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Dual control approach for zone model predictive control

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Abstract— **Over the last few years, model based predictive controller (MPC) has gained popularity in many industrial fields one of which is the building climate control. In order to work properly, the MPC needs a model of a controlled system which describes the reality as accurate as possible. In practice, the model used by the MPC often becomes inapplicable due to the change of either the operating point or other conditions which leads to control performance degradation. In such a situation, it is inevitable to re-identify the model. However, in a majority of cases, the data available for the re-identification are from the closed-loop and they do not contain enough information for the successful re-identification of the model. In this paper, a dual control algorithm based on the maximization of the smallest eigenvalue of the information matrix increase ensuring both the appropriately informative data and satisfaction of the control performance is presented. As the area of the interest is the building climate control, we offer the dual control algorithm for a specific class of the zone MPC which is widely used in this field.**

I. Introduction

A. Motivation

Use of the MPC together with other modern control methods has become very popular in many industries. One of them, the building climate control, witnessing a rapid improvement of these methods as can be seen from a number of research papers [16], [22], [9], [13], [15], [3]. Except for a number of advantages brought by this strategy, there is a drawback present as well – to work properly, the MPC needs a reliable mathematical model of a controlled system. Acquiring such a model is often a delicate task, especially in the case that a system is in a full operation (i.e. is controlled by some controller). A very usual situation is that the system is already operated by the predictive controller and its model starts to be inconvenient because it does not provide accurate predictions degrading thus the control performance. In such a situation, the model needs to be re-identified. However, the data which are available for identification do not contain enough information as the operating controller tries to keep the system output within the required range. This is the motivation to search for methods which are able to deal with the above-mentioned problems and satisfy control requirements and enable a model re-identification if needed. The problem is treated by so-called dual control (DC) [4] concept, when the control and identification problems are solved in parallel. The problem is, however, analytically unsolvable [21] and numerical algorithms are computationally extensive [25], [7], [5]. In the case of DC MPC, several papers were published dealing with this issue. The most usual way is based on the persistent excitation (PE) condition used as a constraint for the MPC optimization [6], [19]. Subsequently the semi-definite programming is used for solving the whole optimization task. In [11], a solution exploiting the fact that the industrially used MPC works with receding horizon (RH), is presented. This work tries to solve DC MPC by using two separate quadratic programming subroutines. However, both of these approaches consider only the approximation of the PE condition which can lead to a situation when the information is not gathered uniformly from all the directions. In [17], a different approach is presented - it brings a balanced excitation in all directions, nevertheless, the computational demands are high. This algorithm was modified in order to reduce demands by another authors' paper [24]. All the mentioned approaches consider only the most frequently used MPC formulation which (except for the supplied energy) penalizes the deviation of the system output from the required reference trajectory. However, in many industrial applications, the reference tracking (in sense of set point) is not very reasonable. A typical example is the temperature control either in various chemical processes (temperature control in the depropanizer column) or in above-mentioned building climate control area - controlling the zone temperature, it is not necessary to track exact temperature profile and keeping the temperature within certain range is sufficient.

B. Contribution of the paper

The objective of this paper is to formulate the Dual Zone MPC algorithm, which, apart from the fulfillment of the control requirements defined by the cost function (including minimization of the consumed energy and output range satisfaction), would be able to ensure sufficient excitation of the system for the needs of the system re-identification. As the primary goal is the industrial use of the algorithm (building climate control), the next requirement is that it must be a simple extension to the already-implemented MPC without a need for extensive revision and, moreover, the computational complexity must be acceptable even in the case of control of a higher-order system. In this paper, the algorithm fulfilling these requirements is introduced. This algorithm extends the algorithm presented by [24] to the zone MPC class and decreases the computational complexity, as it makes use of the industrial MPC with RH.

C. Organization of the paper

This paper proceeds as follows. Section II introduces the MPC formulation and provides a typical set-up. Section III

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shortly explains the Dual Control MPC problem and provides the formulation of the algorithm based on the maximization of the information matrix increase. In Section IV, the case study is presented where the algorithm properties are examined. Finally, the paper is concluded by Section V.

II. Problem formulation

A. Model under investigation

For purposes of predictive control a simple linear time-invariant (LTI) model is considered throughout the paper. The classical ARX structure will be used [8]

$$y_k = Z_k^T \theta + \epsilon_k, \tag{1}$$

where y_k and u_k are the system output and input sequences and ε_k zero-mean white noise. $\theta = \begin{bmatrix} b_{n_d} \dots b_{n_b} - a_1 \dots - a_{n_a} \end{bmatrix}^T$ and $Z_k = \begin{bmatrix} u_{k-n_d} \dots u_{k-n_b} y_{k-1} \dots y_{k-n_a} \end{bmatrix}^T$ are the vector of parameters and the regressor, respectively. Parameters of structure n_a, n_b, n_k specifies numbers of lagged inputs and outputs, respectively a relative delay of the outputs to the inputs ($n_d = 0$ means a direct input-output connection). The representation (1) is equivalent to the state-space description [2]

$$x_{k+1} = Ax_k + Bu_k + W\varepsilon_k, \tag{2}$$

$$y_k = Cx_k + Du_k + \varepsilon_k, \tag{3}$$

where k is the discrete time, $x \in \mathrm{R}^n$, $u \in \mathrm{R}^m$, $w \in \mathrm{R}^v$, $y \in \mathrm{R}^p$ are system state, input, noise and output vectors and A, B, C, D and W are matrices of appropriate dimensions.

B. Model predictive control

MPC is a strategy which has become very popular in various control branches during the last years due to its ability to handle complex multidimensional systems, capability of incorporating the constraints directly into the cost function, etc. In commonly used MPC formulation, a deviation from the required reference trajectory is penalized [10] as well as the norm of the control effort. However, exact reference tracking is not necessary in many industrial applications and the satisfaction of the certain zone requirement is sufficient. As a very illustrative example, building climate control can be chosen - for thermal comfort satisfaction, the exact reference tracking can be replaced by keeping the zone temperature within a chosen temperature range [22], [14], [9]. Thus, the following cost function penalizing control effort u and low reference value violation can be formulated:

$$\begin{aligned} \min: \quad & J_{MPC,k} = \sum_{i=1}^{P} W_1 \|u_{k+i}\| p + \sum_{i=1}^{P} W_2 \|av_{k+i}\| p \\ \text{s.t.}: \quad & \text{linear dynamics (1)} \\ & u_{k+i}^{min} \leq u_{k+i} \leq u_{k+i}^{max}, \quad i = 1, \dots P \\ & \hat{y}_{k+i|k} \leq y_{k+i}^{min} - av_{k+i} \end{aligned} \tag{4}$$

here y^{min} is the minimal allowed value of output and u^{max} and u_{min} are input constraint. Weighting matrices are denoted as W_1, W_2 and P specifies the prediction horizon. Symbol av represents the auxiliary variables used in order to relax constrains y_{min}, and p denotes the norm for the weighting of the particular term in the cost function.

Afterwards, it is possible to rewrite Eq. (4) to the quadratic programming problem [10]:

$$\min E^T H E + j^T E \tag{5}$$

s.t.

$$\text{linear dynamics (1), and}$$

$$\begin{bmatrix} -I_{2P} \\ I_{2P} \\ \begin{bmatrix} \mathbb{B} & I_P \end{bmatrix} \end{bmatrix} E \leq \begin{bmatrix} U^{min} \\ \mathbf{0} \\ U^{max} \\ AV^{max} \\ \mathbb{A}x - Y^{min} \end{bmatrix} \tag{6}$$

here $E = \begin{bmatrix} \mathbf{U} & AV \end{bmatrix}^T$ is the vector of optimizing variables. The controller designed in that way brings (except of many attractive advantages) one practical complication - its good performance is vastly conditioned by the quality of the model providing the predictions. A situation which occurs frequently when applying the MPC to industrial practice is that the model does not produce sufficiently accurate predictions any more (which can be caused by change of operation points, ambient conditions, re-calibration of sensors, etc.) and its re-identification is inevitable as the inaccurate model would degrade the controller performance. However, as the re-identification experiment is usually costly, only the closed-loop data are available. These data suffer from lack of information (as the existing controller maintains the controlled system within certain operation range) as there is a strong correlation between noise and input. In case that the data are sufficiently excited and the signal-to-noise ratio is high enough for every considered frequency, the correlation between noise and input can be omitted. Therefore it is interesting to investigate the methods which are able to ensure the sufficient excitation even in the case when only the closed-loop data. The possibility of incorporation of the persistent excitation condition directly into the cost function of the controller is discussed in Section III.

C. Persistent excitation

To be able to discuss the possibility of the incorporation of the PE condition into the MPC cost function, we shall introduce the basic notation. Considering the model given by Eq. (1), the increment of information matrix from the time k to the time $k + M$ can be written as:

$$\Delta I_k^{k+M} = \sum_{t=k+1}^{k+M} Z_t Z_t^T. \tag{7}$$

Then, the PE condition can be written in the following form:

$$\Delta I_k^{k+M} \geq \gamma I > \mathbf{0} \tag{8}$$

where γ is the scalar specifying the level of the required excitation, I is the unit matrix of the corresponding dimension.

III. Dual Control

A. Control set-up

By now, a few ways of handling the insufficient excitation for the classical MPC formulation can be found in the literature. The most straight-forward idea is to incorporate the PE requirement (8) directly into the cost function (see e.g. [1], [20]). However, such a straightforward approach brings a lot of disadvantages - e.g. this constraint counts (apart from the inputs) with outputs - which are cumbersome to handle. Therefore, the approximation in the form of $\Delta I_k^{k+M} = \sum_{t=k+1}^{k+m} \psi_t \psi_t^T$ with $\psi_t = \begin{bmatrix} u_{t-n_d} & \cdots & u_{t-n_b} \end{bmatrix}^T$ is used in many cases (see e.g. [18]). This approximation incorporates the PE condition into the MPC cost function via constraints, but (as it does not takes outputs into consideration) it does not ensure sufficient excitation in the output directions [17]. Even when such an approximation is used as a constraint, it is in the form of quadratic inequality which turns the original quadratic programming task into a non-convex problem. Linearization of this condition and a transformation of the non-convex task into a simpler semi-definite programming can be found in [19]. In [12], [11], an approach exploiting the fact that the majority of the industrial MPC application works with RH (at each time step, the whole input sequence is computed but only the first input element is applied to the system) is proposed - the use of the RH simplifies the original task into two quadratic programming tasks. However, not even in this case are the outputs considered in the approximation of the information matrix. An alternative which considers the original PE condition (8) was provided by [17]. This approach consists of a two-step procedure: first, the optimal inputs in the sense of the MPC cost function are found and then, the input sequence maximizing the obtained information is searched for such that the original cost function value is degraded by no more than a chosen ΔJ

$$\begin{aligned} U^* &= \arg\max_U \gamma \\ \text{s.t.: } & \Delta I_k^{k+M} \geq \gamma I, \\ & J_{MPC,k}(U) \leq J_{MPC,k} + \Delta J, \\ & u_{k+i}^{min} \leq u_{k+i} \leq u_{k+i}^{max}, \\ & \Delta u_{k+i}^{min} \leq \Delta u_{k+i} \leq \Delta u_{k+i}^{max}, \end{aligned} \tag{9}$$

This formulation enables use of the original PE condition which contains the outputs as well. Besides that, this approach brings even more advantages. While it is quite difficult to choose such a value of the required excitation γ that both the problem is feasible and the excitation is sufficient (in both of the above mentioned formulations), in the latter case, the tuning parameter is the highest allowed perturbation ΔJ which is much more intuitive. The drawback of this approach is that the resulting problem is non-convex and its solution (especially for higher-dimensional systems) can be computationally demanding. In our previous work [24], the algorithm for DC maximizing the information matrix (MIM4DC) was introduced. This algorithm modifies and significantly decreases the computational complexity of the approach suggested by [17]. Like the mentioned algorithm, the MIM4DC algorithm considers the classical MPC with reference tracking . In Section III-B, an extension of the MIM4DC algorithm to the zone MPC class is provided.

B. MIM4DC for zone MPC

The following extension of MIM4DC to the zone MPC (similarly to the original MIM4DC by [24]) makes use of the MPC with RH. The key idea is as follows: first, the optimal input sequence (in the sense of Eq. (4)) is computed and then, only the first element of the optimized sequence is search such that both the supplied information ΔI is maximized and the original value of cost function $J_{k,ZMPC}$ does not increase by more than a certain allowed ΔJ. This algorithm can be divided into two steps as described in the following.

1) Step I: In the first step, the optimal input sequence $U_{ZMPC}^* = \begin{bmatrix} u_{ZMPC,k} & u_{ZMPC,k+1} & \cdots & u_{ZMPC,k+P} \end{bmatrix}$ minimizing the cost function $J_{ZMPC,k}$ and the corresponding cost function value $J_{ZMPC,k}^* = J_{ZMPC,k}(U_{ZMPC}^*)$ are calculated - they are both then used to determine the constraints for the second step of the algorithm where it must hold that $J_{ZMPC,k}(U) \leq J_{ZMPC}^* + \Delta J$. For the needs of optimization, it is convenient to rewrite the constraint into the form of $U^{min} < U < U^{max}$. As RH is applied, only u_k^* maximizing the supplied information is searched for in the second step while all the other elements of the input sequence are fixed as $u_{k+i}^* = u_{ZMPC,k+i}$ for $i = \{1, \cdots, M-2\}$. Then, the only thing that is left to find is the constraint $u_k^{min} < u_k < u_k^{max}$. While for the classical MPC [23] the constraint can be obtained analytically, the task becomes more complicated for the case of the zone MPC. In this case, $J_{ZMPC,k}(u_k)$ cannot be approximated by a single parabola due to the asymmetry which is caused by the penalization of the first term of Eq. (4) at any time. On the other hand, the zone violation is penalized only when $\hat{y}_{k+i} < y_{k+i}^{min}$. The idea is as follows: the relation will be approximated by two different parabolas - the left parabola $J_{ZMPC,k,L} \approx c_{1l}u_k^2 + c_{2l}u_k + c_{3l}$ for $u_k \in \langle u^{min}, u_b \rangle$ and the right parabola $J_{ZMPC,k,R} \approx c_{1r}u_k^2 + c_{2r}u_k + c_{3r}$ for $u_k \in \langle u_b, u^{max} \rangle$ where the following holds for u_b i

$$\begin{aligned} u_b = max\{u_k : \exists i \ : \hat{y}_{k+i|k} - y_{k+i}^{min} \leq 0\}, \\ i \in \{1, 2, \ldots, P-1\}. \end{aligned} \tag{10}$$

Loosely speaking, the breaking point u_b is such value of u_k when some of the predicted outputs $\hat{y}_k \ldots \hat{y}_{k+P}$ are lower for the first time than the lowest required output value y^{min}. The search algorithm for u_b was detailed in [23]. This algorithm computes the second differences of the cost function with respect to u_k in several chosen points to find its change suggesting the break-point. The interval is being gradually shrunk until the u_b is found with sufficient accuracy. Having found u_b, the unknown coefficients $c = \begin{bmatrix} c_{1l} & c_{2l} & c_{3l} & c_{1r} & c_{2r} & c_{3r} \end{bmatrix}$. This can be accomplished by evaluating several values of $J_{ZMPC,k}$ for various u_k (at least three values for each subparabola) and solving two matrix equations to calculate the coefficients c. Having found the formulas for both left and right subparabolas, such values of u_k can be found that the corresponding criterion values are equal to $J_{ZMPC,k} + \Delta J$ which corresponds to solving equations $c_{1l}u_k^2 + c_{2l}u_k + c_{3l} - (J_{ZMPC,k} + \Delta J) = 0$ and $c_{1r}u_k^2 + c_{2r}u_k + c_{3r} - (J_{ZMPC,k} + \Delta J) = 0$. Now, it is necessary to re-arrange the solutions $u_{la}, u_{lb}, u_{ra}, u_{rb}$ to obtain the constraints

$$\bar{u}_k^{min} = \max(u_{min}, \min(u_{la}, u_{lb})) \tag{11}$$

$$\bar{u}_k^{max} = \min(u_{max}, \max(u_{ra}, u_{rb})). \tag{12}$$

2) Step II: The aim of the second step is to find such a value of u_k^*, $\bar{u}^{min} \le u_k^* \le \bar{u}^{max}$ which brings the biggest amount of information and enables the most accurate identification of the parameter θ of Eq. (1). The question is how to quantify this information - as our objective is to identify all the elements of θ, we are looking for such u_k that maximizes the least eigenvalue of the increase of the information matrix. By maximization of the least eigenvalue of the information matrix, we ensure the excitation even in the worst-direction (corresponding to the parameter which is the most difficult to identify). If the trace or the determinant of the information matrix is maximized instead of the least eigenvalue, it can happen that one of the directions is excited significantly less than others causing problem of identifiability of some of the parameters. To find u_k^*, the following task must be solved:

$$u_k^* = \arg\max_{u_k} \left(\lambda_{\min}\left(\Delta \hat{I}_k^{k+M}\right)\right) \tag{13}$$

$$\text{s.t.:}\quad \bar{u}_k^{min} \le u_k \le \bar{u}_k^{max},$$
$$u_{k+i} = u_{ZMPC,k+i}, \qquad \text{for } i \in \{1, \cdots, M-2\}.$$

It can be noticed that this formulation contains $\Delta\hat{I}$ which is the estimate of the increment of the information matrix created analogously to ΔI, but Z, where the following estimate is used:

$$\hat{Z}_t = \begin{bmatrix} u_{t-n_d} & \cdots & u_{t-n_b} & y_{t-1}^{\alpha} & \cdots & y_{t-n_a}^{\alpha} \end{bmatrix}^T, \tag{14}$$

where

$$y_t^{\alpha} = \begin{cases} \hat{y}_{t|k-1} & \text{if} \quad t > k-1, \\ y_t & \text{if} \quad t \le k-1. \end{cases} \tag{15}$$

The multistep predictions $\hat{y}_{t|k-1}$ can be obtained by reccursive application of $\hat{y}_{k+1} = Z_{k+1}^{\mathrm{T}}\theta$. Now, the optimization task Eq. (13) needs to be solved - this problem is nonconvex in general, but (as it is only a single-dimensional optimization) a straight-forward solution can be proposed here. The admissible interval $\hat{u}^{min}, \hat{u}^{max}$ is sampled with sufficient accuracy (with respect to the accuracy of the sensors) and an exhaustive line search is then performed to find u_k maximizing thus the smallest eigenvalue of the increment of the information matrix ΔI. This search might seem to be time-consuming, nonetheless, the resolution of the sensors is quite limited, therefore the density of the samples is bounded from above – in most cases, this search consists in evaluating only a few values.

IV. CASE STUDY

The aim of this case study is to demonstrate the theoretical properties of the algorithm described above and compare its behavior to the ordinary zone MPC (4) and to discuss different settings of the algorithm. For this purpose, a very simple system with ARX structure has been chosen intentionally - thanks to this, the theoretical properties such as ability of parameter estimation can be discussed. In our another paper [23], this algorithm is tested on the example of two-zone control and its favourable theoretical properties which are shown in the following section are demonstrated.

A. Description

Let us consider the following simple system with single input and single output with the ARX structure (1) with $\theta = \begin{bmatrix} 0.002 & 0.001 & 0.002 & 0.966 & -0.5 & 0.49 \end{bmatrix}^{\mathrm{T}}$ and with the noise variance $\sigma_e = 0.06$. The system is controlled by the zone MPC (5) with constraints (6) and $u^{max} = 20, u^{min} = 0, av^{max} = 2, P = 60$ while y^{min} is generated according to the following rules

$$y_k^{min} = \begin{cases} 13 & 10^3 q{+}1 \le k < 10^3(q{+}1), q \text{ is even} \\ 10 & 10^3 q{+}1 \le k < 10^3(q{+}1), q \text{ is odd.} \end{cases} \tag{16}$$

The weighting terms $100/0.1$ and $100/100000$ have been used for penalization of the violation of the required reference y^{min}. The value before the slash represents a quadratic weighting while the value after it represents a linear weighting. With the MPC tuned in this way, several simulations with duration $N = 15000$ samples were performed. First, the classical zone MPC simulations were run and then, the MIM4DC algorithm with $M \in \{1, 2, \ldots, 10\}$ and $\Delta J \in \{5000, 6000, 8000\}$ was tested. Note that the resulting controllers were used for control of the system while the models identified from the excited data provided the necessary predictions. In order to evaluate the performance of our algorithm and compare it to the classical zone MPC Eq. (5),Eq. (6), multiple factors were examined. First of all, the quality of the data generated by the controllers were examined with respect to the amount of supplied information and the ability to identify a model using these data. To accomplish this, 100 models were identified, each out of 700 samples. Regarding the ability to estimate the parameters, two factors were evaluated: q_e expressing the quality of the estimates of all parameters

$$q_E = (E(\hat{\Theta}) - \theta_0^{\mathrm{T}})S(E(\hat{\Theta}) - \theta_0^{\mathrm{T}})^{\mathrm{T}}, \tag{17}$$

with

$$S = \frac{1}{n-1}(\hat{\Theta} - E(\hat{\Theta}))^{\mathrm{T}}(\hat{\Theta} - E(\hat{\Theta})), \tag{18}$$

being a sample covariance matrix. Here, $\hat{\Theta} = \begin{bmatrix} \hat{\theta}_1 & \ldots & \hat{\theta}_n \end{bmatrix}^{\mathrm{T}}$. θ_i specify the parameters identified from the i-th set of data and n is the number of the identified models. The second factor e_{step} expresses how the step responses of the identified models differ from the real system step response,

$$e_{step} = \frac{1}{n}\sum_{i=1}^{n} \|\hat{Y}_{i,step} - Y_{step}\| \tag{19}$$

where $\hat{Y}_{i,step}$ is the step response of the model identified using the i-th data set and Y_{step} is the real system step response. Besides the re-identification features, an important criterion is how the controller satisfies the requirements imposed by Eq. (5) and Eq. (6). Regarding the control performance requirement satisfaction, two factors are compared. The first of them is e_y^+ representing the average minimal reference violation

$$e_y^+ = \overline{\|\max((Y^{min} - Y), 0)\|} \tag{20}$$

while the second one expresses the increase of the MIM4DC algorithm energy consumption for certain M compared to the classical zone MPC

$$I_E = \frac{\sum u_M^2}{\sum u_{MPC}^2}(\%) \tag{21}$$

B. Results

The comparison for various algorithm settings are given in the tables. Regarding the required performance satisfaction, it is obvious that the classical MPC shows superior performance with respect to the cost function. Both the MPC and the MIM4DC algorithm satisfy the output tracking requirement. Moreover, while the classical MPC sometimes does not satisfy the output requirements (the average error 0.005), this situation does not occur when MIM4DC is employed, which is a direct consequence of the weighting - the zone MPC uses asymmetric weighting (III-B.1) and therefore, in the second step, the MIM4DC algorithm often chooses higher input than one originally computed being thus safely above.

TABLE I

COMPARISION OF RESULTS FOR $\Delta J = 5000$

		e_y^+	$I_E(\%)$	$q_E \times 10^9$	e_{step}	time(s)
MIM4DC	M=1	0.000	10.136	3809.000	0.268	1.380
	M=2	0.000	9.337	1325.000	0.150	1.440
	M=3	0.000	8.169	12.400	0.051	1.404
	M=4	0.000	4.127	1.760	0.030	1.584
	M=5	0.001	1.022	3.365	0.046	1.548
	M=6	0.000	1.360	7.076	0.087	1.807
	M=7	0.000	2.644	4.720	0.069	1.728
	M=8	0.000	1.814	52.000	0.074	1.836
	M=9	0.000	2.351	14.260	0.067	1.836
	M=10	0.000	2.759	8.418	0.070	1.900
MPC		0.005	0.000	10902.000	0.305	1.170

As far as the consumed energy is concerned, it can be observed that MIM4DC is very sensitive to the settings. For the small values of M, its consumption is significantly higher, while its increase for larger M is (in comparison to the MPC) less significant, $1.5-8\%$. This slight consumption increase is only a very little cost to pay for the rapid increase of identifiability thanks to a higher amount of the supplied information. In case of well-tuned algorithm, the

TABLE II

COMPARISION OF RESULTS FOR $\Delta J = 6000$

		e_y^+	$I_E(\%)$	$q_E \times 10^9$	e_{step}	time(s)
MIM4DC	M=1	0.000	13.079	1179.000	0.133	1.476
	M=2	0.000	14.263	410.250	0.082	1.440
	M=3	0.000	11.427	1.405	0.040	1.380
	M=4	0.000	5.279	0.057	0.023	1.548
	M=5	0.002	1.230	8.179	0.030	1.908
	M=6	0.001	2.198	4.468	0.058	2.268
	M=7	0.000	3.281	7.210	0.048	1.728
	M=8	0.000	3.466	4.990	0.065	1.728
	M=9	0.000	3.289	11.360	0.061	1.800
	M=10	0.000	3.558	17.600	0.063	1.980
MPC		0.005	0.000	10902.000	0.305	1.170

TABLE III

COMPARISION OF RESULTS FOR $\Delta J = 8000$

		e_y^+	$I_E(\%)$	$q_E \times 10^9$	e_{step}	time(s)
MIM4DC	M=1	0.000	16.868	361.130	0.108	2.160
	M=2	0.000	18.858	214.700	0.070	2.160
	M=3	0.000	15.815	1.120	0.033	1.980
	M=4	0.000	6.586	0.202	0.022	2.167
	M=5	0.002	1.452	2.475	0.030	2.340
	M=6	0.000	4.665	3.043	0.033	2.556
	M=7	0.000	4.718	2.544	0.033	2.268
	M=8	0.000	7.200	7.062	0.102	2.160
	M=9	0.000	6.317	84.900	0.076	2.340
	M=10	0.000	6.109	5.150	0.046	2.088
MPC		0.005	0.000	10902.000	0.305	1.170

parameter q_E is several times lower (lower value of q_E means estimate with higher accuracy). The step responses of the identified models (data generated by MIM4DC) are indeed very accurate. While the average step response error of the models identified from the data generated by the classical MPC is 0.3, the average error of the MIM4DC models is one order of magnitude smaller. The results summarized in the tables suggest that the choice of M is very important for the MIM4DC performance. With $M = 1$, the performance is poor (large energy consumption increase) as the input maximizing only one-step ahead information matrix increase is found. With this setting, the algorithm does not take into

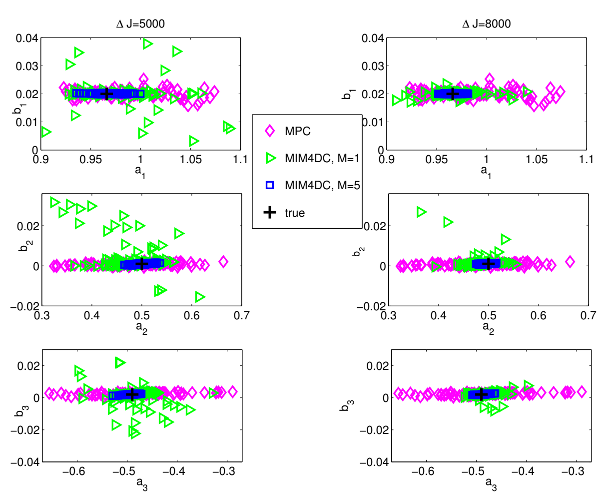

Fig. 1. parameters of identified models

account whether the input change affects the future output which leads to aggressive control effort (causing the energy consumption to increase). Increasing the value of M, the performance improves up to a certain point from which it starts to aggravate. An interesting question can be raised - what is the optimal choice of the M? No doubt, it strongly depends on the application and the controlled system. Yet, in general, it can be said that the appropriate choice of M takes into account the effect of the optimized input on the future outputs within the maximization of the information matrix increment in the second step of the algorithm as described in Section III-B.2 - the excitation in the input directions should cause the excitation in the output directions as well. On the other hand, nor M being too high is the best solution - although the impact of the input on the future outputs is taken into consideration, the impact is predicted by a model with multi-step predictions not being accurate enough and moreover, the fixed inputs u_{k+i} are not actually applied to the system as the RH is used. Another factor to investigate is the time consumption of the algorithm with different settings. The tables list the average duration of one run of the algorithm (the duration includes the time needed to save the data). It is obvious that the duration mostly depends on choice of ΔJ. With higher ΔJ, the computation time rises as more values of u are evaluated and examined. However, even with high ΔJ, one run of the algorithm takes approximately $2\,\text{sec}$. In the industrial applications, the time available for the computation of the input is bounded from above by the input sampling period (the said period makes in the field of zone temperature control approximately tens of minutes), the running time of the algorithm is negligible. The next practical advantage of the MIM4DC algorithm is that this algorithm is fully compatible with the original zone MPC implementation and in case of its use, it is not necessary to change the original implementation in any way.

V. Conclusion

We introduced an algorithm maximizing the information matrix increase for the dual control. In contrast to the other algorithms dealing with parallel identification and control problems using MPC, the presented algorithm is developed specially for the zone MPC which is the favorite industrial approach. The algorithm has very attractive properties: it was able to both obtain data containing enough information for re-identification and to keep the output within the required range at the price of only the negligible energy consumption increase. It appears that the algorithm is an ideal choice for cases when the currently used zone MPC is not working optimally due to the model inaccuracy. In such a case, this algorithm can be used to perform a closed-loop experiment to gather informative data while still satisfying the control quality requirements. The price is increase of energy consumption not more than 5% during the experiment. The original MPC will thus work with major performance improvement.

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