 p + \frac{a^2}{2U}\Big)^{\frac{3}{2}} + \frac{pa}{U}\Big], \quad (16)$$

where

$$u_2(p, a) = \mathrm{sign}\Big(p + \frac{a^2}{2U}\mathrm{sign}(a)\Big). \quad (17)$$

System (14)-(15) is globally stabilized in finite time by the state-feedback control

$$u = -\frac{U + F(\cdot) + \varepsilon^2}{G_1(\cdot)}\mathrm{sign}(S),\ \varepsilon > 0. \tag{18}$$

Remark 2: The control law proposed in [9] is a robust terminal stabilizer for system (14), but it cannot ensure stable sliding mode on a sliding surface, because it uses the switching surface of time-optimal control. To create stable sliding mode, it is required modify the switching surface $S(q,p,a)$ [7]. The authors used in the proof of Theorem 1 [9] results of Filippov [3], but these results are appliable only if functions $F(\cdot)$, $G_1(\cdot)$, $G_2(\cdot)$ are bounded.

In the following, we will need insert constrains on functions $F(\cdot)$, $G_1(\cdot)$, $G_2(\cdot)$:

$$\forall (q,p,a,t) \in \mathcal{R}^3 \times \mathcal{R}_{\geq 0}: \quad \begin{array}{ll} |F(q,p,a,t)| & \leq \bar{F}, \\ G_2(q,p,a,t) & \leq \bar{G}_2, \\ G_1(q,p,a,t) & \geq \bar{G}_1, \end{array} \tag{19}$$

and consider (14) as differential inclusions [3]

$$\forall (q,p,a,t) \in \mathcal{R}^3 \times \mathcal{R}_{\geq 0}: \quad \begin{array}{ll} f(q,p,a,t) & \in [-\bar{F}, \bar{F}] \\ g(q,p,a,t) & \in [\bar{G}_1, \bar{G}_2]. \end{array} \tag{20}$$

According to [8], we can integrate system (14) backward in time, starting from the origin of the state space and using arbitrary constant control. We have two systems:

$$\begin{array}{l} \Gamma_{\pm}(t, u = \mp U_q) = \{U_q(\mp t^3/6, \pm t^2/2, \mp t)^{\mathrm{T}} \in \mathcal{R}^3, t > 0\} \\ \Sigma_{\pm}(t, s, u = \pm U_p, \Gamma_{\pm}) = \{(\pm(U_p s - U_q t), \\ \mp \frac{U_p}{2}s^2 \pm \frac{U_q}{2}(2ts + t^2), \pm\frac{U_p}{6}s^3 \mp \frac{U_q}{6}(3s^2 t + 3st^2 + t^3))^{\mathrm{T}} \\ \in \mathcal{R}^3, t > 0, s > 0\}. \end{array} \tag{21}$$

Eliminating the parameters t and s, it yields the following analytical expressions for $H(\cdot)$ and $\Gamma_U = \Gamma_+ \cup \Gamma_- \cup \{0,0,0\}$

$$\begin{array}{l} H(\cdot) = \frac{a^3}{3U_p^2} + u_2\Big[\Lambda(U_p,U_q)\Big(u_2 p + \frac{a^2}{2U_p}\Big)^{\frac{3}{2}} + \frac{pa}{U_p}\Big], \\ \Lambda(U_p,U_q) = \sqrt{\frac{2}{U_p U_q (U_p + U_q)}}\frac{U_p + 2U_q}{3} > 0, \\ U_q < U_p < 2U_q,\ U_p < U, \end{array} \tag{22}$$

where

$$u_2(p,a,U_q) = \mathrm{sign}\Big(p + \frac{a|a|}{2U_q}\Big). \tag{23}$$

$$\Gamma_U(p,a) = \{(q,p,a) \in \mathcal{R}^3 : q = \frac{a^3}{6U_q}, u_2(p,a,U_q) = 0\}. \tag{24}$$

Note that the function $H(p,a,U_q,U_q)$ is equal to the function $s(\sigma,\dot{\sigma},\ddot{\sigma})$ [10] if $\sigma = q$ and $U_q = U_p = \alpha_r = \bar{G}_1 U - \bar{F}$.

Theorem 2: Let the switching surface be defined as $S_U(q,p,a,U_p,U_q) = q + H(p,a,U_p,U_q)$ and let U be an arbitrary positive constant. System (14), (20) is globally stabilized in finite time by the state-feedback control

$$u = -\frac{U + \bar{F} + \varepsilon^2}{\bar{G}_1}\mathrm{sign}(S_U),\ \varepsilon > 0, \tag{25}$$

and control (25) ensures stable sliding mode on a sliding surface S_U.

Proof of Theorem 2: The proof of this Theorem is similar to the proof of Theorem 1 [9].

Note that the quasi-time-optimal control (25) is time-optimal control for system (14) if $U_q = U_p = U$, $\varepsilon = 0$ and function f and g defind as $f = -\bar{F}\mathrm{sign}(u)$ and $g = \bar{G}_1$.

IV. Finite-time tracking control algorithm for a group of Lagrangian systems with internal dynamics

In this section the dynamics of each agent are described by the Lagrange's equation (1) with local control (2).

Assumption 4: Suppose that $M_i(q_i) = I_i + \Delta M_i$, where the parametric disturbance matrix ΔM_i is bounded

$$\|\Delta M_i\| < \bar{M}_i,\ \dot{M}_i(q_i) = 0. \tag{26}$$

Noise on the right-hand side of (1) is zero; i.e., $\xi_i = 0$. Acceleration u_i of each agent is a solution of equation

$$\dot{u}_i = g_i(q_i,\dot{q}_i,\ddot{q}_i,t)\tau_i + f_i(q_i,\dot{q}_i,\ddot{q}_i,t), \tag{27}$$

where τ_i denotes cooperative control. Functions $f_i(\cdot)$, $g_i(\cdot)$ are bounded:

$$\forall (q_i,\dot{q}_i,\ddot{q}_i,t) \in \mathcal{R}^3 \times \mathcal{R}_{\geq 0}: \quad \begin{array}{ll} f_i(q_i,\dot{q}_i,\ddot{q}_i,t) & \in [-\bar{F}_i, \bar{F}_i] \\ g_i(q_i,\dot{q}_i,\ddot{q}_i,t) & \in [\bar{G}_i, \bar{G}_i]. \end{array} \tag{28}$$

To solve Problem 1 for the new type of agents (1), (2), (27), (28), we apply the pinning control $\tau_i(q_{\mathcal{N}_i}, \dot{q}_{\mathcal{N}_i}, \ddot{q}_{\mathcal{N}_i}, q_0, p_0, \bar{\tau}_0)$, where $\mathcal{N}_i$ was defined in (5). Let us choose the control input as

$$\tau_i = -M(q_i)\pi_i\Big(\sum_{j\in\mathcal{N}_i} a_{ij}\mathrm{sign}(S_U^{ij}) + b_i\mathrm{sign}(S_U^{i0})\Big), \tag{29}$$

where $\pi_i = \bar{\tau}_i/l_i$, $l_i = b_i + \sum_{j\in\mathcal{N}_i} a_{ij}$, $S_U^{ij} = S_U(q_i + \Delta_i - q_j - \Delta_j, \dot{q}_i - \dot{q}_j, u_i - u_j, U_p, U_q)$, $\Delta_0 = 0$, where $\bar{\tau}_i = \frac{U + \bar{F}_i + \varepsilon^2}{\bar{G}_i}$, $\varepsilon > 0$.

We have the following result.

Theorem 3: Consider the leader-follower system (4) and (1), (2), (27), (28) closed by the cooperative control input (29). Let the communication graph $\tilde{\mathcal{G}}(\tilde{\mathcal{V}}, \tilde{\mathcal{E}}, \tilde{\mathcal{A}})$ be a directed tree. Then, the agents construct the finite-time formation with the deviations to define $\mathcal{D}$ if the following inequality hold:

$$\pi_i > \bar{M}^*(\bar{F}_i + \bar{G}_i\bar{\tau}_0), \tag{30}$$

where $\bar{M}^* = \max_{i=1,n}(1 + \bar{M}_i)^{-1}$.

Proof of Theorem 3: The proof of this Theorem is similar to the proof of Theorem 1 if we will set error vectors in the form:

$$z_i^q = q_i + \Delta_i - q_0,\ z_i^p = p_i - p_0,\ z_i^a = a_i - a_0,\ i \in \mathcal{I}_1. \tag{31}$$

V. Example

As an example of a networked Lagrangian system a group of servomechanisms is considered. The group consist of one leader (marked by 5) and four follower.

The communication topology is shown in Fig. 4, where the leader information is available only to follower 1. Suppose that the leader dynamics are $q(t) = q_0\sin(t)$, $p(t) = q_0\cos(t)$, $u_0(t) = -q(t)$, $\tau_0(t) = -p(t)$, $q_0 = \sqrt{2}$. It is

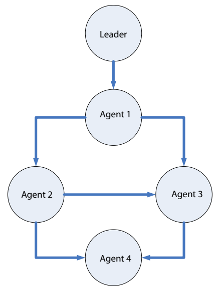

Fig. 1. A leader-follower communication graph

obvious that $\|\tau_0(t)\| \leq q_0$. The control parameter $U = 2+q_0$. Let the initial condition of the four agents be $q_1(0) = 0.3$, $\delta_1 = 0.2$; $q_1(0) = 0.6$, $\delta_1 = 0.5$; Figures 3, 4 and 5 show results of the proposed quasi-time-optimal control algorithm (25). It is seen that the followers can follow the leader in a finite time if the desire deviations are zeros.

VI. CONCLUSIONS

In this paper, we proposed a signum control protocol for formation of networked dynamical systems. Compared with other consensus algorithms, which ignore optimization in time or energy, the proposed algorithm is time-optimal for a pair leader-follower. In the case that there exists a time-varying leader node in the networked Lagrangian systems with internal dynamics, the finite-time tracking control can be achieved by using the signum protocol. By using the networked triple integrators, we illustrated the quasi-time optimal control algorithm derived in this paper.

ACKNOWLEDGMENT

This work was supported by the Department of Power Engineering,

Mechanical Engineering, Mechanics, and Control Process of Russian

Academy of Sciences (Program no. 1 “Scientific foundations of

robotics and mechatronics”).

REFERENCES

[1] Chernous’ko, F.L. et al. Methods of Controlling Non-Linear Mechanical Systems. Moscow. Fizmatlit. 2006

[2] Boltyanskiy, V. G. Mathematical methods of optimal control. Second edition, revised and enlarged. - M.: Nauka, 1969. - 408 P.

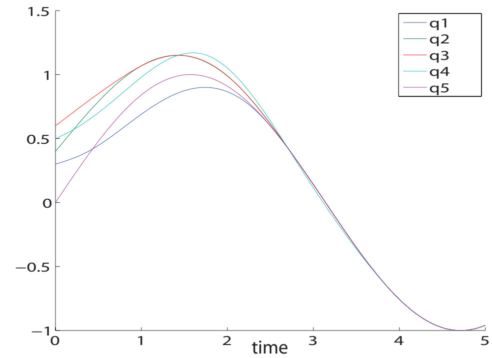

Fig. 2. Position tracking error of each follower

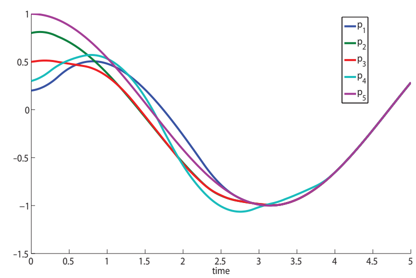

Fig. 3. Velocity tracking error of each follower

[3] Fillipov, A. F. Differential equations with discontinuous right-hand side. M.: Nauka, 1985. - 224 P.

[4] Poznyak, A. S., et al. Analysis of finite-time convergence by the method of Lyapunov functions in systems with second-order sliding modes. // J Appl. Math. Mech., vol. 75, issue 3, 2011. pp. 289P303.

[5] Matyuhin, V. I. Stability of Lagrangian systems in finite time transient // Report of the Academy of Sciences. 1997. vol. 353, no 4. pp. 484P487.

[6] Chen, G., Lewis, F., L.,Distributed Adaptive Tracking Control for Synchronization of Unknown Networked Lagrangian Systems. IEEE Tran. Sys., Man, and Cybernetics, Part B (TSMC) vol. 41, no. 3, 2011. pp. 805P816.

[7] Pao, L. Y., Franklin, G., F.,Proximate Time-Optimal Control of Third-Order Servomechanisms. IEEE Tran. Automatic Control, vol. 38, no. 4, 1993. pp. 560P579.

[8] Lee, E. B. and Markus, L. Foundations of Optimal Control Theory, Wiley. 1967.

[9] Bartolini, G., Pillosu, S., Pisano, A., and Usai, E. Time-optimal stabilization for a third-order integrator: a robust state-feedback imple-

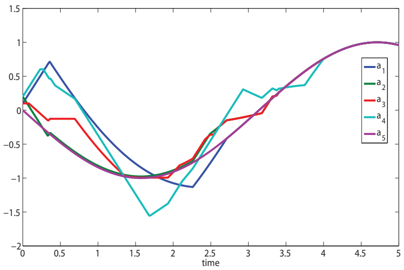

Fig. 4. Acceleration tracking error of each follower

mentation, in Dynamics, Bifurcations and Control, ser. Lecture Notes in Control and Information Sciences, F. Colonius and L. Gruene, Eds. Springer Verlag, 2002, vol. 273, pp. 131P144.

[10] Dinuzzo, F., Ferrara, A. Higher Order Sliding Mode Controllers With Optimal Reaching. IEEE Tran. Automatic Control, vol. 54, no. 9, 2009, pp. 2126P2136 .

[11] Jadbabaie, A., Lin, J., and Morse, A., S. Coordination of groups of mobile autonomous agents using nearest neighbor rules. IEEE Trans. Autom. Control, 2003, vol. 48 no. 6, pp. 988P1001.

[12] Cort's, J. Finite-time convergence gradient flows with applications to network consensus. Automatica, 2006, no. 42, pp1993P2000.

[13] Bingulac, S. P.. on the compatibility of adaptive controllers (Published Conference Proceedings style), in Proc. 4th Annu. Allerton Conf. Circuits and Systems Theory, New York, 1994, pp. 816.

[14] Hui, Q., Haddad, W. M., and Bhat, S. P. Semistability, finite-time stability, differential inclusions, and discontinuous dynamical systems having a continuum of equilibria. IEEE Trans. on Autom. Control, 2009, vol. 54 no. 10, pp. 2465P2470.

[15] Khoo, S., Xie, L. and Man, Z. Robust finite-time consensus tracking algorithm for multirobot systems. IEEE Transactions on Mechatronics, 2009, vol. 14 no. 2, pp. 219P228.

[16] Li, X., Wang, X. F. and Chen, G. R. Pinning a complex dynamical network to its equilibrium. IEEE Transactions on Circuits and Systems, 2004, vol. 51, no. 10, pp. 2074P2087.

[17] Olfati-Saber, R., Murray, R. M. Consensus problems in networks of agents with switching topology and time- delays. IEEE Trans. on Autom. control, vol. 49, no. 9,2004, pp. 1520P1533.

[18] Ananevskii, I. M., Anokhin,N. V. and Ovseevich,A. I. Bounded feedback controls for linear dynamic systems by using common Lyapunov functions. Doklady Mathematics, vol. 82, no. 2, 2010. pp. 831P834