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Observer-based output feedback control of a PEM fuel cell system by high-order sliding mode technique

A. Pisano[1], D. Salimbeni[1], E. Usai[1], S.M. Rakhtala[2], A.R. Noei[2]
1. Department of Electrical and Electronic Engineering (DIEE)
University of Cagliari, Cagliari, Italy.
2. Faculty of electrical and computer engineering, Babol industrial university, Iran.
E-mails: {pisano,dani,eusai}@diee.unica.it sm.rakhtala@gu.ac.ir

***Abstract*— This paper deals with an high-order sliding-mode approach to the observer-based output feedback control of a PEM fuel cell system comprising a compressor, a supply manifold, the fuel-cell stack and the return manifold. The suggested scheme assumes the availability for measurements of readily accessible quantities such as the compressor angular velocity, the load current, and the supply and return manifold pressures. The control task is formulated in term of regulating the oxygen excess ratio (which is estimated by the observer) to a suitable set-point value by using, as adjustable input variable, the compressor supply voltage. The treatment is based on a nonlinear modeling of the PEM fuel cell system under study. Simulations results showing the feasibility and satisfactory performance of the proposed approach are provided.**

Keywords: PEM fuell cell; Nonlinar observers; High order sliding mode.

I. Introduction

Nowadays, Fuel Cells (FCs) technology is considered as a suitable alternative for efficient and environmentally sustainable energy conversion in many applications. However, high cost, low reliability and short lifetime of FCs are still limiting their massive utilization in real applications. Advanced control systems ca be useful to achieve faster dynamic responses, longer lifetime and higher efficiency of energy conversion [10]. In controlling a PEM-FC one of the main problems is to estimate the oxygen excess ratio since its accurate regulation can increase the efficiency significantly [13]. Unfortunately, it depends on the oxygen partial flow in cathode $W_{O_2,in}$ that is an internal unavailable variable of the FC. Since $W_{O_2,in}$ depends on the compressor air mass flow and on the vapor injected by the humidifier, this problem could be circumvented by inferring information on an accessible variable such as the compressor air flow, assuming the humidification at a fixed point [13]. The use of Kalman observer based integral feedback controller has been suggested to improve the management of the oxygen excess ratio during the transients following abrupt changes in the step current [9]. Other types of observers such as Luenberger and adaptive observers have been also considered to estimate the state of a PEM-FC [11].

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In [14], [16], oxygen excess ratio is achieved by controlling the air mass flow W_{cp} delivered by the turbo compressor. This, in fact, allows indirectly controlling $W_{O_2,in}$ (and, therefore, the oxygen excess ratio) once the supply manifold transient expires. The quantity W_{cp} was supposed to be available for measurement in the above work, where no state observation stage was used. Observer based optimal control, based on a linearized PEM-FC model, was suggested in [9].

Sliding mode observers (SMO) do not need linear models and are robust with respect to matched modeling errors and uncertainties as well. Furthermore, they are also simple in structure and can be implemented easily [6]. In [12] a sliding mode observer is designed to estimate the cathode and anode pressures of a PEM-FC system while the other states (i.e., supply manifold pressure, oxygen pressure, hydrogen pressure, return manifold pressure) are estimated by a nonlinear state observer to design a suitable controller. In order to implement the controller, filtering of the state estimates is needed so destroying the finite time convergence property and, consequently, the lack of a separation principle must be taken into account. In [6] authors proposed a higher order sliding mode observer for SISO systems that does not need any transformation since, under the observability condition, it was shown that the output injection can be designed such that the output error is a flat output of the state estimation error dynamics.

In this paper we consider a 6-th order model of the PEM-FC system in which the measured outputs are the compressor angular speed and the supply and return manifold pressures, the load current is a measured disturbance and the compressor motor voltage is the adjustable input. The observer is a replica of the system model in which the three output injection are chosen to guarantee observability in a sufficiently large domain of the state space containing the usual operating conditions and in the presence of stack current variations. Output injections are designed by means of a proper higher order sliding mode algorithm so that the state variable are estimated in a finite time. Taking into account that a kind of separation principle can be stated in the output-feedback control of nonlinear systems by using finite time observers [20], in this paper we design the controller as a state-feedback robust nonlinear controller. In the considered case the relative degree between the oxygen

TABLE I

VARIABLES OF THE NONLINEAR FC MODEL

Variable	Meaning	Unit
x_1	motor angular speed	rad/s
x_2	supply manifold pressure	Pa
x_3	air mass in the supply manifold	kg
x_4	oxygen mass in cathode side	kg
x_5	nitrogen mass in cathode side	kg
x_6	return manifold pressure	Pa
u	motor voltage supply	V
d	stack current	A

excess ratio and the compressor supply voltage is two, but, since the compressor can be considered as a fast actuator with a negligible dynamics, as compared to the typical time constants of the PEM-FC internal variables, a super-twisting sliding mode controller can be implemented as a nonlinear PI regulator. This choice allows us to define a continuous control variable at the cost of a small residual oscillation whose amplitude and frequency can be modulated by introducing linear compensating filters of the lead/lag type [3], [21].

The paper is organized as follows; in Section II a nonlinear model of PEM-FC is introduced. The observer and the controller design are detailed in Sections III and IV, respectively, while in Section V simulation results are given to illustrate the observer/controller performance. Finally some conclusions are resumed in the last Section VI.

II. NONLINEAR DYNAMIC MODEL OF A PEM-FC

We consider a 6-th order nonlinear model of the fuel cell system, obtained after suitable simplification to the 9-th order model proposed by J. Pukrushpan et al. [24], [25] under the assumptions that the anode pressure is constant, the humidity and temperature at the inlet of the FC stack are constant and the electric dynamics of the DC motor driving the compressor can be neglected;

The governing equation for the rotational speed of the compressor is

$$\begin{aligned} J_{cp}\dot{x}_1 &= \tau_{cm} - \tau_{cp} = \eta_{cm}\frac{K_t}{R_{cm}}(u - K_v x_1) \\ &\quad -\frac{C_p}{x_1}\frac{T_{atm}}{\eta_{cp}}\left[\left(\frac{x_2}{P_{atm}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] W_{cp} \end{aligned} \tag{1}$$

where τ_{cm} is the accelerating torque provided by the motor, τ_{cp} is the load torque, the compressor air flowrate W_{cp} depends on the compressor speed and supply manifold pressure according to the approximate static map

$$\begin{aligned} W_{cp} &= B_{00} + B_{10}x_2 + B_{20}x_2^2 + B_{01}x_1 \\ &\quad + B_{11}x_1x_2 + B_{02}x_1^2 \end{aligned} \tag{2}$$

where the values of the B_{ij} constants are $B_{00} = 4.83 \cdot 10^{-5}$, $B_{10} = -5.42 \cdot 10^{-5}$, $B_{20} = 8.79 \cdot 10^{-6}$, $B_{01} = 3.49 \cdot 10^{-7}$, $B_{11} = 3.55 \cdot 10^{-13}$, $B_{02} = -4.11 \cdot 10^{-10}$ (see [15]).

The governing equations for the supply manifold pressure is defined, using the energy conservation principle, as

$$\dot{x}_2 = \frac{\gamma R_a}{V_{sm}} W_{cp} T_{cp} - \frac{\gamma R_a}{V_{sm}} W_{sm,out} T_{sm} \tag{3}$$

where T_{cp} is the air temperature in the compressor, which, using basic thermodynamic relations, can be expressed in the form

$$T_{cp} = T_{atm}\left(1 + \frac{1}{\eta_c}\left[\left(\frac{x_2}{P_{atm}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right), \tag{4}$$

the supply manifold outlet air rate $W_{sm,out}$ is obtained via a linearized nozzle equation as

$$W_{sm,out} = K_{sm,out}(x_2 - p_{ca}) \tag{5}$$

in which the cathode pressure p_{ca} is the sum of three components (oxygen, nitrogen and vapour) according to

$$\begin{aligned} p_{ca} &= p_{O_2,ca} + p_{N_2,ca} + p_{v,ca} = \\ &= \frac{T_{fc}}{V_{ca}}(x_4 R_{O_2} + x_5 R_{N_2} + m_{v,ca} R_v), \end{aligned} \tag{6}$$

and T_{sm}, the air temperature in the supply manifold, depends on x_2 and x_3 according to the ideal gas law

$$T_{sm} = \frac{x_2 V_{sm}}{R_a x_3} \tag{7}$$

The governing equations for the mass of air in the supply manifold, and for the masses of oxygen and nitrogen in the cathode, are defined using the principle of mass conservation as

$$\dot{x}_3 = W_{cp} - W_{sm,out} \tag{8}$$

$$\dot{x}_4 = W_{O_2,in} - W_{O_2,out} - W_{O_2,react} \tag{9}$$

$$\dot{x}_5 = W_{N_2,in} - W_{N_2,out} \tag{10}$$

The oxygen flow rate into the stack from the supply manifold is expressed as

$$W_{O_2,in} = y_{O_2}\frac{1}{1+\Omega_{atm}} W_{sm,out} \tag{11}$$

where

$$y_{O_2} = \frac{y_{O_2,in} M_{O_2}}{y_{O_2,in} M_{O_2} + (1 - y_{O_2,in}) M_{N_2}} \tag{12}$$

is the mass ratio of oxygen in the dry atmospheric air, Ω_{atm} is the humidity ratio of the atmospheric air, and $W_{sm,out}$ was already defined in (5).

The oxygen mass flow rate out of the cathode is

$$\begin{aligned} W_{O_2,out} &= \frac{x_4}{x_4 + x_5 + m_{v,ca}} W_{ca,out} \\ &= \frac{x_4}{x_4 + x_5 + m_{v,ca}} K_{ca,out}(p_{ca}(x_4, x_5) - x_6) \end{aligned} \tag{13}$$

The amount of oxygen consumed in the reaction is $W_{O_2,react} = \frac{nM_{O_2}}{4F} d$. The nitrogen mass flows incoming and outgoing the stack's cathode are defined through similar relations as those just derived for the oxygen, namely

$$W_{O_2,in} = y_{N_2}\frac{1}{1+\Omega_{atm}} W_{sm,out} \tag{14}$$

$$y_{N_2} = \frac{(1-y_{O_2,in})M_{N_2}}{y_{O_2,in}M_{O_2} + (1-y_{O_2,in})M_{N_2}} \tag{15}$$

and

$$\begin{aligned} W_{N_2,out} &= \frac{x_5}{x_4+x_5+m_{v,ca}} W_{ca,out} \\ &= \frac{x_5}{x_4+x_5+m_{v,ca}} K_{ca,out}(p_{ca}(x_4,x_5)-x_6) \end{aligned} \tag{16}$$

The governing equations for the return manifold pressure are defined by means of standard thermodynamic relationships as

$$\dot{x}_6 = \frac{R_a T_{fc}}{M_a V_{rm}}(W_{ca,out} - W_{rm,out}) \tag{17}$$

where $W_{ca,out}$ is given in (13) and $W_{rm,out}$ can be approximated by means of the nonlinear static mapping

$$W_{rm,out} = \sum_{i=0}^{5} P_{a_i} x_6^i \tag{18}$$

whose coefficients values are $P_{a_0} = -1.24 \cdot 10^{-3}$, $P_{a_1} = -1.96 \cdot 10^{-3}$, $P_{a_2} = -1.52 \cdot 10^{-3}$, $P_{a_3} = -2.12 \cdot 10^{-3}$, $P_{a_4} = -27.7 \cdot 10^{-3}$, $P_{a_5} = -78 \cdot 10^{-3}$ (see [15]).

Collecting all the above relations in a unique state-space representation, it yields the compact system

$$\begin{aligned} \dot{x} &= f(x) + g \cdot u + \varphi \cdot d \\ &= \begin{bmatrix} f_1(x_1,x_2) \\ f_2(x_1,x_2,x_3,x_4,x_5) \\ f_3(x_1,x_2,x_4,x_5) \\ f_4(x_2,x_4,x_5,x_6) \\ f_5(x_2,x_4,x_5) \\ f_6(x_2,x_5,x_6) \end{bmatrix} + \begin{bmatrix} g_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u - \begin{bmatrix} 0 \\ 0 \\ 0 \\ \varphi_4 \\ 0 \\ 0 \end{bmatrix} d \end{aligned} \tag{19}$$

$$g_1 = \frac{\eta_{cm} K_t}{J_{cm} R_{cm}}, \quad \varphi_4 = \frac{n M_{O_2}}{4F} \tag{20}$$

III. OBSERVER DESIGN

We assume that the measurable output variables are

$$y = [x_1, x_2, x_6]^T = Cx \tag{21}$$

where the implicitly defined matrix C defines the linear state to output transformation. To design the observer for the PEM-FC (19)-(21) we resort to the approach first described in [5] in which an observer, based on higher-order sliding modes, was proposed for a class of nonlinear system in order to deal with a chaos synchronization problem. In [6] it was extended to a more general class of sufficiently smooth nonlinear uncertain systems with a scalar output, and conditions for its effectiveness under uncertainty were given. The observer is defined as a replica of the system dynamical model (without any state transformation) and the distribution vector of the output injection is defined by means of the inverse of the observability matrix so that the output error is a flat output for the estimation error dynamics. Under some conditions this approach has been recently extended to Multi-Output systems [7] where the approach in [8] for the design of a strong observer for MIMO linear system with unknown inputs was combined with that in [6].

Since the inverse of the observability matrix is needed to design the observer output injection, according to such an approach n independent rows have to be chosen from the $p(n-p+1) \times n$ observability matrix, which for the PEM-FC model (19)-(21) is defined as

$$O(x) = \begin{bmatrix} C \\ C\nabla f(x) \\ C\nabla L_f f(x) \\ C\nabla L_f^2 f(x) \end{bmatrix} \tag{22}$$

where $\nabla f(x) = \frac{\partial f}{\partial x}$ and $L_f^i f(x) = \nabla(L_f^{i-1} f(x)) \cdot f(x)$ $(i = 1, 2, \ldots)$ with $L_f^0 f(x) = f(x)$.

Among the all possible 10 square observability matrices, $O_{sq_i}(x)$ (i=1,…,10), that can be obtained from (22) we choose the one that is not singular in a sufficiently large subset of the state space containing the nominal working conditions of the PEM-FC. The full rank condition of the state dependent matrices $O_{sq_i}(x)$ has been checked by means of a numerical procedure in which the minimum of the squared determinant of the each matrix is searched using the MATLAB function *fminsearch*.

The above procedure gave the following matrix as a proper observability matrix to be used in the observer design,

$$O_{sq}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ & & \nabla f_2(x) & & & \\ 0 & 0 & 0 & 0 & 0 & 1 \\ & & \nabla f_6(x) & & & \\ & & \nabla L_f f_6(x) & & & \end{bmatrix} v \tag{23}$$

The observer is then designed as a replica of the PEM-FC model (19)-(21) driven by an output injection vector function with a gain function designed according to [6], [7]

$$\dot{\hat{x}} = f(\hat{x}) + g \cdot u + \varphi \cdot d + O_{sq}^{-1}(\hat{x}) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} v \tag{24}$$

where $\hat{y} = C\hat{x}$ and $v = [v_1, v_2, v_3]^T$ is the observer injection signal, to be designed. Considering the output error $\tilde{\varepsilon} = \hat{y} - y$ and taking into account (19)-(21) and (24), the following output error dynamics can be derived

$$\begin{aligned} \dot{\tilde{\varepsilon}}_1 &= f_1(\hat{x}) - f_1(x) + v_1 = \varphi_1(\hat{x}, x) + v_1 \\ \ddot{\tilde{\varepsilon}}_2 &= L_f f_2(\hat{x}) - L_f f_2(x) + v_2 = \varphi_2(\hat{x}, x) + v_2 \\ \tilde{\varepsilon}_2^{(3)} &= L_f^2 f_6(\hat{x}) - L_f^2 f_6(x) + v_3 = \varphi_3(\hat{x}, x) + v_3 \end{aligned} \tag{25}$$

Taking into account the real FC behaviour, any bounded error in the state estimation implies bounded drift terms in (25), i.e.,

$$\|\hat{x} - x\| \le \epsilon \Rightarrow |\varphi_i(\hat{x}, x)| \le \Phi_i, (i = 1, 2, 3) \tag{26}$$

and the output error dynamics (25) can be stabilised by resorting to higher order sliding mode control algorithms. We design the output injection as the super-twisting [17] and the sub-optimal [1] control algorithms for the relative degree one and two dynamics, respectively. The relative three dynamics in (25) can be stabilised by using the third order semi-continuous sliding controller [18] with a second-order differentiator [19] embedded in

$$\begin{aligned} v_1 &= -W_1\sqrt{|\tilde{\varepsilon}_1|}sign(\tilde{\varepsilon}_1) + v_{12} \\ \dot{v}_{12} &= -W_2 sign(\tilde{\varepsilon}_1), \quad \dot{v}_{12}(0) = 0 \\ v_2 &= -W_3 sign(\tilde{\varepsilon}_2 - \beta\tilde{\varepsilon}_2^M) \\ v_3 &= -W_4\frac{z_2+2(|z_1|+|\tilde{\varepsilon}_3|^{\frac{2}{3}})^{-\frac{1}{2}}(|z_1|+|\tilde{\varepsilon}_3|^{\frac{2}{3}}sign(\tilde{\varepsilon}_3))}{|z_2|+2(|z_1|+|\tilde{\varepsilon}_3|^{\frac{2}{3}})^{\frac{1}{2}}} \end{aligned} \tag{27}$$

where signals z_1 and z_2 denote the estimates of the first and second derivative of $\tilde{\varepsilon}_3$, computed by means of a finite-time convergent high-order sliding mode differentiator [19], and controller parameters to be set according to the following tuning rules

$$\begin{aligned} &W_1 = 1.1\Phi_1 \qquad W_2 = 0.5\sqrt{\Phi_1} \\ &W_3 \geq \Phi_2 \qquad \beta \in \left(\frac{\Phi_2}{W_3}, 1\right) \qquad W_4 \geq \Phi_3 \end{aligned} \tag{28}$$

the output error dynamics (25) is stabilized in a finite time [17], [1], [19] and the state estimation error as well [6].

IV. CONTROLLER DESIGN

The output variable of interest is the oxygen excess ratio λ_{O_2}, which takes the form

$$\lambda_{O_2} = \frac{W_{O_2,in}}{W_{O_2,react}} = \frac{W_{O_2,in}(x_2, x_4, x_5)}{W_{O_2,react}(d)} \tag{29}$$

and whose relative degree with respect to the DC motor voltage u is two. It is worth noting that the relative degree of the oxygen excess ratio with respect to the compressor speed x_1 is one. The approach taken in this paper is that of neglecting, at the stage of controller design, the compressor dynamics, by assuming that the relationship between u and x_1 is fast enough to be considered as a kind of "fast actuator" dynamics. This allows us to use the super-twisting control to design the oxygen excess ratio feedback loop.

We thus define the next regulation error variable

$$\hat{e}_\lambda = \hat{\lambda}_{O_2} - \lambda_{O_2}^{des} \tag{30}$$

where $\hat{\lambda}_{O_2}$ is estimated using the nominal FC parameters and the state estimates provided by the observer described in the previous section, i.e.

$$\hat{\lambda}_{O_2} = \frac{W_{O_2,in}(x_2, \hat{x}_4, \hat{x}_5)}{W_{O_2,react}(d)} \tag{31}$$

The set-point value $\lambda_{O_2}^{des}$ aims at maximizing the net power generated by the system, considering changes in the current demand to the stack and a wide set of stoichiometry values. However, such an optimum value often presents minor deviations all over the system operation range. Thus, a constant value of can be used. If this condition does not hold for other applications, then a variable $\lambda_{O_2}^{des}$ can be easily obtained as a function of the stack current d. The suggested control is

$$\begin{aligned} u &= u_1 + u_2 \\ u_1 &= -U_1\sqrt{|\hat{e}_\lambda|}sign(\hat{e}_\lambda) \\ \dot{u}_2 &= -U_2 sign(\hat{e}_\lambda), \quad \dot{u}_2(0) = 0 \end{aligned} \tag{32}$$

$$U_1 = 1.1\Phi_4, \quad U_2 = 0.5\sqrt{\Phi_4} \tag{33}$$

where the positive constant Φ_4 should be selected large enough. Being applied to an error dynamics having relative degree two, the utilization of the above controller cannot provide the attainment of an ideal sliding regime $\hat{e}_\lambda = 0$, whereas it can guarantee a permanent, self-sustained, oscillatory motion within a boundary layer of the sliding manifold due to neglected parasitic dynamics [4]. The usage of cascade first order lead or lag filters for reducing the chattering phenomenon (or, equivalently, improving the sliding accuracy) in sliding mode controlled systems with parasitic dynamics was discussed in [3]. The scheme involves pre-filtering the error variable, and it specializes as shown in the Figure 1 within the present application where $C(s)$ denotes

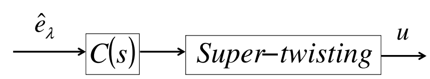

Fig. 1. Pre-filtering of the sliding variable for chattering avoidance

a linear lead/lag filter $C(s) = \frac{1+sT_1}{1+sT_2}$. Optimal tuning of the filter should be made on the basis of a linearized plant model and by carrying out frequency domain considerations (either based on describing function method [1], [4] or the locus of a perturbed system approach [4], [2], [3]). We postpone to next work a thorough treatment of the filter design for the PEM-FC system, which, once specified the linear model, can be carried out by following the guidelines presented in the quoted literature. In the simulation tests presented in the next section, it will be shown that a lead compensator tuned by simple trial and error can remarkably improve the performance of the suggested observer controller scheme.

V. SIMULATION RESULTS

The proposed observer-controller architecture is tested by means of computer simulations. To verify the control system performance in a wide range of operating conditions, the load current is varied between $100A$ and $190A$ according to the profile shown in Fig. 2. The parameters of the output injection observer signals (27) are selected as follows

$$W_1 = W_2 = 1, \quad W_3 = 0.1, \quad \beta = 0.5 \quad W_4 = 1500, \tag{34}$$

In the first simulation test (TEST 1) the observer is built using the actual PEM-FC model (or, in other words, no parameter uncertainty is taken into account). In the second simulation test (TEST 2) parameter errors are included in order to investigate the performance deterioration arising from a mismatch between the actual FC model and the nominal one employed into the observer dynamics. Figure

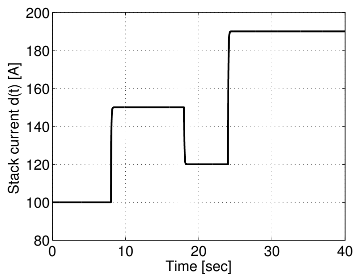

Fig. 2. Stack current d(t)

3 shows the actual and estimated profiles of the unmeasured variables x_3, x_4, x_5, confirming the correct functioning of the observer.

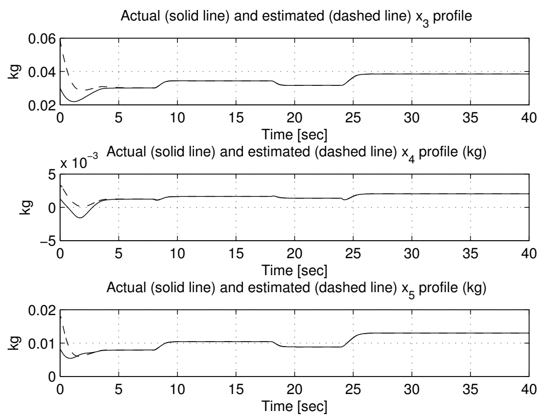

Fig. 3. Actual and observed profiles of variables x_3, x_4, x_5 in TEST 1

The gains of the observer-based controller (32) have been set as $U_1 = 20, \quad U_2 = 30$- Figure 4 shows the actual and estimated profiles of the stoichiometry λ_{O_2} along with the corresponding set-point value. It can be seen that after the abrupt changes of the stack current the stoichiometry deviates from the set point but the controller promptly and accurately restores steers it back to the desired value. Figure 6 shows the compressor supply voltage waveform at the controller output.

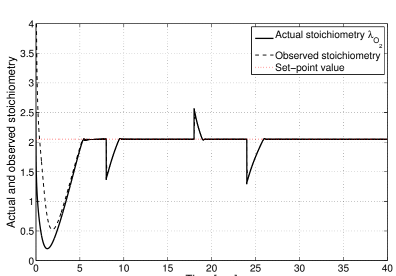

Fig. 4. Actual and observed stoichiometry, and corresponding set–point value, in TEST 1

Since the compressor dynamics has been neglected at the stage of controller design, no ideal sliding mode behaviour

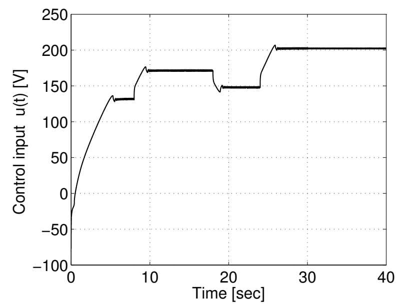

Fig. 5. Compressor supply voltage in TEST 1

can appear in the system and chattering will inevitably affects all the PEM-FC variables in the form of small-magnitude oscillations. Although the amount of chattering observed in this test is negligible, it is of interest to exploit the linear cascade compensators (mentioned and commented in the section IV) to further attenuate the chattering phenomenon. By trial-and-error design the time constants T_1 and T_2 of the compensating filter $C(s)$ have been set as $T_1 = 0.238s$ and $T_2 = 0.011s$, and to better appreciate the beneficial effects of filter introduction (in accordance with the scheme in Figure 1) it has been activated at $t = 30$. Figure 6 clearly shows the remarkable attenuation of chattering provided by the filter on both the system input and output variables.

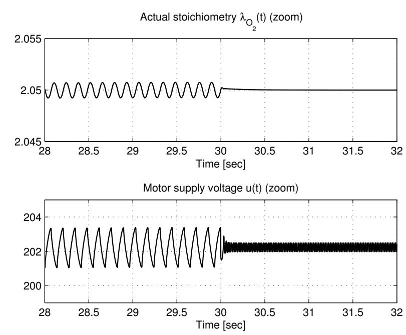

Fig. 6. Zoomed plots of input and output variables across the filter activation

In the successive TEST 2, the performance of the observer/controller system under parameter uncertainty are verified. Following [14], parameter variations up to the 10% have been applied to the PEM-FC system model, while the observer keeps using the nominal parameters of the previous test. Figure 7 shows that the performance of the observer based stoichiometry controller remains satisfactorily accurate.

VI. Conclusions

In this paper a nonlinear observer based on higher order sliding modes is proposed for a PEM-FC. The observer is employed within a feedback high order sliding mode control loop aimed at regulating the oxygen excess ratio to a suitable constant set-point. At the stage of controller design, the compressor dynamics is neglected, to simplify

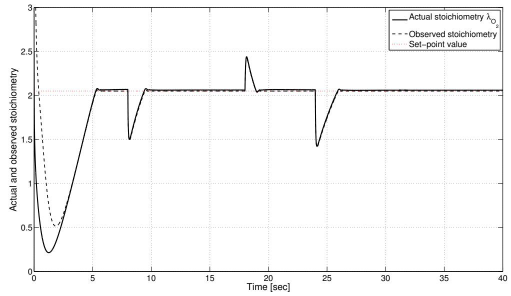

Fig. 7. Actual and observed stoichiometry, and corresponding set–point value, in TEST 2

TABLE II

PHYSICAL PARAMETERS

Parameter	Symbol (unit)	Value
Atmospheric pressure	P_{atm} (Pa)	101325
Atmospheric temperature	T_{atm} (K)	298.15
Air-specific heat ratio	γ	1.4
Air-specific heat	C_p $(J/kg/K)$	1004
Air density	ρ_a (kg/m^3)	1.23
Universal gas constant	R $(J/mol/K)$	8.31451
Air gas constant	R_a $(J/kg/K)$	286.9
Oxygen gas constant	R_{O_2} $(J/kg/K)$	259.8
Nitrogen gas constant	R_{N_2} $(J/kg/K)$	296.8
Vapour gas constant	R_v $(J/kg/K)$	461.5
Hydrogen gas constant	R_{H_2} $(J/kg/K)$	4124.3
Air molar mass	M_a (kg/mol)	28.97 $\times10^{-3}$
Oxygen molar mass	M_{O_2} (kg/mol)	32.0 $\times10^{-3}$
Nitrogen molar mass	M_{N_2} (kg/mol)	28.0 $\times10^{-3}$
Vapour molar mass	M_v (kg/mol)	18.02 $\times10^{-3}$
Hydrogen molar mass	M_{H_2} (kg/mol)	2.0 $\times10^{-3}$
Faradays constant	F (As/mol)	96 487
FC temperature	T_{fc} (K)	353

the controller structure, and this cause the appearance of some negligible chattering. Linear cascade compensators are shown to be potentially useful to attenuate the chattering due to parasitic dynamics. Simulation results, also including model parameter mismatch, are illustrated to validate the proposed observer/controller system.

TABLE III

PEMFC PARAMETERS

Parameter	Symbol (unit)	Value
Motor torque constant	K_t (Nm/A)	0.0153
Motor winding resistance	R_{cm} (Ohm)	0.82
Motor back-emf constant	K_v $(V/rad/s)$	0.0153
Compressor efficiency	η_c	0.8
Motor mechanical efficiency	η_{cm}	0.98
Number of stack cells	n	381
FC active area	A_{fc} (m^2)	280 $\times10^{-4}$
Supply manifold volume	V_{sm} (m^3)	0.02
Single stack cathode volume	V_{ca} (m^3)	0.005
Return manifold volume	V_{rm} (m^3)	0.005
Supply manif. orifice constant	$K_{sm,out}$ $(kg/s/Pa)$	0.36 $\times10^{-5}$
Cath. outlet orifice constant	$K_{ca,out}$ $(kg/s/Pa)$	0.21 $\times10^{-5}$
Compressor diameter	D_c (m)	0.2286
Compressor/motor inertia	J_{cp} (kgm^2)	5 $\times10^{-5}$
Vapour mass in cathode side	$m_{v,ca}$ (kg)	0.0028
O_2 mole fraction at cath. inlet	$y_{O_2,in}$	0.21

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