

Stochastic unit commitment and reserve scheduling: A tractable formulation with probabilistic certificates

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Abstract—The increased penetration of renewable energy sources to the network highlights the necessity of constructing stochastic variants of the standard unit commitment and reserve scheduling problems. Earlier approaches to such problems are either restricted to ad-hoc methodologies (at the expense of a suboptimal solution), or lead to computationally intractable formulations. In this paper we provide a unified framework to deal with such planning problems for systems with uncertain generation, while providing a-priori probabilistic certificates for the robustness properties of the resulting solution. Our methodology is based on a mixture of randomized and robust optimization and leads to a tractable problem formulation. To illustrate the performance of the proposed methodology we apply it to the IEEE 30-bus network, and compare it by means of Monte Carlo simulations against an algorithm based on a deterministic variant of the unit commitment problem.

I. INTRODUCTION

In deregulated power markets unit commitment consists one of the main tasks of the Transmission System Operator (TSO). The objective is to compute a binary vector that corresponds to the “on-off” status of the generating units, and the generation dispatch, which denotes the amount of power that each generator should produce to satisfy a given demand level. In the presence of uncertainty the TSO decides also about corrective actions to avoid any disruption of service. The power corresponding to such actions is referred to as reserves, and is provided by modifying the schedule of the existing generators or by committing fast-start units.

The unit commitment and reserve scheduling problems become more challenging due to the increasing penetration of renewable sources in the power network. This highlights the necessity of formulating stochastic variants of standard day-ahead planning problems, while providing probabilistic guarantees regarding the satisfaction of system constraints. Earlier approaches to such problems are either restricted

to ad-hoc methodologies or raise tractability issues. In [1], [2], the authors follow rule-based approaches for reserve scheduling, whereas in [3] a hybrid scheme that separates the unit commitment problem from the reserve scheduling one is adopted. Following a stochastic approach, [4], [5], [6] (and references therein), formulate stochastic unit commitment and reserve scheduling programs, most of them modeling the uncertain generation by means of scenarios and using reduction techniques to achieve a computationally simpler problem. Nevertheless, no guarantees regarding the robustness of the resulting solution are provided, and only an a-posteriori analysis is conducted. In [7] the authors propose a robust formulation of the unit commitment problem either by arbitrarily considering the uncertainty taking values on a hyper-rectangular set, or by imposing assumptions on the underlying probability distribution.

In earlier work [8], [9], we concentrated on the problem of security constrained reserve scheduling for networks with renewable generation. We formulated a chance constrained optimization program and employed the scenario approach of [10], [11], to achieve a tractable reformulation. Following [10], and if the underlying problem is convex with respect to the decision variables, finite sample guarantees can be provided. In this paper we build on this framework, but follow an alternative methodology [12] that allows to inherit probabilistic guarantees even if the underlying problem is not convex (e.g. mixed integer optimization programs arising in unit commitment problems), while ensuring tractability of the resulting optimization problem. Another contribution of this paper is that unlike [5], [6], following the approach of [9], not only we determine the minimum cost amount of reserves so that robustness, in a probabilistic sense, is guaranteed, but we also determine a reserve strategy that can be deployed in real time operation.

Section II provides a general chance constrained optimization framework for the problem of stochastic unit commitment and reserve scheduling. Section III describes a methodology that ensures tractability while providing probabilistic performance guarantees, whereas in Section IV we demonstrate the efficiency of the proposed approach by means of a numerical example. Finally, Section V provides some concluding remarks and directions for future work.

II. PROBLEM FORMULATION

A. Definitions and preliminaries

We consider a power network comprising N_G generating units, N_L loads, N_l lines, and N_b buses, and base our work

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on the following assumptions: 1) A standard DC power flow approach [13] is adopted. 2) We consider only uncertainty due to wind power generation. 3) No $N - 1$ security constraints are considered. The first assumption is rather standard for this type of problems, whereas the second one is included to simplify the presentation of our results and be can be easily relaxed to include other uncertainty sources as well (e.g. PV, load, etc.). The last assumption refers to the case of component outages emanating from the $N - 1$ security criterion [14]; they could be modelled following the procedure outlined in [9].

Under the DC power flow approximation, and by eliminating the angles by setting the reference one to zero [15], the power flows can be written as $P_f = B_f [(\tilde{B}_{\text{BUS}})^{-1} \tilde{P} \ 0]^T$, whereas the power injection vector $\tilde{P}$ is given by $\tilde{P} = [C_G(P_G + R) + C_w P_w - C_L P_L]_{N_b-1}$. Matrix B_f denotes the imaginary part of the admittance of each network branch, whereas $\tilde{B}_{\text{BUS}}$ and $\tilde{P}$ denote the remaining parts of the nodal admittance matrix and the power injection vector, once the row and column corresponding to the reference angle are removed. $[\cdot]_{N_b-1}$ denotes the first $N_b - 1$ rows of the quantity inside the brackets. $P_G \in \mathbb{R}^{N_G}$, $P_w \in \mathbb{R}$, and $P_L \in \mathbb{R}^{N_L}$ denote the generation dispatch, the wind power in-feed and the load, respectively. Matrices C_G, C_w, C_L are of appropriate dimension, and their element (i, j) is “1” if generator j (respectively wind power/load) is connected to the bus i , and zero otherwise. $\tilde{P}$ implies that the power injection at every bus of the network is equal to the difference between the production and the demand, where the production is given by the sum of the outputs of the conventional units (plus a power correction term) and the wind power output.

The power correction term $R \in \mathbb{R}^{N_G}$ is related to the reserves provided by each generating unit. Following [9], we define R to be a linear function of the total generation-load mismatch, which is the difference of the wind power from its forecast value. This choice is motivated by the fact that any imbalance between load and generation induces frequency deviations and activates the frequency controllers, whose output is distributed in a weighted way to the participating generators. Modeling the steady state behavior of this action,

$$R = d_{\text{up}} \max(- (P_w - P_w^f), 0) - d_{\text{down}} \max(P_w - P_w^f, 0), \quad (1)$$

where $P_w - P_w^f \in \mathbb{R}$ denotes the deviation of the wind power P_w from the forecast P_w^f . This term is directly related to the reserves since for every mismatch, it shows the amount of power with which each generator should adjust its production. Vectors $d_{\text{up}} \in \mathbb{R}^{N_G}$, ($d_{\text{down}} \in \mathbb{R}^{N_G}$) represent the distribution vectors. The sum of their elements is one, and if a generator is not contributing to the frequency control, the corresponding element in the vector is zero. The indices up and down are used to distinguish between the up and down spinning reserves. Note that the distribution vectors, apart from allowing us to determine the amount of required reserves, offer also a reserve strategy that can be deployed in real time operation [9].

B. Chance constrained unit commitment and reserve scheduling

The main objective is to decide about the unit commitment so as to design a minimum cost day-ahead dispatch and reserve schedule. We consider an optimization horizon $N_t = 24$ with hourly steps, and introduce the subscript t in our notation to characterize the value of the quantities defined in the previous section for a given time instance $t = 1, \dots, N_t$.

For each step t of the optimization problem, define the vector of decision variables to be $x_t = [P_{G,t}, d_{\text{up},t}, d_{\text{down},t}, C_t^{\text{SU}}, R_{\text{up},t}, R_{\text{down},t}, z_t]^T \in \mathbb{R}^{7N_G}$, where $C_t^{\text{SU}} \in \mathbb{R}^{N_G}$ is the start-up cost vector, $z_t \in \mathbb{R}^{N_G}$ are auxiliary variables needed to model the minimum up and down times of each generator, and $R_{\text{up},t}, R_{\text{down},t} \in \mathbb{R}^{N_G}$ denote the probabilistically worst case up-down spinning reserves that the system operator needs to purchase. Moreover, $u_t \in \{0, 1\}^{N_G}$ is a binary vector and denotes the “on-off” status of each generator. Let $C_1, C_2, C_{\text{up}}, C_{\text{down}} \in \mathbb{R}^{N_G}$ be generation and reserve cost vectors, and $[C_2]$ denote a diagonal matrix with vector C_2 on the diagonal. The resulting optimization problem is given by

$$\min_{\{x_t\}_{t=1}^{N_t}, \{u_t\}_{t=1}^{N_t}} \sum_{t=1}^{N_t} \left(C_1^T P_{G,t} + P_{G,t}^T [C_2] P_{G,t} + \mathbf{1}^T C_t^{\text{SU}} + C_{\text{up}}^T R_{\text{up},t} + C_{\text{down}}^T R_{\text{down},t} \right), \quad (2)$$

subject to

1) Power balance constraints: For all $t = 1, \dots, N_t$, $\mathbf{1}^T (C_G P_{G,t} + C_w P_w^f - C_L P_{L,t}) = 0$. This constraint encodes the fact that the power balance in the network should be always satisfied when $P_{w,t} = P_w^f$. If load uncertainty is also taken into account, the equality constraint should be satisfied for the forecast load value.

2) Start-up cost constraints: For $t = 1, \dots, N_t$,

$$C_t^{\text{SU}} \geq \lambda_t^{\text{SU}} (u_t - u_{t-1}), \quad C_t^{\text{SU}} \geq 0. \quad (3)$$

Note that C_t^{SU} will always be zero unless the corresponding generator changes status from “off” to “on” within two consecutive periods [5]. In this case, due to (2), C_t^{SU} will become equal to $\lambda_t^{\text{SU}} \in \mathbb{R}^{N_G \times N_G}$, which is a diagonal matrix including the start-up costs.

3) Generation and transmission capacity constraints: For all $t = 1, \dots, N_t$

$$\begin{aligned} u_t P_{\min} &\leq P_{G,t} \leq u_t P_{\max}, \\ -\bar{P}_{\text{line}} &\leq B_f \begin{bmatrix} (\tilde{B}_{\text{BUS}})^{-1} \tilde{P}_t^f \\ 0 \end{bmatrix} \leq \bar{P}_{\text{line}}, \end{aligned} \quad (4)$$

where $P_{\min}, P_{\max} \in \mathbb{R}^{N_G}$ denote the minimum and maximum generating capacity of each unit, $\bar{P}_{\text{line}}$ denotes the line limits, and $\tilde{P}_t^f = [C_G(P_G + R) + C_w P_w^f - C_L P_L]_{N_b-1}$. Note that $\tilde{P}_t^f$ is given by the same expression as $\tilde{P}_t$ but with $P_{w,t} = P_w^f$. These constraints denote the generation and transmission capacity constraints for the deterministic case where the wind power is equal to its forecast value. Following the formulation of

[6] we have

$$-P_{\text{down}} \leq P_{G,t} - P_{G,t-1} \leq P_{\text{up}}, \text{ for } t = 1, \dots, N_t, \quad (5)$$

$$\sum_{j=t-\Delta t_{\text{up}}+1}^t z_j \leq u_j, \text{ for } t = \Delta t_{\text{up}}, \dots, N_t, \quad (6)$$

$$\sum_{j=t+1}^{t+\Delta t_{\text{down}}} z_j \leq 1 - u_j, \text{ for } t = 1, \dots, N_t - \Delta t_{\text{down}}, \quad (7)$$

$$z_t \geq u_t - u_{t-1}, \quad z_t \geq 0, \text{ for } t = 1, \dots, N_t. \quad (8)$$

Equation (5) models the ramping constraints of each generator, whereas by means of the auxiliary variables z_t , $t = 1, \dots, N_t$, (6), (7), (8) encode minimum up and down times for the generating units. $P_{\text{up}}, P_{\text{down}} \in \mathbb{R}^{N_G}$ denote the ramping limits of each unit, and $\Delta t_{\text{up}}, \Delta t_{\text{down}} \in \mathbb{R}_+$ denote the minimum time a unit needs to change status.

4) Distribution vector constraints: For all $t = 1, \dots, N_t$, $\mathbf{1}^T d_{\text{up},t} = 1$, $\mathbf{1}^T d_{\text{down},t} = 1$, encoding the fact that the elements of the distribution vectors should sum up to one.

5) Probabilistic constraints:

$$\begin{aligned} \mathbb{P}\left(\{P_{w,t}\}_{t=1}^{N_t} \in \mathbb{R}^{N_t} \mid -\bar{P}_{\text{line}} \leq B_f \begin{bmatrix} (\tilde{B}_{\text{BUS}})^{-1} \tilde{P}_t \\ 0 \end{bmatrix} \leq \bar{P}_{\text{line}}, \right. \\ u_t P_{\text{min}} \leq P_{G,t} + R_t \leq u_t P_{\text{max}}, \\ \left. -R_{\text{down},t} \leq R_t \leq R_{\text{up},t}, \right. \\ \left. \text{for all } t = 1, \dots, N_t\right) \geq 1 - \varepsilon, \quad (9) \end{aligned}$$

where R_t is given by (1). The first constraint inside the probability encodes the standard transmission capacity constraints, whereas the second one provides guarantees that the scheduled generation dispatch plus the reserve contribution R_t will not result in a new operating point outside the generation capacity limits. The last constraint of (9) is included to determine the reserves $R_{\text{up},t}, R_{\text{down},t}$ that guarantee that the generation and transmission capacity constraints are satisfied with high probability.

III. PROPOSED METHODOLOGY

A. Dealing with the chance constraint

Using a more compact notation, the chance constrained problem of Section II-B can be written as

$$\min_{x \in \mathbb{R}^{N_x}, u \in \{0,1\}^{N_G N_t}} J(x) \text{ subject to} \quad (\text{P}_1)$$

$$\mathbb{P}(\delta \in \Delta \mid A(\delta)x + Bu + c(\delta) \geq 0) \geq 1 - \varepsilon,$$

where $x \in \mathbb{R}^{N_x}$ with $N_x = 7N_G N_t$ is a vector including the decision variables, $u \in \{0,1\}^{N_G N_t}$ is a vector of binary variables, and $\delta \in \Delta \subset \mathbb{R}^{N_t}$ is the vector of uncertain parameters (the wind power prediction error for every hour $t = 1, \dots, N_t$). Variables x, u and δ consist “stacked” versions of $\{x_t\}_{t=1}^{N_t}$, $\{u_t\}_{t=1}^{N_t}$ and $\{P_{w,t} - P_{w,t}^f\}_{t=1}^{N_t}$, respectively. $J(x) \in \mathbb{R}$ is quadratic in x , and for each $\delta \in \Delta$, $A(\delta), B, c(\delta)$ are of appropriate dimension. It is assumed that Δ is endowed with a σ -algebra $\mathcal{D}$, and that $\mathbb{P}$ is a probability measure defined over $\mathcal{D}$. For all $x \in \mathbb{R}^{N_x}$, the constraint functions are assumed

to be measurable with respect to $\mathcal{D}$ and the Borel σ -algebra over $\mathbb{R}^{N_t}$.

To solve (2)-(9) we have to transform the chance constrained problem to a tractable, but in some sense equivalent problem. To avoid introducing arbitrary assumptions on $\mathbb{P}$ and its moments we follow a scenario based methodology. Due to the binary vector u , we can not apply the procedure of [10] and substitute the chance constraint with a finite number of hard constraints corresponding to scenarios of the uncertainty. Following such an approach would not allow us to provide any probabilistic guarantee, since convexity with respect to the decision variables is required. Moreover, even if this condition were satisfied, the number of scenarios that one needs to generate grows linearly with respect to the decision variables [11], thus hampering the applicability of the method to large scale systems. To overcome this difficulty we exploit the recent results of [12], and follow a three step procedure.

Step 1: Let $B(p) = \times_{j=1}^{N_t} [p_j^{\text{min}}, p_j^{\text{max}}]$ be a hyper-rectangle parameterized by $p = (p^{\text{min}}, p^{\text{max}}) \in \mathbb{R}^{2N_t}$, where $p^{\text{min}} = (p_1^{\text{min}}, \dots, p_{N_t}^{\text{min}}) \in \mathbb{R}^{N_t}$ and $p^{\text{max}} = (p_1^{\text{max}}, \dots, p_{N_t}^{\text{max}}) \in \mathbb{R}^{N_t}$. Consider now the problem

$$\begin{aligned} \min_{p \in \mathbb{R}^{2N_t}} \sum_{i=1}^{N_t} (p_i^{\text{max}} - p_i^{\text{min}}) \text{ subject to} \quad (\text{P}_2) \\ \mathbb{P}(\delta \in \Delta \mid \delta_i \in [p_i^{\text{min}}, p_i^{\text{max}}], \text{ for } i = 1, \dots, N_t) \geq 1 - \varepsilon. \end{aligned}$$

By minimizing the sum of the interval lengths which contain every uncertainty element δ_i , P_2 provides an appropriate parametrization p so that $B(p)$ encloses at least an $1 - \varepsilon$ fraction of the probability mass of the uncertainty vector δ . In general $B(p)$ could be any convex set with convex volume, and instead of P_2 we could minimize the volume of $B(p)$ which encloses δ with probability at least $1 - \varepsilon$.

Step 2: Problem P_2 is a convex problem by construction and we can apply the standard scenario approach to obtain a solution. Let $\beta \in (0, 1)$ be a confidence parameter and following [11] choose the number of uncertainty realizations N that need to be extracted according to $N \geq \frac{2}{\varepsilon} (2N_t + \ln \frac{1}{\beta})$. Consider now the scenario program that corresponds to P_2

$$\begin{aligned} \min_{p \in \mathbb{R}^{2N_t}} \sum_{i=1}^{N_t} (p_i^{\text{max}} - p_i^{\text{min}}) \text{ subject to} \quad (\text{P}_3) \\ \delta_i^{(k)} \in [p_i^{\text{min}}, p_i^{\text{max}}], \text{ for } i = 1, \dots, N_t, k = 1, \dots, N. \end{aligned}$$

Following [10], with confidence at least $1 - \beta$, the optimal solution p^* of P_3 is feasible for the chance constrained problem P_2 .

Step 3: Finally, we pose the following robust counterpart of problem P_1

$$\begin{aligned} \min_{x \in \mathbb{R}^{N_x}, u \in \{0,1\}^{N_G N_t}} J(x) \text{ subject to} \quad (\text{P}_4) \\ A(\delta)x + Bu + c(\delta) \geq 0, \text{ for all } \delta \in B(p^*) \cap \Delta. \end{aligned}$$

Note that this is not a randomized program, and we require the constraints to be satisfied for all values of the uncertainty inside $B(p^*) \cap \Delta$. Therefore, P_4 is a robust mixed-integer

quadratic problem. As shown in Proposition 1 of [12], with confidence at least $1 - \beta$, any feasible solution of P_4 is feasible for the initial chance constrained problem P_1 .

B. Tractable robust reformulation

Following the methodology outlined in the previous section, to solve P_1 requires generating N scenarios, where N is independent of the number of decision variables and depends only on the dimension of the parameter vector p , and then solving P_4 which is a robust program. This procedure leads to a tractable problem as long as P_4 is tractable. To ensure this, we follow the approach of [16], [17], whose authors propose a tractable reformulation of robust convex programs, that was subsequently extended in [18] to capture robust mixed-integer problems as well.

For a certain class of uncertainty sets (e.g. hyperrectangular) it is shown in [17] that problems in the form of P_4 are tractable and remain in the same class as the original problems, e.g. robust mixed-integer programs remain mixed-integer programs. This is achieved under the assumption that the constraint functions are concave and homogeneous with respect to the uncertainty vector, and by introducing some additional decision variables and constraints. Notice that the homogeneity assumption is trivially satisfied in our case, but the elements of $A(\delta)$ are not necessarily concave with respect to δ due to the max terms in (1). However, P_4 exhibits a particular structure that allows us to apply the methodology of [17]. Specifically, since the constraints that couple the individual stages (every hour $i = 1, \dots, N_t$) are deterministic, and the constraints of each stage depend only on the uncertainty elements and decision variables that correspond to that stage, we can split the robust constraint in P_4 in two robust constraints as follows. Considering for simplicity $\Delta = \mathbb{R}^{N_t}$, one of these constraints requires $A(\delta)x + Bu + c(\delta) \geq 0$ for all $\delta \in B^+$, where $B^+ = \{\delta \in B(p^*) | \delta_i \geq 0 \text{ for all } i = 1, \dots, N_t\}$, and the other requires $A(\delta)x + Bu + c(\delta) \geq 0$ for all $\delta \in B^-$, where $B^- = \{\delta \in B(p^*) | \delta_i \leq 0 \text{ for all } i = 1, \dots, N_t\}$. In other words, the first constraint should be satisfied for all values of the uncertainty in the positive region $B^+ \subset B(p^*)$, whereas the second should be satisfied for all values of the uncertainty in the negative region $B^- \subset B(p^*)$, thus rendering $A(\delta)$ linear in δ for each robust constraint (to see this inspect (1)).

Similar to the parametrization vector p , let p^+, p^- be parametrization vectors corresponding to the hyperrectangular regions B^+ and B^- , respectively. In the sequel, we follow the methodology of [17] to show how a robust mixed-integer linear constraint in the form of P_4 can be replaced by a list of linear constraints. To avoid unnecessarily complicating the notation, we show this only for a single robust constraint; however, following the aforementioned discussion, the constraint in P_4 should be replaced with two robust constraints corresponding to B^+ and B^- , and for each of them one should apply the procedure presented in the sequel with the parametrization vector p being replaced with p^+ and p^- , respectively.

For all $j = 1, \dots, N_t$ let $e_j \in \mathbb{R}^{N_t}$ be a unit vector whose j -th element is "1", and $p^0 = 0.5(p^{\min} + p^{\max}) \in \mathbb{R}^{N_t}$ be

a vector whose elements are the middle points of each interval $[p_j^{\min}, p_j^{\max}]$, $j = 1, \dots, N_t$. Moreover, denote by N_r the number of rows of $A(\delta)$ and let $y \in \mathbb{R}^{N_r}$, $Q \in \mathbb{R}^{N_r \times N_t}$ (with elements q_{ij}) be additional decision variables. We are now in a position to define an optimization problem which provides a tractable reformulation of P_4 .

$$\begin{aligned} & \min_{x \in \mathbb{R}^{N_x}, u \in \{0,1\}^{N_G N_t}, y \in \mathbb{R}^{N_r}, Q \in \mathbb{R}^{N_r \times N_t}} J(x) \text{ subject to} \quad (P_5) \\ & 1) A_i(e_j e_j^T (p^{\max} - p^0))x + c_i(e_j e_j^T (p^{\max} - p^0)) \geq q_{ij}, \\ & \quad \text{for all } i = 1, \dots, N_r, j = 1, \dots, N_t, \\ & 2) A_i(e_j e_j^T (p^{\min} - p^0))x + c_i(e_j e_j^T (p^{\min} - p^0)) \geq q_{ij}, \\ & \quad \text{for all } i = 1, \dots, N_r, j = 1, \dots, N_t, \\ & 3) \sum_{j=1}^{N_t} q_{ij} \geq y_i, \text{ for all } i = 1, \dots, N_r, \\ & 4) A(p^0)x + Bu + c(p^0) + y \geq 0. \end{aligned}$$

We discuss now the interpretation of the structure of P_5 . Note that we seek to transform P_4 in a tractable form. Its constraints can be equivalently written as

$$A(p^0 + \Delta p)x + Bu + c(p^0 + \Delta p) \geq 0, \quad (10)$$

for all Δp with $\Delta p_j \in [p_j^{\min} - p_j^0, p_j^{\max} - p_j^0]$, $j = 1, \dots, N_t$. Under the concavity and homogeneity assumption for $A(\cdot)$, $c(\cdot)$ (note that these assumptions hold only if we distinguish between B^+ and B^-), we have that for any Δp , $A(p^0 + \Delta p) \geq A(p^0) + A(\Delta p)$ and $c(p^0 + \Delta p) \geq c(p^0) + c(\Delta p)$. Therefore, it suffices to require that for all admissible Δp

$$A(p^0)x + Bu + c(p^0) + A(\Delta p)x + c(\Delta p) \geq 0. \quad (11)$$

To achieve this, we need to bound the term $A(\Delta p)x + c(\Delta p)$. Consider first the worst case perturbation vectors $e_j e_j^T (p^{\max} - p^0) \in \mathbb{R}^{N_t}$, $e_j e_j^T (p^{\min} - p^0) \in \mathbb{R}^{N_t}$, for each $j = 1, \dots, N_t$. Notice that these vectors have all their elements zero and in the j -th position include the maximum (respectively minimum) deviation of this element from the middle point p_j^0 . For each $i = 1, \dots, N_r$, constraints 1), 2), impose a bound q_{ij} to the terms $A_i(e_j e_j^T (p^{\max} - p^0))x + c_i(e_j e_j^T (p^{\max} - p^0))$ and $A_i(e_j e_j^T (p^{\min} - p^0))x + c_i(e_j e_j^T (p^{\min} - p^0))$. Letting now y_i , $i = 1, \dots, N_r$, as set by constraint 3), and considering the worst case superposition of the perturbation vectors, for each $i = 1, \dots, N_r$ we have that $A_i(\Delta p)x + c_i(\Delta p) \geq \sum_{j=1}^{N_t} q_{ij} \geq y_i$. The last equation implies that $A(\Delta p)x + c(\Delta p) \geq y$, and together with (11) justifies constraint 4).

Problem P_5 is a mixed-integer quadratic problem, and compared to P_4 has $2(N_t + 1)N_r$ additional decision variables and $2(2N_t + 1)N_r$ additional constraints, where the factor of 2 accounts for the fact that we need to consider two robust constraints corresponding to B^+ and B^- , respectively. Due to the equivalence between P_5 and P_4 , the optimal solution of P_5 is feasible for P_1 with confidence at least $1 - \beta$. The proposed procedure is summarized in Algorithm 1.

Note that a vertex enumeration scheme can be also applied in the particular set-up without an exponential dependance on the dimension of the uncertainty. The reason is that the

Algorithm 1 Proposed approach.

- 1: **Stochastic unit commitment and reserve scheduling.**
 - 2: Set $u_0 \in \{0, 1\}^{N_G} \triangleright$ initial status of the generating units.
 - 3: Fix $\varepsilon \in (0, 1)$, $\beta \in (0, 1)$.
 - 4: Define $B(p) = \times_{j=1}^{N_t} [p_j^{\min}, p_j^{\max}]$ and generate $N \geq \frac{2}{\varepsilon} (2N_t + \ln \frac{1}{\beta})$ scenarios.
 - 5: Solve P_3 and determine p^* .
 $\triangleright p^*$ is feasible for P_2 with probability at least $1 - \beta$.
 - 6: Solve P_5 .
 $\triangleright P_5$ is equivalent with P_4 and its optimal solution is feasible for P_1 (i.e. (2)-(9)) with probability $1 - \beta$.
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Algorithm 2 Benchmark approach.

- 1: **Deterministic unit commitment.**
 - 2: Set $u_0 \in \{0, 1\}^{N_G} \triangleright$ initial status of the generating units.
 - 3: Define $x_t^1 = [P_{G,t}, C_t^{\text{SU}}, z_t]^T \in \mathbb{R}^{3N_G}$ and set $P_w = P_w^f$,
 $x_t \leftarrow x_t^1$ for all $t = 1, \dots, N_t$.
 - 4: Solve (2)-(8).
 - 5: **Stochastic reserve scheduling.**
 - 6: Fix $\{u_t\}_{t=1}^{N_t}$ according to Step 4, and $\varepsilon \in (0, 1)$, $\beta \in (0, 1)$.
 - 7: Define $x_t^2 = [P_{G,t}, d_{\text{up},t}, d_{\text{down},t}, R_{\text{up},t}, R_{\text{down},t}]^T \in \mathbb{R}^{5N_G}$,
and set $x_t \leftarrow x_t^2$ for all $t = 1, \dots, N_t$.
 - 8: Generate $\tilde{N} \geq \frac{2}{\varepsilon} (5N_G N_t + \ln \frac{1}{\beta})$ scenarios.
 - 9: Solve the scenario program that corresponds to (2)-(9).
 $\triangleright$ the resulting optimal solution is feasible for (2)-(9) with probability $1 - \beta$.
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uncertainty affecting the constraints of each time-step is scalar and does not influence the constraints of the other time-steps. Therefore, for each stage $i = 1, \dots, N_t$ it suffices to enforce the corresponding constraints only for the extreme values of uncertainty interval $(p_i^{\max}, p_i^{\min})$.

IV. NUMERICAL STUDY

A. Simulation set-up

To illustrate the performance of our algorithm we compare it against a hybrid methodology, where the unit commitment and reserve scheduling problems are treated separately. We start by formulating the deterministic variant of the unit commitment problem, for the case where the wind power is equal to its forecast. Note that since this corresponds to the nominal case no reserves are needed (for $P_w = P_w^f$, (1) leads to $R = 0$). Therefore, defining $x_t^1 = [P_{G,t}, C_t^{\text{SU}}, z_t]^T \in \mathbb{R}^{3N_G}$, we need to solve over (2)-(8) with respect to $\{x_t^1\}_{t=1}^{N_t}, \{u_t\}_{t=1}^{N_t}$, with $P_{w,t} = P_w^f$ for all $t = 1, \dots, N_t$.

At a next step, we fix the “on-off” status of the generating units (and also C_t^{SU}, z_t) to the binary vector computed by the deterministic unit commitment program, and formulate a stochastic reserve scheduling problem. This requires solving (2)-(9), where minimization is now carried out with respect to $x_t^2 = [P_{G,t}, d_{\text{up},t}, d_{\text{down},t}, R_{\text{up},t}, R_{\text{down},t}]^T \in \mathbb{R}^{5N_G}$. To deal with the resulting chance constrained problem we follow the standard scenario approach [11], and substitute the chance constraint with $\tilde{N} \geq \frac{2}{\varepsilon} (5N_G N_t + \ln \frac{1}{\beta})$ hard constraints. The term $5N_G N_t$ denotes the number of decision variables in the chance constrained program. The basic steps of this procedure, which we will refer to as “benchmark approach”,

are summarized in Algorithm 2. Note that in the benchmark approach we use the standard scenario approach instead of the proposed methodology, thus generating a different number of scenarios. This is motivated by the fact that our objective is to demonstrate the potential advantage of using Algorithm 1 against an algorithm that does not support chance constrained mixed-integer problems.

To compare the proposed approach with the benchmark one we carried out Monte Carlo simulations. To generate scenarios for the wind power error, we used a Markov chain based model (see [9]). All optimization problems were solved using the solver CPLEX [19] via the MATLAB interface YALMIP [20].

B. Simulation results

Algorithms 1 and 2 are applied to the IEEE 30-bus network [21] with a wind power generator connected to bus 22; numerical data for the reserve, start-up and production cost vectors can be found in [22], whereas $P_{\text{up}} = P_{\text{down}} = P_{\text{max}}/3$ and the elements of the minimum up and down time vectors $(\Delta t_{\text{up}}, \Delta t_{\text{down}})$ corresponding to the first two generators were chosen to be 2 hours.

When attempting to solve the problem using Algorithm 2 we faced memory problems due to the high number of constraints $\tilde{N}$. To carry out the comparison study and illustrate the advantage of a stochastic unit commitment problem, we removed the ramping constraints (5) that couple the optimization stages. The linear dependance of the constraints on the uncertainty elements, together with the fact that the stages in Algorithm 2 are decoupled (the binary variables are fixed), allows us to use for every stage only the minimum and maximum value of the $\tilde{N}$ scenarios. The resulting problem is computationally less expensive, and can be solved by existing tools. At the end of the section we reconsider constraint (5) and repeat our analysis only for Algorithm 1. It should be apparent that, in contrast to the standard scenario approach, the proposed methodology enables us to deal with problems of higher dimension with low computational cost; due the robust problem involved at Step 3 of our method, the resulting solution is not necessarily less conservative.

Fig. 1 shows for one day of the simulated data the total scheduled cost (sum of the production, reserve and start-up costs), and the production cost. The cost pattern follows the load profile that was employed. The proposed algorithm leads to lower total cost, while the production cost is similar for both methods. The improvement in terms of cost is due to the scheduling flexibility offered by the proposed algorithm, where the unit commitment is solved together with the reserve scheduling problem, allowing us to identify more optimal unit commitment schedules.

We repeated the procedure outlined by Algorithms 1 and 2 for 30 days of our data-set, corresponding to different forecast and actual wind power values. Fig. 2 shows for each day the relative cost difference between the costs generated by Algorithms 1 and 2. For all days this difference is always positive (a maximum improvement of 1.29% is encountered), highlighting that the proposed approach leads

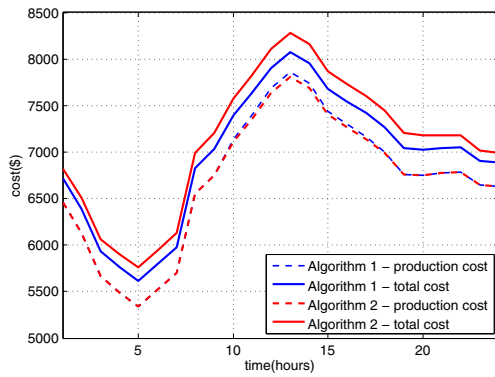


Fig. 1. Total scheduled cost (solid line) as the sum of the production, reserve and start-up costs, and production cost (dashed line) for one day of the simulated data. The “blue” lines correspond to the proposed approach (Algorithm 1), whereas the “red” lines correspond to the benchmark approach (Algorithm 2).

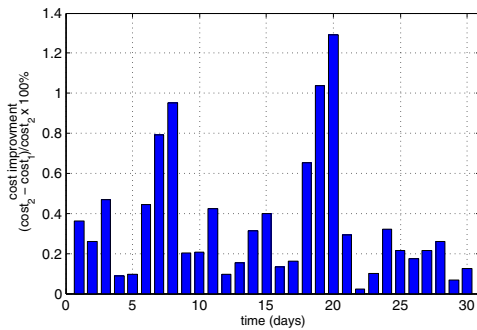


Fig. 2. Relative cost difference between the cost generated by Algorithm 2 and the cost generated by Algorithm 1, normalized with the cost of Algorithm 2, for 30 days of the simulated data.

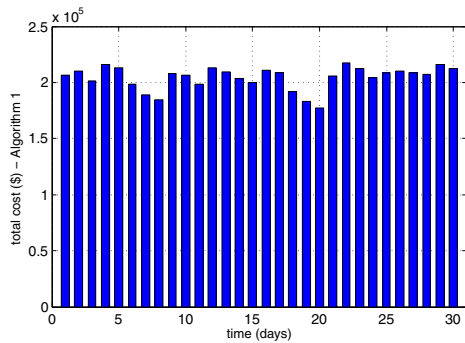


Fig. 3. Total cost per day using Algorithm 1 and including the ramping constraints (5).

to systematically lower cost compared to the benchmark approach. Fig. 3 shows the total cost for every day, obtained after applying Algorithm 1 while considering the ramping constraints (5) of the generating units.

V. CONCLUDING REMARKS

In this paper a unified framework for solving the problem of stochastic unit commitment and reserve scheduling is proposed, while providing a priori guarantees for the probability of constraint satisfaction. Current work concentrates

toward incorporating security constraints in the developed framework and decentralizing the developed scheme.

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