2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Performance Analysis of Relay Feedback Position Regulators for Manipulators with Coulomb Friction

Luis T. Aguilar[1], Leonid B. Freidovich[2], Yury Orlov[3], and Jován Mérida[1]

Abstract— **The purpose of the paper is to analyze the performance of several global position regulators for robot manipulators with Coulomb friction. All the controllers include a proportional-differential part and a switched part whereas the difference between the controllers is in the way of compensation of the gravitational forces. Stability analysis is also revisited within the nonsmooth Lyapunov function framework for the controllers with and without gravity pre-compensation. Performance issues of the proposed controllers are evaluated in an experimental study of a five degrees-of-freedom robot manipulator. In the experiments, we choose two criteria for performance analysis. In the first set of experiments, we set the same gains to all the controllers. In the second set of experiments, the gains of the controller were chosen such that the work done by the manipulator is similar.**

I. Introduction

Many industrial anthropomorphic manipulators are controlled by linear proportional-differential (PD) or proportional-integral-differential (PID) controllers with minor modifications such as additional feedforward terms, compensation terms for friction forces, and anti-windup schemes.

It is well-known that a simple PD feedback regulator with full gravity compensation and constant reference allows achieving global asymptotic stability (GAS) for a rigid manipulator model with revolute joints and without Coulomb friction, see e.g., [1], [2], [3]. Moreover, instead of perfectly canceling the gravitational force, it is possible to use a constant compensation of the gravity at the desired position, as shown in [4]. However, if there is an error in estimation of physical parameters, which is unavoidable in practice, then such a simple feedback under some assumptions leads to GAS of an equilibrium that is shifted from the desired one. To avoid such an off-set, integral action may be invoked. However, adding an integral action in the simplest way typically leads to semiglobal asymptotic stability, as can be seen from singular perturbation theory-based proofs presented e.g., in [5], [6] (note that the proofs in these papers are very similar but the first one mistakenly claims GAS); see also [7].

It is also obvious that including Coulomb friction into the model should lead to an off-set even in the case of present integral action. In fact, due to discontinuous nature of the Coulomb friction model, no continuous feedback is able to achieve GAS for a model including such forces. To deal with this issue, introduction of a discontinuous feedback is necessary. The simplest approach is to add a relay-like feedback on the error as suggested in [8]. A PD controller with full gravity compensation and with a dirty-derivative substitution for the differential action, and with an additional discontinuous term, computed as an amplified signum of the error, allows one to recover GAS as shown in [8] for a model accounting for Coulomb friction.

In this paper, we keep for simplicity differential feedback but show that GAS without an off-set can be achieved not only when the gravity is fully compensated but also when it is compensated only at the target position as in [4]. Moreover, we show that, in fact, errors in gravity compensation do not spoil GAS and therefore such a compensation may be dropped. After that, we present results of our experimental study that has been aimed to see whether it may be still beneficial to compensate gravity or to include an integral action as well.

The paper is structured as follows. Section II presents the dynamic model of a robot manipulator and some its useful properties. Sections III through VI introduce the four controllers under study: relay controller plus gravity compensation, without gravity compensation, with gravity pre-compensation, and with integral action, respectively. Stability analysis is revisited in those Sections. Experimental study made for a five degrees-of-freedom robot manipulator with friction is given in Section VII. Conclusion is given in Section VIII.

Notation: We let $\mathbb{R}$ denote the set of real numbers. The signum function is defined as

$$\operatorname{sign}(x) = \begin{cases} 1 & \text{if } x > 0 \\ [-1, 1] & \text{if } x = 0 \\ -1 & \text{if } x < 0. \end{cases}$$

Here, we define the integral of a real vector function $f = (f_1, \ldots, f_n)$ on the interval $[a, b]$ as the integral of each of

Y. Orlov and L. T. Aguilar gratefully acknowledge the financial support from CONACYT (Consejo Nacional de Ciencia y Tecnología) under Grants 165958 and 127575.

[1]L. Aguilar and J. Mérida are with Instituto Politécnico Nacional, CITEDI, Tijuana, B.C., Mexico; laguilarb@ipn.mx; jmeridao0900@ipn.mx

[2]L. Freidovich is with the Department of Applied Physics and Electronics, Umeå University, Umeå, Sweden; leonid.freidovich@tfe.umu.se

[3]Yu. Orlov is with the Department of Electronics and Telecommunications, Mexican Scientific Research and Advanced Studies Center (CICESE), Ensenada, B.C., México; yorlov@cicese.mx

its components, that is

$$\int_a^b f\,ds = \begin{bmatrix} \int_a^b f_1\,ds \\ \int_a^b f_2\,ds \\ \vdots \\ \int_a^b f_n\,ds \end{bmatrix}, \tag{1}$$

while for a vector $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$ we use the following norm

$$\|x\| = \max_{i=1,\ldots,n} |x_i|.$$

For matrices $A, B \in \mathbb{R}^{n\times n}$ we write

$$A \text{ is positive definite } (A > 0), \qquad A > B$$

to indicate that for every $0 \neq x \in \mathbb{R}^n$ we have

$$x^T A\,x > 0, \qquad x^T A\,x > x^T B\,x$$

respectively.

II. Dynamic Model

The equation of motion of a manipulator with revolute joints and friction is given by [2], [3]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau \tag{2}$$

where $q(t)$ denotes the $n \times 1$ angular position vector, $\dot{q}(t)$ is the $n \times 1$ angular velocity vector, $\tau(t)$ is the $n \times 1$ applied torque vector, $t \in \mathbb{R}$ is the time, $M(q)$ denotes the $n \times n$ inertia matrix, $C(q,\dot{q})$ denotes the $n \times n$ centrifugal and Coriolis force matrix, $G(q)$ is the $n \times 1$ gravitational force vector, and $F(\dot{q})$ represents the friction torques governed by

$$\begin{aligned} F(\dot{q}) &= F_c\,\text{sign}(\dot{q}) \\ \text{sign}(\dot{q}) &= [\text{sign}(\dot{q}_1), \cdots, \text{sign}(\dot{q}_n)]^T \end{aligned} \tag{3}$$

where the Coulomb friction coefficient matrix $F_c \in \mathbb{R}^{n\times n}$ is diagonal and positive semi-definite.

Given a desired ending position $q_d \in \mathbb{R}^n$, the *control objective* for all the regulators under study consists in making the system globally stable and ensuring that

$$\lim_{t\to\infty} \|q(t) - q_d\| = 0 \tag{4}$$

for an arbitrary initial condition $q(0)$.

The following properties, taken from Spong *et al.* [2] and Kelly *et al.* [3], are valid for robots having only revolute joints.

Property 1: The matrix $M(q)$ is symmetric and positive definite for all $q \in \mathbb{R}^n$. The matrix $M^{-1}(q)$ exists and is positive definite as well.

Property 2: The matrix $C(q,\dot{q})$ can be taken related to the inertia matrix $M(q)$ in such a way that

$$x^T \left[\frac{1}{2}\dot{M}(q) - C(q,\dot{q})\right] x = 0 \tag{5}$$

for all $q, \dot{q}, x \in \mathbb{R}^n$.

Property 3: There exist

- a scalar function $U(q)$, called potential energy, such that
$$\frac{\partial U(q)}{\partial q} = G^T(q) \tag{6}$$
- a positive constant k_g such that
$$\|G(q)\| \le k_g \qquad \text{for all } q \in \mathbb{R}^n \tag{7}$$
- and a positive constant α such that
$$\frac{\partial G(q)}{\partial q} \le \alpha\,I \qquad \text{for all } q \in \mathbb{R}^n \tag{8}$$
where I is identity matrix and the inequality is understood in the sense of quadratic forms.

With such properties at hand, we are ready to present some simple modifications of a standard PD regulator.

III. Relay Controller Plus Gravity Compensation

The relay controller with gravitational force compensator is given by

$$\tau = G(q) - K_p\,e - K_d\,\dot{q} - K_s\,\text{sign}(e) \tag{9}$$

where

$$e \triangleq q - q_d \tag{10}$$

is the $n \times 1$ position error vector, K_p and K_d are $n \times n$ diagonal positive definite matrices, and K_s is a $n\times n$ diagonal positive definite matrix whose elements are chosen such that

$$K_{si} > F_{ci} \qquad i = 1, \ldots, n. \tag{11}$$

Conditions of K_p and K_d will be preserved for forthcoming controllers.

Theorem 1: The mechanical manipulator (2)–(3) driven by a relay controller plus gravity compensator (9) has the equilibrium $(e,\dot{q}) = (0,0)$ globally asymptotically stable provided (11) is valid and the PD gains are positive.

Proof: The proof can be obtained by a simple straightforward modification of the proof provided in [8] for a similar system but with differential feedback substituted by a dirty-derivative type feedback. In fact, a stronger claim of finite-time convergence can be verified, see [9]. ■

IV. Relay Controller Plus Gravity Pre-compensation

The relay controller plus gravity pre-compensator is given by

$$\tau = G(q_d) - K_p e - K_d\dot{q} - K_s\,\text{sign}(e) \tag{12}$$

where the diagonal positive definite matrix $K_s \in \mathbb{R}^{n\times n}$ must satisfy (11).

Theorem 2: The mechanical manipulator (2)–(3) driven by a relay controller plus gravity pre-compensator (12) has the equilibrium $(e,\dot{q}) = (0,0)$ globally asymptotically stable provided $K_{si} > F_{ci} + 2\,k_g$, $K_{pi} > \alpha$, and $K_{di} > 0$ for $i = 1, \ldots, n$.

Proof: The proof can be easily deduced from the proof of the next theorem presented below. ■

V. Relay Controller without Gravity Compensation

Consider now the relay controller without gravity compensation

$$\tau = -K_p\, e - K_d\, \dot{q} - K_s\ \mathrm{sign}(e). \qquad (13)$$

Here, $K_s \in \mathbb{R}^{n\times n}$ is a diagonal positive matrix whose elements are chosen such that

$$K_{si} > F_{ci} + k_g \qquad i = 1, \ldots, n. \qquad (14)$$

Theorem 3: The mechanical manipulator (2)–(3) driven by a relay controller plus gravity pre-compensator (13) has the equilibrium $(e, \dot{q}) = (0, 0)$ globally asymptotically stable provided (14) is valid and $K_{pi} > \alpha$, $K_{di} > 0$ for $i = 1, \ldots, n$.

Proof: The closed-loop dynamics (2), (13) in terms of the position errors e and velocities $\dot{q}$ are given by

$$\begin{aligned} \dot{e} &= \dot{q} \\ \ddot{q} &= M(e+q_d)^{-1}[-K_p e - K_d\dot{q} - K_s\,\mathrm{sign}(e) \\ &\quad - F_c\,\mathrm{sign}(\dot{q}) - C(e+q_d,\dot{q})\dot{q} - G(e+q_d)]. \end{aligned} \qquad (15)$$

The equilibria of this system are defined by solutions of the following relations

$$\begin{aligned} &\dot{q} = 0 \qquad \text{and} \quad \text{for} \quad i = 1, \ldots, n \\ &0 \in -K_{pi}\, e_i - K_{si}\,\mathrm{sign}(e_i) + [-F_{ci}, F_{ci}] - G_i(e+q_d). \end{aligned} \qquad (16)$$

It is observed that since (14) implies $k_g \leq K_{si} + F_{si}$, we have that $e = 0$ satisfies this relation.

Now, if there is a solution with $e_i = e_i^+ > 0$, then

$$K_{pi}\, e_i^+ \in [-F_{ci} - K_{si}, F_{ci} - K_{si}] - G_i(e+q_d) \qquad (17)$$

which is impossible since in the left hand-side we have a positive number while the upper bound of the set in the right-hand side is negative due to (14) and (7).

Similarly, if there is a solution with $e_i = e_i^- < 0$, then

$$K_{pi}\, e_i^- \in [-F_{ci} + K_{si}, F_{ci} + K_{si}] - G_i(e+q_d) \qquad (18)$$

which is impossible since in the left hand-side we have a negative number while the lower bound of the set in the right-hand side is positive due to (14) and (7).

Hence, the unique equilibrium of our closed-loop system is $(e, \dot{q}) = (0, 0)$.

Consider now the following Lyapunov function

$$\begin{aligned} V(e,\dot{q}) &= \frac{1}{2}e^T K_p e + U(e+q_d) - U(q_d) \\ &\quad + \frac{1}{2}\dot{q}^T M(e+q_d)\dot{q} + \sum_{i=1}^{n} K_{si}|e_i| \end{aligned} \qquad (19)$$

where $U(q) \in \mathbb{R}$ is the potential energy of the manipulator (2) defined in (6).

Note that $V(0,0) = 0$ and $V(e,\dot{q}) = V(0,\dot{q}) + V(e,0)$.

Computing first partial derivatives yields that the stationary points of this function are solutions of the relations (16) without the multi-valued term $[-F_{ci}, F_{ci}]$. Hence, the only stationary point is $(e,\dot{q}) = (0,0)$. At the same time, matrix of second partial derivatives of $V(0,\dot{q})$ is $M(e+q_d)$, which is positive definite, and the the matrix of second partial derivatives of $V(e,0)$ is equal to $K_p + \frac{\partial G(e+q_d)}{\partial e}$ which is positive definite. It follows that $(e,\dot{q}) = (0,0)$ is the point of global minimum and therefore this function is positive definite and radially unbounded.

The time derivative of (19) along the solution of the closed-loop system (15) is as follows

$$\begin{aligned} \dot{V} &= e^T K_p\dot{q} + \frac{\partial U(e+q_d)}{\partial e}\dot{q} + \frac{1}{2}\dot{q}^T\dot{M}(e+q_d)\dot{q} \\ &\quad + \dot{q}^T M(e+q_d)\ddot{q} + \dot{q}^T K_s\,\mathrm{sign}(e) \\ &= e^T K_p\dot{q} + G^T(e+q_d)\dot{q} + \frac{1}{2}\dot{q}^T\dot{M}(e+q_d)\dot{q} \\ &\quad + \dot{q}^T\left[-K_p e - K_d\dot{q} - K_s\,\mathrm{sign}(e) - F_c\,\mathrm{sign}(\dot{q})\right. \\ &\quad \left. -C(e+q_d,\dot{q})\dot{q} - G(e+q_d)\right] + \dot{q}^T K_s\,\mathrm{sign}(e) \\ &= \dot{q}^T\left[-K_d\dot{q} - F_c\,\mathrm{sign}(\dot{q})\right] < 0 \end{aligned} \qquad (20)$$

for all $\dot{q} \neq 0$. Then, by applying the invariance principle for discontinuous systems from [10] we conclude that the equilibrium is globally asymptotically stable. ■

We see that with the control law (13), one has GAS of the desired equilibrium without introducing an integral action or a compensation of an uncertain force due to gravity. However, it is reasonable to admit that either the integral action or a partial compensation may improve overall performance. Note that (11) is less restrictive than (14) and allows one to have smaller values of the gain in front of the discontinuity, which defines the amplitude of chattering in the control signal. Such chattering may waste the energy and such a reduction may thus be desirable.

Let us introduce simple modifications involving integral action to be used subsequently in our experimental study.

VI. Relay Controller with Integral Action

Consider the relay control with integral action

$$\tau = -K_p\, e - K_d\, \dot{q} - K_s\ \mathrm{sign}(e) - K_I \int_0^t \mathrm{sign}(e(s))\, ds \qquad (21)$$

where K_I is a $n \times n$ diagonal positive definite matrix.

Studying [1], [5], [6], one can assume that such a modification would allow keeping either GAS or at least semi-global asymptotic stability. But the question is whether such a modification is beneficial in terms of an achievable performance.

VII. Experimental Study

A. Experimental Setup

Experimental setup, designed in the research laboratory of CITEDI-IPN and shown in Figure 1, involves a five degrees-of-freedom (5-DOF) industrial robot manipulator manufactured by *Amatrol*. The base of the mechanical robot has a horizontal revolute joint (q_1), whereas two links have vertical revolute joints q_2 and q_3. The rest degrees-of-freedom correspond to the end effector orientation. Nominal parameters of mechanical manipulator are summarized in Table I. Worm gear set, helicon gear set, and roller chain are used for torque transmission to joint q_1, q_2, and q_3, respectively; there is a DC gear motor for each joint with a reduction ratio of 19.7:1 for q_1 and q_2, and 127.8:1 for q_3. These gears are the main source of friction. The PCI multifunction I/O board model 626 from Sensoray Co., Inc. [11] is employed for the real time control system and it consists of four analog outputs (13 bit resolution), 20 digital I/O channels with edge detection and interrupt capability. The controllers are implemented using Simulink from Matlab® running on a Pentium PC. Position measurements of each joint of the robot are obtained using the channels of quadrature encoders available on each DC gear motors which are connected to the I/O card, programmed to provide the encoder signal processing each millisecond. The resolutions of encoders are 52×10^{-3} rad, 62×10^{-3} rad, and 34×10^{-3} rad for q_1, q_2, and q_3, respectively. Linear power amplifiers are installed in each servomotor which applies a variable torque to each joint. These amplifiers accept control inputs from D/A converter in the range of ± 10 V. The obtained gravitational force vector, from Euler-Lagrange formulation, is

$$G(q) = g \begin{bmatrix} 0 \\ (m_1 + m_2)l_1 \cos q_2 + m_2 l_2 \cos(q_2 + q_3) \\ m_2 l_2 \cos(q_2 + q_3) \end{bmatrix}. \tag{22}$$

The physical constant parameters m_i, l_i $(i = 1, 2)$, g, and F_{ci} $(i = 1, \ldots, 3)$ are given in Table I. The interested reader may refer to [8] for the full model.

TABLE I
PARAMETERS OF MECHANICAL MANIPULATOR.

Description	Notation	Value	Units
Length of link 1	l_1	0.297	m
Length of link 2	l_2	0.297	m
mass of link 1	m_1	0.60	kg
mass of link 2	m_2	0.68	kg
Inertia 1	J_1	0.243×10^{-3}	kg m^2
Inertia 2	J_2	0.068×10^{-3}	kg m^2
Inertia 3	J_3	0.015×10^{-3}	kg m^2
Gravity	g	9.8	m/s^2
Friction coeff. 1	F_{c1}	2.1	Nm
Friction coeff. 2	F_{c2}	1.02	Nm
Friction coeff. 3	F_{c3}	0.78	Nm

B. Experimental Results

The manipulator was required to move from the origin $q_1(0) = q_2(0) = q_3(0) = 0$ rad to the desired position $q_{d1} = q_{d2} = q_{d3} = \pi/2$ rad. The initial velocities $\dot{q}_1(0)$, $\dot{q}_2(0)$, and $\dot{q}_3(0)$ were set to zero in the experiment.

We run two experiments:

1) We set all the gains of the controllers to:

$$K_p = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 30 \end{bmatrix}, \quad K_d = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix},$$
$$K_s = \begin{bmatrix} 2.3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad K_I = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}. \tag{23}$$

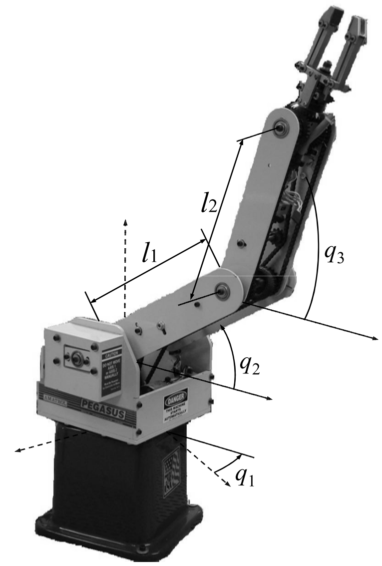

Fig. 1. The five-DOF robot manipulator.

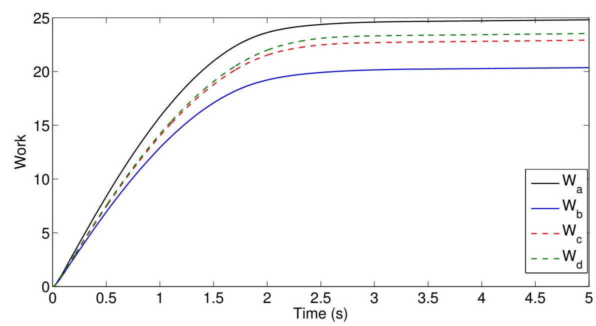

Fig. 2. Experimental work of each controller under gains given in (23): (W_a) Relay controller + gravity compensation, (W_b) Relay controller + gravity pre-compensation, (W_c) Relay controller without gravity compensation, and (W_d) Relay controller with integral action.

Figure 2 shows the work

$$W = \int_0^T |\dot{q}^T(t)\tau(t)|dt \tag{24}$$

done moving the robot. Figure 3 shows the norm of the error $\|e(t)\|_2$ of each controller where it is possible to see the fastest convergence of the error to the origin by using the relay controller with integral action (labeled as p_d). Figure 4 shows the input control where the presence of chattering is evident; it appears when trajectories reach the origin. We would like to remark that another criteria to choose gains (23) was to avoid mechanical resonance.

2) We choose the gains of the controllers such that the work (24) were equivalent, that is,

$$W_a \approx W_b \approx W_c \approx W_d \tag{25}$$

where W_a is the work done using the relay controller plus gravity compensation, W_b is the work done using relay controller plus gravity pre-compensation, W_c is the work

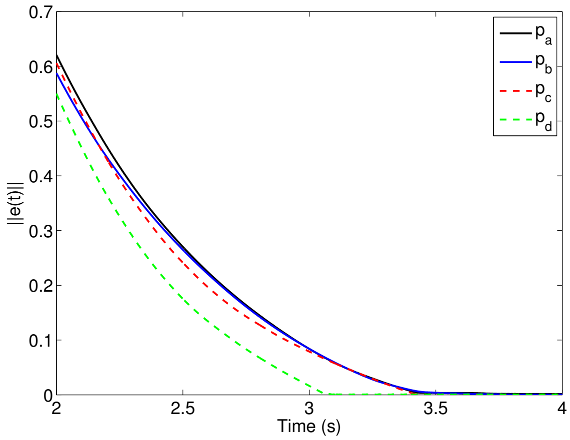

Fig. 3. Norm of the error under gains given in (23).

done using relay controller without gravity compensation, and W_d is the work done using relay controller plus integral action. The obtained gains are

a) Relay controller plus gravity compensation:

$$K_p = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 20 \end{bmatrix}, K_d = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}. \tag{26}$$

b) Relay controller plus gravity pre-compensation:

$$K_p = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix}, K_d = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}. \tag{27}$$

c) Relay controller without gravity compensation:

$$K_p = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 20 \end{bmatrix}, K_d = \begin{bmatrix} 5.65 & 0 & 0 \\ 0 & 5.65 & 0 \\ 0 & 0 & 5.65 \end{bmatrix}. \tag{28}$$

d) Relay controller plus integral action:

$$K_p = \begin{bmatrix} 7.7 & 0 & 0 \\ 0 & 7.7 & 0 \\ 0 & 0 & 20 \end{bmatrix}, K_d = \begin{bmatrix} 6.5 & 0 & 0 \\ 0 & 6.5 & 0 \\ 0 & 0 & 6.5 \end{bmatrix}, \\ K_I = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}. \tag{29}$$

The matrix gain K_s was selected as in (23) for the four controllers ((9), (12), (13), and (21)). Figure 5 corroborates the imposed criterium (25). It is concluded from Figure 6 that the norm of the error of the closed-loop system with controllers (12), (13), and (21) converge to the origin at the same time approximately ($t_s \approx 3.6$ s) while the norm of the error of the closed-loop system with relay controller plus gravity compensation (9) takes one more second to reach the origin approximately. Figure 7 shows the input control for the four controllers.

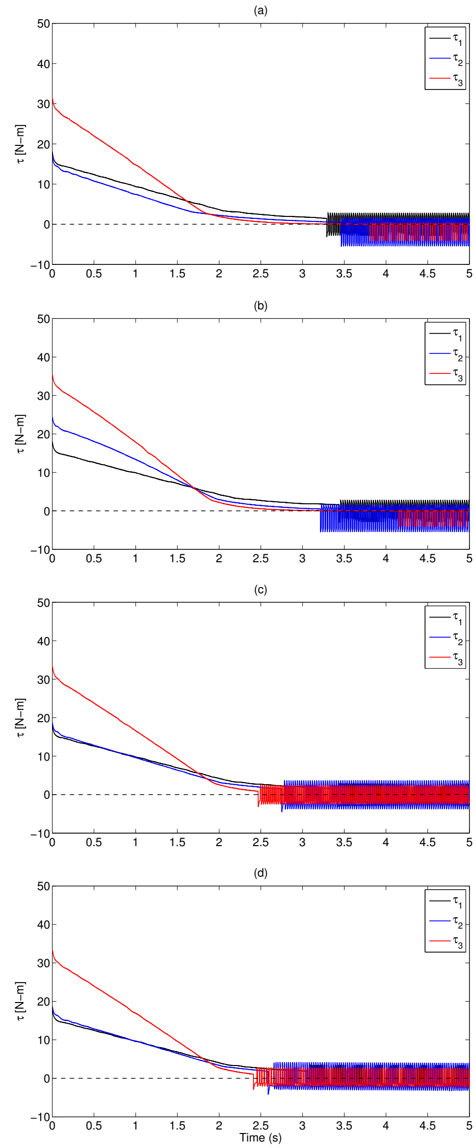

Fig. 4. Control inputs of (a) Relay controller + gravity compensation, (b) Relay controller + gravity pre-compensation (c) Relay controller without gravity compensation, (d) Relay controller with integral action; for experiments under same gains.

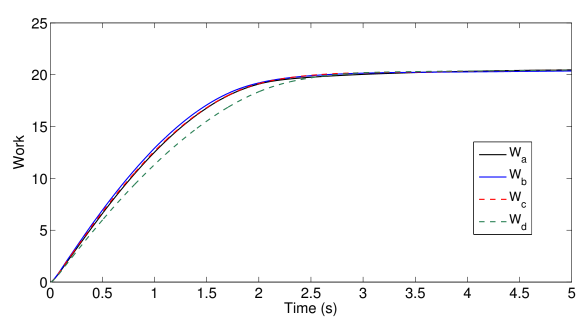

Fig. 5. Experimental work of each controller under gains given in (26)–(29): (W_a) Relay controller + gravity compensation, (W_b) Relay controller + gravity pre-compensation, (W_c) Relay controller without gravity compensation, and (W_d) Relay controller with integral action.

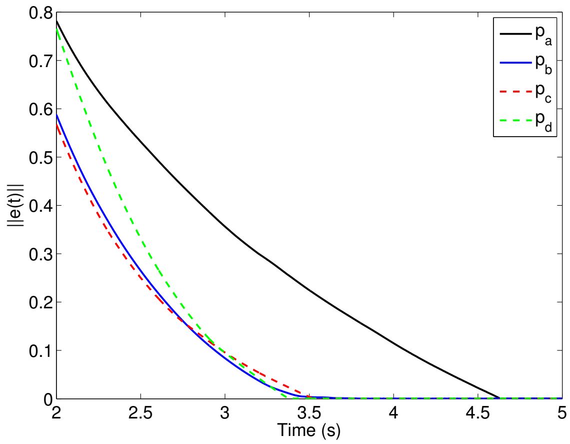

Fig. 6. Norm of the error under gains given in (26)–(29).

VIII. Conclusions

Global asymptotic stability and performance are analyzed for four exact position regulators of a robot manipulator with Coulomb friction. Analyzing the first set of experiments, one can see that all controllers reach the origin without steady-state error but the controller with integral action has fastest convergence while the rest of the controllers converge at the same time approximately but controller with full gravitational compensation does more work than other ones. For the second set of experiments one can also observe that all controllers, except the controller with gravitational compensation, converge at the same time doing the same work. Analyzing both set of experiments we conclude that relay feedback controller with integral action is a suitable option for position regulation of robot manipulators with friction while relay controller with gravitational compensation has worst performance. This conclusion can be interesting for engineers without experience in modelling since that controller with integral part does not need precise model parameters identification but its necessary to find a bound of the friction parameters, at least. Finite-time stability analysis of the studied controllers, except for the controller with gravity compensation (cf. [9]), is left for future work.

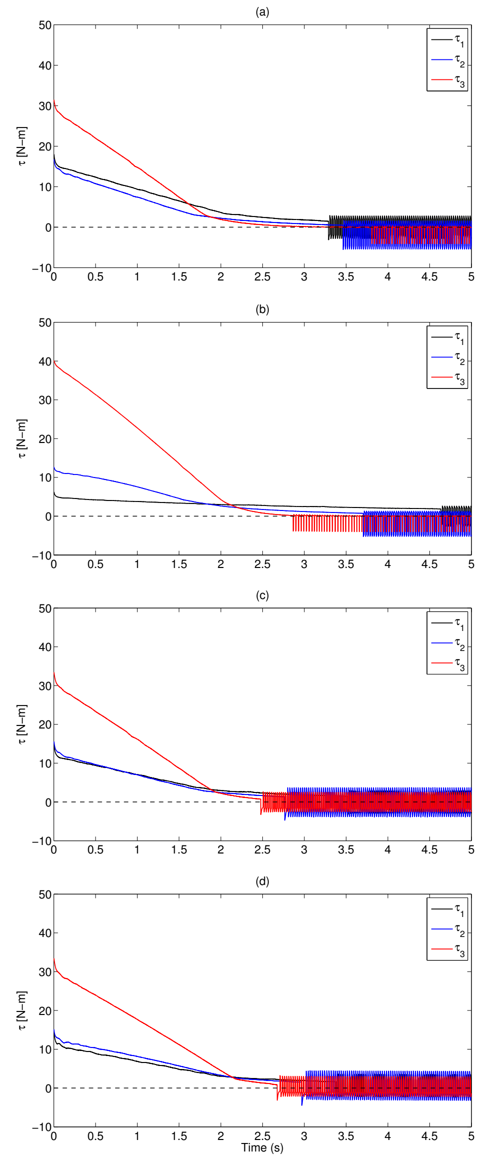

Fig. 7. Control inputs of (a) Relay controller + gravity compensation, (b) Relay controller + gravity pre-compensation (c) Relay controller without gravity compensation, (d) Relay controller with integral action; for experiments under similar work.

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