

# Robust Stabilization via Disturbance Observer with Noise Reduction

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**Abstract**—In this paper, we propose a robust controller using a modified disturbance observer (DOB). This modification is asked because the conventional DOB structure does not provide with any means to tune the high-frequency response. Since the measurement noise is significant in the high-frequency range, the performance against the noise in the conventional DOB control system cannot be better than that of the closed-loop system without DOB. Inspired by the new structure given in (Xie, 2010), we propose a modified disturbance observer structure, and present a necessary and sufficient condition for robust stability of the actual closed-loop system. Illustrative examples show the effectiveness of the proposed design.

## I. INTRODUCTION

The disturbance observer (DOB) based controller has been widely used among control engineers since it has a powerful ability of uncertainty compensation and disturbance attenuation [1]–[4]. While there have been some research works on stability of the DOB control system [2], [5], [6], most of them presented somewhat conservative sufficient conditions. Recently, the authors of [7] studied a necessary and sufficient condition for robust stability of the DOB control system when time constant of the Q-filter is chosen sufficiently small. Under the assumption that actual uncertain plant  $P$  be of minimum phase, it has been shown that, with an appropriate choice of Q-filter, DOB can robustly stabilize the overall system and make the actual plant  $P$  behave as if it is the nominal plant  $P_n$ . Although the uncertainty compensation and disturbance rejection have been successfully handled by the most DOB approaches, there has been little attention as to how to suppress measurement noise effect, except [8] where a modified DOB structure is presented with  $H_\infty$  design.

In this paper, inspired by the new structure of [8], we propose a robust controller using a modified disturbance observer. This modification is asked because the conventional DOB structure does not provide with any means to tune the high-frequency response, which will be seen in Section II. Since the measurement noise is significant in the high-frequency range, the performance against the noise in the conventional DOB control system cannot be better than that of the closed-loop system without DOB. For the modified DOB that will be proposed in Section III, we present a necessary and sufficient condition for robust stability of the

actual closed-loop system in Section IV. Effectiveness of the proposed design is illustrated by simulation in Section V.

*Notation:* Let  $D(s)$  be a polynomial with real coefficients expressed as  $D(s) = d_n s^n + d_{n-1} s^{n-1} + \dots + d_1 s + d_0$ . The polynomial  $D(s)$  is said to be of *degree*  $n$  if  $d_n \neq 0$ , which will be denoted by  $\deg(D) = n$ . For a transfer function  $G(s) = N(s)/D(s)$  (it is assumed that  $N(s)$  and  $D(s)$  are coprime polynomials), the degree and the relative degree of  $G(s)$  are defined as  $\deg(D)$  and  $\deg(D) - \deg(N)$ , respectively, and the latter will be denoted by  $\text{r.deg}(G)$ . A stable transfer function implies that its denominator is a Hurwitz polynomial. LHP (RHP, respectively) stands for the open left (right, respectively) half complex plane.

## II. CONVENTIONAL DISTURBANCE OBSERVER

The conventional DOB control system is illustrated in Fig. 1, which has been actively studied in, e.g., [1], [2], [6], [7]. (Throughout the paper, it will be simply referred to as the DOB control system.) In the figure,  $P(s)$  and  $P_n(s)$  represent the uncertain plant and its nominal model, respectively, and signals  $r$ ,  $d$  and  $n$  represent reference input, input disturbance, and measurement noise, respectively. It is assumed that the design of the ‘nominal controller’  $C(s)$  is carried out for the nominal model  $P_n(s)$ , and thus, it does not require any information of  $P(s)$ . It is also assumed that  $P_n(s)C(s)$  is strictly proper, and  $P(s)$  and  $P_n(s)$  have the same relative degree. The system represented by  $Q(s)$  (called as ‘Q-filter’) is a stable low-pass filter, which has the form of

$$Q(s) = \frac{c_k(\tau s)^k + c_{k-1}(\tau s)^{k-1} + \dots + c_0}{(\tau s)^l + a_{l-1}(\tau s)^{l-1} + \dots + a_1(\tau s) + a_0} \quad (1)$$

where  $\tau > 0$  is the filter time constant, and  $k$  and  $l$  are nonnegative integers. It is assumed that  $a_0 = c_0$  and  $l \geq k + \text{r.deg}(P_n)$ . The former is necessary for the unity dc gain, i.e.,  $Q(0) = 1$  and the latter for  $Q(s)P_n^{-1}(s)$  to be proper. We also assume that there exists an  $\omega_L > 0$  such that, in the low frequency range  $[0, \omega_L]$ , the disturbance  $d(j\omega)$  and the reference  $r(j\omega)$  are significant while the noise  $n(j\omega) \approx 0$ . On the other hand, it is assumed that there exists an  $\omega_H > \omega_L$  such that, in the high frequency range  $[\omega_H, \infty)$ , the noise  $n(j\omega)$  is significant while the disturbance  $d(j\omega) \approx 0$  and the reference  $r(j\omega) \approx 0$ . Note that, by choosing Q-filter (1) appropriately, it is possible that<sup>1</sup>

$$\begin{aligned} |Q(j\omega)| &\approx 1, & \omega &\in [0, \omega_L] \\ |Q(j\omega)| &\approx 0, & \omega &\in [\omega_H, \infty). \end{aligned} \quad (2)$$

<sup>1</sup>For example, let  $Q(s) = 1/(\tau s + 1)^l$  and  $1/\tau = \sqrt{\omega_L \cdot \omega_H}$  and choose  $l$  as large as required to make  $1/(\tau \omega_H + 1)^l$  sufficiently small.

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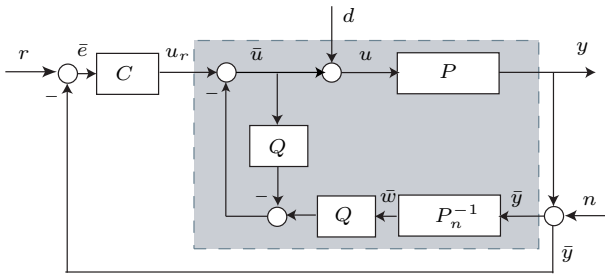


Fig. 1. The conventional disturbance observer (DOB) control system.

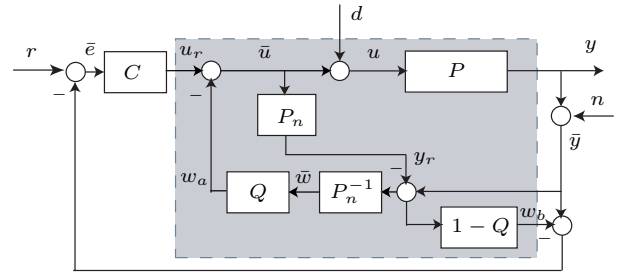


Fig. 2. A noise reduction disturbance observer (NR-DOB) control system.

From Fig. 1, the plant output  $y$  is computed as

$$y(s) = T_{yr}(s)r(s) + T_{yd}(s)d(s) - T_{yn}(s)n(s), \quad (3)$$

where

$$T_{yr} = \frac{P_n P C}{\Delta_D}, \quad T_{yd} = \frac{P_n P (1 - Q)}{\Delta_D}, \quad T_{yn} = \frac{P(Q + P_n C)}{\Delta_D}, \quad (4)$$

$$\Delta_D = P_n(1 + P C) + Q(P - P_n).$$

In the low frequency range  $[0, \omega_L]$ , from (2) we have

$$|T_{yr}(j\omega)| \approx \left| \frac{P_n C}{1 + P_n C}(j\omega) \right|, \quad |T_{yd}(j\omega)| \approx 0, \quad |T_{yn}(j\omega)| \approx 1,$$

which yields

$$y(j\omega) \approx \frac{P_n C}{1 + P_n C}(j\omega) \cdot r(j\omega). \quad (5)$$

Since (5) is the nominal response from the reference to the output, it is expected that, in the low frequency range, the actual plant output of the DOB control system is approximated to the nominal one without DOB.<sup>2</sup>

On the other hand, in the high frequency range  $[\omega_H, \infty)$  where  $d(j\omega) \approx 0$  and  $r(j\omega) \approx 0$ , the output is obtained from (2) and (4) as

$$y(j\omega) \approx -\frac{P C}{1 + P C}(j\omega) \cdot n(j\omega). \quad (6)$$

Note that it is just the same as actual closed-loop system without DOB, and as a result, the noise suppression ability of the DOB controller is not better than the nominal controller.

The control input  $\bar{u}$  is also computed as

$$\bar{u}(s) = T_{\bar{u}r}(s)r(s) - T_{\bar{u}d}(s)d(s) - T_{\bar{u}n}(s)n(s),$$

where

$$T_{\bar{u}r} = \frac{P_n C}{\Delta_D}, \quad T_{\bar{u}d} = \frac{P(P_n C + Q)}{\Delta_D}, \quad T_{\bar{u}n} = \frac{Q + P_n C}{\Delta_D}.$$

Thus, we have from (2), for  $\omega \in [\omega_H, \infty)$ ,

$$|T_{\bar{u}r}(j\omega)| \approx \left| \frac{C}{1 + P C}(j\omega) \right|, \quad |T_{\bar{u}d}(j\omega)| \approx \left| \frac{P C}{1 + P C}(j\omega) \right|,$$

$$|T_{\bar{u}n}(j\omega)| \approx \left| \frac{C}{1 + P C}(j\omega) \right|.$$

This is again the same as for the nominal controller. There-

<sup>2</sup>Using the singular perturbation theory, a rigorous analysis for performance recovery has been done in [10], [11].

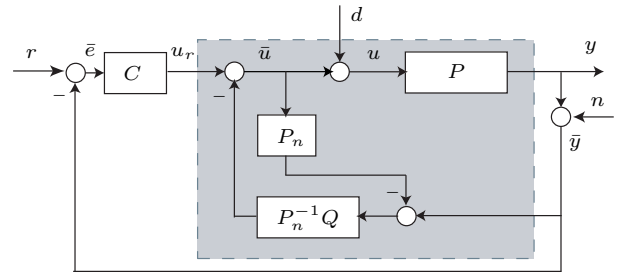


Fig. 3. An equivalent system to the DOB control system shown in Fig. 1.

fore, with respect to the noise suppression ability, the conventional DOB system does not provide any improvement over the nominal controller.

### III. NOISE REDUCTION DISTURBANCE OBSERVER

In this section, we present a modified disturbance observer control system in order to attenuate the measurement noise as well as input disturbance. Fig. 2 shows the control system with a modified disturbance observer, which will be called as noise reduction disturbance observer (NR-DOB). In the low frequency range  $[0, \omega_L]$  where  $|Q(j\omega)| \approx 1$ , the subsystem represented by  $1 - Q(s)$  is almost disconnected and, as a result, the NR-DOB system approximately behaves as shown in Fig. 3. Note that the control system shown in Fig. 3 is equivalent to the DOB system in Fig. 1. Therefore, we expect that, in the low frequency range  $[0, \omega_L]$ , the NR-DOB control system behaves as if it were the DOB control system. Indeed, from Fig. 2, the plant output  $y$  is given by

$$y(s) = T_{yr}(s)r(s) + T_{yd}(s)d(s) - T_{yn}(s)n(s),$$

where

$$T_{yr} = \frac{P_n P C}{(1 + P_n C)(P_n + Q(P - P_n))},$$

$$T_{yd} = \frac{P_n P (1 - Q)}{P_n + Q(P - P_n)}, \quad T_{yn} = \frac{P Q}{P_n + Q(P - P_n)}.$$

In the low frequency range  $[0, \omega_L]$ , we obtain

$$|T_{yr}(j\omega)| \approx \left| \frac{P_n C}{1 + P_n C}(j\omega) \right|, \quad |T_{yd}(j\omega)| \approx 0, \quad |T_{yn}(j\omega)| \approx 1,$$

which yields

$$y(j\omega) \approx \frac{P_n C}{1 + P_n C}(j\omega) \cdot r(j\omega), \quad (7)$$

that is the same as (5). Therefore, in the low frequency range, the actual plant output of the NR-DOB control system is approximated to the nominal one, as is the case for the DOB control system.

On the other hand, in the high frequency range  $[\omega_H, \infty)$ , we have

$$|T_{yr}(j\omega)| \approx \left| \frac{PC}{1 + P_n C}(j\omega) \right|, \quad |T_{yd}(j\omega)| \approx |P(j\omega)|,$$

and

$$|T_{yn}(j\omega)| \approx \left| \frac{PQ}{P_n}(j\omega) \right|. \quad (8)$$

Here, we see a clear contrast between (6) and (8). While (6) has no design freedom to suppress the high-frequency noise, a suitable design of Q-filter may suppress it in (8).

Likewise, we can also analyze how much the control input is affected by the measurement noise. From Fig. 2, the control input  $\bar{u}$  can be computed as

$$\bar{u}(s) = T_{\bar{u}r}(s)r(s) - T_{\bar{u}d}(s)d(s) - T_{\bar{u}n}(s)n(s),$$

where

$$T_{\bar{u}r} = \frac{P_n C}{(1 + P_n C)(P_n + Q(P - P_n))},$$

$$T_{\bar{u}d} = \frac{PQ}{P_n + Q(P - P_n)}, \quad T_{\bar{u}n} = \frac{Q}{P_n + Q(P - P_n)}.$$

In the high frequency range  $[\omega_H, \infty)$ , we have

$$|T_{\bar{u}r}(j\omega)| \approx \left| \frac{C}{(1 + P_n C)}(j\omega) \right|,$$

$$|T_{\bar{u}d}(j\omega)| \approx \left| \frac{P}{P_n}Q(j\omega) \right|, \quad |T_{\bar{u}n}(j\omega)| \approx \left| \frac{1}{P_n}Q(j\omega) \right|.$$

Comparing  $T_{\bar{u}n}$  with that of the previous section, the appearance of  $Q$  enables reduction of noise effect on the control input  $\bar{u}$ .

It should be emphasized that the analysis up until this point is based on the assumption that all the transfer functions are stable. Therefore, in the next section, we study robust stability of the NR-DOB control system.

#### IV. STABILITY ANALYSIS OF NR-DOB

In this section, we study robust stability of the proposed NR-DOB control system, in the perspective of [7]. From Fig. 2, nine transfer functions from  $[r, d, n]^T$  to  $[\bar{e}, \bar{u}, y]^T$  are given by

$$\frac{1}{\Delta(s)} \begin{bmatrix} Q(P - P_n) + P_n, & 0, & 0 \\ P_n C, & \bar{T}, & -Q(1 + P_n C) \\ PP_n C, & P\bar{T}, & \bar{T} \end{bmatrix} \quad (9)$$

where  $\Delta(s) = (1 + P_n C)(P_n + Q(P - P_n))$  and  $\bar{T} = P_n(1 + P_n C)(1 - Q)$ . If the above nine transfer functions are stable, then the NR-DOB control system is internally stable. In order to analyze the stability, let  $P$ ,  $P_n$ ,  $C$ , and  $Q$  be represented

by some ratios of coprime polynomials, that is,

$$P(s) = \frac{N(s)}{D(s)}, \quad P_n(s) = \frac{N_n(s)}{D_n(s)}, \quad (10)$$

$$C(s) = \frac{N_c(s)}{D_c(s)}, \quad Q(s) = \frac{N_Q(s; \tau)}{D_Q(s; \tau)},$$

where  $N_Q(s; \tau) = c_k(\tau s)^k + c_{k-1}(\tau s)^{k-1} + \dots + c_0$  and  $D_Q(s; \tau) = (\tau s)^l + a_{l-1}(\tau s)^{l-1} + \dots + a_1(\tau s) + a_0$ . With them, (9) becomes

$$\frac{1}{\delta} \begin{bmatrix} R_{11}, & 0, & 0 \\ D_n D D_Q N_n N_c, & R_{22}, & -D_n D N_Q (D_n D_c + N_n N_c) \\ D_n D_Q N N_n N_c, & R_{32}, & R_{22} \end{bmatrix},$$

where

$$\delta := (D_c D_n + N_c N_n)(N_n D D_Q + N_Q (N D_n - D N_n)),$$

$$R_{11} := D_n D_c [N_Q (N D_n - N_n D) + D D_Q N_n],$$

$$R_{22} := D N_n (D_n D_c + N_n N_c) (D_Q - N_Q),$$

$$R_{32} := N N_n (D_n D_c + N_n N_c) (D_Q - N_Q).$$

Then the NR-DOB control system is internally stable if and only if the characteristic polynomial  $\delta(s; \tau)$  is Hurwitz. Let

$$m = \deg(DD_c N_n D_n).$$

Then, for given  $\tau > 0$ , the highest power of  $s$  in  $\delta(s; \tau)$  depends on the term  $D_c D_n N_n D D_Q$  (since  $\text{r.deg}(P) = \text{r.deg}(P_n) = r$  and  $\text{r.deg}(Q) \geq r$ ) and as a result

$$\deg(\delta(s; \tau)) = \deg(D_c D_n N_n D D_Q) = m + l,$$

which implies that  $\delta(s; \tau)$  has  $m+l$  roots. To proceed further, we make the following assumption.

**Assumption 1:** Let the set  $\mathcal{P}$  of transfer functions be

$$\mathcal{P} = \left\{ P(s) = \frac{\beta_{n-r} s^{n-r} + \beta_{n-r-1} s^{n-r-1} + \dots + \beta_0}{\alpha_n s^n + \alpha^{n-1} s^{n-1} + \dots + \alpha_0} \right. \\ \left. : \alpha_i \in [\underline{\alpha}_i, \bar{\alpha}_i], \beta_i \in [\underline{\beta}_i, \bar{\beta}_i] \right\}$$

where all  $\underline{\alpha}_i$ ,  $\bar{\alpha}_i$ ,  $\underline{\beta}_i$ , and  $\bar{\beta}_i$  are known constants such that  $0 \notin [\underline{\alpha}_n, \bar{\alpha}_n]$  and  $0 \notin [\underline{\beta}_{n-r}, \bar{\beta}_{n-r}]$ . Both the actual plant  $P(s)$  and its nominal model  $P_n(s)$  are contained in  $\mathcal{P}$ .  $\diamond$

Let

$$p_s(s) := (D_c(s)D_n(s) + N_c(s)N_n(s))N(s)D_n(s),$$

$$p_f(s) := D_Q(s; 1) + \left( \lim_{s \rightarrow \infty} \frac{P(s)}{P_n(s)} - 1 \right) N_Q(s; 1).$$

Then, since  $\text{r.deg}(P) = \text{r.deg}(P_n)$ , it follows that  $\deg(N D_n) = \deg(N_n D)$ , which implies that  $\deg(p_s) = \deg(D_c D_n N D_n) = \deg(D_c D_n N_n D) = m$ . Thus, the polynomial  $p_s(s)$  has  $m$  roots.

The following lemma shows that, as  $\tau$  approaches zero, all roots of  $\delta(s; \tau) = 0$  are related with those of  $p_s(s)$  and  $p_f(s)$ .

**Lemma 1:** Let  $s_1^*, \dots, s_m^*$  and  $s_{m+1}^*, \dots, s_{m+l}^*$  be the roots of  $p_s(s) = 0$  and  $p_f(s) = 0$ , respectively. Then,  $m + l$  roots of  $\delta(s; \tau) = 0$ , say  $s_i(\tau)$ ,  $i = 1, \dots, m + l$ , satisfy that (a)  $\lim_{\tau \rightarrow 0} s_i(\tau) = s_i^*$  for  $i = 1, \dots, m$ , and (b)  $\lim_{\tau \rightarrow 0} \tau s_i(\tau) = s_i^*$ , for  $i = m + 1, \dots, m + l$ .  $\diamond$

*Proof:* Since  $D_Q(s; 0) = N_Q(s; 0) = a_0 \neq 0$ , we have

$$\begin{aligned} \delta(s; 0) &= (D_c(s)D_n(s) + N_c(s)N_n(s))N(s)D_n(s)a_0 \\ &= a_0 p_s(s). \end{aligned}$$

Thus,  $m$  roots of  $\delta(s; \tau) = 0$  converge to those of  $p_s(s) = 0$  as  $\tau$  approaches zero. Therefore, the claim (a) directly follows.

On the other hand, the remaining  $l$  roots of  $\delta(s; \tau) = 0$  goes to infinity as  $\tau$  approaches zero. To investigate those roots, let

$$\bar{\delta}(s; \tau) := \tau^m \delta\left(\frac{s}{\tau}; \tau\right).$$

Then, we have

$$\bar{\delta}(s; \tau) = \gamma_1(s; \tau)D_Q\left(\frac{s}{\tau}; \tau\right) + \gamma_2(s; \tau)N_Q\left(\frac{s}{\tau}; \tau\right),$$

where

$$\begin{aligned} \gamma_1(s; \tau) &:= \tau^m [(D_c D_n + N_c N_n) N_n D] |_{s=s/\tau}, \\ \gamma_2(s; \tau) &:= \tau^m [(D_c D_n + N_c N_n) (N D_n - D N_n)] |_{s=s/\tau} \end{aligned}$$

Using the fact that  $D_Q(s/\tau; \tau) = D_Q(s; 1)$  and  $N_Q(s/\tau; \tau) = N_Q(s; 1)$ ,  $\bar{\delta}(s; \tau)$  can be written as

$$\bar{\delta}(s; \tau) = \gamma_1(s; \tau)D_Q(s; 1) + \gamma_2(s; \tau)N_Q(s; 1).$$

Since  $P_n C$  is strictly proper, it follows that  $\deg(D_c D_n) > \deg(N_c N_n)$  and thus  $m = \deg(D_c D_n N_n D) > \deg(N_c N_n N_n D)$ , which implies that

$$\begin{aligned} \lim_{\tau \rightarrow 0} \gamma_1(s; \tau) &= \lim_{\tau \rightarrow 0} \tau^m [D_c D_n N_n D] |_{s=s/\tau} = \bar{\gamma}_1 s^m, \\ \lim_{\tau \rightarrow 0} \gamma_2(s; \tau) &= \bar{\gamma}_2 s^m, \end{aligned}$$

for some constants  $\bar{\gamma}_1$  and  $\bar{\gamma}_2$ . Note that  $\bar{\gamma}_1 \neq 0$  since  $\deg(D_c D_n N_n D) = m$ . Thus, we obtain

$$\begin{aligned} \bar{\delta}(s; 0) &= \lim_{\tau \rightarrow 0} \bar{\delta}(s; \tau) \\ &= \lim_{\tau \rightarrow 0} \gamma_1(s; \tau)D_Q(s; 1) + \lim_{\tau \rightarrow 0} \gamma_2(s; \tau)N_Q(s; 1) \\ &= \bar{\gamma}_1 s^m \left[ D_Q(s; 1) + \frac{\bar{\gamma}_2}{\bar{\gamma}_1} N_Q(s; 1) \right]. \end{aligned}$$

Since

$$\begin{aligned} \frac{\bar{\gamma}_2}{\bar{\gamma}_1} &= \frac{\lim_{\tau \rightarrow 0} \tau^m [D_c D_n (N D_n - D N_n)] |_{s=s/\tau}}{\lim_{\tau \rightarrow 0} \tau^m (D_c D_n N_n D) |_{s=s/\tau}} \\ &= \lim_{s \rightarrow \infty} \frac{P(s)}{P_n(s)} - 1, \end{aligned}$$

it follows that

$$\bar{\delta}(s; 0) = \bar{\gamma}_1 s^m p_f(s),$$

which implies that  $\bar{\delta}(s; 0) = 0$  has  $m$  roots at the origin and  $l$  roots at  $s_{m+1}^*, \dots, s_{m+l}^*$ . Therefore, applying Lemma 1 of [7], we see that there exist  $l$  roots of  $\bar{\delta}(s; \tau) = 0$ , say  $\bar{s}_i(\tau)$ ,  $i = m+1, \dots, m+l$ , such that  $\lim_{\tau \rightarrow 0} \bar{s}_i(\tau) = s_i^*$ . Since  $\bar{s}_i(\tau)/\tau$  are roots of  $\delta(s; \tau) = 0$ , the claim (b) is proved. ■

Based on Lemma 1, we present an almost necessary and sufficient condition for robust stability of the NR-DOB system.

**Theorem 1:** Under Assumption 1, there exists a constant

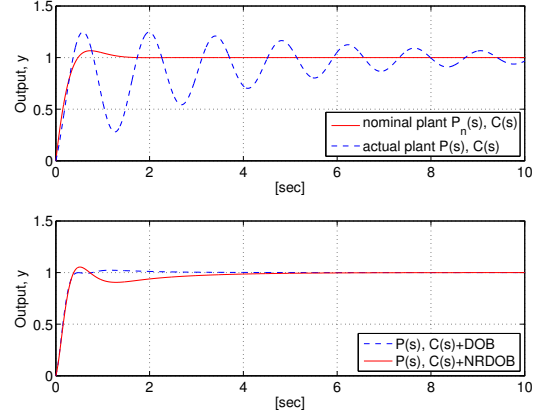


Fig. 4. Step response of the closed loop system. Upper: nominal controller responses for nominal plant  $P_n(s)$  and actual plant  $P(s)$ ; lower: the DOB and the NR-DOB controller responses for actual plant  $P(s)$ . In this simulation, Q-filter is chosen as (13).

$\tau^* > 0$  such that, for all  $0 < \tau \leq \tau^*$ , the NR-DOB control system is internally stable for all  $P(s) \in \mathcal{P}$  if the following four conditions hold;

- 1)  $P_n C / (1 + P_n C)$  is stable,
- 2)  $P_n$  is stable,
- 3)  $P(s)$  is of minimum phase for all  $P(s) \in \mathcal{P}$ ,
- 4)  $p_f(s)$  is Hurwitz for all  $P(s) \in \mathcal{P}$ .

On the contrary, for given  $P(s) \in \mathcal{P}$ , there is  $\tau^* > 0$  such that, for all  $0 < \tau \leq \tau^*$ , the closed loop system is unstable if at least one of the conditions (1)–(4) is violated in the sense that  $P_n C / (1 + P_n C)$  or  $P_n$  has some poles in the RHP, or some zeros of  $P(s)$  or some roots of  $p_f(s) = 0$  are located in the RHP. ◊

*Proof:* Note that the denominator of  $P_n C / (1 + P_n C)$ , the numerator of  $P(s)$ , and the denominator of  $P_n(s)$  are  $(D_c(s)D_n(s) + N_c(s)N_n(s))$ ,  $N(s)$ , and  $D_n(s)$ , respectively. Thus, three conditions 1), 2), and 3) are equivalent to the condition that the polynomial  $p_s(s)$  is stable. Therefore, the proof directly follows from Lemma 1. ■

**Remark 1:** Theorem 1 indicates that the NR-DOB control system can be robustly stabilized provided that  $P_n(s)$  is stable,  $C(s)$  stabilizes the nominal model  $P_n(s)$ , and uncertain plant  $P(s)$  is of minimum phase. In contrast to the DOB control system [7], the NR-DOB control system additionally requires that  $P_n(s)$  is stable, which can be understood as a price for the noise suppression. ◊

## V. EXAMPLE AND SIMULATION

To illustrate the effectiveness of the NR-DOB control system, we consider, for example, the plant whose transfer function is given by

$$P_n(s) = \frac{s + 3}{s^2 + 6s + 8}. \quad (11)$$

It is also supposed that the primary control goal is to achieve zero steady-state error (to step reference) with overshoot less than 20% and settling time less than 4 seconds. To this end,

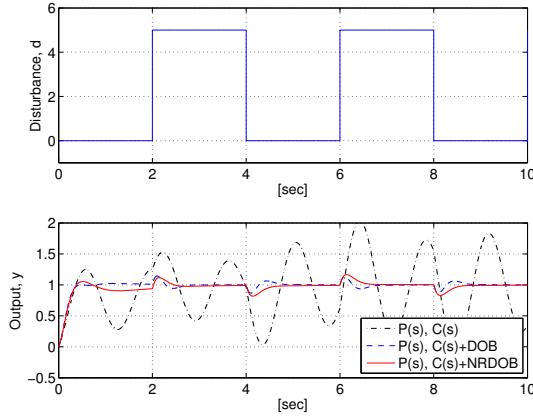


Fig. 5. Disturbance attenuation ability. Upper: disturbance signal  $d(t)$ ; lower: responses of nominal controller, DOB controller, and NR-DOB controller for actual plant  $P(s)$  in the presence of input disturbance  $d(t)$ . In this simulation, Q-filter is chosen as (13).

a nominal controller is designed as

$$C(s) = \frac{4s + 20}{s}.$$

The step response of the closed loop system consisting of  $P_n(s)$  and  $C(s)$  is depicted in Fig. 4 (upper figure, solid line), where it is seen that the control objective is clearly achieved.

Now, suppose that, probably due to parameter changes, the transfer function of the plant is no longer equal to  $P_n(s)$  but changed to

$$P(s) = \frac{0.5(s + 1)}{s^2 - s + 8}, \quad (12)$$

which will hereafter be referred to as actual plant. We also assume that  $P(s)$  itself is not known but it is known that  $0.5 \leq \gamma(P) := \lim_{s \rightarrow \infty} \frac{P(s)}{P_n(s)} \leq 2$ . Fig. 4 (upper figure, dashed line) shows that the step response becomes much worse for the actual plant, and hence the control objective is not be accomplished. In order to achieve the control objective in spite of the plant uncertainty, we introduce the conventional DOB and the NR-DOB controllers (for comparison). With Q-filter

$$Q(s) = \frac{1}{(\tau s + 1)^4}, \quad \tau = 0.01, \quad (13)$$

the robust stability condition of both Theorem 1 and that in [7] holds. (Indeed, the polynomial  $p_f(s)$  becomes  $s^4 + 4s^3 + 6s^2 + 4s + \gamma(P)$ . Since both polynomials  $s^4 + 4s^3 + 6s^2 + 4s + 0.5$  and  $s^4 + 4s^3 + 6s^2 + 4s + 2$  are Hurwitz, it follows from Kharitonov theorem that  $p_f(s)$  is also Hurwitz for all  $P(s)$  satisfying  $0.5 \leq \gamma(P) \leq 2$ . Thus, according to Theorem 1 and that in [7], two DOB control systems are robustly stable with sufficiently small  $\tau$ , and computer simulation shows that both control systems are stable with  $\tau = 0.01$ .)

The step responses of the DOB and the NR-DOB control systems are shown in the lower figure of Fig. 4, where it is seen that those are much similar to that of 'nominal

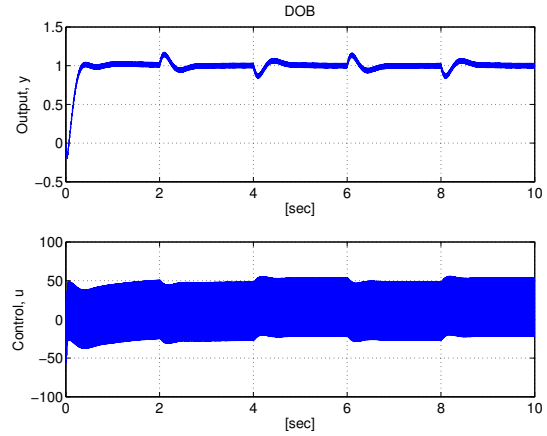


Fig. 6. Noise reduction ability of the DOB controller for actual plant  $P(s)$ . Upper: plant output,  $y(t)$ ; lower: controller output,  $\bar{u}(t)$ . Input disturbance is chosen as in Fig. 5, measurement noise is selected as (14), and Q-filter is chosen as (13).

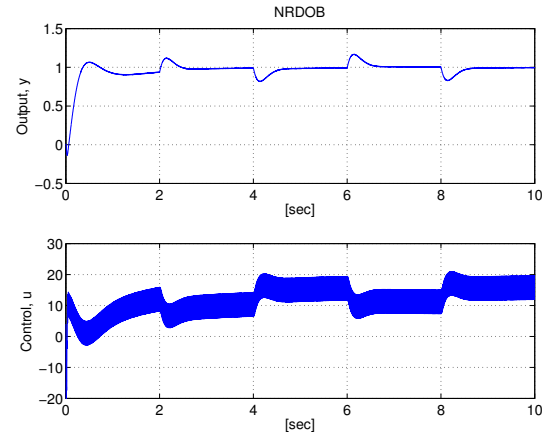


Fig. 7. Noise reduction ability of the NR-DOB controller for actual plant  $P(s)$ . Upper: plant output,  $y(t)$ ; lower: controller output,  $\bar{u}(t)$ . Input disturbance, measurement noise, and Q-filter are chosen the same as in Fig. 6.

response' (upper figure of Fig. 4, solid line) in spite of plant uncertainty. This implies that the performance of the nominal closed-loop system is recovered for both cases of two DOBs. Next, we verify the disturbance attenuation ability of the NR-DOB controller. To this end, we consider an input disturbance  $d(t)$  shown in the upper figure of Fig. 5. The step responses in the presence of disturbance  $d(t)$  are shown in the lower figure of Fig. 5, where it is seen that the step responses of the DOB and the NR-DOB control systems are quite similar to the nominal response although there exist modeling error and input disturbance. This verifies the performance recovery property (5) or (7).

From now on, we compare the noise reduction ability of the DOB and the NR-DOB control systems. We assume that the measurement noise is given by

$$n(t) = 10 \sin(200\pi t). \quad (14)$$

The simulation results for the DOB and the NR-DOB sys-

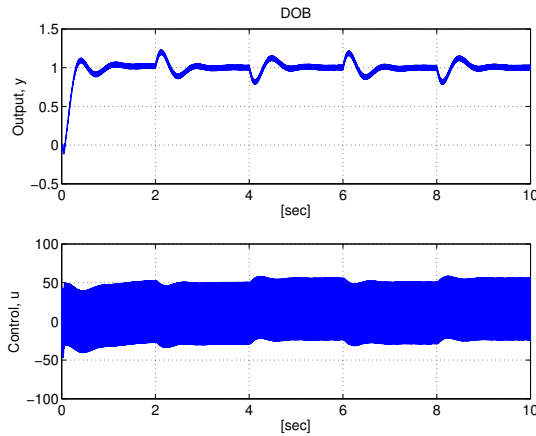


Fig. 8. Noise reduction ability of the NR-DOB controller using Q-filter (15). Upper: plant output,  $y(t)$ ; lower: controller output,  $\bar{u}(t)$ . Input disturbance is chosen as in Fig. 5, measurement noise is selected as (14), and Q-filter is chosen as (15).

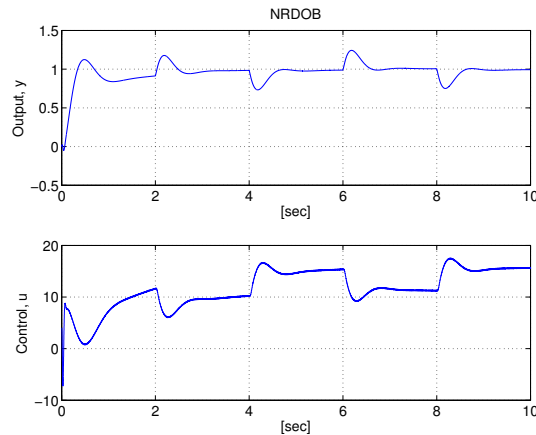


Fig. 9. Noise reduction ability of the NR-DOB controller for actual plant  $P(s)$ . Upper: plant output,  $y(t)$ ; lower: controller output,  $\bar{u}(t)$ . Input disturbance, measurement noise, and Q-filter are chosen the same as in Fig. 8.

tems are depicted in Fig. 6 and 7, respectively. As discussed in Section III, it is seen that the noise suppression ability of the NR-DOB system is superior to that of the DOB system. In particular, with the help of NR-DOB controller, the noise effect is almost removed from  $y$ .

However, the noise effect is still apparent in the control input  $\bar{u}$  for both cases. In order to attenuate it further, we may redesign  $Q(s)$  as, for example,

$$Q(s) = \frac{1}{(\tau s + 1)^6}, \quad \tau = 0.01. \quad (15)$$

(With this new  $Q(s)$ , it can be checked that the polynomial  $p_f(s)$  is Hurwitz for all  $P(s)$  satisfying  $0.5 \leq \gamma(P) \leq 2$ , and the closed-loop system is stable with both the DOB and the NR-DOB controller with  $\tau = 0.01$ .) The simulation result using the Q-filter (15) is shown in Fig. 8 and Fig. 9, which indicates that the noise reduction performance of NR-DOB becomes much better than that of (13). It is also

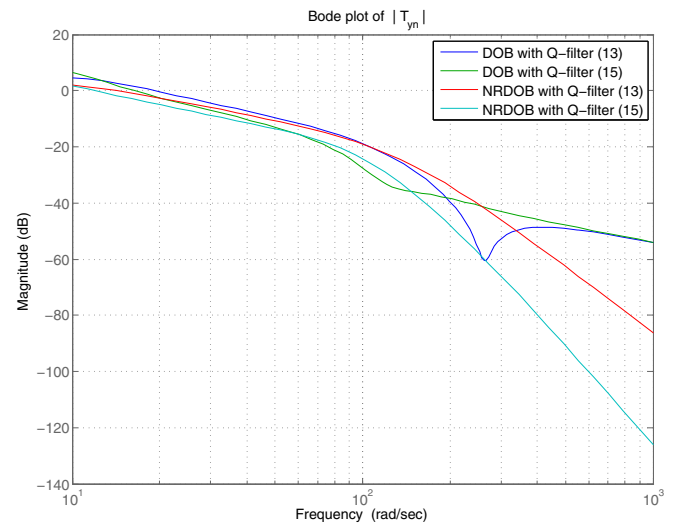


Fig. 10. Bode magnitude plots of  $T_{yn}(j\omega)$  for the cases of the conventional DOB and the NR-DOB using (13) and (15).

seen that the noise reduction of Fig. 8 is not better than that of Fig. 6, which recalls the fact that the conventional DOB has no control to tune the high-frequency response (see (6)). Indeed, this fact is also seen from the Bode magnitude plots of  $T_{yn}(j\omega)$  in Fig. 10 where all four cases are drawn. As expected, the magnitudes of  $T_{yn}(j\omega)$  with the Q-filter of (13) and (15) are similar to each other in high-frequency range.

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