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Adaptive model predictive control for constrained linear systems

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***Abstract*— A novel adaptive output feedback control technique for uncertain linear systems is proposed, able to cope with input and output constraints and measurement noise. At each time step, the collected input-output data are exploited to refine the set of models that are consistent with the available information on the system. Then, the control input is computed according to a receding horizon strategy, which guarantees recursive constraint satisfaction for all the admissible models, hence also for the actual plant. The technique relies only on the solution of linear and quadratic programs. The effectiveness of the approach is illustrated in a numerical example.**

I. Introduction

The idea of adaptive control is to carry out controller adjustments in real time, on the basis of input-output data collected on-line, in order to control an initially unknown or time varying dynamic system. Although adaptive control has been studied extensively for decades and a well established theory has been developed, there are few results for adaptive control of systems that are subject to constraints [1]. This limits the use of adaptive control techniques in practical applications, where often both uncertain dynamics and input and/or output constraints are present. The two main existing approaches to adaptive control of systems with input constraints are pole placement control with anti-windup compensation [2] and one-step-ahead predictive control [3], combined with recursive least square identification. Both approaches have limitations, such as deterioration of the reference tracking performance in the anti-windup schemes or the need for exact knowledge on some characteristics of the controlled system, like transport delay and sign of the first nonzero impulse response coefficient, in the case of one-step-ahead predictive control. In addition, these control strategies are not able to handle output constraints.

It is interesting to note that model predictive control (MPC) originated as a control algorithm for adaptive control schemes, and in its initial form it did not take constraints into consideration [4]. Over the years, a lot of progress has been made in the MPC theory, outside the adaptive context, and as a result MPC became a powerful approach to control systems with constraints [5]. However, the topic of adaptive MPC for constrained systems received little attention, mainly due to the difficulty in guaranteeing stability and recursive feasibility when adaptive schemes are adopted [6].

In [7], simultaneous MPC and identification based on recursive least squares was studied. Additional constraints were added to the MPC formulation in order to guarantee persistence of excitation. However this gives rise to a difficult optimization problem. In [8] an adaptive MPC algorithm based on modified recursive least squares identification and tube-like robust MPC was proposed. The algorithm guarantees stability and recursive feasibility, but the condition for persistence of excitation is not considered and noise free measurements of the plant states are required, which might be a significant limitation. Nonlinear adaptive MPC for a specific class of systems was considered in [9]. Set membership (SM) identification was used for adaptive MPC in [10], where an explicit MPC law is repeatedly re-calculated off-line when new information on the controlled plant becomes available. Outside the context of adaptive control, the idea of combining SM identification with MPC as a tool for controlling a known linear time invariant plant with states that can not be directly measured was proposed in [11].

We propose here a new adaptive model predictive control approach for linear systems, subject to input and output constraints as well as measurement noise. In particular, we consider a class of single input, single output systems that are time invariant, but uncertain. By using a novel real-time SM identification algorithm, the set of all possible models that are consistent with the initial information, named the Feasible System Set (FSS), is refined on-line, on the basis of the measured plant output. The MPC controller then uses the computed FSS in order to control the plant output to track a desired reference, while at the same time enforcing the input and output constraints. The proposed control algorithm guarantees recursive feasibility and it requires the solution of only linear and quadratic programs at each time step.

II. Problem Statement

We consider a single input, single output (SISO), discrete time, strictly proper linear time invariant (LTI) system S in the infinite impulse response (IIR) form

$$S \doteq \{h_S\}_1^\infty, \tag{1}$$

where $\{h_S\}_1^\infty = \{h_S(1), h_S(2), \ldots\}$ are the impulse response coefficients. At any time step $t \in \mathbb{Z}$, the output y of system (1) is given by:

$$y(t) = \sum_{i=1}^{\infty} h_S(i)u(t-i) \doteq \mathbf{h_S} * \mathbf{u}[t], \tag{2}$$

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and the measured system output is:

$$\tilde{y}(t) = y(t) + v(t) \tag{3}$$

where $*$ is the convolution operator, $u(t) \in \mathbb{R}$ is the control input and $v(t) \in \mathbb{R}$ is the measurement noise. The system and the disturbances are characterized by the following assumptions.

Assumption 1: (Prior assumption on noise) v is bounded as:

$$|v(t)| \quad \leq \quad \epsilon, \forall t \in \mathbb{Z}, \tag{4}$$

where ϵ is a positive scalar. ■

Assumption 2: (Prior assumption on the system)

$$S \in \mathcal{K}(L, \rho, \mu), \tag{5}$$

where, for given ρ, $L \in \mathbb{R}$: $\rho \in (0, 1)$, $L > 0$ and $\mu \in \mathbb{N}$:

$$\mathcal{K}(L, \rho, \mu) \doteq \left\{ \{h\}_1^\infty : \begin{array}{ll} |h(i)| \leq L & i = 1, \ldots, \mu \\ |h(i)| \leq L\rho^{i-\mu} & i = \mu + 1, \ldots \end{array} \right\}. \tag{6}$$

This means that the impulse response coefficients of the true system $\{h_S\}_1^\infty$ belong to a set of all impulse response coefficients $\{h\}_1^\infty$ that satisfy the inequalities in (6). ■

Under these assumptions, the goal is to design a controller to track a known desired output $y_{\text{des}}(t)$, while satisfying both input and output constraints. The latter are given as:

$$\begin{array}{lll} |u(t)| & \leq & \overline{u} \\ |\Delta u(t)| & \leq & \overline{\Delta u} \\ |y(t)| & \leq & \overline{y} \end{array}, \forall t \in \mathbb{Z}, \tag{7}$$

where $\Delta u(t) = u(t) - u(t-1)$ is the rate of change of the control input.

III. Real-time set membership identification

The first step we take to solve the proposed control problem is to devise a recursive algorithm which, based on the past sequence of applied control inputs and measured plant outputs, returns a set containing all the models that are consistent with the available information, as well as a nominal model of the system (1). Moreover, for any given sequence of inputs $\mathcal{U}_{-\infty}^{t-1}$, where $\mathcal{U}_{t_1}^{t_2} \doteq \{u(t_1), \ldots, u(t_2)\}$, we provide a way to calculate guaranteed bounds on the possible future plant output.

A. Computation of the Feasible System Set

Following a SM approach [12], we first consider the set of all systems that are consistent with the problem settings. In particular, let us denote the infinite sequence of collected input–output data as:

$$\mathcal{M}_{-\infty}^t : \tilde{\mathcal{U}}_{-\infty}^{t-1}, \{\tilde{y}(i)\}_{-\infty}^t, \tag{8}$$

where $\tilde{\mathcal{U}}_{-\infty}^{t-1}$ is an infinitely long sequence of applied control inputs and $\{\tilde{y}(i)\}_{-\infty}^t$ is its corresponding sequence of measured plant outputs. On the basis of Assumptions 1 and 2, for an infinite sequence of input-output data (8), the tightest set which is guaranteed to contain the system S is:

$$\mathcal{F}(\mathcal{M}_{-\infty}^t) \doteq \left\{ \begin{array}{r} \{h\}_1^\infty \in \mathcal{K}(L, \rho, \mu) : |\tilde{y}(i) - \mathbf{h} * \tilde{\mathbf{u}}[i]| \leq \epsilon, \\ \forall i \in (-\infty, t] \end{array} \right\}, \tag{9}$$

The set $\mathcal{F}(\mathcal{M}_{-\infty}^t)$ can not be directly exploited, since it contains infinite-dimensional elements and it is defined by infinitely many constraints. In order to have a tractable problem, we trade-off some identification accuracy and consider a set $\mathcal{K}_m(L, \rho, \mu)$ whose elements are FIR models of length $m > \mu$.:

$$\mathcal{K}_m(L, \rho, \mu) \doteq \left\{ \{h_m\}_1^m : \begin{array}{ll} |h_m(i)| \leq L & i = 1, \ldots, \mu \\ |h_m(i)| \leq L\rho^{i-\mu} & i = \mu+1, \ldots, m \end{array} \right\}. \tag{10}$$

The output y_m for any model $\{h_m\}_1^m \in \mathcal{K}_m(L, \rho, \mu)$ depends on a finite sequence of control inputs:

$$y_m(t) = \mathbf{h_m} * \mathbf{u}[t] = \sum_{i=1}^{m} h_m(i) u(t-i). \tag{11}$$

Note that, for any sequence $\mathcal{U}_{-\infty}^{t-1}$ satisfying the input constraints in (7) and any element $\{h\}_1^\infty \in \mathcal{F}(\mathcal{M}_{-\infty}^t)$, there exists an element $\{h_m\}_1^m \in \mathcal{K}_m(L, \rho, \mu)$ such that $\{h_m\}_1^m = \{h\}_1^m$, and the truncation error is bounded by:

$$|\mathbf{h} * \mathbf{u}[t] - \mathbf{h_m} * \mathbf{u}[t]| \leq \eta_m, \tag{12}$$

where $\eta_m = \overline{u} L \rho^{m-\mu} \frac{\rho}{1-\rho}$. Moreover, we consider the information given by a finite sequence of past input-output data, measured from an initial time step (taken to be equal to zero without loss of generality) up to a finite time $t \geq m$:

$$\mathcal{M}_0^t : \tilde{\mathcal{U}}_0^{t-1}, \{\tilde{y}(i)\}_{i=m}^t, \tag{13}$$

where $\tilde{\mathcal{U}}_0^{t-1}$ denotes a finite length sequence of known, past control inputs. We can now define, at a given time step t, the set $\mathcal{F}_m(\mathcal{M}_0^t)$, as the set containing all system models of the form (10), that are consistent with the available prior information and collected input–output data (13):

$$\mathcal{F}_m(\mathcal{M}_0^t) \doteq \left\{ \begin{array}{l} \{h_m\}_1^m \in \mathcal{K}_m(L, \rho, \mu) : \\ |\tilde{y}(i) - \mathbf{h_m} * \tilde{\mathbf{u}}[i]| \leq \epsilon + \eta_m, \\ \forall i = m, \ldots, t \end{array} \right\}, \tag{14}$$

By defining the matrices:

$$\tilde{U}(t) \doteq \begin{bmatrix} \tilde{u}(t-1) & \ldots & \tilde{u}(t-m) \\ \tilde{u}(t-2) & \ldots & \tilde{u}(t-m-1) \\ \vdots & \vdots & \vdots \\ \tilde{u}(m-1) & \ldots & \tilde{u}(0) \end{bmatrix} \in \mathbb{R}^{(t-m) \times m}$$

$$\overline{Y}(t) \doteq \begin{bmatrix} \tilde{y}(t) + \epsilon + \eta_m \\ \vdots \\ \tilde{y}(m) + \epsilon + \eta_m \end{bmatrix} \in \mathbb{R}^{t-m}$$

$$\underline{Y}(t) \doteq \begin{bmatrix} \tilde{y}(t) - \epsilon - \eta_m \\ \vdots \\ \tilde{y}(m) - \epsilon - \eta_m \end{bmatrix} \in \mathbb{R}^{t-m}$$

$$H^{\max} \doteq [\underbrace{L, \ldots, L}_{\mu} \underbrace{L\rho, \ldots, L\rho^{m-\mu}}_{m-\mu}]^T$$

$$H^{\min} \doteq [\underbrace{-L, \ldots, -L}_{\mu} \underbrace{-L\rho, \ldots, -L\rho^{m-\mu}}_{m-\mu}]^T$$

where ^{T} stands for the matrix transpose operation, the set $\mathcal{F}_m(\mathcal{M}_0^t)$ (14) can be equivalently written as

$$\mathcal{F}_m(\mathcal{M}_0^t)=\left\{ H_m \in \mathbb{R}^m : \begin{array}{lll} H_m & \leq & H^{\max} \\ H_m & \geq & H^{\min} \\ \tilde{U}(t)H_m & \leq & \overline{Y}(t) \\ \tilde{U}(t)H_m & \geq & \underline{Y}(t) \end{array} \right\}, \quad (15)$$

where $H_m = [h_m(1), \ldots, h_m(m)]^T$ and $\leq, \geq$ are to be interpreted as element-wise inequalities. The set $\mathcal{F}_m(\mathcal{M}_0^t)$ (15) is a compact set defined by linear inequalities, i.e. a polytope, containing all possible FIR models of the system (1), which are consistent with Assumptions 1 and 2 and with the information given by the finite sequence of measured data (13). Such a polytope can be described in a compact form by a set of non-redundant inequalities:

$$\mathcal{F}_m(\mathcal{M}_0^t) = \left\{ H_m \in \mathbb{R}^m : A(\mathcal{M}_0^t)H_m \leq b(\mathcal{M}_0^t) \right\}, \quad (16)$$

where $A(\mathcal{M}_0^t) \in \mathbb{R}^{n(t)\times m}$ and $b(\mathcal{M}_0^t) \in \mathbb{R}^{n(t)}$ and $n(t)$ is the number of non-redundant inequalities. Based on the definition of the set $\mathcal{F}_m(\mathcal{M}_0^t)$, we can now define the feasible system set as:

$$FSS(\mathcal{M}_0^t) \doteq \left\{ \{h\}_1^\infty \in \mathcal{K}(L, \rho, \mu) : \{h\}_1^m \in \mathcal{F}_m(\mathcal{M}_0^t) \right\}. \quad (17)$$

Unlike the set $\mathcal{F}(\mathcal{M}_{-\infty}^t)$, the feasible system set $FSS(\mathcal{M}_0^t)$ can be exploited due to the finite dimensionality of the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$. However, this is compensated by the fact that the feasible system set $FSS(\mathcal{M}_0^t)$ is more conservative than the set $\mathcal{F}(\mathcal{M}_{-\infty}^t)$, i.e. $\mathcal{F}(\mathcal{M}_{-\infty}^t) \subseteq FSS(\mathcal{M}_0^t), \forall t$.

B. Real-time refinement of the feasible system set

In order to be able to incorporate the knowledge of $FSS(\mathcal{M}_0^t)$ in an adaptive control algorithm, the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$ has to be updated recursively. To this end, we consider the fact that in principle the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$ can be calculated as the intersection of the polytope $\mathcal{F}_m(\mathcal{M}_0^{t-1})$ with the two half spaces defined by the newly measured plant output:

$$\begin{aligned} \mathcal{F}_m(\mathcal{M}_0^t) = \; & \mathcal{F}_m(\mathcal{M}_0^{t-1}) \\ & \cap \{H_m \in \mathbb{R}^m : \tilde{\varphi}(t)^T H_m \leq \tilde{y}(t) + \epsilon + \eta_m\} \\ & \cap \{H_m \in \mathbb{R}^m : -\tilde{\varphi}(t)^T H_m \leq -\tilde{y}(t) + \epsilon + \eta_m\}, \end{aligned} \quad (18)$$

where $\tilde{\varphi}(t) = [\tilde{u}(t-1) \ldots \tilde{u}(t-m)]$ is the regressor vector of the m past control input values and $\tilde{y}(t)$ is the measured plant output. The matrix $A(\mathcal{M}_0^t)$ and the vector $b(\mathcal{M}_0^t)$ can be calculated accordingly, see e.g. [13]. The procedure for calculating $A(\mathcal{M}_0^t)$ and $b(\mathcal{M}_0^t)$ amounts to solving a Linear Program (LP) for each face of the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$ in order to determine whether it is redundant. This computation can be parallelized. However, with this recursive update the number of faces of $\mathcal{F}_m(\mathcal{M}_0^t)$ in general grows linearly with time and therefore $\mathcal{F}_m(\mathcal{M}_0^t)$ can become arbitrarily complex making the memory needed to store $A(\mathcal{M}_0^t)$ and $b(\mathcal{M}_0^t)$ impractical. In order to overcome this problem, a polytope update algorithm with bounded complexity, as in [14] can be used. This algorithm guarantees that the dimensions of the matrix$A(\mathcal{M}_0^t)$ and the vector $b(\mathcal{M}_0^t)$ remain bounded over time. In addition, the algorithm guarantees that $\mathcal{F}_m(\mathcal{M}_0^t) \subseteq \mathcal{F}_m(\mathcal{M}_0^{t-1})$, which is necessary for recursive output constraint satisfaction.

C. Candidate model selection

Once the FSS has been updated, for the purpose of control computation, a nominal model $\{h_c(t)\}_1^\infty$ should be selected. To this end, based on the knowledge of the set $FSS(\mathcal{M}_0^t)$, at each time step we select the nominal model such that $\{h_c(t)\}_{m+1}^\infty = 0$, while the first m coefficients are the elements of a vector $H_c(t) = [h_{c1}(t) \ldots h_{cm}(t)]^T \in \mathcal{F}_m(\mathcal{M}_0^t)$. In particular the vector $H_c(t)$ is computed as the center of the maximum volume l_2-norm ball inscribed in the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$. This can be done by solving an LP, however the solution might not be unique. Therefore, we introduce an additional regularization term that penalizes the deviation from the previously calculated point $H_c(t-1)$, giving rise to the following LP:

$$\begin{gathered} \max_{\phi(t), H_c(t)} \phi(t) - \alpha \|H_c(t-1) - H_c(t)\|_1 \\ \text{subject to} \\ a_i(\mathcal{M}_0^t)H_c(t) + \phi(t)\|a_i(\mathcal{M}_0^t)\|_2 \leq b_i(\mathcal{M}_0^t), \forall i, \end{gathered} \quad (19)$$

where $\phi(t) \in \mathbb{R}$ is the radius of the maximum volume ball inscribed in $\mathcal{F}_m(\mathcal{M}_0^t)$, $\alpha > 0$ is a design variable, and $a_i(\mathcal{M}_0^t)$ and $b_i(\mathcal{M}_0^t)$ stand for the i^{th} row of the matrix $A(\mathcal{M}_0^t)$ and the vector $b(\mathcal{M}_0^t)$.

Remark 3.1: The level of initial knowledge required by the presented identification algorithm is in general not higher than the one needed in the case of least squares identification. In fact, in our approach, a bound on the magnitude of the measurement noise, and loose bounds on the impulse response coefficients of the system are required, while in the least-squares identification the exact model structure is assumed to be known. The most important feature of the presented approach, which is not present in standard techniques, is the capability to provide guaranteed bounds on the future system's output, as we show in the next section. ■

D. Computation of the guaranteed bounds on the system output

In order to define bounds on the possible future system output $y(t)$, we first define for any given sequence of inputs $\mathcal{U}_{-\infty}^{t-1}$ satisfying constraints (7), the (local) upper and lower bounds of $FSS(\mathcal{M}_0^t)$ as the tightest maximal and minimal values of the possible plant output that are compatible with the prior information:

$$\begin{aligned} \overline{y}(t, \mathcal{U}_{-\infty}^{t-1}) &= \sup_{\{h\}_1^\infty \in FSS(\mathcal{M}_0^t)} \mathbf{h} * \mathbf{u}[t] \\ \underline{y}(t, \mathcal{U}_{-\infty}^{t-1}) &= \inf_{\{h\}_1^\infty \in FSS(\mathcal{M}_0^t)} \mathbf{h} * \mathbf{u}[t], \end{aligned} \quad (20)$$

The bounds (20) are "local", because they are referred to a specific control sequence $\mathcal{U}_{-\infty}^{t-1}$. These bounds depend on the infinite control input sequence and are therefore very hard to calculate in general. However, over approximations of (20) can be computed by constructing the local upper and lower bounds with respect to the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$ as:

$$\begin{aligned} \overline{y}_m(t,\mathcal{U}_{t-m}^{t-1}) &= \max_{H_m \in \mathcal{F}_m(\mathcal{M}_0^t)} \varphi(t)^T H_m + \eta_m \\ \underline{y}_m(t,\mathcal{U}_{t-m}^{t-1}) &= \min_{H_m \in \mathcal{F}_m(\mathcal{M}_0^t)} \varphi(t)^T H_m - \eta_m, \end{aligned} \tag{21}$$

where $\varphi(t)^T = [u(t-1) \dots u(t-m)]$. Note that the optimization problems in (21) are LPs and therefore the bounds (21) are in general easy to calculate for a given input sequence $\mathcal{U}_{t-m}^{t-1}$.

For any given sequence of admissible inputs, the actual value of the system's future output, $y(t)$, is bounded by the corresponding local upper and lower bounds (20) and (21) as:

$$\begin{aligned} \underline{y}_m(t,\mathcal{U}_{t-m}^{t-1}) \leq \quad & \underline{y}(t,\mathcal{U}_{-\infty}^{t-1}) \leq \quad y(t,\mathcal{U}_{-\infty}^{t-1}) \\ y(t,\mathcal{U}_{-\infty}^{t-1}) \leq \quad & \overline{y}(t,\mathcal{U}_{-\infty}^{t-1}) \leq \quad \overline{y}_m(t,\mathcal{U}_{t-m}^{t-1}). \end{aligned} \tag{22}$$

In the following, for the sake of notational simplicity, we will denote the upper and lower bounds on the predicted output as $\overline{y}_m(t)$ and $\underline{y}_m(t)$, by omitting the dependence on $\mathcal{U}_{t-m}^t$, with the knowledge that the bounds are indeed local, i.e. related to a specific input sequence.

IV. Constrained adaptive model predictive control

Following an MPC approach, at each time step t a sequence of $N \geq 1$ future inputs is calculated, according to an optimality criterion that accounts for the aim of the control problem at hand, subject to the input and output constraints (7) and taking the uncertainty described by $FSS(\mathcal{M}_0^t)$ into account. Then, the first element in the optimal sequence is applied as the actual control input, and the procedure is repeated at the next sampling time, in a receding horizon fashion. More specifically, let $U = [u(t|t), \dots, u(t+N-1|t)]^T \in \mathbb{R}^N$ be a vector containing the predicted control moves (where the notation $i|t$ indicates the prediction at step $i \geq t$ given the information at the current step t), and let $\Delta U = [u(t|t) - \tilde{u}(t-1), u(t+1|t) - u(t|t), \dots, u(t+N-1|t) - u(t+N-2|t)]^T$ be a vector of predicted control input increments. Moreover, we define the vectors $V(i|t) \in \mathbb{R}^m$, $i \in [t, t+N+m-2]$ that consist of past known inputs $\tilde{\mathcal{U}}_{t-m}^{t-1}$ and potential future control inputs $\mathcal{U}_t^{t+N-1}$ as follows:

$$V(i|t) = \{v(k|t)\}_{k=i-m+1}^{i}, \ i = t, t+1, \dots, t+N+m-2,$$

where

$$v(k|t) = \begin{cases} \tilde{u}(k) & \text{if } k < t \\ u(k|t) & \text{if } t \leq k \leq t+N-1 \\ u(t+N-1|t) & \text{if } k > t+N-1 \end{cases}$$

Then, we consider the following cost function:

$$J(\Delta U, \tilde{\mathcal{U}}_{t-m}^{t-1}) \doteq \sum_{i=t}^{t+N-1} \left(V(i|t)^T H_c(t) - y_{\text{des}}(i+1|t) \right)^2 + \xi \|\Delta U\|_2^2. \tag{23}$$

In (23), ΔU is the decision variable, while $\tilde{\mathcal{U}}_{t-m}^{t-1}$ is a known parameter. $y_{\text{des}}(i|t)$, $i \in [t+1, t+N]$, are the predicted values of the desired output, and $\xi > 0$ is a weighting factor chosen by the control designer. The leftmost term in the cost function J penalizes the deviation between the nominal predicted output and the desired one. The second term in J, $\|\Delta U\|_2^2$, penalizes the input variation, and the weight ξ can be tuned to achieve a trade-off between these two aspects.

Robust satisfaction of both input and output constraints can be enforced by the following set of inequalities:

$$\|U\|_\infty \leq \overline{u} \tag{24}$$

$$\|\Delta U\|_\infty \leq \overline{\Delta u} \tag{25}$$

$$\overline{y}_m(i+1|t) \leq \overline{y}, \ \forall i \in [t, t+N+m-2] \tag{26}$$

$$\underline{y}_m(i+1|t) \geq -\overline{y}, \ \forall i \in [t, t+N+m-2], \tag{27}$$

where $\overline{y}_m(i+1|t)$ and $\underline{y}_m(i+1|t)$ denote the predicted local bounds (21), based on the past input sequence applied up to the time step t and on the predicted control moves up to time step i, and are given by:

$$\begin{aligned} \overline{y}_m(i+1|t) &= \max_{H_m \in \mathcal{F}_m(\mathcal{M}_0^t)} V(i|t)^T H_m + \eta_m \\ \underline{y}_m(i+1|t) &= \min_{H_m \in \mathcal{F}_m(\mathcal{M}_0^t)} V(i|t)^T H_m - \eta_m. \end{aligned} \tag{28}$$

Constraints (24) and (25) follow directly from (7), while the constraints (26) and (27) follow from (7) and (22).

For fixed values of N and ξ we can define the following finite horizon optimal control problem (FHOCP) at time t:

$$\begin{aligned} &\min_{\Delta U} J(\Delta U, \tilde{\mathcal{U}}_{t-m}^{t-1}) \\ &\text{subject to } (24) - (27). \end{aligned} \tag{29}$$

By using the ideas of robust linear programing [15], constraints (26) and (27) can be replaced by an equvalent set of linear constraints and therefore the optimization problem (29) can be turned into a quadratic program (QP). We embede the resulting FHOCP in the following receding horizon scheme:

Algorithm 4.1: (Adaptive MPC)

1) At time step t, update the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$ based on $\tilde{\mathcal{U}}_{t-m}^{t-1}$ and $\tilde{y}(t)$;
2) Find $H_c(t)$ by solving the LP (19);
3) Solve the problem (29), let U^* be the computed control sequence;
4) Apply $u(t) = u^*(t|t)$, set $t = t+1$, go to 1).

■

The proposed control algorithm is an indirect adaptive controller, as it involves the estimation of the plant model on which the control computation is based. The algorithm guarantees robust satisfaction of both input and output constraints, as shown by the following result, for which we omit the proof.

Theorem 4.1: Let the Assumptions 1–2 hold, and assume that the problem (29) is feasible at time $t = 0$. Then the closed-loop system obtained by applying the Algorithm 4.1

to the system (1) is guaranteed to satisfy input and output constraints $\forall t \geq 0$. ∎

In practice, the condition that the problem is feasible for $t = 0$ means that the initial assumptions are selected such that, if the system is initially at rest, there exists a nonzero input sequence that does not violate the input and output constraints for all the plants in the initial FSS, which is a reasonable condition. This condition, together with the fact that the initial nominal model is selected as a nonzero vector in $\mathcal{F}_m(\mathcal{M}_0^t)$, ensures that the Algorithm 4.1 does not result in a trivial control law of always applying zero control input. In addition to the theoretical guarantee of robust constraint satisfaction, the proposed adaptive control algorithm exhibits good tracking performance, as shown in the numerical results of section V.

The proposed control algorithm is computationally tractable as it only requires the solution of LPs that can be parallelized and of a QP. Moreover, all the optimization problems that have to be solved are guaranteed to have bounded size. These problems can be solved very efficiently with available software tools for fast optimization as in [16].

V. Numerical example

The performance of the proposed adaptive control algorithm is illustrated via a numerical example. We consider a system of two masses connected by springs with dampers as shown in Fig. 1. The differential equations describing the

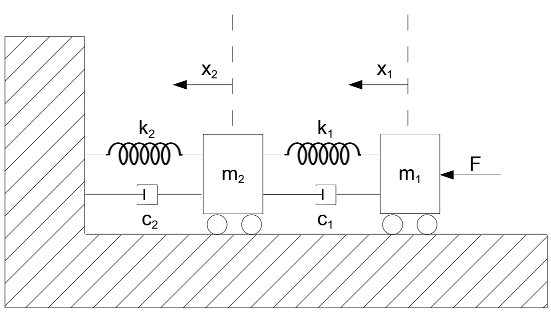

Fig. 1. Numerical example: system layout.

motion of masses m_1 and m_2 are given by:

$$m_1\ddot{x}_1 = F - k_1(x_1 - x_2) - c_1(\dot{x}_1 - \dot{x}_2) \quad (30)$$

$$m_2\ddot{x}_2 = k_1(x_1 - x_2) + c_1(\dot{x}_1 - \dot{x}_2) - k_2x_2 - c_2\dot{x}_2 \quad (31)$$

Taking the state vector to be $x = [x_1\ \dot{x}_1\ x_2\ \dot{x}_2]^T$, the plant input u to be the force F and the plant output y to be the position of the second mass x_2, the following representation of the model is obtained:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du, \end{aligned} \quad (32)$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{c_1}{m_1} & \frac{k_1}{m_1} & \frac{c_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{c_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{c_1+c_2}{m_2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D = 0$$

In the simulations, the numerical values of Table I have been used. The system's impulse response, obtained by discretizing the system model with the impulse response preserving method and sampling time $T_s = 0.8$, is shown in Fig. 2. It is assumed that the maximal force that can

TABLE I

Computational example: parameters for the two-mass-spring system.

k_1	k_2	c_1	c_2	m_1	m_2	T_s
50	1.2	0.9	0.86	1	0.5	0.8

be applied to the mass m_1 and its rate are limited by $\overline{u} = 1$ and $\overline{\Delta u} = 0.2$ respectively, and that the position of the mass m_2 is constrained as well by $\overline{y} = 2$, due to mechanical limitations. The design parameters used for the control algorithm are listed in Table II. In the simulations a stochastic measurement noise, uniformly distributed in the interval $[-\epsilon, \epsilon]$ was used. In Fig. 2 we show that the initial

TABLE II

Computational example: design parameters of the control system.

ϵ	L	μ	ρ	m	α	N	ξ
0.1	1.2	4	0.86	24	0.01	26	1

bounds (6) for the impulse response coefficients are selected quite conservatively. In particular, the tight values of the relevant parameters in (6) for the plant are $L = 0.52$, $\mu = 2$ and $\rho = 0.8$ (compare Table II and Fig. 2). The initial plant model is taken as a random nonzero point inside the set $\mathcal{K}_m(L, \rho, \mu)$. The simulation results of Fig. 3 show that good tracking of the desired reference is obtained, with quite tight worst-case bound on the output.

Moreover, as intuitively expected, the tracking performance improves over time, as the size of the feasible system set is reduced with the increasing amount of input-output data. This is also illustrated in Fig. 4, where the time evolution of the intersection of the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$ and the hyperplane defined by the true values of coefficients $\{h_m\}_{i=3}^m = \{h_S\}_{i=3}^m$ is given. It can be seen that $\mathcal{F}_m(\mathcal{M}_0^t)$ shrinks with time, and that $\mathcal{F}_m(\mathcal{M}_0^t) \subseteq \mathcal{F}_m(\mathcal{M}_0^{t-1})$, as imposed by the polytopic update algorithm.

VI. Conclusion

We proposed a novel adaptive model predictive control for linear systems subject to both input and output constraints. The method relies on real-time SM identification to provide guaranteed bounds on the predicted system output. These bounds are used to design a receding horizon controller

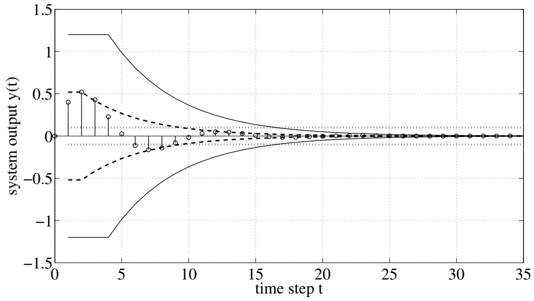

Fig. 2. Numerical example: discrete plant impulse response (bars ending with o) compared with the used initial bounds on the impulse response coefficients (solid lines), the tightest possible initial bounds on the impulse response coefficients (dashed lines) and measurement noise bounds (dotted lines)

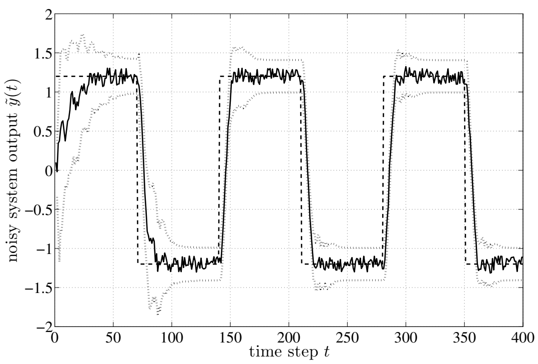

Fig. 3. Numerical example: simulation results for the case when the future reference changes are not known in advance. The set reference $y_{\text{des}}(t)$ (dashed line) is compared with the noisy system output $\tilde{y}(t)$ (solid line). Dotted lines represent the worst-case bounds on the range of possible outputs corresponding to all the models in the set $FSS(\mathcal{M}_0^t)$.

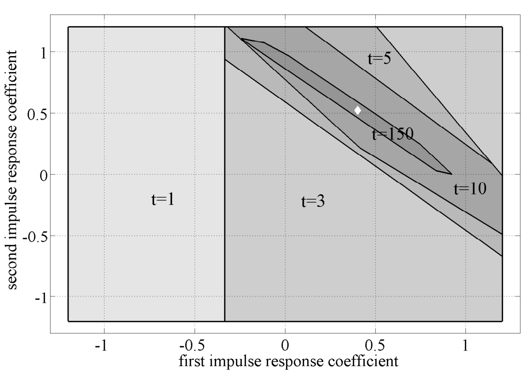

Fig. 4. Numerical example: time evolution of the intersection of the polytope $\mathcal{F}_m(\mathcal{M}_0^t)$ and the hyperplane defined by the actual values of the impulse response coefficients $\{h_m\}_{i=3}^m = \{h_S\}_{i=3}^m$. The white diamond represents the real values of the first two impulse response coefficients.

able to robustly satisfy output constraints. The proposed adaptive control algorithm requires only the solution of standard convex optimization problems. Moreover, all of the optimization problems are guaranteed to be recursively feasible. Simulations on a numerical example show that the system's output prediction uncertainty is reduced with time, which results in practice in a good reference tracking performance. The current research is aimed at deriving theoretical guarantees for the convergence of the tracking error to zero.

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