

Discrete-time Sliding Mode Control of GMAW Systems using Infrequent Output Measurements

Manas Kr. Bera, P. S. Lal Priya, B. Bandyopadhyay and A. K. Paul

Abstract—Gas metal arc welding (GMAW) is the most popular industrial welding process today, preferred for its versatility, speed and relative ease of adapting to robotic automation. This process has been used extensively by many industrial environments especially, the automobile industry. The advantages of automated welding process with feedback controllers include increased productivity, consistency in welding quality, as well as health and safety benefits for the welder. This paper proposes a discrete time sliding mode controller, a highly robust controller, to control the welding current and arc voltage of a GMAW system using output feedback. The concept of multirate output feedback technique, with infrequent output measurements has been employed to design the robust controller. A linear multi input multi output (MIMO) system model has been considered here for the design. Since the proposed algorithm is based on output feedback, it is more practical in comparison to any state feedback based control algorithms. The performance of the controller is analyzed in the presence of model parameter uncertainties and the simulation results prove the efficiency of the controller proposed.

Keywords: Gas Metal Arc Welding (GMAW); Discrete-time Sliding Mode Control (DSMC); Periodic Output Feedback (POF)

I. INTRODUCTION

Welding technology has made dramatic technical progress in the recent past. Today, welding plays an indispensable role in the fields of construction, atomic energy, automation, electronics, aircraft, guided missile development and much more. Maintenance, repair and fabrication are the major functions of arc welding process in industry.

Gas Metal Arc Welding (GMAW) is the most widely used process in many industrial and manufacturing operations. Consistency and high quality welding procedures are the key issues to maintain and increase the overall product quality. Industrial welding heavily depends on the skill and experience of the welder, and as such, at present it is more of an art than science. In the last two decades significant efforts have been made to introduce the ideas of feedback control to control the welding process. A good weld is characterized by its microstructure as well as factors like the amount of spatter or

the amount of overfill/underfill. These factors can be directly related to the cooling rate of the weld pool, the metal transfer mode, the bead/ groove geometry, workpiece defects, etc., though they are not easily measured or quantified. Likewise, many of these characteristics can be related to the mass and heat, transferred from the tip of the electrode to the weld pool. To achieve automation in the welding process a model based control strategy is essential for controlling the heat and mass transfer from the process to the weld pool. For such a control strategy, it is important to have a mathematical model which will replicate the physical system. Many researchers have worked on the modeling of the GMAW process [1], [2], [3] and [4].

To improve the control performance to a certain extent, the feedback linearization has been exploited in [4], [5], [6] and [7] which eliminates the nonlinear dynamics of the process and decouples the control variables. In [6], a procedure for the design of a model-based nonlinear controller based on feedback linearization has been proposed for the GMAW systems where, the linearized system is then controlled by PI-controllers. In [7], the GMAW process was considered as a nonlinear single input single output (SISO) system and a feedback linearization controller was proposed. A MIMO direct model reference adaptive control based on a simplified and linearized model has been proposed in [1], [2] for a GMAW process where the system considered is of second order and with two inputs and two outputs. The main drawback of this method is that it requires online estimation or tuning of the parameters.

There have been several approaches till date, using sliding mode control (SMC) algorithms [8], [9] also, to control the arc welding systems like in [10], [11] and [12]. In [10] and [11] a feedback linearization controller based on sliding mode control action for a GMAW system is presented. Sliding mode control [8] is a well accepted technique for the systematic design of robust controllers for complex nonlinear dynamic systems operating under uncertainty conditions. In the recent past, considerable efforts have also been put in the research of Discrete-time Sliding Mode Control [13]. It has been shown in the recent past [14], [15] that, sliding mode may be achieved in discrete-time systems without the usage of a switching function. The control does not need the switching function and it brings the state trajectory to the sliding surface in one sampling time. It is well known that in most of the physical systems, the states are seldom available. Therefore, the state feedback sliding mode control cannot be directly used for implementations. Output feedback sliding mode control is

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where C_1 and C_2 are the melting rate constants, ρ is the resistivity in ohm/m of the electrode. The droplet radius r_d is defined by $r_d = \left(\frac{3X_3(t)}{4\pi\rho_w}\right)^{1/3}$ where ρ_w is the electrode density.

The above third order MIMO nonlinear model is valid under the assumption that the stick-out distance ($l_s = X_4$) is much larger than the sum of the droplet radius r_d and the drop displacement X_1 , it can be written as

$$X_4 \gg \left[\frac{1}{2} \left(\left(\frac{3X_3(t)}{4\pi\rho_w} \right)^{1/3} + X_1(t) \right) \right]$$

which simplifies R_L to

$$R_L = \rho X_4(t) + \rho \left[\frac{1}{2} \left(\left(\frac{3X_3(t)}{4\pi\rho_w} \right)^{1/3} + X_1(t) \right) \right] \cong \rho X_4(t)$$

The approximate nonlinear model is linearized about an operating point to obtain the linearized model of the GMAW system. The operating point chosen here are at an arc voltage of 25V and at a welding current of 250A by considering that the GMAW process works in globular-spray melted droplets transfer mode. The state space representation of the linearized model is,

$$\dot{x} = Ax + Bu, \quad y = Cx \quad (5)$$

$$\text{where, } x = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \delta l_s \\ i \\ s \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} v_{arm} \\ v_{oc} \end{bmatrix}$$

$$\text{and } y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} v_{arc} \\ i \end{bmatrix}$$

x_4 , x_5 and x_6 represent the linearized variables of X_4 , X_5 and X_6 respectively. The linearized model matrices are,

$$A = \begin{bmatrix} \frac{-C_2\rho}{\pi r_e^2} \bar{X}_5^2 & \frac{-C_1+2C_2\rho\bar{X}_4\bar{X}_5}{\pi r_e^2} & 1 \\ L_s^{-1}(E_a - \rho\bar{X}_5) & -L_s^{-1}(R_a + \bar{R}_s + \rho\bar{X}_4) & 0 \\ 0 & 0 & -\tau_m^{-1} \end{bmatrix} \quad (6)$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & L_s^{-1} \\ -k_m\tau_m^{-1} & 0 \end{bmatrix} \quad \text{and } C = \begin{bmatrix} -E_a & R_a & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

δl_s , s , i , v_{arm} , v_{arc} and v_{oc} represent the respective linearized variables. \bar{X}_4 , \bar{X}_5 are stick out and welding current at the operating point, mentioned above. y_1 and y_2 are the outputs, u_1 and u_2 are the inputs of the linearized model.

III. DISCRETE-TIME SLIDING MODE CONTROL WITH PERIODIC OUTPUT FEEDBACK FOR A GMAW SYSTEM

The motivation of discrete-time controllers stemmed from the advent and use of computers and microcontrollers for the implementation of control algorithms to any system. Sliding mode control is a well established technique for controlling any uncertain dynamical system. Moreover, it has been proved [16] that, the closed loop stability of a system can be guaranteed by multirate output feedback methods, a feature not assured by static output feedback [24]. So, here a discrete-time

sliding mode control with one of the multirate output feedback techniques has been developed to control an uncertain GMAW system.

In the periodic output feedback control technique, the control input is implemented in the following manner. The output of the system is measured at the time instants $t = k\tau, k = 0, 1, \dots$, using a sample and hold system. The output sampling interval τ is divided into N subintervals of length $\Delta = \tau/N$, where N is an integer greater than or equal to the controllability index of the system. The hold function is assumed constant on these subintervals. Thus the control law becomes,

$$\begin{aligned} u(t) &= K_l y(k\tau), \\ k\tau + l\Delta < t \leq k\tau + (l+1)\Delta, \\ K_{l+N} &= K_l, l = 0, 1, 2, \dots, (N-1). \end{aligned} \quad (7)$$

A sequence of N gains $\{K_0, K_1, \dots, K_{N-1}\}$ when substituted in (7) generates a time-varying, piecewise constant output feedback gain $K(t)$ for $0 \leq t \leq \tau$. Define,

$$\mathbf{K}^T = [K_0 \quad K_1 \quad \dots \quad K_{N-1}] \quad (8)$$

The detailed background of this technique is given in references cited [20], [21], [22].

A. Main Result

This section discusses the main result which is the periodic output feedback based discrete-time sliding mode control for a GMAW system.

The third order linear model of an uncertain GMAW system with bounded parametric uncertainties is considered here for the control law development. The system is both controllable and observable. The inputs to the system are open circuit voltage, and armature voltage and the outputs of the system are welding current and arc voltage.

Let us consider the n -th order continuous-time linear time invariant system, represented as,

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) \\ y(t) &= Cx(t) \end{aligned} \quad (9)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^p$ is the output of the system. A , B and C are constant matrices of appropriate dimensions, here for the GMAW system it is given by (6). ΔA and ΔB are the uncertainty matrices. The system (9) can be simplified as,

$$\dot{x}(t) = Ax(t) + Bu(t) + F(x, t) \quad (10)$$

where, $F(x, t) = \Delta Ax(t) + \Delta Bu(t)$ which accounts for the bounded parametric uncertainties of the system. Let the system (10) be sampled at τ sec given as,

$$\begin{aligned} x(k+1)\tau &= \Phi_\tau x(k) + \Gamma_\tau u(k) + d_\tau(x, k) \\ y(k) &= Cx(k) \end{aligned} \quad (11)$$

where Φ_τ , Γ_τ and C are constant matrices of appropriate dimensions, given by, $\Phi_\tau = e^{A\tau}$, $\Gamma_\tau = \int_0^\tau e^{A\omega} B d\omega$,

$$d_\tau(x, k) = \int_0^\tau e^{A\omega} F((k+1)\tau - \omega) d\omega.$$

Now, we are considering the nominal system of (11) given by

$$\begin{aligned} x(k+1) &= \Phi_\tau x(k) + \Gamma_\tau u(k) \\ y(k) &= Cx(k) \end{aligned} \quad (12)$$

Assumption 1: The system (12) is completely controllable and observable. Let us consider the dual system of (12)

$$\begin{aligned} \tilde{x}(k+1) &= \Phi_\tau^T \tilde{x}(k) + C^T u(k) \\ y(k) &= \Gamma_\tau^T \tilde{x}(k) \end{aligned} \quad (13)$$

A stable sliding surface is designed [8], [20], [21], [22] for this system (13) as,

$$\bar{S}(k) = \tilde{M}\tilde{x}(k) \quad (14)$$

To reach the sliding surface in one sampling period, the reaching law, $\bar{S}(k+1) = 0$ is used. Thus the discrete time sliding mode control of (13), using the sliding surface (14) is obtained as

$$\begin{aligned} \bar{S}(k+1) &= \tilde{M}\tilde{x}(k+1) = 0 \\ \tilde{M}\Phi_\tau^T \tilde{x}(k) + \tilde{M}C^T \tilde{u}(k) &= 0 \end{aligned}$$

Therefore,

$$\begin{aligned} \tilde{u}(k) &= -(\tilde{M}C^T)^{-1} \tilde{M}\Phi_\tau^T \tilde{x}(k) \\ &\equiv F_{eq} \tilde{x}(k) \end{aligned} \quad (15)$$

The control (15) brings the trajectory of the system (13) to the surface, in one sampling time and thereafter makes it slide along the surface given in (14).

Remark 1: It is trivial to verify that the control law (15) ensures the existence of discrete time sliding mode.

The eigenvalues during sliding is obtained from the following closed loop system.

$$\begin{aligned} \tilde{x}(k+1) &= \Phi_\tau^T \tilde{x}(k) + C^T \tilde{u}(k) \\ &= (\Phi_\tau^T + C^T F_{eq}) \tilde{x}(k) \end{aligned} \quad (16)$$

The eigenvalues of (16) are the eigenvalues chosen to design the sliding surface of the dual system.

Let the Δ system representation of (12) for a sampling interval of $\Delta = \tau/N$ sec, where N is an integer greater than or equal to the controllability index of the system be

$$\begin{aligned} \hat{x}(k+1) &= \Phi \hat{x}(k) + \Gamma u(k) \\ \hat{y}(k) &= C \hat{x}(k) \end{aligned} \quad (17)$$

where Φ , Γ and C are constant matrices of appropriate dimensions of the Δ system.

Assumption 2: The system (17) is completely controllable and observable.

When the periodic output feedback control (7), is applied to this Δ system (17), it brings a sliding mode motion along a surface and the eigenvalues during sliding is the same as that of system (16) which are stable by design. The closed loop τ system under the application of (7) in (17) is

$$\hat{x}(k+1) = (\Phi_\tau + \mathbf{K}C) \hat{x}(k). \quad (18)$$

where,

$$\mathbf{K} = \begin{bmatrix} \Phi^{N-1}\Gamma & \Phi^{N-2}\Gamma & \dots & \Gamma \end{bmatrix} \quad (19)$$

The periodic output feedback gain \mathbf{K} is obtained by equating the closed loop system (18) with $(\Phi_\tau^T + C^T F_{eq})$ as

$$\mathbf{K}\mathbf{K} = F_{eq}^T \quad (20)$$

As, the systems (12) and (17) are controllable, the existence of \mathbf{K} is guaranteed for $N \geq$ controllability index of the systems (12) and (17).

Remark 2: The eigenvalues of (16) are stable by design and the eigenvalues of (18) during sliding are same as that of (16). Hence, the closed loop system (18), which is obtained under the application of the feedback control law (7) in (17) is also stable.

Now, let us consider the Δ system representation of (10) given by,

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) + d(x, k) \\ y(k) &= Cx(k) \end{aligned} \quad (21)$$

where Φ , Γ and C are constant matrices of appropriate dimensions of the Δ system. It is assumed that $d(x, k < 0) = 0$ and $d(x, k)$ is bounded. When the control input (7) is applied to (21), then the closed loop τ system becomes,

$$x(k+1) = (\Phi^N + \mathbf{K}C)x(k) + \sum_{i=0}^{N-1} \Phi^i d(x, k) \quad (22)$$

where, \mathbf{K} and \mathbf{K} are defined as (19) and (8) respectively. The periodic output feedback gain \mathbf{K} is obtained from (20).

B. Design of sliding surface with POF control

In this section the design procedure of sliding surface with POF control for a general n^{th} order system with m inputs is explained in detail. Let the sliding surface be $S(k) = [M \ I_m]x(k) + \zeta(k)$, $M \in \mathbb{R}^{m \times (n-m)}$ and $\zeta(k) \in \mathbb{R}^m$, which accounts for the parametric uncertainty terms.

Let the system (22) be transformed by $x = U\tilde{x}$ with a non singular transformation matrix $U \in \mathbb{R}^{n \times n}$, to

$$\tilde{x}(k+1) = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 & 0 \\ 0 & \lambda_2 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \lambda_{n-m} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tilde{x}(k) + \Psi d(x, k) \quad (23)$$

where, $\Psi = U^{-1}\bar{\Phi}$ and $\lambda_1, \lambda_2, \dots, \lambda_{n-m}$ and m number of zeros are the eigenvalues of the system. It is assumed that, the eigenvalues are real and distinct.

Let us define

$$\Psi = U^{-1}\bar{\Phi} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \dots & \Omega_{1n} \\ \Omega_{21} & \Omega_{22} & \dots & \Omega_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn} \end{bmatrix} \quad (24)$$

Let $d(x, k)$ be represented as

$$d(k) = [d_{11}(k) \ d_{21}(k) \ \cdots \ d_{n1}(k)]^T \quad (25)$$

where $d(k)$ is bounded. The solution of (23) for a general i -th state is,

$$\begin{aligned} \tilde{x}_i(k) = & (\lambda_i)^k \tilde{x}_i(0) + (\lambda_i)^{k-1} (\Omega_{i1} d_{11}(0) + \cdots \\ & + \Omega_{in} d_{n1}(0)) + \cdots + \lambda_i (\Omega_{i1} d_{11}(k-2) + \cdots \\ & + \Omega_{in} d_{n1}(k-2)) + (\Omega_{i1} d_{11}(k-1) + \cdots \\ & + \Omega_{in} d_{n1}(k-1)) \end{aligned} \quad (26)$$

The higher power terms of λ can be neglected in the uncertain terms, as all the eigenvalues are stable and within the unit circle. Therefore, the approximated value of $\tilde{x}_i(k)$ can be written as,

$$\begin{aligned} \tilde{x}_i(k) = & (\lambda_i)^k \tilde{x}_i(0) + \lambda_i (\Omega_{i1} d_{11}(k-2) + \cdots + \\ & \Omega_{in} d_{n1}(k-2)) + (\Omega_{i1} d_{11}(k-1) + \cdots + \\ & \Omega_{in} d_{n1}(k-1)) \end{aligned} \quad (27)$$

Let us define for any i -th state,

$$\begin{aligned} Di(k-2) & \equiv (\Omega_{i1} d_{11}(k-2) + \cdots + \Omega_{in} d_{n1}(k-2)) \\ Di(k-1) & \equiv (\Omega_{i1} d_{11}(k-1) + \cdots + \Omega_{in} d_{n1}(k-1)) \end{aligned} \quad (28)$$

Therefore, $\tilde{x}_i(k)$ can be written as

$$\tilde{x}_i(k) = (\lambda_i)^k \tilde{x}_i(0) + \lambda_i Di(k-2) + Di(k-1) \quad (29)$$

The sliding surface for the uncertain system is $S(k) = [M \ I_m]x(k) + \zeta(k)$, $\zeta(k) \in \mathbb{R}^m$, accounts for the uncertainty terms.

Let

$$\zeta(k) = [\zeta_1(k) \ \zeta_2(k) \ \cdots \ \zeta_n(k)]^T \quad (30)$$

Therefore from $[M \ I_m]x(k) + \zeta(k) = 0$ we can rearrange the equations as,

$$\begin{aligned} m_{11}x_1(k) + m_{12}x_2(k) + \cdots + m_{1(n-m)}x_{n-m}(k) = \\ -x_{n-m+1}(k) - \zeta_1(k) \end{aligned} \quad (31)$$

$$\begin{aligned} m_{m1}x_1(k) + m_{m2}x_2(k) + \cdots + m_{m(n-m)}x_{n-m}(k) = \\ -x_n(k) - \zeta_n(k) \end{aligned} \quad (32)$$

substituting for $x_1(k)$, $x_2(k) \cdots x_{n-m}(k) \cdots$ and $x_n(k)$ in each of the above equations and comparing the coefficients of $\tilde{x}_1(0)$, $\tilde{x}_2(0)$, \cdots , and $\tilde{x}_{n-m}(0)$, from the left hand side and right hand side of equations (31), and (32), we obtain the scalar values m_{11} , $m_{12} \cdots$ and $m_{1(n-m)}$, m_{21} , $m_{22} \cdots$ and $m_{2(n-m)}$, m_{n1} , $m_{n2} \cdots$ and $m_{n(n-m)}$. Any general n -th element of $\zeta(k)$ is as shown below. From equation (32), the value of ζ_n , is calculated as

$$\begin{aligned} \zeta_n(k) = & (m_{m1}u_{11} + \cdots + m_{m(n-m)}u_{(n-m)1} + u_{(n-m+1)1}) \\ & [\lambda_1 D_1(k-2) + D_1(k-1)] \\ & + (m_{m1}u_{12} + \cdots + m_{m(n-m)}u_{(n-m)2} + u_{(n-m+1)2}) \\ & [\lambda_2 D_2(k-2) + D_2(k-1)] \\ & \vdots \\ & + (m_{m1}u_{1n} + \cdots + u_{(n-m+1)(n-m)}) [D_n(k-1)] \end{aligned}$$

Since the uncertainties are bounded, $\zeta(k)$ is also bounded. Thus there exist a sliding mode in the ζ vicinity of the sliding hyperplane $S(k) = 0$, or $S(k) = \sigma x(k) - \zeta(k) = 0$, where $\sigma = [M \ I_m]$ when the periodic output feedback control is applied to the system (21). From (23), it is seen that all the states are affected by the disturbance. So, the ultimate boundedness of the state trajectories need to be proved and the detailed proof is presented in the reference cited [21].

IV. SIMULATION RESULTS

The performance analysis of an uncertain GMAW system with parametric uncertainties using discrete-time sliding mode controller with periodic output feedback has been carried out in this Section. Here, a third order linearized MIMO model (5) has been used to perform the simulations. The parameter values are chosen for the simulation study of the GMAW system as referred in [23].

The control objective here is to track the reference set point values of arc voltage and welding current, which are chosen to be square waves with $\pm 0.5V$ and $\pm 10A$ amplitudes respectively around their corresponding operating points $25V$ and $250A$. Here for tracking, the error dynamics between the desired output and actual output has been stabilized with a discrete time sliding mode control using POF technique as explained in Section III. Fig. 2 and Fig. 3 illustrate the variation of the arc voltage and the welding current respectively of the GMAW system using the designed controller. The uncertainty system matrices are chosen to be 30% of its nominal values for the simulation study and it has been observed that the outputs of the system with such parametric variations, could perfectly track the set point, which proves the effectiveness of the controller used. Fig. 4 and Fig. 5 show the sliding surfaces. Fig. 6 and Fig. 7 show the control inputs of the system. The discrete sliding mode controller is designed by discretizing the continuous-time system (5) by sampling it at $\tau = 0.1\text{sec}$ and $\Delta = 0.05\text{sec}$ to obtain the corresponding discrete τ system and Δ system models. The number of gain changes for the periodic output feedback is chosen as $N = 2$ since the controllability index of the system is 2. The value of periodic output feedback gain \mathbf{K} obtained is

$$\mathbf{K} = \begin{bmatrix} 1 & 1 \\ 26.4253 & 27.8827 \\ -0.3678 & -0.3686 \\ -0.2196 & -0.2421 \end{bmatrix}$$

The sliding surface matrix is

$$[M \ I_m] = 10^3 \times \begin{bmatrix} -6.6544 & 0.0010 & 0 \\ -0.0057 & 0 & 0.0010 \end{bmatrix}$$

V. CONCLUSIONS

The advent and use of computers has necessitated the requirement of discrete controllers for controlling any dynamic system. To ensure the quality of welding, in a welding process a highly robust controller is required and sliding mode control

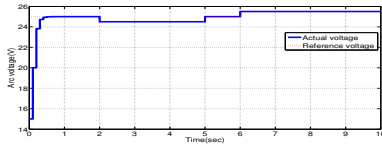


Fig. 2. Tracking of the arc voltage

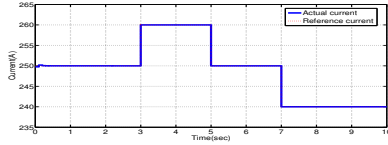


Fig. 3. Tracking of the welding current

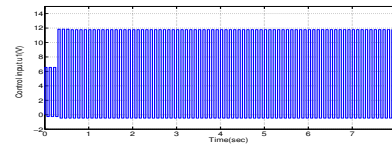


Fig. 6. First control input

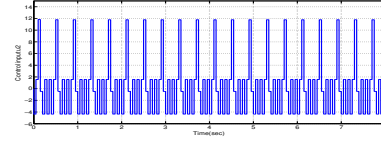


Fig. 7. Second control input

is one such robust control technique. In this paper, a discrete-time sliding mode control approach for a GMAW system has been presented which makes use of the concept of a multirate output feedback method by considering a third order linearized model of the process. The proposed algorithm is based on output feedback, and hence it is more practical in comparison to any state feedback based control algorithms. The simulation results show the effectiveness of the robust controller proposed.

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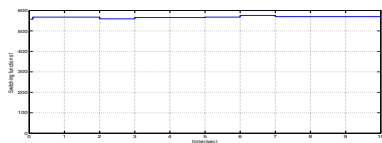


Fig. 4. First sliding surface

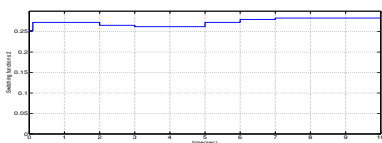


Fig. 5. Second sliding surface