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Dynamic Analysis and Control of dc/dc Boost Converters used in Stand-Alone PV Systems

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***Abstract*—A stand-alone photovoltaic (PV) system connected to a resistance load through a dc/dc boost converter is considered. Taking into account the PV source voltage-current nonlinear dependence as well as the dc/dc boost nonlinear dynamic model, an extensive system analysis is conducted. It is proven that as the load power becomes smaller than the maximum power input provided by the PV source, the system acquires the minimum phase property with respect to the output voltage, while the domain of attraction of the stable zero dynamics of the system becomes larger. Exploiting this property, a simple adaptive-based direct output voltage controller is implemented, which acts independently from the system parameters and the equilibrium guaranteeing stability and convergence to the desired operating point. Extensive simulations results verify the theoretical analysis and the system good performance.**

***Index Terms*—Voltage regulation, dc/dc boost converter, nonlinear dynamic analysis, stability analysis, adaptive control.**

I. Introduction

THE use of Renewable Energy Sources (RES) is continuously increasing worldwide [1] because of their environmental friendly operation. Among the RES, photovoltaic (PV) systems belong to the most promising emerging technologies due to their high reliability caused by the absence of mechanical moving parts and their suitability with respect to distributed power systems [2]. A significant penetration of PV energy systems into distributed power networks has been already taken place [3], since governments and utility companies support programs for grid connected or stand-alone PV systems. As this penetration increases its dynamic influence on the power system performance and reliability becomes crucial. Especially, in cases where stand-alone PV systems are developed their stability and transient performance is essential. However, a complete dynamic and stability analysis of such systems is not an easy task due to the nonlinear characteristics of the PV modules and the power converters dynamics.

The most common scheme used in stand-alone PV applications comprises a PV source which feeds a resistance load through a dc/dc boost converter [4]. The controller task in these cases is to regulate the output voltage at a desired reference. This is a challenging problem due to the fact that the dc/dc boost converter is non-minimum phase with respect to the output to be regulated. For the analysis and the controller design, a simple technique used in PV systems is based on the eigenvalue analysis or the small signal analysis [5]-[8]. The main drawback of these methods is that they are developed near a specific operating point and thus can predict the behavior of the system only close to the preselected equilibrium. Therefore, in order to obtain a more complete analysis of the dynamic behavior of the system, the analysis must be repeated for a wide range of different operating points. In other research works [9], [10] a stability analysis was carried using bode-plots and root locus diagrams. However, this technique requires the system description in the frequency domain. Thus, again the nonlinear model of the system cannot be considered. A more advanced and precise analysis has been conducted in [11] which overcomes the zero dynamics constraints by exploiting the fact that, with respect to the inductor current, the system is minimum-phase and there is a one-to-one corresponding between the output voltage and the inductor current equilibria. The voltage is then indirectly controlled via regulation of the current. An important drawback of this approach is the high sensitivity to the circuit parameters and in particular to the load resistance, while the PV nonlinearities are not taken into account.

In this paper, we start by considering the nonlinear model of the PV source, i.e. it is taken into account the input voltage-current nonlinear dependence of the PV modules [12]. For the dc/dc boost power converter description the average model is used, which includes the accurate nonlinear dynamics [13]. Dynamic and stability analysis is carried out using Lyapunov theory and zero dynamics [14]. The controller task is to regulate the output voltage at a desired value independently from the sun irradiance variations or load changes. The nonlinear analysis addressed in this paper allows us to get more insight in the dynamic performance of the dc/dc boost converter when it is fed by a PV source and to observe some interesting properties. Particularly, it is proven that the zero-dynamics of the system can be both: stable or unstable depending on the operating point of the system. This twofold dynamic behavior appears due to the nonlinearities of the PV voltage source. The above analysis indicates that under some mild constraints a direct voltage control law can be applied. A simple adaptive based

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implementation is proposed that makes possible the controller performance to be independent from the system parameters and load. Furthermore, as it is proven in the paper, this controller is capable to guarantee system stability and convergence to the desired operating point. Simulation results verify the theoretical analysis.

The remaining of the paper is organized as follows. In Section II, we describe the nonlinear dynamic model of the PV system. In Section III and IV, steady state and dynamic analysis is obtained. In Section V, an adaptive state feedback controller is proposed and a stability analysis of the closed-loop system is provided. In Section VI simulations results are carried out and finally, in Section VII some conclusions are drawn.

II. SYSTEM MODELING

The system, as shown in Fig. 1, consists of a PV source, actually a PV string, connected to a dc/dc boost power converter, which feeds a resistance load. This structure represents a classical stand-alone PV system.

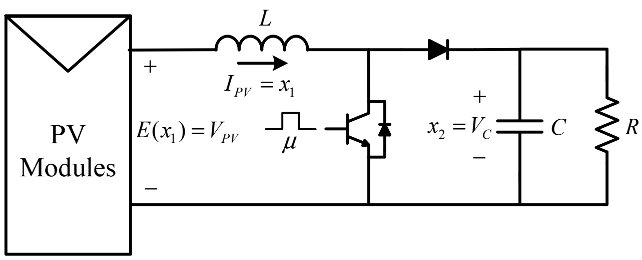

Fig. 1. Model of the PV System.

A. PV module circuit model and equations

A PV module, which is consisting by a number of PV cells, can be modeled as shown in Fig. 2 [12]. According to this model a photo-current source I_{SC}, that produces a current proportional to the sun irradiance, is in parallel with a diode and a resistor R_P, which in turn is in series with the resistor R_s. The equations of the PV module output current $I_{PV,\text{mod}}$ and voltage $V_{PV,\text{mod}}$ are given by the following equations:

$$I_{PV,\text{mod}} = I_{SC} - I_D - \frac{V_D}{R_p}$$

$$I_D = I_0\left(e^{V_D/V_T} - 1\right)$$

$$V_{PV,\text{mod}} = V_D - R_s I_{PV,\text{mod}}$$

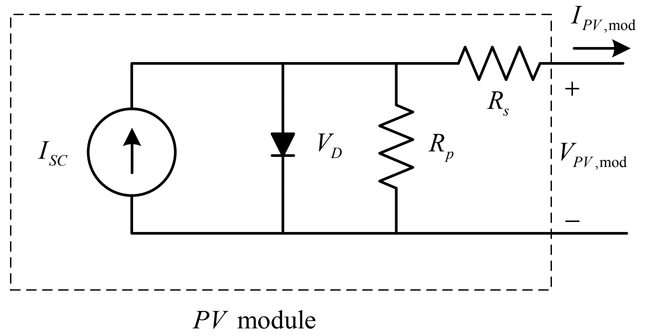

Fig. 2. PV module circuit model.

B. PV module integration in a string with the dc/dc boost converter

A PV string consisting of m PV modules in series is considered. We define the output voltage and current of the PV string as V_{PV} and I_{PV} respectively, where $V_{PV} = mV_{PV,\text{mod}}$ and $I_{PV} = I_{PV,\text{mod}}$. A voltage-current ($V_{PV} - I_{PV}$) and a power-current ($P_{PV} - I_{PV}$) characteristics are shown in Fig. 3. It is pointed out that the power produced by the PV string for a given irradiance has a unique maximum point defined as maximum power point (MPP). The maximum power produced, as well as, the current and voltage at that point, are denoted as P_{PV}^{MPP}, I_{PV}^{MPP} and V_{PV}^{MPP} respectively.

As shown in Fig.1, the PV module string is considered as input to a dc/dc boost converter. Thus, the input of the dc/dc boost converter, denoted by E in Fig. 1, equals to the PV string output voltage V_{PV}, i.e. $E = V_{PV}$. Similarly, for the current of the converter it holds $I = I_{PV}$. Now, taking into account the average model of the boost converter [13], the following state-space model of the system is obtained

$$\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -(1-\mu) \\ (1-\mu) & -\dfrac{1}{R} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} E(x_1) \\ 0 \end{bmatrix} \tag{1}$$

where x_1 is the input inductor current $(x_1 = I)$, x_2 is the output voltage of the capacitor and $E(x_1)$ is the input voltage of the dc/dc converter. The converter inductance and capacitance are represented by L, C respectively while R is the resistance load. Finally μ is the duty-ratio representing the switching cycle ratio in a period.

It should be noted that μ is a continuous function that takes values in the range $[0,1]$ and constitutes the controlled input of the system while $E(x_1)$ is considered as an unknown disturbance input.

It is significant to remark that in the present case where the voltage input of the dc/dc boost converter is provided by a PV string, this input is a function of the system state x_1 as one can see from the $V_{PV} - I_{PV}$ characteristic (Fig. 3).

Furthermore, as it is well known in the control systems community [11] the dc/dc boost converter with a constant input voltage is a non-minimum phase system with respect to the output voltage state. Particularly, only the zero dynamics with respect to the inductor current x_1 are stable, which implies that the output voltage x_2 should be indirectly controlled via the regulation of the inductor current x_1. This constitutes an inherent system obstacle in designing voltage regulators which is overcomed by a cascaded PI controller scheme. This scheme includes a fast inner-loop consisting of a PI current regulator, whose reference value is determined by a slower outer-loop PI voltage controller [15]. Also, sliding mode controller schemes can substitute the inner-loop PI current controller [16]. However, in this case the controller appears to have a strong dependence from the system parameters or from the steady state equilibrium.

In this study, it is proven that the non-minimum phase property of the system can be eliminated under some mild constraints on the power amount extracted from the system. This is possible in this case since the input voltage $E(x_1)$ is not a constant or piecewise constant input but it is strongly dependent from the input current x_1.

Thus, feeding the dc/dc boost converter by a PV system a main advantage results on the stability properties that permits the direct output voltage regulation. Therefore, our purpose is to analyze the conditions under which the minimum phase property of the system appears for this case, in order to apply an effective simple adaptive based voltage regulator having an independent performance from the system parameters or the equilibrium point.

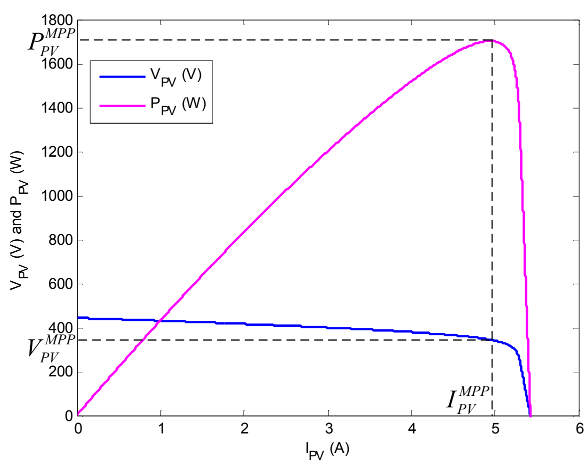

Fig. 3. $V_{PV}-I_{PV}$ and $P_{PV}-I_{PV}$ characteristics of a PV string.

III. STEADY STATE SYSTEM ANALYSIS

To proceed with our analysis, we can easily find the equilibrium points of the nonlinear system (1) by setting the vector $[\dot{x}_1 \quad \dot{x}_2]^T = 0$ and solving the resulting nonlinear algebraic system of equations. Thus, we obtain

$$x_1^* = \frac{E(x_1^*)}{(1-\mu)^2 R} \quad , \quad x_2^* = \frac{E\left(\dfrac{x_2^*}{R(1-\mu)}\right)}{1-\mu} = \frac{E\left(x_2^*,\mu\right)}{1-\mu} \qquad (2)$$

It is obvious that for $\mu = 1$ the equilibrium points cannot be defined. Therefore, the permitted range of μ is $[0\ 1)$. Now, eliminating the duty-ratio input μ from (2) we arrive at

$$E\left(x_1^*\right)x_1^* = \frac{\left(x_2^*\right)^2}{R} \qquad (3)$$

which simply means that

$$P_{PV}\left(x_1^*\right) = P_{out}\left(x_2^*\right) \qquad (4)$$

where P_{PV} represents the power produced by the PV string and P_{out} the power consumed by the resistance load.

Equation (4) expresses the simple fact that at steady state the power produced by the PV string (P_{PV}) equals to the power consumed by the resistance load (P_{out}). This nonlinear algebraic expression cannot be solved analytically. However, using a graphical method, Fig. 4 is obtained that provides a graphical solution of (4). As one can easily observe, there exist two solutions for the state $x_1 : x_{11}^*, x_{12}^*$ that satisfy the equation (4) for a given x_2^*, when $P_{out} < P_{PV}^{MPP}$. Therefore the equilibrium points for this case are $\left(x_{11}^*, x_2^*\right)$, $\left(x_{12}^*, x_2^*\right)$. Only when $P_{out} = P_{PV}^{MPP}$ there exists one equilibrium $\left(x_1^{MPP}, x_2^*\right)$.

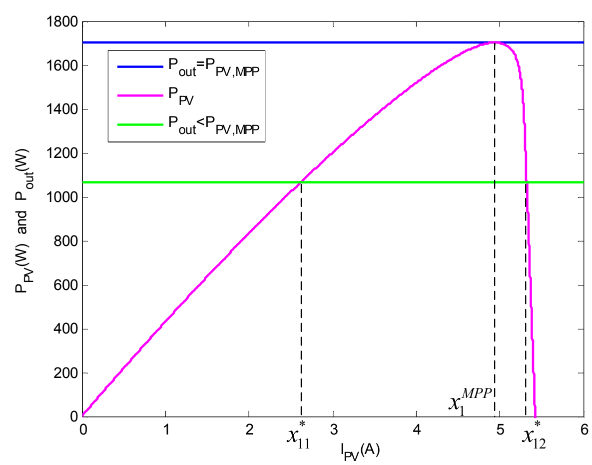

Fig. 4. Graphs of the functions $P_{PV}\left(I_{PV}\right)$ and $P_{out}\left(I_{PV}\right)$ for constant output voltage of the PV system.

However, in the case of operating at the output voltage regulation mode, the stability with respect to these two equilibrium points is essential since always this operation is far from the MPP. In the following section such an analysis is addressed.

IV. DYNAMIC ANALYSIS OF THE SYSTEM

For the dynamic analysis, we consider as output of the system the voltage of the capacitor: $y = h(x) = x_2$.

Eliminating x_1 from the differential equations of system (1) we arrive after some manipulations to the following nonlinear input-output differential representation:

$$\ddot{x}_2 + \left(\frac{1}{RC} + \frac{\dot{\mu}}{1-\mu}\right)\dot{x}_2 + \frac{1}{LC}\left((1-\mu)^2 + \frac{L}{R}\frac{\dot{\mu}}{(1-\mu)}\right)x_2 = \frac{(1-\mu)}{LC}E(x_1) \tag{5}$$

Now since $x_1 = \dfrac{C\dot{x}_2}{(1-\mu)} + \dfrac{x_2}{R(1-\mu)}$, input $E(x_1)$ can be considered as function of $x_2, \dot{x}_2, \mu$, i.e. $E(x_2, \dot{x}_2, \mu)$. Thus (5) becomes

$$\ddot{x}_2 + \left(\frac{1}{RC} + \frac{\dot{\mu}}{1-\mu}\right)\dot{x}_2 + \frac{1}{LC}\left((1-\mu)^2 + \frac{L}{R}\frac{\dot{\mu}}{(1-\mu)}\right)x_2 = \frac{(1-\mu)}{LC}E(x_2, \dot{x}_2, \mu) \tag{6}$$

In order to obtain the zero dynamics of the system at a desired equilibrium point $x_2 = x_2^*$, we let $\ddot{x}_2 = 0$ and $\dot{x}_2 = 0$. Then we arrive at the following dynamic expression for μ:

$$\dot{\mu} = \frac{R(1-\mu)^2}{Lx_2^*}\left(E(x_2^*, \mu) - (1-\mu)x_2^*\right) \tag{7}$$

from which the following equilibrium points are obtained for μ:

$$\mu = 1, \quad \mu = 1 - \frac{E\left(x_2^*, \mu\right)}{x_2^*} \tag{8}$$

The graphical solution of (7) in phase plane for different levels of P_{out} are shown in Fig. 5. As P_{out} becomes smaller than the maximum power extracted from the PV system, i.e. $P_{out} : P_0 > P_1 > P_2 > P_3$ etc, where $P_0 = P_{PV}^{MPP}$, the μ, $\dot{\mu}$ phase portrait provides two equilibriums lying more to the left and right from the unique equilibrium $\mu = 1 - \dfrac{E\left(x_1^{MPP}\right)}{x_2^*}$ of the P_0 curve. For the reader convenience, in Fig 6, a graphical solution is presented for a particular case where $P_{out} < P_{PV}^{MPP}$. One can then observe that after excluding the unstable solution , $\mu = 1$ there exist two equilibrium points $\mu_{\min}$ and $\mu_{\max}$. One of them is an unstable node and one is a stable point. Fortunately, the locally stable point is an admissible equilibrium point. Hence, we conclude that the boost converter with a PV input voltage and output represented by the capacitor voltage $y = h(x) = x_2$ is a minimum phase system if:

$P_{out} < P_{PV}^{MPP}$ and $\mu > \mu_{\min}$ for all t

In the case when $P_{out} = P_{PV}^{MPP}$ the admissible equilibrium for the duty ratio μ is a saddle point, hence unstable as shown in Fig. 5.

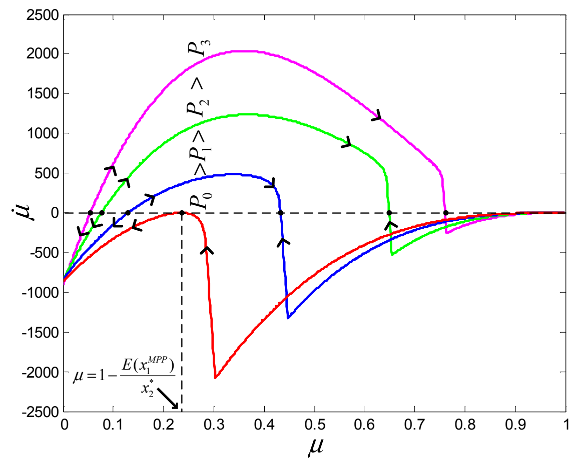

Fig. 5. Zero dynamics of the PV system corresponding to constant output voltage for different levels of P_{out}.

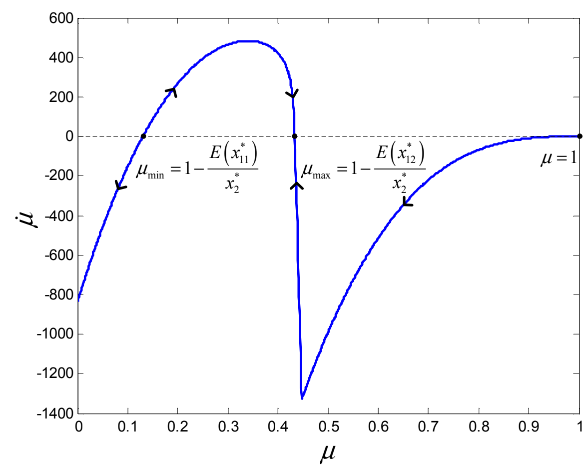

Fig. 6. Zero dynamics of the PV system corresponding to constant output voltage when $P_{out} < P_{PV}^{MPP}$.

V. PROPOSED CONTROL OF THE PV SYSTEM AND STABILITY ANALYSIS

A. Control law

The previous dynamic analysis indicates that when the system operates far from the MPP the zero dynamics are stable, hence direct output voltage regulation can be applied. Also it should be taken into account that the resistance load is usually unknown and can change. Therefore, we propose the following adaptive controller for the photovoltaic system, which regulates the output voltage to a reference value:

$$\mu = 1 - \frac{1}{x_1}\left(\hat{G}x_2^{ref} - \lambda\tilde{x}_2\right) \quad (9)$$

$$\dot{\hat{G}} = -\gamma x_2^{ref}\tilde{x}_2 \quad (10)$$

where we define: $G = \frac{1}{R}$ the conductance, $\hat{G}$ the estimated conductance, while $\tilde{x}_2 = x_2 - x_2^{ref}$ and λ, γ are some positive constants.

To avoid limiters for the controlled input, restriction $0 \le \mu < 1$, with μ given by (9), can provide suitable bounds for λ and γ.

B. Stability analysis of the closed-loop system

Let the case where $P_{out} < P_{PV}^{MPP}$. Substituting the proposed control law (9) in the second differential equation of (1) we obtain after some manipulations

$$C\dot{\tilde{x}}_2 + \left(\lambda + G\right)\tilde{x}_2 = \tilde{G}x_2^{ref} \quad (11)$$

where $\tilde{G} = \hat{G} - G$ with $\hat{G}$ dynamics given by (10).

Now defining for (11) and (10) the following Lyapunov function,

$$V(x) = \frac{1}{2}C\tilde{x}_2^2 + \frac{1}{2\gamma}\tilde{G}^2 \quad (12)$$

its derivative is calculated as

$$\dot{V}\left(x\right) = -(\lambda + G)\tilde{x}_2^2 \quad (13)$$

From (13) asymptotic stability of the output voltage is guaranteed, since

$$\lim_{t\to\infty}\tilde{x}_2 = 0 \Rightarrow \lim_{t\to\infty}x_2 = x_2^{ref}$$

One can easily see from (11) that also $\hat{G}$ will be stabilized at G. Under the initial assumption $P_{out} < P_{PV}^{MPP}$, if $\mu > \mu_{\min}$ for all t, then the stabilization of the PV current x_1 and the duty ratio μ at an admissible equilibrium point is guaranteed, due to the minimum phase property of the system established in the previous Section.

VI. Simulation Results

In order to verify the theoretical analysis of the photovoltaic system, the response of the system is simulated for two different cases. The system consists of a string of 20 PV modules which is used as the input voltage of the dc/dc boost converter. The parameters of the dc/dc boost converter are $C = 200\mu F$ and $L = 20mH$. The controller gains are chosen $\lambda = 0.02$ and $\gamma = 0.000003$.

In the first case, at time instant $t = 1\sec$, the irradiance increases from $1000W/m^2$ to $1200W/m^2$, while at time instant $t = 2\sec$, the resistance load changes from $R = 350\Omega$ to $R = 250\Omega$. The reference value for the output voltage is set at $x_2^{ref} = 600V$. At time instant $t = 3\sec$ the reference value changes from $600V$ to $630V$ and at time instant $t = 4\sec$ to $580V$. In this case the output power consumed by the resistance is adequately less than the maximum power that can be produced by the PV string.

Aim of the proposed control is to stabilize the output voltage at the reference value, regardless from the irradiance or resistance load variations. At the same time the PV current and the duty-ratio of the dc/dc boost converter must be stabilized at an admissible equilibrium point. In Fig. 7, we can observe, that after a fast transient, the output voltage is stabilized at the desired steady-state value. In Figures 8 and 9, one can see that the I_{PV}, V_{PV}, $\hat{G}$ and μ exhibit satisfactory transient performance and convergence to the admissible equilibrium point.

In the second case we examine the response of the PV system when it operates very close to the MPP. The reference value of the output voltage is set at $x_2^{ref} = 600V$, while the irradiance and resistance are $R = 250\Omega$ $1000W/m^2$ respectively. The maximum power that can be produced by the PV string under this value of solar irradiance can be calculated to be equal with $P_{PV} = 1702.8W$. At time instant $t = 1\sec$ the resistance changes to $R = 212\Omega$. For this load the consumed power is $P_{out} = 1698.1W$. We can see from Figure 10, that the system is stabilized at the desired equilibrium point. However, at time instant $t = 2\sec$ the resistance changes to $R = 211.4165\Omega$ corresponding to a power $P_{out} \approx P_{PV}^{MPP}$. In this case the zero dynamics become unstable, and as expected the system cannot be stabilized. It is however remarkable that for appropriate controller gains, the system stability and good performance are maintained for output power very close to the maximum power extracted from the PV source. In any case, the simulation results verify the controller effectiveness for almost all the operating points different from the MPP.

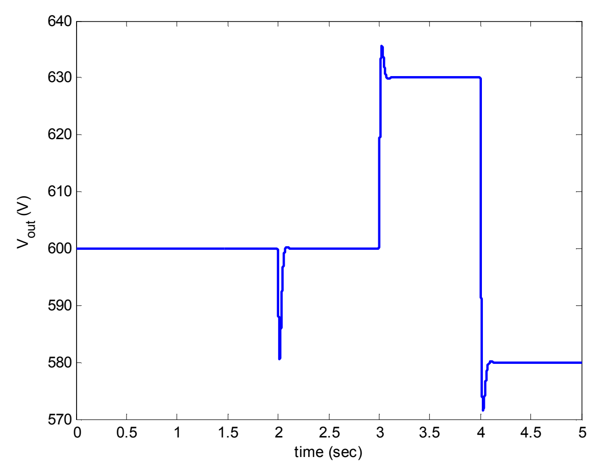

Fig. 7. Output voltage of the dc/dc boost converter for the first case.

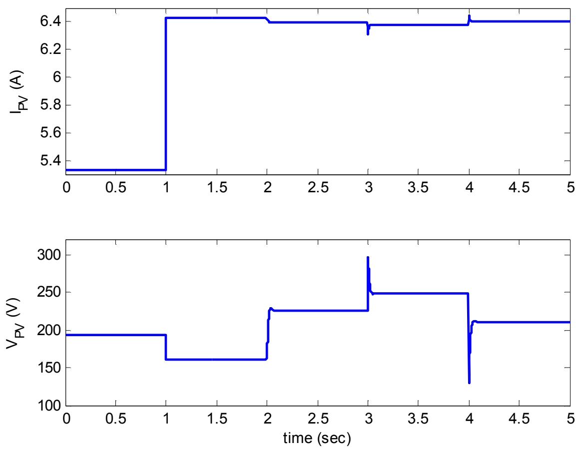

Fig. 8. Current of the PV string and input voltage of the dc/dc boost converter for the first case.

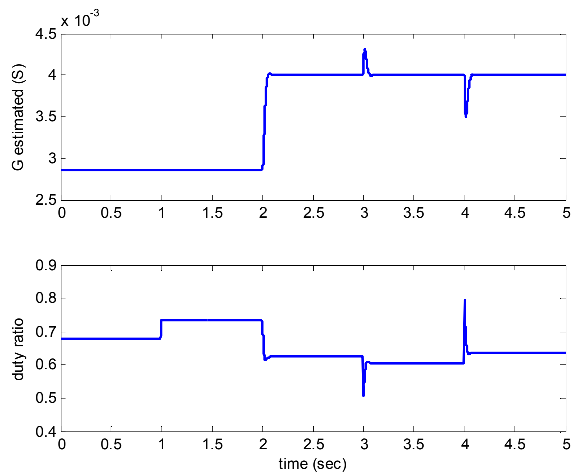

Fig. 9. Estimated conductance of the load and duty ratio of the dc/dc boost converter for the first case.

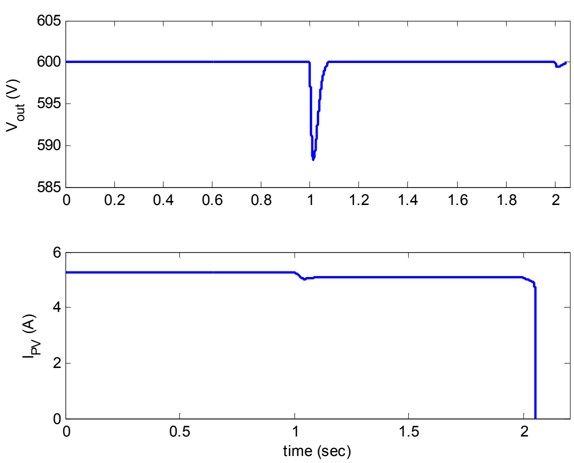

Fig. 10. Output voltage of the dc/dc boost converter and current of the PV string for the second case.

VII. CONCLUSIONS

In this paper an extensive dynamic analysis is presented for a stand-alone PV system. It is shown that the nonlinear PV source plays a significant role in the system behavior. In particular, a minimum phase property with respect to the output voltage is established, under some mild constraints, permitting a direct output voltage controller to be designed. As expected from the theoretical analysis and verified by the simulation results this controller can regulate fast and precisely the output dc voltage at the desired level independently from the load changes or the sun irradiance.

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