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A New Generalized Vector Observation for Discrete-Time Delay Systems Identification

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***Abstract*—This paper suggests a new approach for simultaneous recursive identification of an unknown time delay and dynamic parameters of discrete-time delay systems. The proposed algorithm is based on a new formulations allowing to admit the unknown time delay in the parameter vector. The recursive least-squares method is then used to solve the obtained system. A simulation study is included to illustrate the merit of our algorithm.**

I. INTRODUCTION

Recursive identification of time delay systems has received great attention in the last years since a broad class of physical systems includes time delay phenomena in their dynamics. Time delays arise in physical, chemical and economic systems, as well as in the process measurement, communication and control [20]. Neglecting its presence may lead to a the emanation of complex behavior [23], [16].

Several approaches have been proposed in the literature for the identification of time delay system identification [1], [2], [6], [4], [3], [5], [7], [8], [9], [10], [11], [12], [13], [14], [15], [18], [19], [22], [27], [26], [25]. These approaches can be classified into two classes of models: continuous time models and discrete-time models.

This paper addressed the problem of simultaneous recursive identification of discrete-time systems. Indeed, this approach consist in constructing a new generalized regression vector which defined the time delay and the rational dynamic parameters in the same vector and in using the recursive least squares algorithm to solve the obtained system. This paper is organized as follows. Section 2 presents the model and its assumptions. In Section 3, we propose the new algorithm for simultaneous recursive identification of an unknown time delay and parameters of discrete-time delay systems. Moreover, we develop the statistics properties of the estimates in order to show that the obtained estimates are unbiased. Simulation results are presented in Section 4.

II. PROBLEM STATEMENT

In the following, we address the problem of estimating the time delay and the parameters of the following system:

$$A(q^{-1})y(k) = q^{-d(k)}B(q^{-1})u(k) + v(k) \tag{1}$$

where $u(k)$ and $y(k)$ are the system input and output, respectively, $v(k)$ is a white noise, $d(k)$ is the time delay, $A(q^{-1})$ and $B(q^{-1})$ are two polynomials in the unit backward shift operator q^{-1}, [*i.e.* $q^{-1}y(k) = y(k-1)$], defined by:

$$A(q^{-1}) = 1 + a_1(k)q^{-1} + \ldots + a_{n_a}(k)q^{-n_a}$$

$$B(q^{-1}) = b_1(k)q^{-1} + \ldots + b_{n_b}(k)q^{-n_b}$$

The following assumptions are made:

- A1. The polynomials $A(q^{-1})$ and $B(q^{-1})$ are coprime.
- A2. The orders n_a and n_b of the model are known.
- A3. The input sequence $\{u(k)\}$ is a stationary ergodic process, independent of $v(k)$ and is persistently exciting.
- A4. The disturbance $v(k)$ is a sequence of independent, identically distributed random variable with zero mean and finite variance σ_v^2.
- A5. The input, the output and the noise are causal, i.e. $u(k) = 0$, $y(k) = 0$ and $v(k) = 0$ for $k \leq 0$.

Problem statement: The goal is to develop a recursive algorithm to estimate, simultaneously, the time delay $d(k)$ and the parameters $\{a_i(k), b_i(k)\}$ using the input/output measurement data $\{u(k), y(k)\}$.

In the following, we present two necessary definitions:

Definition 1. Operator $round(d)$ is defined by:

$$round(d) = \begin{cases} int(d) + 1 & if\ d - int(d) \geq 0.5 \\ int(d) & if\ d - int(d) < 0.5 \end{cases} \tag{2}$$

where $int(d)$ denote the integer part of d.

Definition 2. Operator $\tilde{d}(.)$ is defined by:

$$\tilde{d}(.) = round(\hat{d}(.)) \tag{3}$$

III. THE PROPOSED APPROACH

This paragraph proposes an alternative solution for the purpose of simultaneous recursive identification of an unknown time delay and parameters of discrete time systems.

Equation (1) can be rewritten as:

$$y(k) = \varphi(k, d(k))\theta + v(k) \tag{4}$$

where θ is the parameter vector and $\varphi(k,d(k))$ is the observation vector which are defined as:

$$\begin{aligned}\varphi(k,d(k)) = [&-y(k-1),-y(k-2),...,-y(k-n_a),\\ &u(k-d(k)-1),...,u(k-d(k)-n_b)] \quad (5)\\ \theta = \big[&a_1(k),a_2(k),...,a_{n_a}(k),b_1(k),b_2(k),...,b_{n_b}(k)\big]^T\end{aligned}$$

Then, the estimated output is described by the following relation:

$$\hat{y}(k) = \hat{\varphi}(k,\tilde{d}(k))\hat{\theta} \quad (6)$$

where $\hat{\theta}$ and $\tilde{d}$ represent the estimated parameter vector and the estimated time delay.

Now, let consider the prediction error:

$$e(k) = y(k) - \hat{y}(k) = y(k) - \hat{\varphi}(k,\tilde{d}(k))\hat{\theta} \quad (7)$$

This formulation does not admit the unknown time delay in the parameter vector and consequently it is not directly applicable to achieve our objective which is simultaneous recursive identification of the time delay and the parameters of time delay systems. To overcome this problem, we suggest to consider the time delay in the vector of parameters to be estimated. Indeed, the new vector, called generalized vector of parameters, is given by:

$$\theta_G = \big[\theta^T, d(k)\big]^T$$

Moreover, we propose the use of the negative gradient of the error to obtain an appropriate observation vector which is given by :

$$\phi(k,\hat{\theta}_G) = -\frac{\partial e}{\partial \hat{\theta}_G} \quad (8)$$

Then,

$$\phi(k,\hat{\theta}_G) = \left[\varphi^T(k,\hat{d}(k)), -\frac{\partial e}{\partial \hat{d}(k)}\right]^T \quad (9)$$

Using the approximation of $Ln(q) \approx 1 - q^{-1}$, we obtain:

$$\phi(\hat{\theta}_G) = \left[\varphi^T(k,\tilde{d}(k)), -\sum_{i=1}^{n_b} \hat{b}_i q^{-\tilde{d}(k)} u(k-i)(1-q^{-1})\right]^T \quad (10)$$

Replacing $\varphi^T(k,\tilde{d}(k))$ by its expression, the generalized vector parameters is obtained :

$$\phi(k,\hat{\theta}_G) = \begin{bmatrix} -y(k-1) \\ \vdots \\ -y(k-n_a) \\ q^{-\tilde{d}(k)}u(k-1) \\ \vdots \\ q^{-\tilde{d}(k)}u(k-n_b) \\ -\sum_{i=1}^{n_b} \hat{b}_i(k) q^{-\tilde{d}(k)} \Delta u(k-i) \end{bmatrix}^T \quad (11)$$

where $\Delta u(k) = u(k) - u(k-1)$.

An estimation $\hat{\theta}_G$ of θ_G is obtained by the minimization of the following criterion:

$$J(k,\theta_G) = \frac{1}{2}\sum_{i=0}^{k} \lambda^{k-i} e(i)^2 \quad (12)$$

where $0 < \lambda \leq 1$.

Then, the partial derivative of the criterion with respect to the generalized vector parameter is:

$$\frac{\partial J}{\partial \hat{\theta}_G} = \sum_{i=0}^{k} \lambda^{k-i} \frac{\partial e(i)}{\partial \hat{\theta}_G} e(i) = -\sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G) e(i) \quad (13)$$

So,

$$\begin{aligned}\frac{\partial J}{\partial \hat{\theta}_G} &= -\sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G)\left[y(i) - \hat{\theta}\varphi(k,\tilde{d}(k))\right] \\ &= -\sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G)\left[y(i) - \hat{\theta}\varphi(k,\tilde{d}(k)) + \psi - \psi\right]\end{aligned} \quad (14)$$

where

$$\psi = -\sum_{i=1}^{n_b} \tilde{d}(k)\hat{b}_i(k) q^{-\tilde{d}(k)} \Delta u(k-i)$$

Then,

$$\begin{aligned}\frac{\partial J}{\partial \hat{\theta}_G} &= -\sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G)\left[y(i) - \phi^T(i,\hat{\theta}_G)\hat{\theta}_G + \psi\right] \\ &= -\sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G)\left[y(i) + \psi - \phi^T(i,\hat{\theta}_G)\hat{\theta}_G\right]\end{aligned}$$

Canceling the partial derivative of the criterion, we obtain:

$$\begin{aligned}\hat{\theta}_G(k) = &\left[\sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G)\phi^T(i,\hat{\theta}_G)\right]^{-1} \\ &\sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G)\left[y(i) + \psi\right]\end{aligned} \quad (15)$$

Let,

$$R(k) = \sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G)\phi^T(i,\hat{\theta}_G) \quad (16)$$

Under the PE assumptions $R(k)^{-1}$ exist ([24]), then,

$$\hat{\theta}_G(k) = R(k)^{-1} \sum_{i=0}^{k} \lambda^{k-i} \phi(i,\hat{\theta}_G)\left[y(i) + \psi\right]$$

$$\begin{aligned}\hat{\theta}_G(k) = R(k)^{-1} &\left(\sum_{i=0}^{k-1} \lambda^{k-i} \phi(i,\hat{\theta}_G)\left[y(i) + \psi\right]\right. \\ &\left. + \phi(k,\hat{\theta}_G)\left[y(k) + \psi\right]\right)\end{aligned}$$

Using equation (15), we have:

$$\begin{aligned}\hat{\theta}_G(k) = R(k)^{-1} &\left(\lambda R(k-1)\hat{\theta}_G(k-1) + \phi(k,\hat{\theta}_G)\right. \\ &\left.\left[y(k) + \psi\right]\right)\end{aligned}$$

Hence,

$$R(k)=\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\hat{\theta}_G)\phi^T(i,\hat{\theta}_G)$$
$$=\lambda R(k-1)+\phi(k,\hat{\theta}_G)\phi^T(k,\hat{\theta}_G)$$

Then,

$$\hat{\theta}_G(k)=R(k)^{-1}\left(R(k)\hat{\theta}_G(k-1)-\phi(k,\hat{\theta}_G)\phi^T(k,\hat{\theta}_G)\hat{\theta}_G(k-1)+\phi(k,\hat{\theta}_G)\left[y(k)+\psi\right]\right)$$

So,

$$\hat{\theta}_G(k)=\hat{\theta}_G(k-1)+R(k)^{-1}\phi(k,\hat{\theta}_G)\left(-\phi^T(k,\hat{\theta}_G)\hat{\theta}_G(k-1)+[y(k)+\psi]\right) \quad (17)$$

It follows from (17) that:

$$\hat{\theta}_G(k)=\hat{\theta}_G(k-1)+R(k)^{-1}\phi(k,\hat{\theta}_G)\left(y(k)-\hat{\theta}\varphi(k,\tilde{d}(k))\right)$$

Thus,

$$\hat{\theta}_G(k)=\hat{\theta}_G(k-1)+P(k)\phi(k,\hat{\theta}_G)e(k) \quad (18)$$

Where,

$$P(k)=\left[\lambda R(k-1)+\phi(k,\hat{\theta}_G)\phi^T(k,\hat{\theta}_G)\right]^{-1}$$

Using the matrix inversion lemma ([24]) given by:

$$\left[B+CD^T\right]^{-1}=B^{-1}-B^{-1}CD^TB^{-1}\left[1+D^TB^{-1}C\right]^{-1} \quad (19)$$

Let $B=\lambda R(k-1)$, $C=\phi(k,\theta_G)$ and $D=\phi(k,\theta_G)^T$, then, we have:

$$P(k)=\frac{1}{\lambda}\left(P(k-1)-\frac{P(k-1)\phi(k,\hat{\theta}_G)\phi^T(k,\hat{\theta}_G)P(k-1)}{\lambda+\phi^T(k,\hat{\theta}_G)P(k-1)\phi(k,\hat{\theta}_G)}\right) \quad (20)$$

The application of the proposed algorithm is summarized by the following step by step algorithm:

Step1: set $\hat{\theta}_G=\theta_{G_0}=[0_{(1,na)},0_{(1,nb)},0]$ and $P=\beta I_{(n_a+n_b+1)}$ where β is a scalar and $I_{(n_a+n_b+1)}$ is the identity matrix of size $(na+nb+1)$ and $k=0$.

Step2: Increment k and construct the observation vector $\varphi(k,\tilde{d}(k))$, the generalized observation vector $\phi^T(k,\hat{\theta}_G)$ using (4) and (10).

Step3: Computing the $\hat{\theta}_G$ using (21, 22, 23):

$$e(k)=y(k)-\hat{\varphi}(k,\tilde{d})\hat{\theta}(k-1) \quad (21)$$

$$P(k)=\frac{1}{\lambda}\left(P(k-1)-\frac{P(k-1)\phi(k,\hat{\theta}_G)\phi^T(k,\hat{\theta}_G)P(k-1)}{\lambda+\phi^T(k,\hat{\theta}_G)P(k-1)\phi(k,\hat{\theta}_G)}\right) \quad (22)$$

$$\hat{\theta}_G(k)=\hat{\theta}_G(k-1)+P(k)\phi(k,\hat{\theta}_G)e(k) \quad (23)$$

Step4: Return to step 2 until $k=N$ where$N$is the number of input/output data.

Lemma

For the estimate (15) with the assumption A4, the following proprieties are hold:

P1. *$\hat{\theta}_G$ is an unbiased estimate of θ_G.*

P2. *The covariance matrix of $\hat{\theta}_G$is given by:*

$$E\left[(\hat{\theta}_G-\theta_G)(\hat{\theta}_G-\theta_G)^T\right]=\left(\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\phi(i,\theta_G)^T\right)^{-1}\sigma_v^2 \quad (24)$$

Proof

If we replace the equation (4) in (15), we have:

$$\hat{\theta}_G=\left[\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\phi^T(i,\theta_G)\right]^{-1}\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\left[y(i)+\psi\right]$$

Then,

$$\hat{\theta}_G=\left[\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\phi^T(i,\theta_G)\right]^{-1}\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\left[\varphi(i,d)\theta+v(i)+\psi\right]$$

$$\hat{\theta}_G=\left[\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\phi^T(i,\theta_G)\right]^{-1}\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\left[\varphi(i,d)\theta+\psi\right]+\left[\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\phi^T(i,\theta_G)\right]^{-1}\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)v(i)$$

So,

$$E\left[\hat{\theta}_G\right]=\theta_G+E(\left[\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)\phi^T(i,\theta_G)\right]^{-1}\sum_{i=0}^{k}\lambda^{k-i}\phi(i,\theta_G)v(i)) \quad (25)$$

Since $\phi(i,\theta_G)$ (10) is independent of $v(i)$, then:

$$E\left[\hat{\theta}_G\right]=\theta_G \quad (26)$$

Which proves (P1).□

Consider the following first order Taylor series expansion around the real parameter of θ_G

$$\frac{\partial J(k,\hat{\theta}_G)}{\partial\hat{\theta}_G}=\frac{\partial J(k,\theta_G)}{\partial\theta_G}+\frac{\partial^2 J(k,\theta_G)}{\partial\theta_G^2}(\hat{\theta}_G-\theta_G) \quad (27)$$

Since $\frac{\partial J(k,\hat{\theta}_G)}{\partial\hat{\theta}_G}=0$, it derives from (27)

$$(\hat{\theta}_G-\theta_G)(\hat{\theta}_G-\theta_G)^T=\left[\frac{\partial^2 J(k,\theta_G)}{\partial\theta_G^2}\right]^{-1}\frac{\partial J(k,\theta_G)}{\partial\theta_G}\left[\frac{\partial J(k,\theta_G)}{\partial\theta_G}\right]^T\left[\left(\frac{\partial^2 J(k,\theta_G)}{\partial\theta_G^2}\right)^{-1}\right]^T \quad (28)$$

The second partial derivative of the criterion with respect to the generalized vector parameter is

$$\frac{\partial^2 J(k,\theta_G)}{\partial\theta_G^2}=\sum_{i=0}^{k}\lambda^{k-i}\left(e(i)\frac{\partial^2 e(i)}{\partial^2\theta_G}-\frac{\partial e(i)}{\partial\theta_G}\phi(i,\theta_G)\right)$$

So,

$$\frac{\partial^2 J(k,\theta_G)}{\partial \theta_G^2} = \sum_{i=0}^{k} \lambda^{k-i}\left(e(i)\frac{\partial^2 e(i)}{\partial^2 \theta_G} + \phi(i,\theta_G)\phi^T(i,\theta_G)\right)$$

Using *the small residual algorithms*, the following term can be neglecting ([21]), then:

$$\sum_{i=0}^{k} \lambda^{k-i} e(i)\frac{\partial^2 e(i)}{\partial^2 \theta_G} \to 0$$

Hence, an approached of $\frac{\partial^2 J(k,\theta_G)}{\partial \theta_G^2}$ is obtained:

$$\frac{\partial^2 J(k,\theta_G)}{\partial \theta_G^2} \simeq \sum_{i=0}^{k} \lambda^{k-i}\big(\phi(i,\theta_G)\phi^T(i,\theta_G)\big)$$

Applying the mean value of $\frac{\partial J(k,\theta_G)}{\partial \theta_G}\frac{\partial J(k,\theta_G)}{\partial \theta_G}^T$, we get:

$$E\left[\frac{\partial J(k,\theta_G)}{\partial \theta_G}\frac{\partial J(k,\theta_G)}{\partial \theta_G}^T\right] = \sum_{i=0}^{k} \lambda^{k-i}\phi(i,\theta_G)\phi(i,\theta_G)^T E(e(i)e(i)^T)$$

So,

$$E\left[\frac{\partial J(k,\theta_G)}{\partial \theta_G}\frac{\partial J(k,\theta_G)}{\partial \theta_G}^T\right] = \sum_{i=0}^{k} \lambda^{k-i}\phi(i,\theta_G)\phi(i,\theta_G)^T \sigma_v^2$$

Then, we have:

$$E\left[(\hat{\theta}_G-\theta_G)(\hat{\theta}_G-\theta_G)^T\right] = E\left[\left(\frac{\partial^2 J}{\partial \theta_G^2}\right)^{-1} \sum_{i=0}^{k} \lambda^{k-i}\phi(i,\theta_G)\phi(i,\theta_G)^T \sigma_v^2 \left(\left(\frac{\partial^2 J}{\partial \theta_G^2}\right)^{-1}\right)^T\right] \tag{29}$$

Finally, we obtain:

$$E\left[(\hat{\theta}_G-\theta_G)(\hat{\theta}_G-\theta_G)^T\right] = \left(\sum_{i=0}^{k} \lambda^{k-i}\phi(i,\theta_G)\phi(i,\theta_G)^T\right)^{-1}\sigma_v^2 \tag{30}$$

Which proves P2. □

A. The initial value choice of P

The covariance matrix P is given by:

$$P(k) = \left[\sum_{i=0}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)\phi^T(i,\hat{\theta}_G)\right]^{-1}$$

$$R(k) = \left[\lambda^k R(0) + \sum_{i=1}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)\phi^T(i,\hat{\theta}_G)\right]$$

Let consider,

$$x(k) = R(k)\hat{\theta}_G(k) \tag{31}$$

Replace $\hat{\theta}_G(k)$ by (17):

$$x(k) = R(k)\left[\hat{\theta}_G(k-1) + R(k)^{-1}\phi(k,\hat{\theta}_G)e(k)\right]$$

Then,

$$x(k) = R(k)\hat{\theta}_G(k-1) + \phi(k,\hat{\theta}_G)e(k)$$

So,

$$x(k) = \left[\lambda R(k-1) + \phi(k,\hat{\theta}_G)\phi^T(k,\hat{\theta}_G)\right]\hat{\theta}_G(k-1) + \phi(k,\hat{\theta}_G)e(k)$$

$$x(k) = \lambda R(k-1)\hat{\theta}_G(k-1) + \phi(k,\hat{\theta}_G)\phi^T(k,\hat{\theta}_G)\hat{\theta}_G(k-1) + \phi(k,\hat{\theta}_G)e(k)$$

Adding and subtracting ψ:

$$x(k) = \lambda R(k-1)\hat{\theta}_G(k-1) + \phi(k,\hat{\theta}_G)\phi^T(k,\hat{\theta}_G)\hat{\theta}_G(k-1) + \phi(k,\hat{\theta}_G)(y(k) - \hat{\theta}\varphi(k-1) \pm \psi)$$

So,

$$x(k) = \lambda x(k-1) + \phi(k,\hat{\theta}_G)(y(k)+\psi)$$

Then,

$$x(k) = \lambda^k x(0) + \sum_{i=1}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)(y(i)+\psi) \tag{32}$$

It follows from (31) that:

$$\hat{\theta}_G(k) = \left[\lambda^k R(0) + \sum_{i=1}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)\phi^T(i,\hat{\theta}_G)\right]^{-1} \left(\lambda^k x(0) + \sum_{i=1}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)(y(i)+\psi)\right)$$

$$\hat{\theta}_G(k) = \left[\lambda^k R(0) + \sum_{i=1}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)\phi^T(i,\hat{\theta}_G)\right]^{-1} \left(\lambda^k R(0)\hat{\theta}_G(0) + \sum_{i=1}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)(y(i)+\psi)\right) \tag{33}$$

If $R(0)$ is small, ($P(0)$ is large), then $\hat{\theta}_G(k)$ is expressed by,

$$\hat{\theta}_G(k) \simeq \left[\sum_{i=1}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)\phi^T(i,\hat{\theta}_G)\right]^{-1} \left(\sum_{i=1}^{k} \lambda^{k-i}\phi(i,\hat{\theta}_G)(y(i)+\psi)\right) \tag{34}$$

Then, to ensure the parameters convergence, we choose a gain $P(k)$ initially high ($P(0) \gg$).

IV. Simulation results

Now, we present a simulation example to illustrate the performance of the proposed approach for simultaneous and recursive identification of the unknown time delay and the parameters of time-varying delay systems.

The simulations are performed under the following conditions:

- The input $\{u(k)\}$ is a persistent excitation signal sequence with zero mean and unit variance.

- The additive noise $\{v(k)\}$ is a white noise sequence with zero mean and constant variance σ_v^2 computing to obtain the desired Signal-to-Noise Ratio (SNR):

$$SNR(db) = 10Log[\frac{\sigma_x^2}{\sigma_v^2}] \tag{35}$$

where σ_x^2 is the variance of the noise free output sequence $\{x(k)\}$.
- The estimation starts with zero initial values for the parameters and the time delay.

Consider a first-order plus time delay system given by:

$$T_p\dot{y}(t) + y(t) = K_p u(t - \tau(t)) \tag{36}$$

where $\tau(t) = 0.5 + |sin(\frac{t}{500})|$, $K_p = 1$ and $T_p = 2$.

The evolution of the time delay is given by:

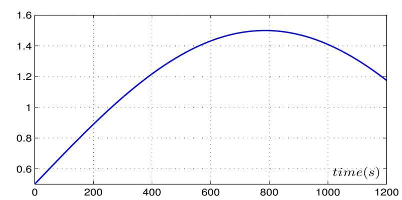

Fig. 1. Variation law of the time delay τ.

Since $\tau = dT_e + \varepsilon$, $0 < \varepsilon < T_e$, T_e is the sampling period and d is a non-negative integer, it can be easily shown using Zero Order Holder (ZOH) that (36) is given by:

$$A(q^{-1})y(k) = q^{-d(k)}B(q^{-1})u(k-1) + v(k) \tag{37}$$

where

$$A(q^{-1}) = 1 + a_1 q^{-1}$$

$$B(q^{-1}) = b_1 q^{-1} + b_2(k) q^{-2}$$

Thus, the fractional delay ε gives rise in the z-domain to a pole at the origin and to a real negative zero. Suppose the sampling frequency is $2Hz$, we can obtain the plant model in the discrete time domain, The system's output is subject to additive zero mean white noise. The Signal-to-Noise Ratio (SNR) is equal to 15dB on system's output.

Fig. 2– 5 show the evolution of the real and the estimated parameters and the time delay:

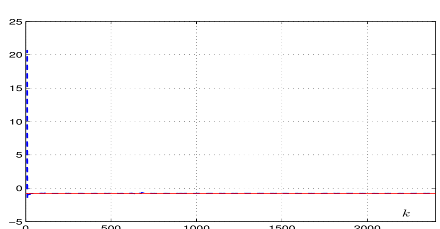

Fig. 2. The evolution of the real(–) and the estimated (- -) parameters $a(k)$.

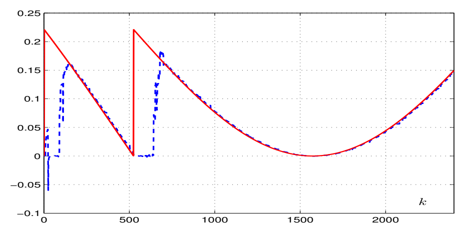

Fig. 3. The evolution of the real(–) and the estimated (- -) parameters $b_1(k)$.

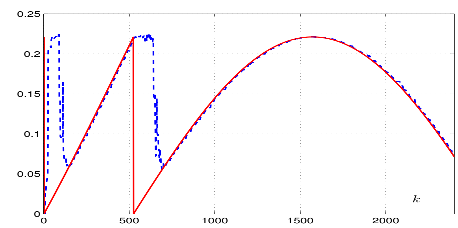

Fig. 4. The evolution of the real(–) and the estimated (- -) parameters $b_2(k)$.

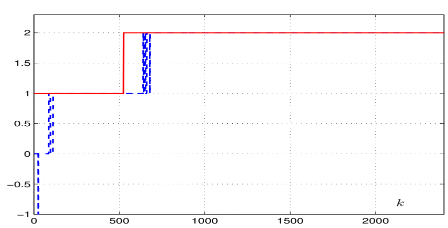

Fig. 5. The evolution of the real(–) and the estimated (- -) time delay $d(k)$.

From Fig. 2– 5, we can remark that the proposed method gives acceptable precision. Indeed, The estimates converge reasonably fast to the true values.

A validation of the model is realized. Fig. 6 gives the evolution of real output $y(k)$ and estimated $\hat{y}(k)$.

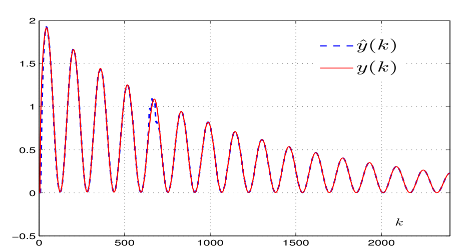

Fig. 6. The evolution of the real(–) and the estimated (- -) outputs signals.

This figure shows that the estimated output $\hat{y}(k)$, which is generated by the proposed approach, tracks fastly and accurately the true output.

V. CONCLUSION

In this paper, we have addressed the problem of identification of discrete-time delay systems. In fact, we have proposed a new approach for the simultaneous identification of the unknown time delay and the parameters of these systems based on recursive algorithm. The proposed approach consists in constructing a linear-parameters formulation that will used to estimate the time delay and the parameters using recursive least squares algorithm. The obtained estimates were shown to be unbiased and an expression for their covariance matrix was given. This efficiency of the considered algorithm is illustrated by a simulation example of time delay varying systems.

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APPENDIX

Let consider the shift operator and the backward difference given, respectively, by (38) and (39)

$$qu(k) = u(k+1) \tag{38}$$

$$\Delta u(k) = u(k) - u(k-1) \tag{39}$$

So,

$$\Delta u(k) = (1 - q^{-1})u(k)$$

We can infer the identity between the shift operator and the backward difference ([17]), then:

$$\Delta = 1 - q^{-1}$$

It is equivalent to

$$q^{-1} = 1 - \Delta \tag{40}$$

Applying the Logarithm function of both side of (38), we get:

$$Ln(q) = -Ln(1-\Delta)$$

Using the series expansion of $Ln(1-x)$, we have:

$$Ln(q) = \Delta + \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 + ... \tag{41}$$

Finally, we use a first order approximation of the shift operator given by:

$$Ln(q) = \Delta = 1 - q^{-1} \tag{42}$$