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Model-Predictive On-Off Control of a Combustion-Heating-System for Vehicles

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***Abstract*— In this paper, a control-oriented model of a combustion-heating system for vehicles such as buses, coaches and trucks is developed. Based on this model, a novel model-predictive on-off strategy is proposed for the control of the combustion-heating system. Experiments are carried out to obtain a fair comparison of the predictive on-off controller with a classical on-off controller. The classical on-off controller and the predictive on-off controller are evaluated with respect to fuel economy, reduction in the emission of pollutants and the number of on-off switching cycles of the combustion valve connected to the heater. A discrete-time Extended Kalman Filter is employed for the estimation of state variables, heat losses and the volumetric flow. Experimental results show that the predictive on-off control strategy leads to a superior performance in terms of fuel economy and switching action of the valve as compared to the classical on-off controller.**

I. Introduction

Recent demands in public transport for efficient fuel consumption and reduction in the emission of pollutants require innovative control strategies for all auxiliaries that require fuel for their operation. Besides the engine, a combustion-heating unit is a major component that requires fuel supply for its operation [1]. For such auxiliary combustion-heaters, the fuel consumption increases by 30 % [2]. The high thermal output power provided by burning fuel is used to keep the heating fluid within a desired temperature range. The thermal energy from the preheated fluid is transmitted by radiation and convection to the vehicle chamber using heat exchangers installed at various locations. This energy provides thermal comfort to the driver as well as the passengers during winter conditions. Heating systems with a water circuit are advantageous due to their environment-friendly nature [3]. A detailed modeling of the vehicle chamber has been reported in [4], [5] for buses.

In the literature, only little attention has been paid to the control of the heater temperature. In [1], an optimal feedback law is proposed to provide efficient fuel consumption and thermal comfort to the passengers. In the given paper, however, two on-off control strategies are considered: 1) a classical on-off control based on the relay principle and 2) a predictive on-off control. Criteria for the evaluation of these control strategies are an improved fuel economy, a reduction in the emission of pollutants, and an avoidance of fast switching control actions. The avoidance of fast switching control actions contributes to the longevity of the combustion valve connected to the combustion-heater.

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In a classical on-off control strategy, specified threshold limits of the heater temperature are used as hard constraints. In contrast to the classical on-off strategy, a novel model-predictive on-off controller is proposed in this paper. As the name implies, this controller explicitly uses a model of the system. In such a controller, the upper threshold limit is used as a hard constraint, whereas the lower threshold limit is treated as a soft constraint.

The outline of this paper is as follows: in Sec. II, a control-oriented model of the overall heating system is developed. To estimate both the heat losses as well as the volumetric flow, a discrete-time Extended Kalman Filter is designed in Sec. III. Sec. IV describes the classical on-off control strategy as well as the model-predictive control strategy for the on-off control. A test rig used for the experimental investigation of the above mentioned control strategies is described in Sec. V. Experimental results are depicted in Sec. VI. Finally, conclusions are given in Sec. VII.

II. Control-Oriented Model of Heating System in a Vehicle

The block diagram of the heating system is shown in Fig. 1.

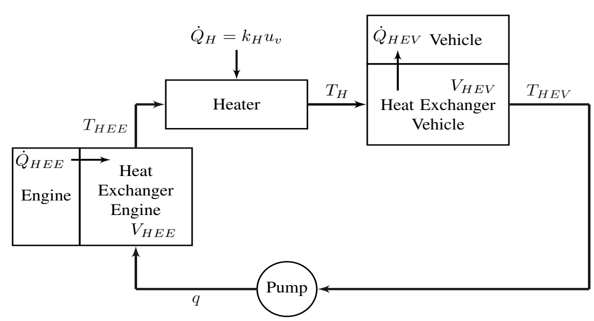

Fig. 1. Block diagram of the overall heating system.

As depicted in Fig. 1, the heating system contains a pump providing the volumetric flow rate q and a combustion-heater. Furthermore, two heat exchangers are part of this system. The heat exchangers in contact with the vehicle and with the engine are denoted as HEV and HEE, respectively. The variables T_H, T_{HEV} and T_{HEE} represent the temperature of the fluid in the heater, and the heat exchangers, HEV and HEE, respectively. The heating power $\dot{Q}_H$ provided by the combustion-heater

$$\dot{Q}_H = k_H u_v, \qquad u_v \in \{0, 1\} \tag{1}$$

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is directly proportional to the combustion heat k_H provided by the burning fuel. Here, u_v denotes the opening and closing of the combustion valve. A heat balance leads to the dynamics of the combustion-heater

$$\frac{d}{dt}T_H = \frac{q}{V_H}(T_{HEE} - T_H) + \frac{\dot{Q}_H}{\rho c_p V_H}, \quad (2)$$

where ρ and c_p are the density and the specific heat capacity of water. The volume of the heater is denoted by V_H.
The heat losses within the internal space of the subsystem vehicle are denoted as $\dot{Q}_{HEV}$. The energy balance for the heat exchanger HEV of volume V_{HEV} yields

$$\frac{d}{dt}T_{HEV} = \frac{q}{V_{HEV}}(T_H - T_{HEV}) - \frac{\dot{Q}_{HEV}}{\rho c_p V_{HEV}}. \quad (3)$$

The back-flowing water to the heater passes through the pipes that are in contact with the coolant fluid of the engine. Two scenarios can occur at the engine: During the start-up phase of the engine, the circulating water is cooled due to heat losses into the colder engine chamber. On the other hand, during normal engine operation, the high temperature of the engine causes the water temperature to raise. The heat balance equation for the heat exchanger HEE during the normal operation phase is given by

$$\frac{d}{dt}T_{HEE} = \frac{q}{V_{HEE}}(T_{HEE} - T_{HEE}) + \frac{\dot{Q}_{HEE}}{\rho c_p V_{HEE}}. \quad (4)$$

The state-space representation of the control-oriented model Eqs. (2)-(4) becomes

$$\underbrace{\begin{bmatrix} \dot{T}_H \\ \dot{T}_{HEV} \\ \dot{T}_{HEE} \end{bmatrix}}_{\dot{\underline{x}}} = \underbrace{\begin{bmatrix} -\frac{q}{V_H} & 0 & \frac{q}{V_H} \\ \frac{q}{V_{HEV}} & -\frac{q}{V_{HEV}} & 0 \\ 0 & \frac{q}{V_{HEE}} & -\frac{q}{V_{HEE}} \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} T_H \\ T_{HEV} \\ T_{HEE} \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} \frac{k_H}{\rho c_p V_H} \\ 0 \\ 0 \end{bmatrix}}_{\underline{b}} \underbrace{u_v}_{u} + \underbrace{\begin{bmatrix} 0 & 0 \\ \frac{-1}{\rho c_p V_{HEV}} & 0 \\ 0 & \frac{1}{\rho c_p V_{HEE}} \end{bmatrix}}_{\underline{E}} \underbrace{\begin{bmatrix} \dot{Q}_{HEV} \\ \dot{Q}_{HEE} \end{bmatrix}}_{\underline{z}} \quad (5)$$

with the corresponding measurement vector $\underline{y}_m$

$$\underline{y}_m = \underline{x} = [T_H \quad T_{HEV} \quad T_{HEE}]^T. \quad (6)$$

The predictive on-off controller requires full state information. These states are estimated using a discrete-time Extended Kalman Filter (EKF) as proposed in the next section.

III. Discrete-time Extended Kalman Filter

The model-predictive on-off controller requires not only the state information but also the unmeasured disturbances acting on the combustion heating system. In order to estimate the volumetric flow rate as well as the heat losses, the state-vector of the control-oriented model in Eq. (5) is extended by $\underline{x}_z = [q \quad \dot{Q}_{HEV} \quad \dot{Q}_{HEE}]^T$. Assuming the slowly varying nature of the heat losses and the volumetric flow rate, an integrator disturbance model is introduced as follows:

$$\frac{d}{dt}\underline{x}_z = \underline{0}. \quad (7)$$

The extended state-space description of the system represents a continuous-time nonlinear model

$$\dot{\underline{x}}_e = \underline{f}(\underline{x}_e, u), \quad (8)$$

with $\underline{x}_e = [\underline{x} \quad \underline{x}_z]^T$ and the corresponding output vector is given by

$$\underline{y}_m = \underline{C}_{m,e}\underline{x}_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \underline{x}_e. \quad (9)$$

An explicit Euler-discretisation of Eq. (8)-(9) leads to the discrete-time model used for the EKF implementation

$$\underline{x}_{e,k+1} = \underline{f}_k(\underline{x}_{e,k}, u_k) + \underline{w}_k \quad (10)$$
$$\underline{y}_{m,k} = \underline{C}_{m,e}\underline{x}_{e,k} + \underline{v}_k, \quad (11)$$

where $\underline{x}_{e,k}$, u_k and $\underline{y}_{m,k}$ denote the extended state vector, the input and the outputs at discrete-time t_k. Furthermore, white noise processes affecting the state variables as well as the outputs are denoted by $\underline{w}_k$ and $\underline{v}_k$, respectively. The covariance matrices corresponding to the process noise $\underline{w}_k$ and the measurement noise $\underline{v}_k$ are denoted by $\underline{Q}_k$ and $\underline{R}_k$, respectively, assuming zero cross-correlation.
Fig. 2 shows that the discrete-time EKF implementation can be divided into two stages, namely, the prediction stage and the update stage [6].

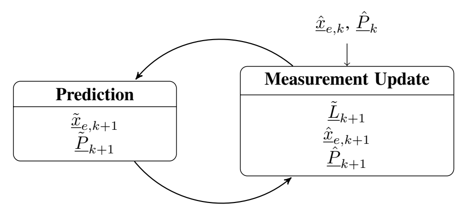


Fig. 2. Operation of the Extended Kalman Filter.

Here, $\underline{P}_k$ is the error covariance matrix. The implemented algorithm for the discrete-time EKF can be stated as follows [6]:

- State prediction

$$\tilde{\underline{x}}_{e,k+1} = \underline{\varphi}(\hat{\underline{x}}_{e,k}, u_k) \quad (12)$$

- Prediction of the error covariance matrix $\tilde{\underline{P}}_{k+1}$.

$$\tilde{\underline{P}}_{k+1} = \underline{\phi}_k \hat{\underline{P}}_k \underline{\phi}_k^T + \underline{Q} \text{ , with } \underline{\phi}_k = \frac{\partial \underline{\varphi}(\partial \hat{\underline{x}}_{e,k}, u_k)}{\partial \underline{x}_{e,k}}\Big|_{\hat{\underline{x}}_{e,k}} \quad (13)$$

- Update of the gain matrix $\tilde{\underline{L}}_{k+1}$

$$\tilde{\underline{L}}_{k+1} = \tilde{\underline{P}}_{k+1}\underline{C}_{m,e}^T\left(\underline{C}_{m,e}\tilde{\underline{P}}_{k+1}\underline{C}_{m,e}^T + \underline{R}\right)^{-1} \quad (14)$$

- Update of the state vector $\hat{\underline{x}}_{e,k+1}$

$$\hat{\underline{x}}_{e,k+1} = \tilde{\underline{x}}_{e,k+1} + \tilde{\underline{L}}_{k+1}\big(\underline{y}_{m,k+1} - \underline{C}_{m,e}\tilde{\underline{x}}_{e,k+1}\big) \tag{15}$$

- Update of the error covariance matrix for the next sampling interval

$$\hat{\underline{P}}_{k+1} = \big(\underline{I} - \tilde{\underline{L}}_{k+1}\underline{C}_{m,e}\big)\tilde{\underline{P}}_{k+1} \tag{16}$$

In the next section, two control strategies are proposed for controlling the heater temperature.

IV. Control Design

The main goals of the control strategies are better fuel economy, low emission of pollutants, and a small number of switchings in the control signal u_v for the longevity of the combustion valve. In the following, two controller implementation schemes are considered: 1) a classical on-off controller and 2) a novel model-predictive on-off controller.

For both control strategies, the upper threshold limit $T_{H,max}$ of the combustion-heater temperature is used as a hard constraint. This keeps the heating operation within the safety limits. The lower threshold limit $T_{H,min}$ is fixed in the case of the classical on-off controller. The lower threshold limit for the predictive controller, however, is implemented as a soft constraint, i.e., it is allowed to be violated.
The working principles of both control strategies are described in the following subsections.

A. Classical On-Off-Controller

The classical on-off controller is based on the relay principle and involves only a feedback of the measured heater temperature T_H. Fig. 3 depicts the state-automata for the classical on-off controller.

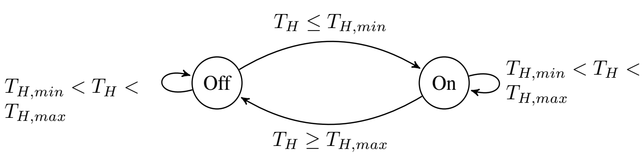


Fig. 3. State-automata of a classical on-off controller.

1) Off-State: When the heater temperature attains its upper limit, $T_H \geq T_{H,max}$, the heater is switched off, i.e., $u_v = 0$. The off-state is retained until the lower threshold limit is reached. This is achieved by using the following criteria in the off-state
$$T_{H,min} < T_H < T_{H,max}\,.$$
2) On-State: When $T_H \leq T_{H,min}$ holds, the combustion valve is opened, i.e., $u_v = 1$. This in turn raises the temperature of the water until the condition $T_{H,min} < T_H < T_{H,max}$ for the combustion-heater temperature T_H remains valid.

Dramatic changes in the heat losses during extreme weather conditions could lead to too frequent switching actions of the combustion valve. This situation can be avoided with the help of a predictive on-off controller as proposed in the next subsection.

B. Model-Predictive On-Off Controller

As the name implies, this approach explicitly uses the model of the heating system. The control decision is based on the model information, e.g., the current state variables of the system, the estimated heat losses and the estimated volumetric flow rate. This situation, on the one hand, avoids rapid on-off cycles and allows for more flexibility in selecting the on-time for the combustion-heater. On the other hand, it also leads to higher fuel economy.
The state-automata for the predictive controller is shown in Fig. 4. The working principle of the model based predictive

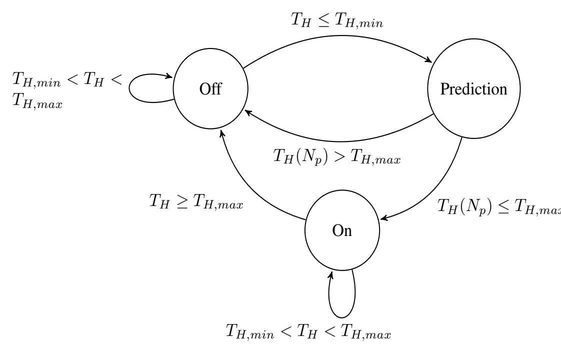


Fig. 4. State-automata of a predictive on-off controller.

on-off controller is described as follows:

1) Off-State: The off-state is initiated, when the condition
$$(T_H \geq T_{H,max}) \,\|\, (T_H(N_p) > T_{H,max})$$
is met. In this automata-state, the combustion valve remains closed, i.e., $u_v = 0$ until $T_{H,min} < T_H < T_{H,max}$ is fulfilled. This condition ensures that the heater is not switched on until it reaches the lowest threshold limit. Hence, intermediate on-off switching actions are avoided.
2) Prediction-State: The motivation of the prediction-state is to guarantee that the heater is not turned on before the upper threshold limit is not achieved within a fixed waiting time. The length of this waiting interval corresponds to the prediction horizon N_p.
The prediction-state is invoked only when the heater temperature T_H becomes less than or equal to the lower threshold limit $T_{H,min}$, i.e., $T_H \leq T_{H,min}$.
In this stage, a discrete-time control-oriented model of the heating system given by
$$\underline{x}_{k+1} = \underline{x}_k + T_s\big(\underline{A}(q)\underline{x}_k + \underline{b}u_k + \underline{E}\,\underline{z}_k\big) \tag{17}$$
$$\underline{y}_{m,k} = \underline{C}_m\underline{x}_k, \tag{18}$$
is utilized. Here, $k = \{0, \cdots, N_p - 1\}$ holds, and T_s is the sampling time. The discrete-time model is evaluated for the length of the prediction horizon N_p. Moreover, when the model is in the prediction-state, the heat losses $\dot{Q}_{HEV}$ and $\dot{Q}_{HEE}$ are assumed to be constant during the whole length of the prediction horizon N_p. The controller is switched either to the on-state or the off-state depending on the transition

condition.

Here, it is worth pointing out that the lower threshold limit $T_{H,min}$ is used as a soft constraint.

3) On-State: This automata-state is initiated only when the condition $T_H(N_p) \leq T_{H,max}$ is fulfilled. The controller will keep the heater on until the following condition

$$T_{H,min} < T_H < T_{H,max}$$

remains valid. As soon as this condition is violated, the combustion valve will be closed and a transition is made to the off-state.

The procedure described above will be repeated for the subsequent predictive on-off operations.

Fig. 5 depicts the implementation scheme for the model-predictive on-off controller.

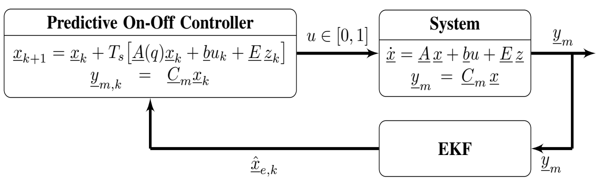


Fig. 5. Implementation scheme for the predictive on-off controller.

In Fig. 5, the predictive on-off controller uses full-state information provided by the discrete-time EKF. The state vector $\hat{\underline{x}}_{e,k}$ is given by

$$\hat{\underline{x}}_{e,k} = \left[\hat{T}_{H,k}\ \hat{T}_{HEV,k}\ \hat{T}_{HEE,k}\ \hat{q}_k\ \hat{\dot{Q}}_{HEV,k}\ \hat{\dot{Q}}_{HEE,k}\right]^T.$$

In contrast to the predictive on-off strategy, the classical on-off controller uses only the measured heater temperature T_H for the computation of the control action.

V. Test-rig

A prototype test-rig developed at the Chair of Mechatronics considered in [7] has been enhanced to realize the heating system within a vehicle as depicted in Fig. 6.

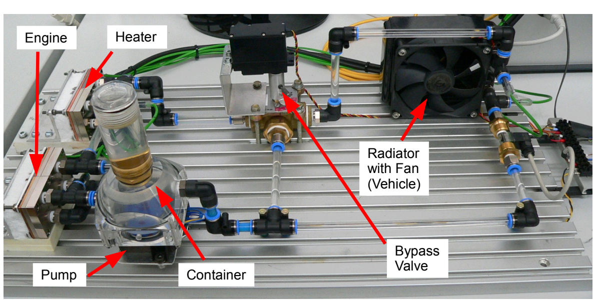


Fig. 6. Prototypic test-rig at the Chair of Mechatronics.

Here, two heating units are shown: According to an analogy between the test-rig and the combustion-heating system, one of the heating units represents the heater, the other heating unit the engine. A cooling fan connected to the radiator serves as vehicle heat exchanger with variable dissipative rates. The amount of heat dissipated by the radiator-fan-combination is proportional to the speed of the fan.

In the next section, experimental results using the implementation of the above mentioned controllers as well as the discrete-time EKF on the test-rig are presented.

VI. Experimental Results

For real-time implementation, a sampling time $T_s = 0.1\,\mathrm{s}$ is selected for the discrete-time model evaluation. The parameters V_H, V_{HEV} and V_{HEE} have been chosen based on the physical dimensions of the system. The length of the prediction horizon is set to $N_p = 120\,\mathrm{s}$. The upper limit and the lower limit for the heater temperature are given as follows:

$$308\,\mathrm{K} \leq T_H \leq 310.8\,\mathrm{K}.$$

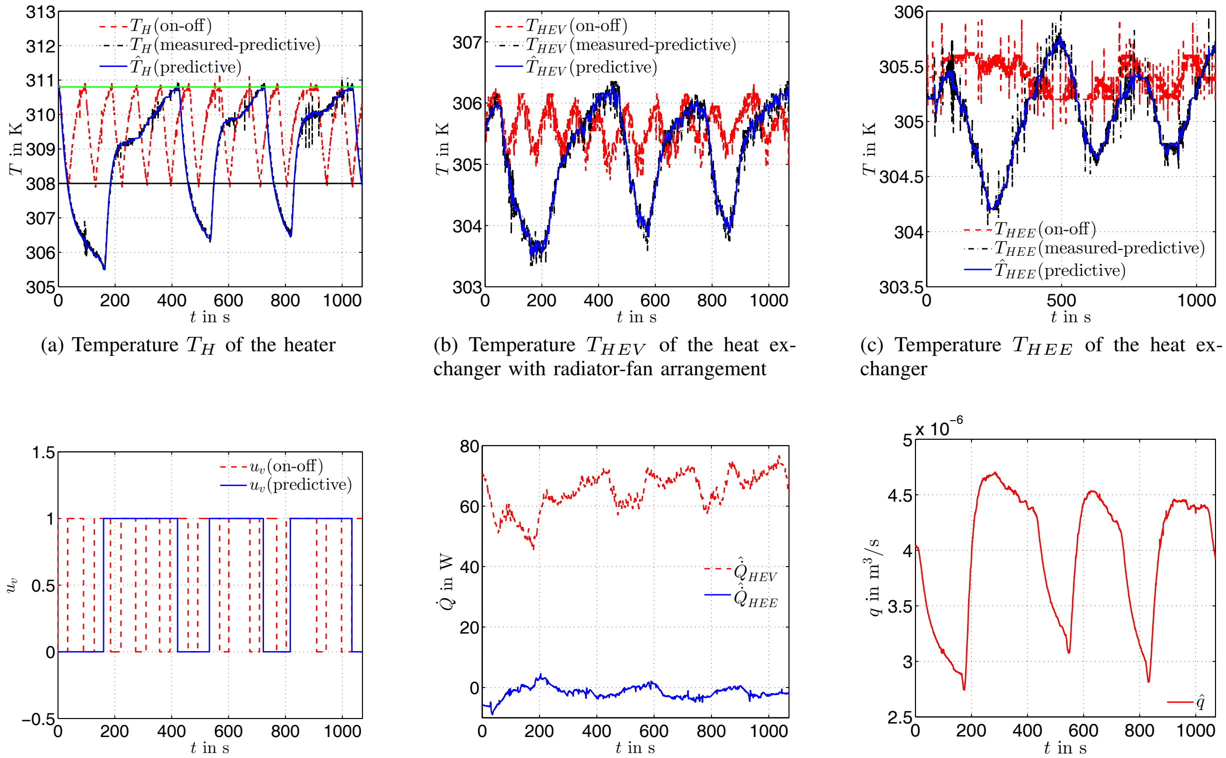
The initial value of the covariance matrices $\underline{Q}$, $\underline{R}$ and $\underline{P}_{k,0}$ are selected such as to obtain a fast error convergence. The performance criteria for the evaluation of the classical on-off and the predictive on-off controllers are based on the number of switching cycles for the control signal u_v and heat energy expenditure. Two scenarios are considered for the performance evaluation. In both scenarios, the heater representing the internal combustion engine is turned-off. Thus, representing the situation when heat is dissipated into the engine chamber, i.e., negative $\dot{Q}_{HEE}$.

1) Constant fan speed: In this case, the speed of the fan is kept constant for both control strategies. This leads to constant heat losses from the radiator. This operation is similar to vehicles like coaches and trucks having few stops. During the journey, heat losses inside the vehicles chamber can be considered as constant.

Fig. 7 shows a comparison of the classical on-off and the predictive on-off controllers. The number of switching cycles of the control signal u_v is higher for the classical on-off controller as compared to the predictive on-off controller, see Fig. 7d. In the case of the classical on-off controller, the heater temperature T_H remains within the upper and the lower bounds. However, in the case of the predictive on-off controller, the lower bound acts only as a soft constraint. Furthermore, when the heater temperature drops below the lower threshold limit, it means that the controller is in the prediction-stage.

It can be seen from Fig. 7d that the duration of the on-time for the heater is longer than the length of the prediction horizon $N_p = 120\ s$. This can be explained as follows:

At the end of the prediction-stage, there is a considerable decrease in the temperatures T_H, T_{HEV} and T_{HEE} (cf. Fig. 7a, 7b, 7c) and the heat loss $\dot{Q}_{HEV}$, cf. Fig. 7e. The future control action is calculated based on these current values. As soon as the control action $u_v = 1$ is taken, the thermal energy provided by the heater has to compensate for the heat-losses within the heating system. This results in a longer duration to reach the upper threshold limit. Moreover, the negative heat losses $\dot{Q}_{HEE}$ shows that the heat is dissipated into the engine chamber, cf. Fig. 7e. Furthermore, it can be seen that the estimated temperatures with the help of the EKF used in the predictive controller are in close proximity to the measured ones.



(a) Temperature T_H of the heater

(b) Temperature T_{HEV} of the heat exchanger with radiator-fan arrangement

(c) Temperature T_{HEE} of the heat exchanger

(d) Heater on-off signal

(e) Estimated heat losses

(f) Estimated volumetric flow rate

Fig. 7. Comparison of classical on-off and predictive on-off control approaches with fixed fan speed.

Parameter	On-Off Controller	Predictive Controller
No. of switching cycles	11	3
On-time	685.1 s	666.1 s
Average On-time	62.28 s	222 s
Off-time	385.3 s	404.3 s
$T_{H,avg}$	309.66 K	308.90 K
$T_{HEV,avg}$	305.63 K	305.17 K
$T_{HEE,avg}$	305.41 K	305.08 K

TABLE I

CONTROLLER EVALUATION WITH FIXED FAN SPEED

Table I shows that the energy efficiency in terms of on-time is improved by 3 % in the predictive on-off controller as compared to the classical on-off controller. The average on-time, defined as the ratio of the on-time and the number of switching cycles, indicates the average time for the heater temperature to reach its maximum threshold limit. Furthermore, the average values of the temperatures are in close proximity to each other.

2) Variable fan speed: In this case, the speed of the fan ω_f is switched between $0.1\,\omega_{max}$ and $0.9\,\omega_{max}$. Here, ω_{max} denotes the maximum speed of the fan. A variation in the fan speed is related to a change in the heat losses. This situation corresponds to an operation of an urban bus with frequent stops. Here, the changes in the heat losses within the vehicle chamber are caused by frequent opening and closing of the doors.

The comparison between the classical on-off and the predictive on-off controllers is shown in Fig. 8. The estimated temperatures used in the predictive on-off controller are in close proximity to the measured ones. The number of switching cycles in the predictive on-off controller is lower as compared to that of the classical on-off controller.

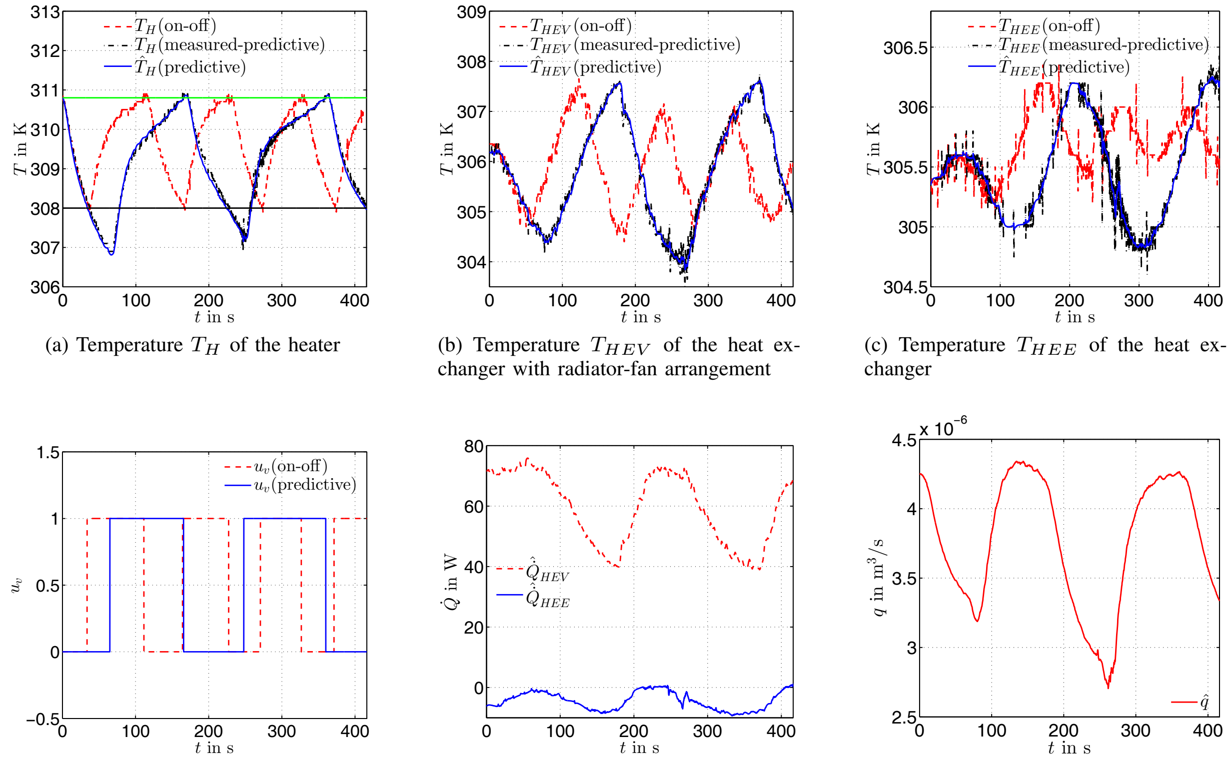
Furthermore, the variable speed of the fan enhances the rate of increase/decrease in the heat losses, cf. Fig. 8e. This causes the duration of the control signal u_v to be in close proximity to the prediction horizon $N_p = 120\,$s as shown in Fig. 8d. The negative heat losses $\dot{Q}_{HEE}$, cf. Fig. 8e, show that the heat energy is dissipated into the engine chamber. With the help of the predictive on-off controller,

Parameter	On-Off Controller	Predictive Controller
No. of switching cycles	4	2
On-time	241.35 s	212.7 s
Average On-time	60.34 s	106.4 s
Off-time	174.65 s	203.30 s
$T_{H,avg}$	309.59 K	309.21 K
$T_{HEV,avg}$	305.87 K	305.80 K
$T_{HEE,avg}$	305.69 K	305.48 K

TABLE II

CONTROLLER EVALUATION WITH VARIABLE FAN SPEED

the energy efficiency in terms of the on-time is increased by 12 % as compared to that of the classical on-off controller. Furthermore, the average values of the temperatures are in



(a) Temperature T_H of the heater

(b) Temperature T_{HEV} of the heat exchanger with radiator-fan arrangement

(c) Temperature T_{HEE} of the heat exchanger

(d) Heater on-off signal

(e) Estimated heat losses

(f) Estimated volumetric flow rate

Fig. 8. Comparison of classical on-off and predictive on-off controller with variable fan speed.

close proximity to each other.
The estimated volumetric flow for the predictive on-off controller is shown in Fig. 8f. The volumetric flow rate drops to a low value when the heater is switched off. But as soon as the heater is turned on, the estimated value of the volumetric flow rate tracks the actual value. For implementation purposes on the test-rig, a memory block is used to cope with these large changes in the volumetric flow rate when the heater is turned off. This memory block stores the value of the volumetric flow rate when the control signal is $u_v = 1$. As soon as the control signal drops to $u_v = 0$, the last value of q stored in the memory block is used for the model evaluation in the prediction-stage.

VII. Conclusions and Future Outlook

In this paper, a control-oriented model of the combustion-heating system for vehicles is developed. In order to keep the heater temperature between specified upper and lower bounds, two control strategies are considered: The first strategy is a classical on-off controller based on the relay principle, the second one a novel model predictive on-off controller. In case of the predictive on-off controller, the control action is taken only if the upper threshold limit is achievable within the length of the prediction horizon. An experimental analysis on a test-rig is carried out to compare these controllers. The states as well as the volumetric flow rate and the heat losses estimated by a discrete-time EKF are used in the predictive on-off controller. The predictive on-off controller, on the one hand, yields few switching cycles for the combustion valve connected to the heater. On the other hand, the fuel consumption for the heater is also reduced.

Future work will address a sensitivity analysis of the model with respect to parameter variations. Moreover, the online identification for the parameters will be addressed. This enables an adaptation of control strategies with respect to the different combustion-heaters and types of vehicles.

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