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On Quantifying Tolerable Closed-Loop Uncertainty in Frequency Domain

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Abstract— **Given a linear plant and a feedback controller it is natural to ask: *How much uncertainty can be tolerated by the closed-loop while achieving a specified level of performance*? In this paper, a characterization of this question is formulated in terms of a constrained optimization problem; the cost reflects the size of non-constant weights used to quantify system uncertainty across frequency and the constraint is a structured singular value characterization of the required level of robust performance. In the case of unstructured uncertainty the problem can be solved via a family of problems that are convex pointwise in frequency. An iterative algorithm is developed for the case of structured uncertainty.**

I. INTRODUCTION

The following two questions, representing two application directions of robust control, arise naturally in industry.

1) For a given uncertain plant set (characterized by some fixed uncertainty weight), what level of robust performance can be achieved in closed-loop?
2) How much uncertainty can be tolerated by a nominal closed-loop, while achieving a given level of robust performance (characterized by some fixed performance weight)?

The first question is addressed in [11], where the idea of skewed-μ [5], [15] is extended to accommodate non-constant scaling over frequency. A corresponding algorithm for the synthesis of a controller and a weight function that reflects the achievable level of robust performance is also presented in [11]. The second question, which may arise within the context of controller certification and model validation [6], is considered here. In fact, it is a topic related to the recent work on the control performance assessment subject to multi-objective user-specified performance characteristics [1], [3], [9], [10], [12], [13].

As is well-known, the structured singular value (introduced by [4]) can be used to assess the level of closed-loop performance achieved by a feedback controller with all plants in a specified set, when *weighted* $\mathcal{H}_\infty$ norms are employed to quantify performance and the size of the uncertain plant set, which may be structured [14], [15], [16]. More specifically, with reference to the interconnection structure of Figure 1, let the generalized plant transfer function

$$G = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{pmatrix} = \begin{pmatrix} w \\ d \\ u \end{pmatrix} \mapsto \begin{pmatrix} z \\ e \\ y \end{pmatrix}$$

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be constructed from a nominal model of the plant and so-called *performance and uncertainty weights* so that:

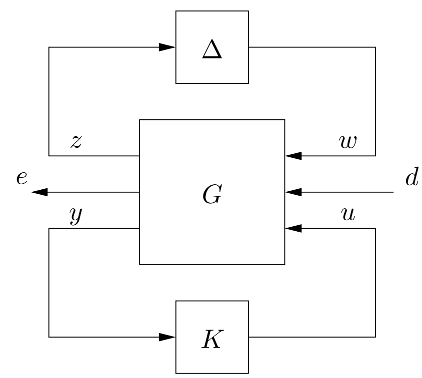

Fig. 1. General LFT Interconnection Structure

1) the upper linear fractional transformation (LFT)

$$\mathcal{F}_u\left(\begin{pmatrix} G_{11} & G_{13} \\ G_{31} & G_{33} \end{pmatrix}, \Delta\right) := G_{33}(s) + G_{31}\Delta(I - G_{11}\Delta)^{-1}G_{13}$$

describes the uncertain plant set as Δ varies over the unit ball

$$\mathcal{B}(\boldsymbol{\Delta}) := \{\Delta \in \mathcal{H}_\infty^{r\times r} : \|\Delta\|_\infty < 1; \; \Delta(s) \in \boldsymbol{\Delta} \text{ for all } s \in \bar{\mathbb{C}}_+\}, \qquad (1)$$

with the block diagonally structured set

$$\boldsymbol{\Delta} := \{\operatorname{diag}_{i=1}^{f}(I_{\alpha_i} \otimes \Delta_i) : \Delta_i \in \mathbb{C}^{\beta_i \times \beta_i} \text{ and } \textstyle\sum_{i=1}^{f} \alpha_i\beta_i = r\} \subset \mathbb{C}^{r\times r}, \qquad (2)$$

where the Kroneker matrix product is defined by $A \otimes B := [a_{ij}B]$, $\mathcal{H}_\infty$ denotes the standard Hardy ∞-space (i.e. the set of stable transfer functions) and with norm $\|\cdot\|_\infty$; and

2) for a given controller K, the lower LFT

$$\mathcal{F}_\ell\left(\begin{pmatrix} G_{22} & G_{23} \\ G_{32} & G_{33} \end{pmatrix}, K\right) := G_{22} + G_{23}K(I - G_{33}K)^{-1}G_{32} = \mathcal{F}_u(\mathcal{F}_\ell(G,K), 0)$$

comprises the *nominal* (weighted) closed-loop transfer-functions used for gauging performance.

Then the following result holds [14]:

Proposition 1.1: $\mathcal{F}_u(\mathcal{F}_\ell(G,K),\Delta) \in \mathcal{H}_\infty^{m\times m}$ and

$$\|\mathcal{F}_u(\mathcal{F}_\ell(G,K),\Delta)\|_\infty \leq 1 \text{ for all } \Delta \in \mathcal{B}(\boldsymbol{\Delta})$$

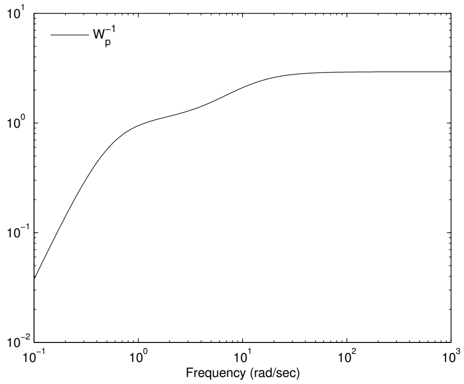

Fig. 3. The performance weight W_p

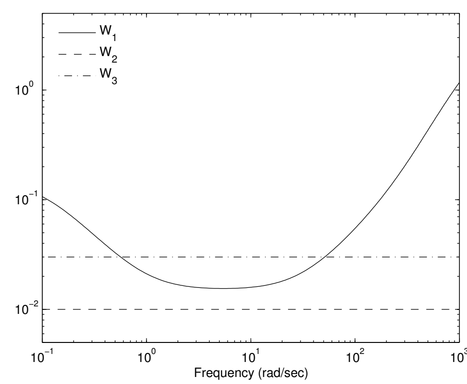

Fig. 4. The uncertainty weight W_Δ

if, and only if, $\mathcal{F}_\ell(G,K) \in \mathcal{H}_\infty^{(m+r)\times(m+r)}$ and

$$\sup_{\omega\in\mathbb{R}\cup\{\infty\}} \mu_{\mathbf{\Delta}_{\mathrm{T}}}\left(\mathcal{F}_\ell(G,K)(j\omega)\right) \le 1,$$

where $\mu_{\mathbf{\Delta}_{\mathrm{T}}}$ denotes the structured singular value taken with respect to the structured set

$$\mathbf{\Delta}_{\mathrm{T}} := \{\mathrm{diag}(\Delta,\Delta_p) \,:\, \Delta\in\mathbf{\Delta} \text{ and } \Delta_p\in\mathbb{C}^{m\times m}\}.$$

Remark 1: With respect to the particular performance and uncertainty weights in G, the closed-loop uncertainty $\Delta \in \mathcal{B}(\mathbf{\Delta})$ is said to be tolerable if the corresponding μ-curve does not exceed unity over the entire frequency range, in the sense that a level of robust performance is achieved.

Consider, for example the uncertain closed-loop system in Figure 2, where P is a model for the longitudinal dynamics of the NASA X-29A air vehicle at approximately 30° angle of attack [2]. The controller K is designed in [8] so that (among other things) with the performance weight W_p shown in Figure 3, for all $\Delta := \mathrm{diag}\left(\Delta_1,\Delta_2,\Delta_3\right) \in \mathcal{H}_\infty$ with $\|\Delta\|_\infty < 1$, the uncertain closed-loop sensitivity $S_\Delta := (I - P_\Delta K)^{-1}$ is stable with $\|W_p S_\Delta\|_\infty \le 1$, where P_Δ denotes the uncertain plant model in Figure 2 for the uncertainty weight $W_\Delta = \mathrm{diag}\left(W_1,W_2,W_3\right)$ shown in Figure 4.[1] In particular, the corresponding μ-curve, which is shown in Figure 5, is below unity over all frequencies.

It is interesting to note that for the particular performance and uncertainty weights used for the example above, the value of μ is significantly less than unity, especially over the high frequency range. What might one conclude from this? In particular, the question 2) introduced at the beginning of the paper is studied.

As discussed in Section II, the question of how much uncertainty a nominal closed-loop (i.e. fixed P and K) can tolerate while achieving robust performance with respect to a given performance weight (i.e. fixed W_p), is formulated in terms of a constrained optimization problem; the cost reflects

[1]In Figure 2, d models pitch rate reference or the high bandwidth disturbances. The additive uncertainty (Δ_1 with weight W_1) is used to account for the unmodeled high frequency (> 300 rad/sec) modes. The multiplicative input uncertainty (Δ_2 with weight W_2) and output uncertainty (Δ_3 with weight W_3) are selected to account for actuator errors and sensor errors respectively.

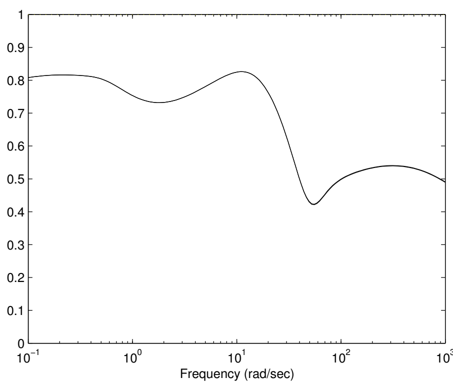

Fig. 5. μ-curve for the uncertain closed loop in Figure 2, with W_p of Figure 3 and W_Δ of Figure 4

the size of (non-constant) weights used to quantify system uncertainty across frequency and the structured singular-value constraint characterizes the required level of robust performance. Solving the optimization problem can yield a μ-curve that is close to unity across frequency, which is necessary for the resulting uncertainty weight to represent an answer to the question considered, see Sections III and IV respectively for the cases of structured uncertainty and unstructured uncertainty. In Section IV, the results are finally applied on the uncertain closed-loop system illustrated in Figure 2.

II. FORMULATION OF THE OPTIMIZATION PROBLEM

Consider the LFT interconnection structure shown in Figure 6. Here the uncertainty weight W has been purposefully omitted from the construction of the generalized plant G, which includes the nominal plant model and performance weights. Furthermore, it is assumed that K achieves nominal closed-loop stability, in that $\mathcal{F}_\ell(G,K) \in \mathcal{H}_\infty^{(m+r)\times(m+r)}$.

Now, let the structured sets $\mathbf{\Delta}$ and $\mathcal{B}(\mathbf{\Delta})$ be as defined in (1) and (2). Then from Proposition 1.1, $\mathcal{F}_u\left(\mathcal{F}_\ell(G,K),\Delta\right) \in \mathcal{H}_\infty^{m\times m}$ and

$$\|\mathcal{F}_u\left(\mathcal{F}_\ell(G,K),\Delta\right)\|_\infty \le 1 \text{ for all } \Delta\in\mathcal{B}(\mathbf{\Delta}) \qquad (3)$$

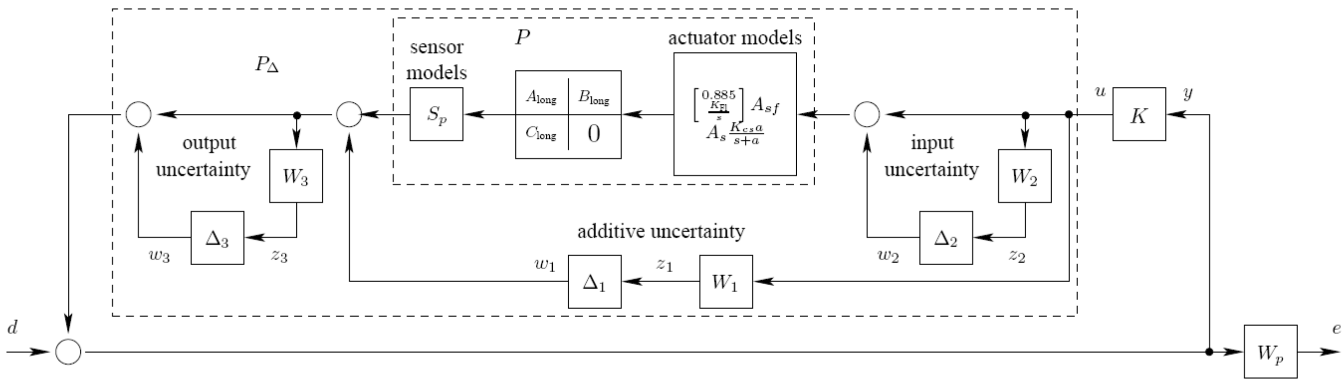

Fig. 2. Block diagram of uncertainty analysis problem formulation

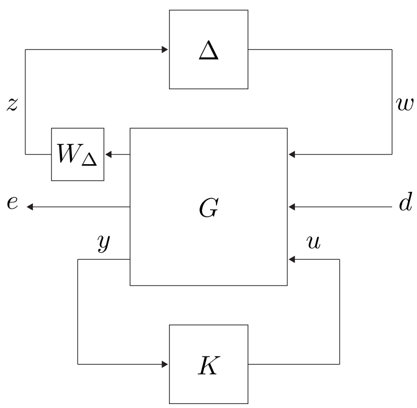

Fig. 6. LFT configuration for quantifying tolerable uncertainty

if, and only if,

$$\sup_{\omega\in\mathbb{R}\cup\{\infty\}} \mu_{\boldsymbol{\Delta}_{\mathrm{T}}}\left(\begin{pmatrix} W_\Delta & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)(j\omega)\right) \leq 1, \tag{4}$$

where

$$\boldsymbol{\Delta}_{\mathrm{T}} := \{\mathrm{diag}(\Delta,\Delta_p) : \Delta\in\boldsymbol{\Delta} \text{ and } \Delta_p\in\mathbb{C}^{m\times m}\}. \tag{5}$$

The level of tolerable uncertainty could, as such, be quantitatively determined by appropriately maximizing some measure of the size of the uncertainty weight W, while ensuring that (4) is satisfied. To this end, consider the following optimization problem:

Problem 2.1: Given an optimization directionality function

$$V \in \mathcal{V} := \{\mathrm{diag}_{i=1}^r(v_i) \ : \ v_i \in \mathcal{H}_2\},$$

where $\mathcal{H}_2$ denotes the standard Hardy 2-space with norm $\|\cdot\|_2$, solve

$$\begin{gathered} \min_{W_\Delta\in\mathcal{W}} \|VW_\Delta^{-1}\|_2^2 \\ \text{subject to} \\ \sup_{\omega\in\mathbb{R}\cup\{\infty\}} \mu_{\boldsymbol{\Delta}_{\mathrm{T}}}\left(\begin{pmatrix} W_\Delta & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_l(G,K)(j\omega)\right) \leq 1, \end{gathered} \tag{6}$$

the set of permissible uncertainty weights is defined by

$$\mathcal{W} := \{\mathrm{diag}_{i=1}^r(w_i) \ : \ w_i, \frac{1}{w_i} \in \mathcal{H}_\infty\}.$$

As it stands, this problem is not computationally straightforward, since the μ-constraint is difficult to handle. Before discussing this aspect of the problem, it is instructive to briefly discuss the role of the optimization directionality function V. First observe that

$$\|VW_\Delta^{-1}\|_2^2 = \int_{-\infty}^{\infty}\sum_{i=1}^{r}\left|\frac{v_i(j\omega)}{w_i(j\omega)}\right|^2 d\omega, \tag{7}$$

where $w_i(j\omega)$ (resp. $v_i(j\omega)$) is the i-th diagonal element of $W_\Delta(j\omega)$ (resp. $V(j\omega)$). From this decomposition, it can be seen that the cost function $1/\|VW^{-1}\|_2^2$ is a cumulative measure of the frequency-dependent size of the uncertainty weights $w_i(j\omega)$. Each uncertainty weight $w_i(j\omega)$ is itself weighted across frequency by an optimization directionality $v_i(j\omega)$. This can be used to steer the optimization by choosing $v_i(j\omega)$ to be large (resp. small) where it is expected that the corresponding uncertainty weight $w_i(j\omega)$ should be large (resp. small).[2]

III. Solving the optimization problem

As mentioned above, the optimization Problem 2.1 is difficult to solve because of the μ-constraint. In this section, it is argued that the μ-constraint can be replaced by one that is more amenable to computation. This may or may not introduce some conservatism as discussed below.

For a given matrix $M \in \mathbb{C}^{r\times r}$, it can be shown that [14]

$$\mu_{\boldsymbol{\Delta}}(M) \leq \inf_{D\in\mathcal{C}(\boldsymbol{\Delta})} \bar{\sigma}(DMD^{-1}), \tag{8}$$

where $\bar{\sigma}(\cdot)$ denotes the maximum singular value, $\boldsymbol{\Delta}$ is the structured set defined in (2) and[3]

$$\begin{aligned} \mathcal{C}(\boldsymbol{\Delta}) := \quad & \{D \in \mathbb{C}^{r\times r} : \det(D) \neq 0 \text{ and} \\ & D\Delta = \Delta D \text{ for all } \Delta \in \boldsymbol{\Delta}\}. \end{aligned} \tag{9}$$

In general, equality in (8) does not hold, but there are situations in which it does [14]. Similarly, with $\boldsymbol{\Delta}_{\mathrm{T}}$ as

[2] As also pointed out in [11], the direction of steepest descent of $\sum_{i=1}^r |\frac{v_i}{w_i}|^2$ is dominated by the smallest ratio $\left|\frac{w_i}{v_i}\right|$.

[3] In the definition of $\mathcal{C}(\boldsymbol{\Delta})$ it is possible to replace the constraint $\det(D) \neq 0$ with the constraint $D = D^* > 0$, without loss of generality [16].

defined in (5), it follows that

$$\sup_{\omega\in\mathbb{R}\cup\{\infty\}} \mu_{\boldsymbol{\Delta}_{\mathrm{T}}}\left(\begin{pmatrix} W_\Delta & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)(j\omega)\right) \tag{10}$$
$$\leq \inf_{\substack{D_1\in\mathcal{D}(\boldsymbol{\Delta})\\ d_2,\frac{1}{d_2}\in\mathcal{H}_\infty}} \left\| \begin{pmatrix} D_1W_\Delta & 0 \\ 0 & d_2 I_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} D_1^{-1} & 0 \\ 0 & \frac{1}{d_2}I_m \end{pmatrix}\right\|_\infty$$
$$= \inf_{D\in\mathcal{D}(\boldsymbol{\Delta})} \left\| \begin{pmatrix} DW_\Delta & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} D^{-1} & 0 \\ 0 & I_m \end{pmatrix}\right\|_\infty,$$

where

$$\mathcal{D}(\boldsymbol{\Delta}) := \{D\in\mathcal{H}_\infty^{r\times r} \;:\; D^{-1}\in\mathcal{H}_\infty^{r\times r} \text{ and } D(s)\in\mathcal{C}(\boldsymbol{\Delta}) \text{ for all } s\in\bar{\mathbb{C}}_+\}. \tag{11}$$

Note that the upper bound in (10) is more amenable to computation than the structured singular value itself. As such, one possible approach is to substitute this for the constraint in the following optimization problem:

Problem 3.1: Given an optimization directionality function $V\in\mathcal{V}$, solve

$$\min_{W_\Delta\in\mathcal{W}} \|VW_\Delta^{-1}\|_2^2$$
$$\text{subject to} \tag{12}$$
$$\inf_{D\in\mathcal{D}(\boldsymbol{\Delta})} \left\| \begin{pmatrix} DW_\Delta & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} D^{-1} & 0 \\ 0 & I_m \end{pmatrix}\right\|_\infty \leq 1.$$

In general, the constraint is not convex. However, as shown in the Section III-A, in the case of unstructured uncertainty, the problem can be reformulated in terms of a problem that is convex pointwise in frequency. An iterative algorithm is developed in Section III-B for the non-convex case of structured uncertainty.

A. The case of unstructured uncertainty

Consider the case of unstructured uncertainty; i.e. in the definition (2) of $\boldsymbol{\Delta}$, $f=\alpha_1=1$ and $\beta_1=r$. Then the relationship (10) becomes [14]

$$\sup_{\omega\in\mathbb{R}\cup\{\infty\}} \mu_{\boldsymbol{\Delta}_{\mathrm{T}}}\left(\begin{pmatrix} W_\Delta & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)(j\omega)\right)$$
$$= \inf_{d_1,\frac{1}{d_1}\in\mathcal{H}_\infty} \left\| \begin{pmatrix} d_1W_\Delta & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} \frac{1}{d_1}I_r & 0 \\ 0 & I_m \end{pmatrix}\right\|_\infty$$
$$= \inf_{d,\frac{1}{d}\in\mathcal{H}_\infty} \left\| \begin{pmatrix} W_\Delta & 0 \\ 0 & dI_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} I_r & 0 \\ 0 & \frac{1}{d}I_m \end{pmatrix}\right\|_\infty. \tag{13}$$

In this case, the optimization Problems 2.1 and 3.1 are equivalent. Furthermore, they can be reformulated in terms of a problem that is convex pointwise in frequency:

Problem 3.2: Given an optimization directionality function $V=\mathrm{diag}_{i=1}^r(v_i)\in\mathcal{V}$, solve

$$\min_{W_\Delta=\mathrm{diag}_{i=1}^r(w_i)\in\mathcal{W}} \int_{-\infty}^{\infty}\sum_{i=1}^r \frac{|v_i(j\omega)|^2}{|w_i(j\omega)|^2}\,d\omega$$
$$\text{subject to} \tag{14}$$
$$\forall\,\omega\in\mathbb{R}\;\exists\,\delta_\omega \text{ such that}$$
$$\mathcal{F}_\ell(G(j\omega),K(j\omega))\begin{pmatrix} I_r & 0 \\ 0 & \delta_\omega I_m \end{pmatrix}\mathcal{F}_\ell(G(j\omega),K(j\omega))^*$$
$$\leq \begin{pmatrix} \mathrm{diag}_{i=1}^r(1/|w_i(j\omega)|^2) & 0 \\ 0 & \delta_\omega I_m \end{pmatrix}.$$

Note that at each frequency the constraint in Problem 3.2 is convex in δ_ω and $1/|w_i(j\omega)|^2$, $i=1,\dots,r$. Indeed, since the sum in the cost is always non-negative, Problem 3.2 can be approximately solved pointwise in frequency using standard LMI tools [7]. That is, in the frequency range of interest, set sufficiently dense, but finite grid and solve Problem 3.2 at each grid frequency. Let the optimal value of δ_ω and w_i, $i=1,\dots,r$ at $\omega=\omega_k$ be denoted by $\delta_{\omega_k}^\star$ and $w_i^\star(j\omega_k)$. Then if required, transfer function characterizations of D and W_Δ can be obtained via interpolation of the pointwise solutions $\delta_{\omega_k}^\star$ and $w_i^\star(j\omega_k)$ for the entire grid, followed by spectral factorization. State-space methods, similar to those described in [11] can then be used to refine the pointwise solution.

B. An iterative algorithm for the case of structured uncertainty

In the case that $\boldsymbol{\Delta}$ is structured, the following iterative approach could be employed to obtain a local solution to the optimization Problem 3.1:

1) Set $i=0$ and $\lambda_0^\star=\|V(W_\Delta)_0^{-1}\|_2^2$, with $(W_\Delta)_0\in\mathcal{W}$ taken to satisfy
$$\inf_{D\in\mathcal{D}(\boldsymbol{\Delta})} \left\| \begin{pmatrix} D(W_\Delta)_0 & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} D^{-1} & 0 \\ 0 & I_m \end{pmatrix}\right\|_\infty < 1,$$
where $\mathcal{D}(\boldsymbol{\Delta})$ is defined (11).
2) Solve
$$\theta_i^\star := \mathrm{argmin}_{0<\theta\leq 1}\theta$$
$$\text{subject to} \tag{15}$$
$$\inf_{D\in\mathcal{D}(\boldsymbol{\Delta})} \left\| \begin{pmatrix} D(W_\Delta)_i & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} D^{-1} & 0 \\ 0 & I_m \end{pmatrix}\right\|_\infty < \theta,$$
and select $D_i\in\mathcal{D}(\boldsymbol{\Delta})$ so that
$$\left\| \begin{pmatrix} D_i(W_\Delta)_i & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} D_i^{-1} & 0 \\ 0 & I_m \end{pmatrix}\right\|_\infty \leq \theta_i^\star.$$
3) Set $i=i+1$ and solve
$$(W_\Delta)_i := \mathrm{argmin}_{W_\Delta\in\mathcal{W}}\|VW_\Delta^{-1}\|_2^2$$
$$\text{subject to} \tag{16}$$
$$\left\| \begin{pmatrix} D_{i-1}W_\Delta & 0 \\ 0 & I_m \end{pmatrix}\mathcal{F}_\ell(G,K)\begin{pmatrix} D_{i-1}^{-1} & 0 \\ 0 & I_m \end{pmatrix}\right\|_\infty \leq 1,$$
and let $\lambda_i^\star := \|V(W_\Delta)_i^{-1}\|_2^2$.
4) If $|\lambda_i^\star-\lambda_{i-1}^\star|$ is sufficiently small then stop, else return to Step 2).

Observe that the intermediate optimization problems (15) and (16) can both be reformulated as convex problems, either pointwise in frequency, as in Problem 3.2 above, or using state-space techniques similar to those described in [11]. Furthermore, note that since $(W_\Delta)_{i-1}$ is always feasible for Step 3), $\lambda_i^\star \leq \lambda_{i-1}^\star$ at each iteration.

IV. Concluding Example

Consider again the uncertain closed-loop system illustrated in Figure 2. We now try to quantify the additional uncertainty that can be tolerated with the controller K and the performance weight W_p considered in Section I.

A. Structured uncertainty

In the case of structured uncertainty ($\Delta = \begin{bmatrix} \Delta_1 & & \\ & \Delta_2 & \\ & & \Delta_3 \end{bmatrix}$), applying the iterative approach in Section III-B for a local solution to the optimization Problem 3.1, the calculated W is as shown in Figure 7. Note the directionality function is set as $V = \frac{1}{10^{-5}s+1} W_\Delta$, where W_Δ is taken to be the original uncertainty weights with frequency responses shown in Figure 4. This yields the 'largest' tolerable uncertainty weights shown in Figure 7. The corresponding μ-curve is shown in Figure 8.

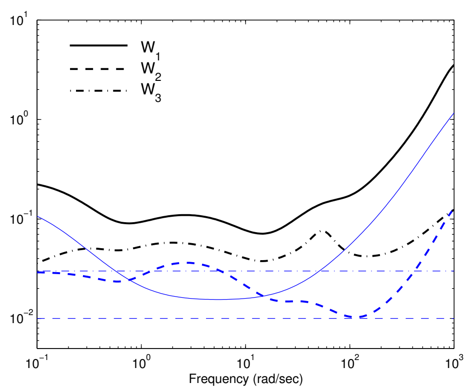

Fig. 7. Thick lines - the 'largest' tolerable structured uncertainty weights; Thin lines - the original structured uncertainty weights (and optimization directionality) for reference

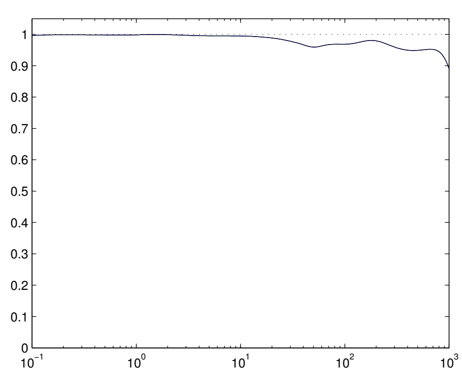

Fig. 8. μ-curve with the 'largest' tolerable uncertainty weights

B. Unstructured uncertainty

Assume the input and output uncertainties are constant across frequency ($W_2 = 0.01$, $W_3 = 0.03$). We just consider the additive uncertainty Δ_1. Solving the optimization Problem 3.2 yields the tolerable uncertainty weight W_1 as shown in Figure 9. As expected, in the unstructured case,

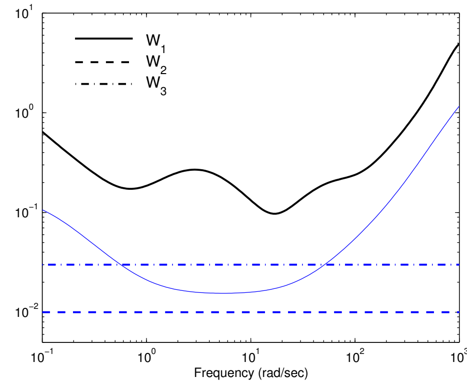

Fig. 9. The tolerable additive uncertainty weight W_1 compared with that in Fig. 4

the μ-curve with the optimized uncertainty weight is close to unity across frequency (see Figure 10).

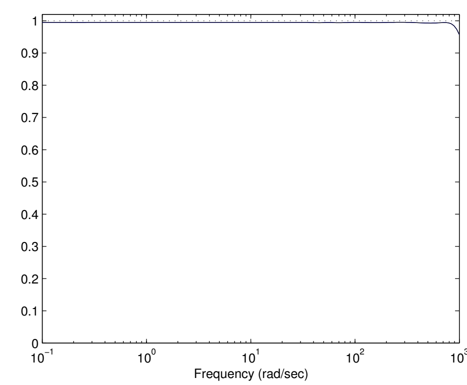

Fig. 10. μ-curve for tolerable unstructured uncertainty

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