

Dynamic Pricing in Consolidated Ancillary Service Markets

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Abstract—Regulation is a costly aspect of power system operation, which has long been inadequately and inconsistently incentivized via ill-formulated pricing schemes, as recently acknowledged by regulators. We propose a pricing formulation in which regulation prices are the optimal dual multipliers or costate of an optimal control problem. More precisely, we use the linear quadratic regulator to formulate a regulation pricing policy. We then construct a Vickrey-Clarke-Groves mechanism to induce honest participation among selfish agents. We apply the formulation to a scenario combining traditional frequency regulation and the California Independent System Operator’s Flexible Ramping Product.

I. INTRODUCTION

In a recent ruling, the Federal Energy Regulatory Commission mandated that current frequency regulation payments be improved so that “providers of frequency regulation receive just and reasonable and not unduly discriminatory or preferential rates” [1]. While practices vary across ISOs and RTOs, regulation providers have commonly received a capacity payment followed by a market price payment for the net energy they transact through regulation. The new ruling indicates that the latter component does not adequately capture the cost or quality of regulation provided. ISOs have responded by introducing various forms of mileage payments, in which providers are payed in proportion to the integral of some norm of the regulation signal rather than just its integral [2].

While such formulations do better account for the total amount of regulation provided, they do not incorporate dynamic couplings and network effects or the local value and costs of regulation. To address this shortcoming, we revisit an idea from [3] in the context of regulation payments. Specifically, we propose using the optimal control costate as a price for regulation. Similar pricing schemes have been proposed in [4]–[6], and the volatility of such scenarios has been investigated in [7]. In the past, our approach could only have been of theoretical interest because any system level feedback control required state information that was

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not centrally available at the time. However, synchophasors now overcome this shortcoming [8].

We consider prices associated with the linear quadratic regulator (LQR), which itself has long been well known to power systems [9], [10], but was also formerly unimplementable due to communication limitations. Indeed, the LQR is the solution to a convex optimization problem in the same sense as economic dispatch, and its associated dual multipliers admit price interpretations similar to locational marginal prices [11]. A novel insight of this work is that because the LQR is the analytical rather than numerical solution of an equality constrained quadratic program, the prices are a linear function of the system state, and thus constitute a linear pricing policy.

The cost and value of regulation is the private information of selfish agents, and thus constitutes a potential venue for gaming and market power. However, any framework that accounts for systemic costs and benefits must rely on such information, as with the dependence of economic dispatch on supply functions and their attendant strategic interaction [12]. To counter strategic behavior, we construct a Vickrey-Clarke-Groves mechanism, which makes truthful revelation of private information a dominant strategy [13]. A similar approach is applied in [6] to induce honest reporting of dynamic costs.

The formulation mechanistically applies to any dynamic scenario, and can thus unify different dynamic services such as voltage and frequency regulation. In an example, we apply our approach to a regulation scenario containing traditional frequency regulation and the California Independent System Operator’s new “flexible ramping product” [14].

II. OPTIMAL CONTROL

A. Notation

Let $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and define $x \in \mathbb{R}^n$, $n = \sum_i n_i$ and $u \in \mathbb{R}^m$, $m = \sum_i m_i$ to be x_i and u_i respectively stacked column-wise. Let their associated cost matrices be denoted $Q \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{m \times m}$, with terminal costs denoted by a superscripted T . The state transition and control input matrices are given by $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$. The state disturbance is denoted $w \in \mathbb{R}^n$ with covariance $W \in \mathbb{R}^{n \times n}$.

We use single subscripts to denote column blocks of a matrix and double subscripts to denote diagonal blocks; for example, $B_i \in \mathbb{R}^{n \times m_i}$ and $B_{ii} \in \mathbb{R}^{n_i \times m_i}$ are the i^{th} column block and the i^{th} diagonal block of B , respectively. We assume that $Q_{ij} = 0$ and $R_{ij} = 0$ for all $i \neq j$.

To reduce notation and because many of the following expressions are found in standard optimal control textbooks,

we suppress time indexing of dynamic quantities whenever it would be generically t .

B. The classical linear quadratic regulator

Consider the controllable LTI system

$$\dot{x} = Ax + Bu + w, \quad w \sim \mathcal{N}(0, W), \quad x(0) = x_0 \quad (1)$$

and the objective

$$J = \min_u \frac{1}{2} E \sum_i \int_0^T (x_i' Q_{ii} x_i + u_i' R_{ii} u_i) dt + x_i(T)' Q_{ii}^T x_i(T). \quad (2)$$

The optimal control policy is given by

$$u = -R^{-1} B' P x, \quad (3)$$

where $P \in \mathbb{R}^{n \times n}$ is the solution to the matrix Riccati equation

$$\dot{P} + A'P + PA - PBR^{-1}B'P + Q = 0, \quad P(T) = Q^T. \quad (4)$$

Lemma 1: The optimal control policy (3) is attained if each agent i independently solves

$$\min_{u_i} \frac{1}{2} u_i' R_{ii} u_i - (x' PB)_i u_i. \quad (5)$$

Proof: Differentiating (5) gives the result. ■

Remark 1 (Lagrangian duality): Lemma 1 can be straightforwardly derived using Lagrangian duality, paralleling standard locational marginal pricing [11] and the development of [3].

At optimality, the expectation of (2) may be written

$$\text{Tr} P(0) m_0 m_0' + \text{Tr} P(0) X_0 + \text{Tr} \int_0^T P W dt.$$

We will require similar expressions for generic objectives and dynamics. Suppose that the state evolves according to $\Phi \in \mathbb{R}^{n \times n}$. Let the state's mean and covariance be $m \in \mathbb{R}^n$ and $X \in \mathbb{R}^{n \times n}$, which evolve under Φ as

$$\dot{m} = \Phi m, \quad m(0) = 0 \quad (6)$$

$$\dot{X} = \Phi X + X \Phi' + W, \quad X(0) = 0. \quad (7)$$

Lemma 2 ([15]): Define

$$\Lambda(\Phi, \Psi) = e^{\Phi'(T-t)} \Psi^T e^{\Phi(T-t)} + \int_t^T e^{\Phi'(T-\tau)} \Psi e^{\Phi(T-\tau)} d\tau,$$

which is the solution to the Lyapunov differential equation

$$\dot{\Lambda} + \Lambda \Phi + \Phi' \Lambda + \Psi = 0, \quad \Lambda(T) = \Psi^T. \quad (8)$$

Then

$$\frac{1}{2} E \left[\int_t^T x' \Psi x dt + x(T)' \Psi^T x(T) \right] = \text{Tr} \int_0^T \Lambda(\Phi, \Psi) W dt.$$

III. ECONOMIC INTERPRETATION

A. Pricing regulation

The objective (2) represents resource costs, for example factory production losses, battery life degradation, or fuel consumption. As in [3], Lemma 1 has a pricing interpretation, which motivates the following definition:

Definition 1 (Pricing policy): The quantity $(x' PB)_i$ is agent i 's locational marginal regulation price.

This price has a number of desirable properties:

- It supports a competitive equilibrium, which is the LQR control policy.
- It is a pricing policy that is valid at any system state.
- It is valid under zero mean Gaussian disturbances due to the certainty equivalence of LQR.
- It is valid if x is estimated using a Kalman filter due to the separation principle.

While prices are often used in other contexts to actively moderate supply and demand [16], here we are not exclusively suggesting that regulation be provided in response to a price, which may be unrealistic at the time scales frequency excursions occur over. Rather, $(x' P)_i B_{ii}$ could also determine a regulation provider's ex-post settlement, so that they ultimately receive

$$\int_0^T (x' PB)_i u_i dt, \quad (9)$$

where x and u are the actual state and control trajectories in $[0, T]$.

Remark 2 (Non-LQR control): In (9), u_i can be any control chosen by agent i , rather than exclusively the LQR control policy (3).

B. The value of regulation

We now use Lemma 2 to evaluate various costs associated with regulation. We consider realized and expected quantities, in the case of the latter assuming that the LQR policy is employed by each agent, i.e. $u = -R^{-1} B' P x$. In this section we only consider instantaneous quantities, the integral of which yields corresponding ex-post quantities.

Define $F = A - BR^{-1} B' P$, $U = PBR^{-1} B' P$, and $U_{ii} = PB_i R_{ii}^{-1} B_i' P$. Let $Q_{ii}^0 \in \mathbb{R}^{n \times n}$ be equal to Q_{ii} on the i^{th} block of its main diagonal and be zero elsewhere. To simplify our exposition, we also assume that the system begins exactly at its equilibrium point so that $m_0 = 0$ and $X_0 = 0$.

Consider for example the expected instantaneous regulation payment to agent i :

$$\frac{1}{2} E x' U_{ii} x.$$

Lemma 2 tells us that it is given by the following definition:

Definition 2 (Cost of regulation): The expected and actual regulation payments to agent i are

$$\begin{aligned} \rho_i &= \text{Tr} \Lambda(F, U_{ii}) W \quad \text{and} \\ \hat{\rho}_i &= (x' PB)_i u_i. \end{aligned}$$

We may proceed similarly for other quantities of interest.

Definition 3 (Benefit of regulation): The expected and actual reduction in agent i 's operating cost due to regulation are

$$\nu_i = \text{Tr}(\Lambda(A, Q_{ii}^0) - \Lambda(F, Q_{ii}^0))W \quad \text{and} \quad (10)$$

$$\hat{\nu}_i = \text{Tr}(\Lambda(A, Q_{ii}^0)W - x'Q_{ii}^0x). \quad (11)$$

Now consider the contribution of disturbances from agent i to the cost of regulation. The total cost of regulation may be written

$$\begin{aligned} \text{Tr}\Lambda(F, U)W &= \sum_i \text{Tr}\Lambda(F, U)_{ii}W_{ii} \\ &+ 2 \sum_{j \neq i} \text{Tr}\Lambda(F, U)_{ij}W_{ij} \end{aligned}$$

The first summation may be unambiguously divided index-wise among the agents, but a sharing rule must be defined for the latter. A naive scheme is to attribute $\text{Tr}\Lambda(F, U)_{ij}W_{ij}$ each to agents i and j , yielding the net contribution $\text{Tr}\Lambda(F, U)_i W_i$; this however may disproportionately penalize small disturbance sources for being correlated with larger disturbance sources. We instead suggest the following measure:

Definition 4 (Expected cost of disturbances): The proportionally fair expected cost of disturbances from agent i is

$$\begin{aligned} \mu_i &= \text{Tr}\Lambda(F, U)_{ii}W_{ii} \\ &+ 2 \sum_{j \neq i} \frac{\text{Tr}W_{ii}}{\text{Tr}W_{ii} + \text{Tr}W_{jj}} \text{Tr}\Lambda(F, U)_{ij}W_{ij}. \end{aligned} \quad (12)$$

Because the disturbance vector w does not appear explicitly in (2) and because the actual trajectory may not correspond to the idealized system dynamics, we cannot give an exact expression for $\hat{\mu}_i$, but rather use μ_i to design statistically correct payments. If we assume a symmetric imbalance fee γ_i , we should have

$$\int_0^T \gamma_i E|w_i| dt = \int_0^T \mu_i dt. \quad (13)$$

The imbalance fee may either be time-varying or constant, which respectively yield

$$\gamma_i = \frac{\mu_i}{E|w_i|} \quad \text{and} \quad \gamma_i = \frac{\int_0^T \mu_i dt}{\int_0^T E|w_i| dt}, \quad (14)$$

where

$$E|w_i| = 2\sqrt{\frac{2W_{ii}}{\pi}}.$$

Definition 5 (Imbalance cost): The actual imbalance-wise cost of disturbances from agent i is

$$\hat{\mu}_i = \gamma_i |w_i|. \quad (15)$$

IV. DESIGNING MARKETS

We now address how to finance (9). We assume here that the system operator is revenue neutral.

A. Payments

We respectively consider disturbance and regulation payments of the form

$$\alpha\phi_i^\mu(\hat{\mu}) \quad \text{and} \quad (1-\alpha)\phi_i^\nu(\hat{\nu}), \quad (16)$$

that satisfy

$$E \sum_i \phi_i^\mu(\hat{\mu}) = E \sum_i \phi_i^\nu(\hat{\nu}) = \chi, \quad (17)$$

where $\chi = \sum_i \rho_i$ and $\alpha \in [0, 1]$.

Definition 6: Agent i 's aggregate instantaneous cost is

$$\begin{aligned} \hat{J}_i &= \frac{1}{2}(x'_i Q_{ii} x_i + u'_i R_{ii} u_i) - (x'PB)_i u_i \\ &+ \alpha\phi_i^\mu(\hat{\mu}) + (1-\alpha)\phi_i^\nu(\hat{\nu}). \end{aligned} \quad (18)$$

We now discuss this payment.

- The parameter α determines what portions of (9) should be paid by the disturbance sources versus the regulation beneficiaries; determining the correct value of α is outside of our current scope.
- We have not specified a particular payment structure because no single format may be uniformly appropriate. For example, if wind farms are to be directly accountable for their intermittency, those producing larger disturbances should pay more, and the proportionally fair payment $\phi_i^\mu(\hat{\mu}) = \hat{\mu}$ is appropriate. On the other hand, it could be difficult and contentious to audit household demand variability, in which case the constant $\phi_i^\mu(\hat{\mu}) = \chi/n$ is more appropriate. Similar analogies hold for charging beneficiaries, where $\phi_i^\nu(\hat{\nu}) = \chi\hat{\nu}/\sum_i \nu_i$ and $\phi_i^\nu(\hat{\nu}) = \chi/n$ are the proportionally fair and constant payments, respectively.
- Since neither of the latter two terms in (18) explicitly depend on u_i , the LQR competitive equilibrium is supported by the local objectives \hat{J}_i as with (5).
- Payments balance in expectation but not necessarily in realization, placing the onus on the system operator to absorb and potentially socialize temporary financial imbalances.

B. Preempting strategic behavior

Q_{ii} and R_{ii} are provided by selfish agents, for whom it may be profitable to be dishonest. To counter such behavior, we derive a Vickrey-Clarke-Groves (VCG) [13], which specifies an additional tax that depends on Q_{ii} and R_{ii} , which we assume are available at time $t = 0$.

Define

$$\begin{aligned} J_i &= E \int_0^T \hat{J}_i dt \\ &= \int_0^T \text{Tr}\Lambda(F, Q_{ii}^0 + U_{ii})W - \rho_i \\ &+ E[\alpha\phi_i^\mu(\hat{\mu}) + (1-\alpha)\phi_i^\nu(\hat{\nu})] dt, \end{aligned}$$

and note that $\sum_i J_i = J$.

Define R_{-i} to be R with the i^{th} row and column block removed, and let $P_{-i} \in \mathbb{R}^{n \times n}$ be the solution to the Riccati equation

$$\dot{P}_{-i} + A'P_{-i} + P_{-i}A - P_{-i}B_{-i}R_{-i}^{-1}B_{-i}'P_{-i} + Q - Q_{ii}^0 = 0,$$

where

$$\begin{aligned} B_{-j} &= [B_1, \dots, B_{j-1}, B_{j+1}, \dots, B_n] \\ u_{-j} &= [u_1, \dots, u_{j-1}, u_{j+1}, \dots, u_n]' \\ F_{-i} &= A - B_{-i}(R_{-i})^{-1}(B_{-i})'P_{-i} \end{aligned}$$

Consider the LTI system

$$\dot{x} = Ax + B_{-j}u_{-j} + w,$$

and the objective

$$\begin{aligned} J_{-i} &= \min_{u_{-i}} E \frac{1}{2} \sum_{j=-i}^T \int_0^T (x_j' Q_{jj} x_j + u_j' R_{jj} u_j) dt \\ &= \int_0^T \text{Tr} \Lambda (F_{-i}, Q - Q_{ii}^0 + U - U_{ii}) W dt. \end{aligned}$$

Definition 7: Let (Q_{ii}, R_{ii}) , $i = 1, \dots, n$ be the pairs cost matrices reported by each agent. The VCG tax for agent i is given by

$$\beta_i = \sum_{j=-i} J_j - J_{-i}. \quad (19)$$

Lemma 3: Given the tax function of (19), truthful reporting of the matrices Q_{ii}, R_{ii} is a dominant strategy for each agent i .

Proof: Firstly, observe that the term J_{-i} does not depend on the matrices reported by agent i . Now, suppose agent i reports $(\tilde{Q}_{ii}, \tilde{R}_{ii})$ that are different from the true (Q_{ii}, R_{ii}) . Then, the locational marginal regulation prices based on the mis-reported values will result in a (socially) sub-optimal control policy and a net social cost of $\tilde{J} = \sum_k \tilde{J}_k \geq J$. Further, the tax for agent i would be $\sum_{j=-i} \tilde{J}_j - J_{-i}$. Agent i 's net cost, $\tilde{J}_i + \beta_i$, would then be

$$\begin{aligned} \tilde{J}_i + \sum_{j=-i} \tilde{J}_j - J_{-i} &= \tilde{J} - J_{-i} \\ &\geq J - J_{-i}. \end{aligned} \quad (20)$$

On the other hand, if $(\tilde{Q}_{ii}, \tilde{R}_{ii}) = (Q_{ii}, R_{ii})$, then the inequality in (20) would become an equality. Therefore, truthful reporting is a dominant strategy for agent i . ■

Remark 3: Q_{ii} may be much harder to quantify than R_{ii} , in which case it is viable that only R_{ii} be reported and that Q_{ii} be set by a regulating body, i.e. an ISO.

V. EXAMPLE: FREQUENCY REGULATION

Renewable variability has increased need for active power regulation resources that can make large adjustments to their output levels over short time intervals. In recognition, the California Independent System Operator has introduced a ‘‘flexible ramping product’’ [14], in which ramping energy and shadow prices are determined via optimal power flow [17]. In this example, we consolidate conventional frequency regulation and flexible ramping by appropriately defining the

control input, and numerically examine the ramping cost due to variability.

Define ω_i , θ_i , and u_i to be the nominal frequency deviation, voltage angle, and real power injection at bus i . We regard u_i as a state, and define the control input v_i to be the derivative of u_i . Let $I, D, H \in \mathbb{R}^{n \times n}$ respectively be the identity matrix, a positive, diagonal damping matrix, and a diagonal matrix of rotor inertias. Define

$$L_{ij} = \begin{cases} b_{ij} & i \neq j \\ -\sum_k b_{ik} & i = j \end{cases}, \quad (21)$$

where $b_{ij} \in \mathbb{R}^+$ is the inductance of line ij . The linearized system dynamics are given by

$$\begin{bmatrix} \dot{\omega} \\ \dot{\theta} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} -H^{-1}D & H^{-1}L & H^{-1} \\ \omega_0 I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega \\ \theta \\ u \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I^n \end{bmatrix} v + \begin{bmatrix} w \\ 0 \\ 0 \end{bmatrix}, \quad w \sim \mathcal{N}(0, W).$$

One can straightforwardly verify that this system is controllable. Consider the objective

$$\min_v \frac{1}{2} E \sum_i \int_0^T (\omega_i' Q_{ii} \omega_i + u_i' R_{ii}^u u_i + v_i' R_{ii}^v v_i) dt. \quad (22)$$

The first and second term are standard and penalize frequency deviations and regulation power injections. The last term is a ramping cost, and penalizes rapid changes in control input.

If we naively apply the pricing scheme of Lemma 1, agent i 's total regulation payment is $\int_0^T (x' P B)_i v_i dt$, which, if all agents apply LQR, is equal in expectation to the third term of (22). Agent i is thus being paid for their nominal control action v_i , but not the actual regulation they provide, v_i and u_i . The inconsistency stems from defining u_i as a state rather than a control, implicit in which is the assumption that agent i is not directly aware of its influence on the cost $u_i' R_{ii} u_i$. A solution consistent with this assumption is to simply also pay agent i $\int_0^T u_i' R_{ii} u_i dt$; indeed, since u_i is not a control variable, this retains all of the desirable properties of the locational marginal regulation price, albeit in a narrow technical sense. A perhaps more consistent solution is found in discrete time, in which case the a control variable may be incorporated into both the regulation and ramping term; this will be explored in future work.

We may nevertheless examine some characteristics of our approach in this example. We consider a nine-bus system in which the ramping control input v is the LQR solution, and prices are from the infinite horizon case. Parameter values are given by $\omega_0 = 120\pi$, $H = 8I$, $D = 5I$, $Q = \text{diag}[1 \ 2 \ 3 \ 1 \ 2 \ 3 \ 1 \ 2 \ 3]/10$, $R^u = \text{diag}[3 \ 2 \ 1 \ 3 \ 2 \ 1 \ 3 \ 2 \ 1]/10$, $R^v = 2 \times \text{diag}[1 \ \dots \ 9]$, and $W = \text{diag}[1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 3 \ 3 \ 3]/100$. Line inductances are taken from the nine-bus test case in MATPOWER [18].

Fig. 1 shows ramping control input, the corresponding ramping price, and their product, the regulation payment, at

buses two and seven for a sample disturbance trajectory. As is easily shown analytically, the payment is always positive if each bus employs LQR, but the price and optimal control may be considerably different at different buses, as seen at 0.4 seconds.

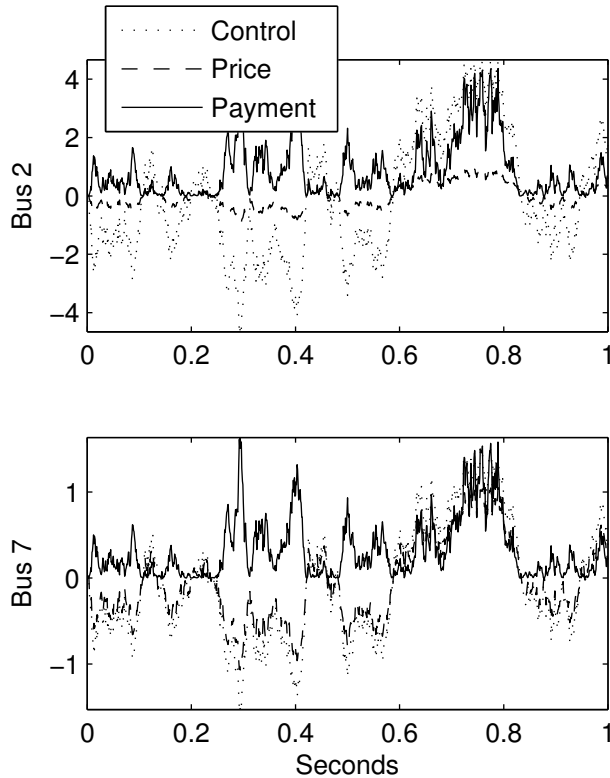


Fig. 1. Ramping input, price, and payment at buses two and seven.

Fig. 2 shows the expected average regulation payment and VCG tax for each bus on top and the expected average disturbance cost and imbalance fee on bottom. The VCG tax is smaller than the expected payment, but of the same order; this is among many known issues with VCG mechanisms, justifying further work in more practical schemes [19]. As expected, the imbalance fee is proportional to the square root of the expected average cost of disturbances, which is desirable because it reduces the spread of imbalance fees across buses.

VI. CONCLUSIONS AND FUTURE WORK

The ideas of [3] were premature at the time of introduction, but are ideally suited to the technological and economic landscape of the power system today. In this work, we have explored optimal control-based pricing in the context of current regulation issues [1], [14]. By working within an LQR framework and employing tools from mechanism design, we have made the original approach considerably more versatile and practical. The new formulation is desirable in that it

- Rigorously incorporates dynamic modeling,
- Is simple and scalable,
- Can consolidate formerly diverse regulation services.

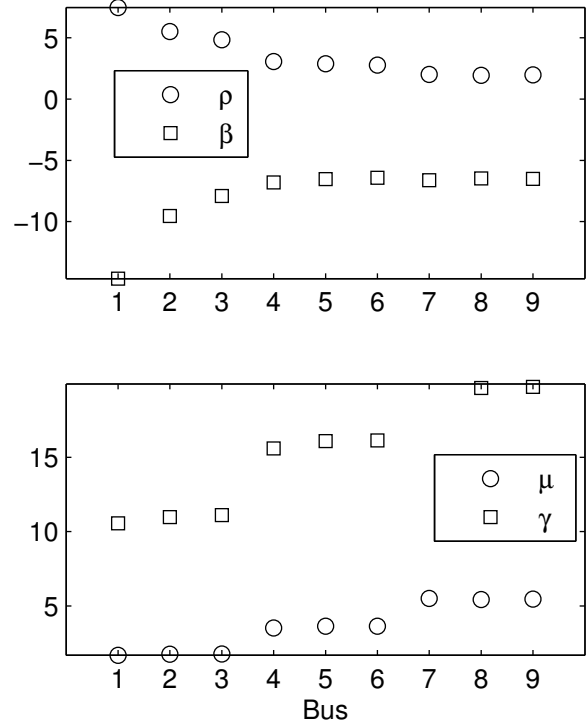


Fig. 2. Top: Bus-wise expected payments ρ and VCG taxes β , bottom: expected costs of disturbances μ and imbalance fee γ .

Future directions include the incorporation of dead-zones and hard constraints, e.g. via hybrid control and constrained linear quadratic regulation [20], non-Gaussian noise models for contingencies and other disturbances, and modeling the state dependencies introduced by linearization about a changing operating points.

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