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Adaptive control for state dependent switched systems in Brunovsky form

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Abstract— **In this work an adaptive strategy for state dependent switching systems in Brunovsky form is presented. Lyapunov direct method is used to derive the adaptive control law to track a given reference signal. The resulting controller is equipped with a set of adaptive gains able to tackle plant commutations. Control gains are adapted through a feedback law depending on the state space region visited by the system trajectories. A consistent proof of convergence of the tracking error is given and a numerical example is used to illustrate the analytical result.**

I. Introduction

Models in Brunovsky form are useful to describe the behavior of many plants of practical interest [1]. Several nonlinear systems can be described via dynamical systems in Brunovsky form, e.g., some relevant second order plants see [2], [3], [4], [5], and systems where some part of the nonlinear behavior is represented by means of function approximation technique [6], [7], [8], [9], [10], [3], [11], [12]. Moreover, generic plants with highly nonlinear behavior can be recast by means of function approximation techniques (e.g. series expansions and parametric models), in Brunovsky form with unknown constant coefficients and additional nonlinear functions [13]. To steer their dynamics in a desired manner, different control approaches, including backstepping and model reference adaptive control [9], [14], [3], [15], [16], [17] have been shown to be effective in guaranteeing stability or asymptotic convergence of the tracking error under a proper choice of the update and control laws. Other control methods to handle this type of plant model include those in [9], [18], [12] and Nussbaum gain technique [19], [20]. Typically, most of the available techniques involve a control law with compensation terms and either a projection modification of the update law, as in [9], [18], or a σ modification, as in [12]. Some estimates of lower or upper bounds of the plant coefficients are required to be known to achieve asymptotic convergence of the tracking error to a residual set of user–defined size. This requirement can be overcome if the controller is properly formulated, as can be concluded from what reported in [19]. Some hybrid plants can also be described as state dependent switched systems in Brunovsky form with additional nonlinear terms [21], [22], [23]. Most of the existing publications concerning the control of hybrid plants are about switching systems governed by arbitrary or time-dependent switching signals (see, for example, [24], [25] and reference therein), while only few control approaches have been dedicated to deal specifically with the discontinuities that characterize the evolution of state-dependent switched systems dynamics. Among these techniques, different control schemes have been proposed in the literature to extend to some specific classes of state dependent switched systems the features of control techniques previously developed for smooth dynamical systems. For example, L_2-gain is presented in [26], robust stability is investigated in [27]. Model reference adaptive control (MRAC) for piecewise affine (PWA) systems in control canonical form can be found in [28], [29], [30], [31]. Their applicability to generic PWA plant can be achieved after linear transformation of the state space as those proposed in [32] while some experimental validations can been found in [33], [34]. We note that recently those strategies have been applied to the problem of the master-slave synchronization of PWA systems [35] and their parameter identification [36], [37]. A different approach to model reference adaptive control for this class of switched systems can be fond in [38] where the reference model switches synchronously with the plant, while in [39], the MRAC problem has been solved by using time depend-switched adaptive controllers.

Here we propose a switched adaptive control approach to achieve asymptotic tracking of a reference trajectory for a n-dimensional multimodal state dependent switching nonlinear system in Brunovsky form. The resulting hybrid control strategy consists of a full state feedback action whose gains switch according to the phase space regions visited by the plant. The adaptive laws are designed for updating the controller parameters when the parameters characterizing each of the modes of the plant are unknown. Under the typical hypothesis of smoothness of the reference trajectory [40], asymptotic convergence of the output tracking error and the closed-loop signal boundedness are ensured. The control design is based on the idea of finding an appropriate

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common Lyapunov function [24]. Simulation results on a representative example complement the theoretical derivation and show the effectiveness of the proposed approach. There are several advantages with respect to other controllers in the literature as for example that lower or upper bounds of the plant coefficients are not required to be known a priori.

II. Problem statement and definitions

Assume that the Euclidian space $I\!R^n$ is partitioned into M domains with smooth and known boundaries, say $\{\Omega_i\}_{i\in\mathcal{M}}$ with $\mathcal{M}\triangleq\{1,2,\ldots,M\}$ ($\bigcup_{i=1}^{M}\Omega_i=I\!R^n$ and $\Omega_{i_1}\cap\Omega_{i_2}=\emptyset$ with $i_1,\, i_2\in\mathcal{M}$, $i_1\neq i_2$ and $\Omega_{i_1},\ \Omega_{i_2}$).

Consider a state dependent switched system so that, in each domain Ω_i ($i=1,2,\ldots,M$), the plant dynamics are described by an n-dimensional nonlinear system in Brunovky form as:

$$\dot{x}_v = x_{v+1},\quad v=1,2,\ldots,n-1, \tag{1a}$$

$$\dot{x}_n = -\sum_{k=1}^{n} a_k^{[i]}x_k + c^{[i]}\phi\left(x,t\right)+b^{[i]}u,\quad \text{if } x\in\Omega_i, \tag{1b}$$

$$y = x_1, \tag{1c}$$

where $y\in I\!R$ is the system output, u is the control input, x is the state vector defined as

$$x=\left[\begin{array}{cccc} x_1 & x_2 & \cdots & x_n\end{array}\right]^T=\left[\begin{array}{cccc} y & \dot{y} & \cdots & y^{(n-1)}\end{array}\right]^T\in I\!R^n \tag{2}$$

and $\phi\left(x,t\right)$ is a known and smooth nonlinear function of state variables and time. Note that the parameters $\left\{a_k^{[i]}\right\}$ and $c^{[i]}$ ($k=1,2,\ldots,n$) of the i-th Brunovsky model are assumed to be completely unknown, while only the sign of parameters $\left\{b^{[i]}\right\}$ is meant to be known *a priori*. Note that, through the rest of the paper, we will assume without loss of generality that system (1) can exhibit only a finite number of switching times on every bounded time interval [24].

Assume there exists a reference signal y_d so that:

$$x_d := \left[\begin{array}{cccc} y_d & \dot{y}_d & \cdots & y_d^{(n-1)}\end{array}\right]^T\in C^0\left[0,+\infty\right[, \tag{3}$$

$$\bar{x}_d := \left[\begin{array}{ccccc} y_d & \dot{y}_d & \cdots & y_d^{(n-1)} & y_d^{(n)}\end{array}\right]^T\in L^\infty\left[0,+\infty\right[; \tag{4}$$

the problem is to find a piecewise-smooth full state feedback adaptive control law $u(t)$ so that the output of the system (1) tracks asymptotically y_d, i.e.:

$$\lim_{t\to+\infty}\left(y\left(t\right)-y_d\left(t\right)\right)=\lim_{t\to+\infty}e(t)=0, \tag{5}$$

with bounded adaptive gains and a bounded control action.

III. Control law and Main result

As detailed in the rest of the paper, the control problem described in Section II can be solved according to the following result.

Theorem 1: Consider a generic Hurwitz polynomial:

$$P\left(s\right)=s^{n-1}+\lambda_{n-2}s^{n-2}+\lambda_{n-3}s^{n-2}+\ldots+\lambda_1 s+\lambda_0. \tag{6}$$

Given a switching system of the form (1) and a reference output trajectory y_d so that conditions (3)-(4) hold, then the following full state feedback switching adaptive strategy guarantees asymptotic stability of the output tracking error (i.e., $e\to 0$ as $t\to+\infty$), boundness of the closed loop system, namely $x(t)$, $\widehat{\theta}_i(t)$, and of the control action $u(t)$ (i.e. $x(t)$, $\widehat{\theta}_i(t)$, $u(t)\in L^\infty[0,\infty[$). Specifically, it suffices to choose:

$$u=\varphi^T\widehat{\theta}_i,\quad \text{if}\quad x\in\Omega_i\quad i=1,2,\ldots,M, \tag{7}$$

where

$$\varphi^T=\left[\begin{array}{ccccc} x_n & \ldots & x_1 & -\phi & \varphi_a-\gamma\mathcal{S}\end{array}\right], \tag{8}$$

$$\varphi_a=-y_d^{(n)}+\lambda_{n-2}e^{(n-1)}+\ldots+\lambda_1\ddot{e}+\lambda_0\dot{e}, \tag{9}$$

and γ is a positive scalar constant and $\widehat{\theta}_i$ is chosen as:

$$\dot{\widehat{\theta}}_i=\begin{cases} -\Gamma_i\text{sgn}\left(b^{[i]}\right)\mathcal{S}\varphi, & \text{if } x\in\Omega_i,\\ 0 \quad \text{elsewhere}\end{cases} \tag{10}$$

with

$$\mathcal{S}=e^{(n-1)}+\lambda_{n-2}e^{(n-2)}+\lambda_{n-3}e^{(n-3)}+\ldots+\lambda_1\dot{e}+\lambda_0 e, \tag{11}$$

and $\Gamma_i\in I\!R^{(n+2)\times(n+2)}$ being a set of generic strictly positive matrices.

Remark 1: Conventional control laws for control schemes based on the Lyapunov function consist of the product between a vector of adaptive gains and a regression vector, where the adaptive gains are provided by an updating mechanism whereas the entries of the regression vector are known and depend on the plant states, see [40] pp. 327, 333, [41] pp. 137, [42] pp. 207. From this viewpoint, the control law (7) is similar to these control laws because it involves a regression vector φ and adaptive gains $\hat{\theta}_i(i=1,\cdots,M)$ that are provided by updating mechanisms. Nevertheless, it is substantially different from those presented in the literature as it uses a different adaptive gain vector $\hat{\theta}_i(i=1,\cdots,M)$ according to the region in which the state x lies at a given time instant.

Remark 2: According to the adaptation law (10), a generic adaptive gain $\widehat{\theta}_i$ evolves only when the plant trajectory enters into the domain Ω_i and it is kept constant to the last value it assumed when it left region Ω_i.

Remark 3: The reference trajectory y_d can be also originated as the output of a reference model. Specifically, in order to fulfill properties (3)-(4), the reference model can be chosen either as an asymptotically stable n-dimensional LTI system or as a BIBS (Bounded-Input Bounded-State) state dependent switching system, again in Brunovsky form:

$$\dot{\bar{x}}_v=\bar{x}_{v+1},\quad v=1,2,\ldots,n-1, \tag{12a}$$

$$\dot{\bar{x}}_n=-\sum_{k=1}^{n}\bar{a}_k^{[i]}\bar{x}_k+\bar{b}^{[i]}r,\ \text{ if } \bar{x}\in\bar{\Omega}_{\bar{i}}, \tag{12b}$$

$$y_d=\bar{x}_1, \tag{12c}$$

where $r\in I\!R$ is the command signal generating the reference trajectory, $\left\{\bar{\Omega}_{\bar{i}}\right\}$ are the $\bar{M}$ domains partitioning the Euclidian space of the reference model and $\left\{\bar{a}_v^{[\bar{i}]}\right\}$, $\left\{\bar{b}^{[\bar{i}]}\right\}$ are the parameters of the reference model in each domain, being

$\bar{i} = 1, 2, \ldots \bar{M}$. We further remark here that the reference model can be characterized by a number and geometry of phase space regions that can be entirely different from those of the plant.

IV. Proof of Theorem 1

Before giving a detailed proof of Theorem 1 in what follows some further mathematical details are provided.

A. Mathematical preliminaries

From the definition of the system output (1c) and assumption (3), it follows that signals $y_d,\ y \in C^{(n-1)}\,[0,\ +\infty[$, thus function $\mathcal{S}$ defined as in (11) is continuous and differentiable and its derivative can be computed as:

$$\begin{aligned}\dot{\mathcal{S}} &= y^{(n)} - y_d^{(n)} + \lambda_{n-2} e^{(n-1)} + \ldots + \lambda_1 \ddot{e} + \lambda_0 \dot{e} \\ &= y^{(n)} + \varphi_a. \end{aligned} \tag{13}$$

Furthermore, from equation (1b), in every domain Ω_i the above expression can be written as:

$$\dot{\mathcal{S}} = -\sum_{k=1}^{n} a_k^{[i]} x_k + c^{[i]}\phi + b^{[i]} u + \varphi_a, \ \text{ if } \ x \in \Omega_i. \tag{14}$$

Note that, in the ideal case of perfect knowledge of the plant, it would be possible to design an ideal control law, say $u^\star$, based on the system parameters as:

$$u^\star = \varphi^T \theta_i^\star, \quad \text{if} \quad x \in \Omega_i, \tag{15}$$

with

$$\theta_i^\star = \left[\begin{array}{ccccc} a_1^{[i]} & \ldots & a_n^{[i]} & c^{[i]} & 1 \end{array}\right]^T. \tag{16}$$

Since the control law (7) has the same structure of (15), the adaptive gains $\widehat{\theta}_i$ $(i = 1, \ldots, M)$ can be considered as an estimate of the plant parameter vectors $\theta_i^\star$.

After some algebraic manipulation, the derivative of the function $\mathcal{S}$ in (17) can be now rewritten in terms of $\theta_i^\star$ as:

$$\dot{\mathcal{S}} = -\gamma S + b^{[i]} \left(u - \varphi^T \frac{\theta_i^\star}{b^{[i]}} \right), \quad \text{if} \quad x \in \Omega_i. \tag{17}$$

We remark that, in the case where plant parameters are perfectly known, asymptotic stability of the tracking error under the control action (15) can be easily shown taking into account expression (17). Specifically, stability can be achieved by choosing $V = \mathcal{S}^2$ as a candidate Lyapunov function. Under this choice, from expression (17), setting $u = u^\star / b^{[i]}$, it follows that $\dot{V} = -2\gamma \mathcal{S}^2$.

Substituting (7) into (17) the derivative of $\mathcal{S}$ can be now written as:

$$\dot{\mathcal{S}} = -\gamma S + b^{[i]} \varphi^T \widetilde{\theta}_i, \quad \text{if} \quad x \in \Omega_i, \tag{18}$$

where

$$\widetilde{\theta}_i = \widehat{\theta}_i - \frac{\theta_i^\star}{b^{[i]}}, \quad i = 1, \ldots, M. \tag{19}$$

The dynamics of the parameter's error vectors $\widetilde{\theta}_i$ can then be easily obtained by differentiating expression (19), yielding:

$$\dot{\widetilde{\theta}}_i = \dot{\widehat{\theta}}_i, \quad i = 1, \ldots, M. \tag{20}$$

Hence, the adaptation dynamics of gains $\widetilde{\theta}_i$ are defined as in (10).

B. Proof of Theorem 1

To show asymptotic convergence of the tracking error e and boundedness of the state vector x, the adaptive gains $\widehat{\theta}_i$ and the control law u, we consider the extended switching dynamical system (18)-(20), whose state vector is

$$\xi = \left[\begin{array}{cccc} \mathcal{S} & \widetilde{\theta}_1^T & \ldots & \widetilde{\theta}_M^T \end{array}\right]^T. \tag{21}$$

Note that this system exhibits an equilibrium point at the origin, $\xi = 0$.

To prove stability we choose the following common candidate Lyapunov function [24]:

$$\begin{aligned} V(\xi) &= V_{\mathcal{S}} + V_{\widetilde{\theta}}, & (22)\\ V_{\mathcal{S}} &= \frac{1}{2}\mathcal{S}^2, & (23)\\ V_{\widetilde{\theta}} &= V_{\widetilde{\theta}_1} + V_{\widetilde{\theta}_2} + \ldots + V_{\widetilde{\theta}_M}, & (24)\\ V_{\widetilde{\theta}_i} &= \frac{1}{2}\left| b^{[i]} \right| \widetilde{\theta}_i^T \Gamma_i^{-1} \widetilde{\theta}_i, \ \ i = 1 \ldots M. & (25) \end{aligned}$$

Note that $V(\xi)$ is radially unbounded and it is positive for all ξ different from the null vector while it is null in the origin. Furthermore it is continuous and differentiable almost everywhere.

From (18), the time derivative of $V_{\mathcal{S}}$ is:

$$\dot{V}_{\mathcal{S}} = -\gamma \mathcal{S}^2 + b^{[i]} \mathcal{S} \varphi^T \widetilde{\theta}_i, \ \text{ if } \ x \in \Omega_i, \ \ i = 1, \ldots, M. \tag{26}$$

Furthermore, from expressions (25), (20) and (10), we have

$$\dot{V}_{\widetilde{\theta}_i} = \begin{cases} -b^{[i]} \mathcal{S} \varphi^T \tilde{\theta}_i, & \text{if} \quad x \in \Omega_i, \\ 0, \quad \text{elsewhere.} \end{cases} \tag{27}$$

Combining expressions (26) and (27), it is then possible to compute the derivative of the Lyapunov function (22) along the solutions of the system (18)-(20) as:

$$\dot{V} = \dot{V}_{\mathcal{S}} + \dot{V}_{\widetilde{\theta}} = -\gamma \mathcal{S}^2. \tag{28}$$

Since (28) is semi–definite negative then, from Lyapunov theory, it is possible to conclude that the origin is a globally stable equilibrium point with all trajectories of the system bounded (i.e. $\xi \in L^\infty\,[0,\ +\infty[$ or, equivalently, $\mathcal{S} \in L^\infty\,[0,\ +\infty[$ and $\widetilde{\theta}_i \in L^\infty\,[0,\ +\infty[$, independently from the initial condition).

From the boundedness of $\mathcal{S}$ and $\widetilde{\theta}_i$ $(i = 1, 2 \ldots M)$ the following implications follow:

1) (*boundedness of the adaptive gains*) Since $\widetilde{\theta}_i \in L^\infty\,[0,\ +\infty[$, from (19) we have that $\widehat{\theta}_i \in L^\infty\,[0,\ +\infty[$. Notice that expression (28) does not involve quadratic forms with respect to the parameter error vectors $\tilde{\theta}_1,\ \cdots,\ \tilde{\theta}_M$. This property is common in adaptive control schemes based on the Lyapunov function, as can be noticed in [40] pp. 325, 329, [42] pp. 207, [41] pp. 135. This implies that the parameter error vectors $\tilde{\theta}_1, \cdots, \tilde{\theta}_M$ do not converge towards zero, or equivalently, the estimated parameters $\hat{\theta}_1,\ \cdots,\ \hat{\theta}_M$ do not converge to the plant parameters $\theta_1^*/b^{[1]},\ \cdots,\ \theta_M^*/b^{[M]}$.
2) (*boundedness of plant state trajectories*) Since $\mathcal{S} \in L^\infty\,[0,\ +\infty[$, we have $\left[\begin{array}{cccc} e & \dot{e} & \ldots & e^{(n-1)} \end{array}\right]^T =$

$(x - x_d) \in L^\infty [0, +\infty[$ according to the approach in [40]. Hence, since x_d is bounded (see assumption (4)), it follows that $x(t) \in L^\infty [0, +\infty[$. Note that boundedness of the state vector x implies that $\phi(x,t)$ is also bounded for all time.

3) Since $\left[\begin{array}{cccc} e & \dot{e} & \ldots & e^{(n-1)} \end{array} \right]^T \in L^\infty [0, +\infty[$ and $y_d^{(n)}$ is bounded (again according to the assumption in (4)), φ_a is bounded and, therefore, φ, defined in (8), is bounded too (all of its components are bounded).

4) (*boundedness of the control law*) Since $\varphi, \theta_i \in L^\infty [0, +\infty[$, then the adaptive control law $u(t)$ in (7) remains bounded for all time.

5) (*boundedness of* $\dot{\mathcal{S}}$) From the boundedness of $\mathcal{S}$, φ, and $\widetilde{\theta}_i$, it follows that the time derivative $\dot{\mathcal{S}}$ defined in (18) belongs to $L^\infty [0, +\infty[$.

Based on the above derivation, it is now possible to prove that $e(t) \to 0$ when $t \to +\infty$.

From (28) we have

$$\gamma \mathcal{S}^2 = -\dot{V}, \tag{29}$$

hence, by integrating both sides of (29), we have

$$\gamma \int_{t_0}^{t} \mathcal{S}^2 d\tau = V(t_0) - V(t), \tag{30}$$

that in turn implies:

$$\gamma \int_{t_0}^{t} \mathcal{S}^2 d\tau \leq V(t_0). \tag{31}$$

Inequality (31) holds for any generic time instant, hence $\mathcal{S} \in L^2 [0, +\infty[$. Furthermore, since $\mathcal{S}, \dot{\mathcal{S}} \in L^\infty [0, +\infty[$ (see the considerations made above) the function $\mathcal{S}$ asymptotically converges to zero according to Barbalat's Lemma (see e.g.[40] for a discussion and applications), i.e. $\mathcal{S} \to 0$ as $t \to +\infty$.

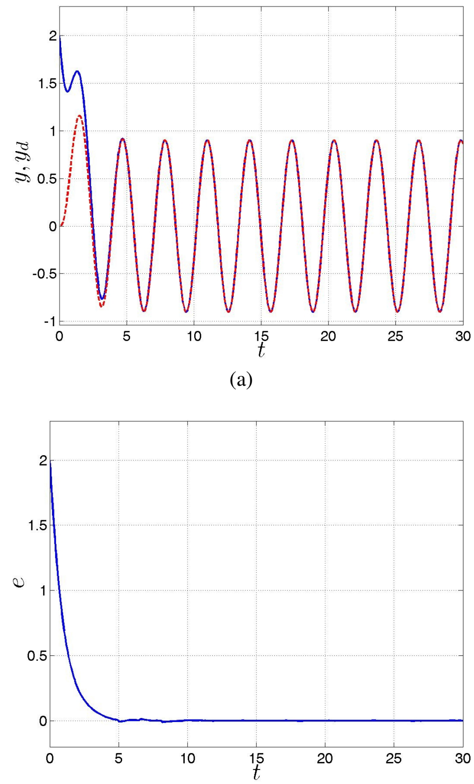
Now, to conclude the proof, it is sufficient to follow the steps of the standard mathematical derivation presented e.g. in [[40], page 279] noticing that, from the definition of $\mathcal{S}$ in (11), the error signal $e(t)$ can be regarded as the output of a BIBO linear filter whose transfer function is $G(s) = 1/P(s)$, being $P(s)$ the Hurwitz polynomial defined in (6), when excited by the input signal $\mathcal{S}(t)$. Hence, according to linear filter theory [43], it easily follows that:

$$\lim_{t \to +\infty} e(t) = G(0) \cdot \lim_{t \to +\infty} \mathcal{S}(t) = G(0) \cdot 0 = 0. \tag{32}$$

V. Numerical Example

In what follows the performance of the proposed control strategy is analyzed numerically on an representative case study. Specifically, the plant under study is a 4-modal, bi-dimensional switching system in Brunovsky form as (1), where the Euclidean space $I\!R^2$ is partitioned into the following $M = 4$ domains defined as:

$$\begin{array}{l} \Omega_1 \triangleq \left\{ x \in I\!R^2 : {H_0}^T x < 0 \text{ and } {H_1}^T x \geq 0 \right\}, \\ \Omega_2 \triangleq \left\{ x \in I\!R^2 : {H_0}^T x \geq 0 \text{ and } {H_1}^T x \geq 0 \right\}, \\ \Omega_3 \triangleq \left\{ x \in I\!R^2 : {H_0}^T x > 0 \text{ and } {H_1}^T x < 0 \right\}, \\ \Omega_4 \triangleq \left\{ x \in I\!R^2 : {H_0}^T x \leq 0 \text{ and } {H_1}^T x < 0 \right\}, \end{array} \tag{33}$$



(a)

(b)

Fig. 1: Tracking results. (a) reference trajectory (red dashed line), plant output (blue solid line), (b) tracking error.

with ${H_0}^T = [-1 \quad 1]$ and ${H_1}^T = [1 \quad 1]$, while the parameters describing the dynamics of each subsystem in each state space region are set as:

$$\begin{array}{l} a_1^{[1]} = -6,\ a_2^{[1]} = -5,\ c^{[1]} = -1,\ b^{[1]} = 2; \\ a_1^{[2]} = -10,\ a_2^{[2]} = -6.5,\ c^{[2]} = 1,\ b^{[2]} = 1; \\ a_1^{[3]} = -1.5,\ a_2^{[3]} = -2.5,\ c^{[3]} = 2,\ b^{[3]} = 4; \\ a_1^{[4]} = -37.75,\ a_2^{[4]} = -12,\ c^{[4]} = 4,\ b^{[4]} = 2.5. \end{array} \tag{34}$$

Moreover $\phi = x_1^2$ is the smooth nonlinear term and the initial conditions are set to $x(0) = [2 \quad 2]^T$. Note that parameters values are given here for the sake of completeness and they are not exploited for control design.

The signal to be tracked, satisfying assumptions (3) and (4), is generated as the output of the following asymptotically stable continuous model (see Remark 3):

$$\dot{\bar{x}} = \left[\begin{array}{cc} 0 & 1 \\ -4 & -5 \end{array} \right] \bar{x} + \left[\begin{array}{c} 0 \\ 4.5 \end{array} \right] r, \tag{35}$$

$$y_d = \bar{x}_1, \tag{36}$$

when the input signal $r(t)$ is sinusoidal, namely $r(t) = 2\sin(2t)$, and the initial conditions are assumed to be null, $\bar{x}(0) = [0 \quad 0]^T$.

The control action is selected according to the statement of Theorem 1 choosing $P(s) = s+1$, $\gamma = 10$ and $\Gamma_i = 100 \cdot I$ $(i = 1, \ldots, 4)$, with I being the identity matrix.

Results in figure 1 confirm the effectiveness of the control strategy that guarantees asymptotic stability of the output

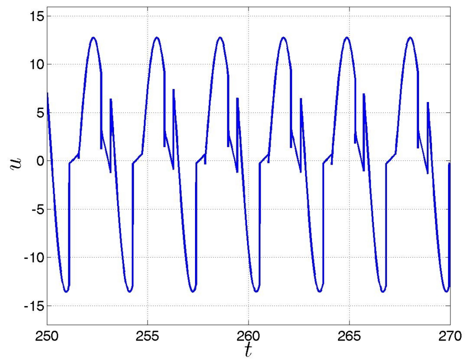
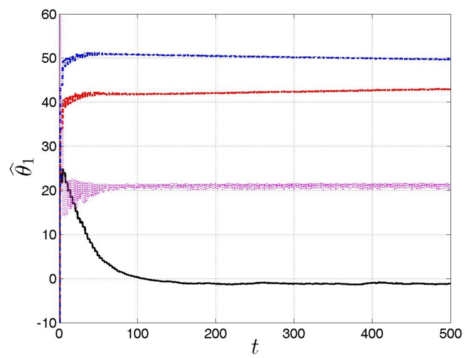
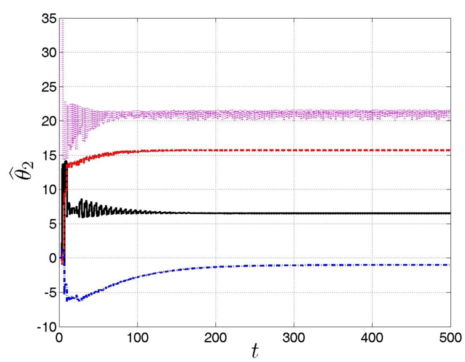


Fig. 2: Control Action.



(a)



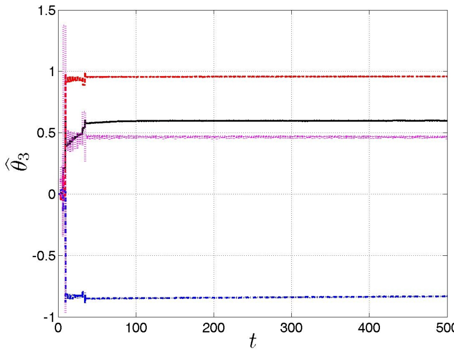
(b)

Fig. 3: Adaptive Gains with $\widehat{\theta}_{i1}$ in black solid line, $\widehat{\theta}_{i2}$ in red dashed line, $\widehat{\theta}_{i3}$ in blue dashed-dotted line, $\widehat{\theta}_{i4}$ in magenta dotted line, $i = 1, 2$. (a) $\widehat{\theta}_1$,(b) $\widehat{\theta}_2$.

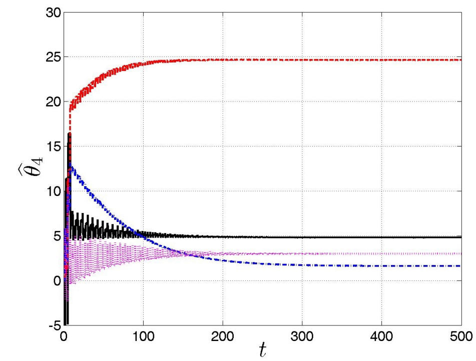
tracking error under the action of the bounded control signal reported in figure 2. As expected, the evolutions of the adaptive control gains is also bounded, with all gains converging to finite steady-state values as the tracking error reduces to zero (see figures 3 and 4).

VI. Conclusions

In this paper a novel adaptive algorithm to solve the tracking problem for state dependent switching systems in Brunovsky form has been presented. The main challenge, when controlling this class of hybrid plants, is to guarantee stability and convergence of the tracking error in the presence of sudden and persistent plant parameter variations originated



(a)



(b)

Fig. 4: Adaptive Gains with $\widehat{\theta}_{i1}$ in black solid line, $\widehat{\theta}_{i2}$ in red dashed line, $\widehat{\theta}_{i3}$ in blue dashed-dotted line, $\widehat{\theta}_{i4}$ in magenta dotted line, $i = 3, 4$. (a) $\widehat{\theta}_3$ (b) $\widehat{\theta}_4$.

by the plant switching from one state space region to another. Here, to achieve the fast reconfiguration of the controller, the problem has been solved by adapting the control-gains vectors according to the state space regions visited by the plant state trajectories. Convergence of the closed-loop error, as well as boundness of gains evolutions, have been proved by using a common Lyapunov function. Numerical results confirmed the effectiveness of the theoretical derivation.

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