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Attitude synchronization of spacecraft formation with adaptation of consensus penalty terms

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***Abstract*— The main thrust of this work is on the time variation of the consensus weights used to enforce agreement (synchronization) amongst a network of spacecraft. The main idea is to allow a flexibility in the consensus gains to adapt in proportion to the Euclidean distance of the spacecraft states. A prelude to the time adaptation of the consensus controllers is also provided and takes the form of considering different gains in the synchronization signal thereby arriving in two cases: node-dependent and edge-dependent synchronization gains. A way to choose these fixed gains is in terms of the initial mismatch between the pairwise difference of the spacecraft states. Further, an adaptation of the synchronizing controllers is proposed and compared to the fixed gains. Extensive numerical studies are provided to further support the theoretical predictions and provide insights on the choice of consensus gains in synchronization control used for spacecraft formation.**

***Index Terms*— Attitude synchronization; spacecraft formation; gain adaptation.**

I. INTRODUCTION

The main focus of this work is on the time adaptation of the consensus gains in the attitude synchronization control. The problem at hand assumes that a network of N spacecraft can exchange information through an appropriate communication topology. The objective is to synchronize the spacecraft via the appropriate choice of the control inputs so that each spacecraft has identical orientation. This is quantified via the pairwise difference of the orientation states, chosen here as the Modified Rodrigues Parameters [1], [2].

The work follows earlier work [3], [4], with the difference that it does not parameterize the plant parameters and does not attempt to use adaptation of the parameters to design the control laws, but instead adapts the consensus (local) weights in the control signal. The basic idea behind the time variation of the consensus weights is that the weights used to penalize the pairwise mismatch of the orientation states amongst the spacecraft should in some sense be proportional to the difference between them. This approach of local gain adaptation was first explored in [5], [6], [7], [8] for general finite dimensional systems and in [9] for a class of infinite dimensional systems.

Extending the case of fixed-gains in the synchronization signal, we consider two cases in which the local weights of the synchronization signal can differ for each spacecraft (node-dependent gains) and differ for each spacecraft and for each of its communication neighbors (edge-dependent gains). Both cases of static gains are examined and an algorithm for choosing these gains in proportion to the initial pairwise difference of spacecraft states is presented.

The case of adaptive adjustment of the gains leads to combined control and adaptation laws. The proposed modification assumes that the synchronization signal contains a fixed gain but the torque inputs are weighted by a local adaptive gain. Using Lyapunov redesign methods, the adaptation laws for the local gains are derived and the synchronization objective can subsequently be established.

We first formulate the problem and state the synchronization objective in the next section. In Section III, we present the results on the fixed gains and summarize the stability and convergence properties of the resulting closed-loop systems. Additionally, an algorithm for choosing the consensus gains is provided. The proposed adaptive scheme is presented in Section IV along with the stability and convergence results. Simulation studies are then presented in Section V and conclusions follow in Section VI.

II. PROBLEM FORMULATION

Standard notation is used throughout this manuscript. Since the background material on graph theory is by now considered standard, we provide a summary with details presented in [10], [11]. We consider both a directed and undirected graph $\mathcal{G}$ since the gain adaptation immediately removes coupling due to any information on communication topology at the local level. To facilitate the convergence of the adaptive scheme, it is implicitly assumed that when a direct graph is a considered then the communication digraph is fixed and has a spanning tree. We assume N spacecraft with the set of neighbors of a given agent (spacecraft) denoted by N_i. The graph adjacency matrix is denoted by $\mathcal{A}$ and the degree matrix is denoted by $\mathcal{D}$. The graph Laplacian is then given by $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

We consider a network of N spacecraft, having identical dynamics given by

$$\mathbf{J}\dot{\boldsymbol{\omega}} - (\mathbf{J}\boldsymbol{\omega}) \times \boldsymbol{\omega} = \mathbf{u} + \mathbf{d},$$

where $\mathbf{J}$ denotes the inertia matrix and the signals $\mathbf{u} \in \mathbb{R}^3$ and $\mathbf{d} \in \mathbb{R}^3$ denote the control and disturbance torques, respectively. Associated with the above Euler equations of motion are the appropriate orientation states. A set of rigid body orientations that are conducive to transformations that would render the above equations in a form more amenable to parametrization and attitude synchronization are the Modified Rodriques Parameters [1]. The attitude vector $\mathbf{q} \in \mathbb{R}^3$ is related to the angular velocity vector via

$$\dot{\mathbf{q}} = Z(\mathbf{q})\boldsymbol{\omega},$$

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where

$$Z(\mathbf{q}) = \frac{1}{2}\left(\frac{1}{2}(1-\mathbf{q}^T\mathbf{q})\mathbf{I}_3 + \mathbf{q}\mathbf{q}^T + [\mathbf{q}^\times]\right).$$

Using the Modified Rodriques Parameters, one arrives at the Euler-Lagrange equation for each spacecraft

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i = \boldsymbol{\tau}_i + \boldsymbol{\tau}_{ext,i}, \tag{1}$$

for $i = 1,\dots,N$, where

$$\begin{aligned} &\boldsymbol{\tau}_i = Z^{-T}(\mathbf{q}_i)\mathbf{u}_i, \quad \boldsymbol{\tau}_{ext,i} = Z^{-T}(\mathbf{q}_i)\mathbf{d}_i, \\ &\mathbf{M}_i(\mathbf{q}_i) = Z^{-T}(\mathbf{q}_i)\mathbf{J}Z^{-1}(\mathbf{q}_i), \\ &\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) = -Z^{-T}(\mathbf{q}_i)\mathbf{J}Z^{-1}(\mathbf{q}_i)\dot{Z}(\mathbf{q}_i)Z^{-1}(\mathbf{q}_i) \\ &\qquad\qquad -Z^{-T}(\mathbf{q}_i)[(\mathbf{J}_i\omega_i)^\times]Z^{-1}(\mathbf{q}_i). \end{aligned}$$

The above formulation enjoys several fundamental properties, which we summarize below.

Properties:

- The inertia matrix $\mathbf{M}_i(\mathbf{q}_i)$ is lower and upper bounded.
- The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ is skew symmetric.
- The term $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i$ is norm bounded by $|\dot{\mathbf{q}}_i|^2$

Notation simplification: We henceforth make the following simplification:

$$\mathbf{M}_i \equiv \mathbf{M}_i(\mathbf{q}_i), \quad \mathbf{C}_i \equiv \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$$

which drops the dependence of the matrices $\mathbf{M}_i$ and $\mathbf{C}_i$ on their arguments. Additionally, we drop the explicit dependence of the state vectors (position and velocity) on time. However, time dependence will be included for the adaptive gains in order to distinguish from the constant gains. Finally, we define the pairwise difference as

$$\mathbf{q}_{ij} = \mathbf{q}_j - \mathbf{q}_i, \quad j \in N_i, i = 1,\dots,N.$$

We can now state the *synchronization objective*. The objective is to design control torques $\boldsymbol{\tau}_i$ that regulate the angular velocities of each spacecraft and synchronize the spacecraft attitudes in the following sense

$$\begin{aligned} &\lim_{t\to\infty}|\mathbf{q}_i(t) - \mathbf{q}_j(t)| = 0 \\ &\lim_{t\to\infty}|\boldsymbol{\omega}_i(t)| = 0, \end{aligned} \quad \forall j \in N_i, \forall i = 1,\dots,N.$$

For the current work, we assume that there are no parametric uncertainties and that the external torques $\boldsymbol{\tau}_{ext,i}$ are known, This then does not require the adaptive parameter estimation considered in [3]. Additionally, it is assumed that each spacecraft knows its own state and has access to the states $(\mathbf{q}_j, \dot{\mathbf{q}}_{ji})$, $j \in N_i$ of its neighbors.

III. Node-dependent and edge-dependent fixed synchronization gains

In order to provide additional flexibility in the weighting of the penalty terms that enforce synchronization, the synchronization gains are allowed to differ for each spacecraft. Possible modifications include a *node-dependent* gain in which each spacecraft has one gain that differs from the synchronization gain of all other spacecraft. The other modification considers an *edge-dependent* gain where each pairwise difference of the spacecraft states has its own gain.

Using the *node-dependent* modification, the synchronization signal is

$$\mathbf{v}_i(t) \triangleq \dot{\mathbf{q}}_i(t) - \mu_i \sum_{j\in N_i}(\mathbf{q}_j(t) - \mathbf{q}_i(t)), \tag{2}$$

and using the *edge-dependent* modification, the synchronization signal is

$$\mathbf{w}_i(t) \triangleq \dot{\mathbf{q}}_i(t) - \sum_{j\in N_i}\mu_{ij}(\mathbf{q}_j(t) - \mathbf{q}_i(t)). \tag{3}$$

The former uniformly penalizes all pairwise differences between the state of the ith spacecraft and its neighbors $(\mathbf{q}_j(t) - \mathbf{q}_i(t))$, $\forall j \in N_i$ using a common gain, while the latter has a different gain for each pairwise difference between the state of the ith spacecraft and its neighbors $(\mathbf{q}_j(t) - \mathbf{q}_i(t))$, $\forall j \in N_i$.

Remark 1: One can easily observe that the node-dependent modification is a special case of edge-dependent modification with $\mu_{ij} = \mu_i$, $\forall j \in N_i$.

We summarize the results for the edge-dependent modification and provide an algorithm for choosing the gains μ_{ij}. Using the control law

$$\boldsymbol{\tau}_i = \sum_{j\in N_i}\mu_{ij}(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}) - \boldsymbol{\tau}_{ext,i} + \overline{\boldsymbol{\tau}}_i \tag{4}$$

in (1), the closed-loop system is given by

$$\mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{C}_i\dot{\mathbf{q}}_i = \overline{\boldsymbol{\tau}}_i + \sum_{j\in N_i}\mu_{ij}(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}).$$

Using the definition of the synchronization signal (3), the above closed-loop equation can be expressed in terms of $\mathbf{w}_i$

$$\mathbf{M}_i\dot{\mathbf{w}}_i + \mathbf{C}_i\mathbf{w}_i = \overline{\boldsymbol{\tau}}_i. \tag{5}$$

A Lyapunov-based control design for (5) would provide the expression for $\overline{\boldsymbol{\tau}}_i$. Using the fact that $\dot{\mathbf{M}}_i - 2\mathbf{C}_i$ is skew symmetric, then the following Lyapunov function

$$V_i(\mathbf{w}_i) = \frac{1}{2}\mathbf{w}_i^T\mathbf{M}_i\mathbf{w}_i \tag{6}$$

has a derivative along the trajectories of (5) given by

$$\dot{V} = \sum_{i=1}^N \dot{V}_i(\mathbf{w}_i) = \sum_{i=1}^N \mathbf{w}_i^T\overline{\boldsymbol{\tau}}_i.$$

The choice $\overline{\boldsymbol{\tau}}_i = -\mathbf{K}_i\mathbf{w}_i$ with $\mathbf{K}_i = \mathbf{K}_i^T > 0$, results in

$$\dot{V} = -\sum_{i=1}^N \mathbf{w}_i^T\mathbf{K}_i\mathbf{w}_i < 0.$$

One immediately has $\lim_{t\to\infty}|\mathbf{w}_i| = 0$, $i = 1,\dots,N$. In fact, the convergence is exponential. Re-writing (3), one has

$$\dot{\mathbf{q}}_i = \sum_{j\in N_i}\mu_{ij}\mathbf{q}_{ij} + \mathbf{w}_i$$

or in vector form

$$\dot{\mathbf{q}} = -(\mathcal{N}\otimes\mathbf{I}_3)\mathbf{q} + \mathbf{I}_{3\times N}\otimes\mathbf{w},$$

where $\mathcal{N}$ denotes the graph Laplacian weighted by the weights μ_{ij} and defined via

$$\mathcal{N} = \mathcal{D}_\mu - \mathcal{A}_\mu, \qquad [\mathcal{A}_\mu]_{ij} = \mu_{ij}[\mathcal{A}]_{ij}, \;\; j \in N_i,$$
$$[\mathcal{D}_\mu]_{ii} = \sum_{j \in N_i} \mu_{ij}, \quad [\mathcal{D}_\mu]_{ij} = 0, \;\; \forall j \neq i,$$

and the concatenated vectors $\mathbf{q}, \mathbf{w} \in \mathbb{R}^{3N}$ are given by

$$\mathbf{w}^T = \left[\begin{array}{cccc} \mathbf{w}_1^T & \mathbf{w}_2^T & \dots & \mathbf{w}_N^T \end{array} \right],$$
$$\mathbf{q}^T = \left[\begin{array}{cccc} \mathbf{q}_1^T & \mathbf{q}_2^T & \dots & \mathbf{q}_N^T \end{array} \right].$$

Using the fact that $\mu_{ij} > 0$, then following [3], one has $|\mathbf{q}_i(t) - \mathbf{q}_j(t)| \to 0$ as $t \to \infty$. The boundedness of $\mathbf{q}_i$ and $\dot{\mathbf{q}}_i$ along with $\dot{\mathbf{q}}_i = Z(\mathbf{q}_i)\boldsymbol{\omega}_i$ immediately yield

$$\lim_{t \to \infty} |\boldsymbol{\omega}_i(t)| = 0, \quad i = 1, \dots, N$$

and therefore the attitude synchronization objective is attained.

The closed loop systems (5) along with the synchronization signals can be written in a compact form

$$\left\{ \begin{array}{l} \mathbb{M}\dot{\mathbf{w}} + \mathbb{C}\mathbf{w} = \mathbb{T}, \\ \dot{\mathbf{q}} = -(\mathcal{N} \otimes \mathbf{I}_3)\mathbf{q} + \mathbf{I}_{3\times N} \otimes \mathbf{w}, \end{array} \right. \tag{7}$$

where $\mathbb{T} = -\mathbb{K}\mathbf{w}$ and

$$\mathbb{M} = \text{diag}\,\{\mathbf{M}_1, \dots, \mathbf{M}_N\},$$
$$\mathbb{C} = \text{diag}\,\{\mathbf{C}_1, \dots, \mathbf{C}_N\},$$
$$\mathbb{K} = \text{diag}\,\{\mathbf{K}_1, \dots, \mathbf{K}_N\},$$

with $\dot{\mathbb{M}} - 2\mathbb{C} \in \mathbb{R}^{3N \times 3N}$ a skew symmetric matrix. This formulation can be used for the optimization of the consensus gains comprising the entries of $\mathcal{N}$

A. Guidelines for choosing the edge-dependent gains

When one considers fixed gains μ_{ij}, $j \in N_i$ in (3), a good choice of these gains for a given i should be proportional to the initial distance of $\mathbf{q}_i(0)$ from its neighbors $\mathbf{q}_j(0)$, $j \in N_i$. Therefore, the μ_{ij} in (3) can be chosen as

$$\mu_{ij} = |\mathbf{q}_i(0) - \mathbf{q}_j(0)|, \quad j \in N_i, \quad i = 1, \dots, N, \tag{8}$$

resulting in the closed-loop systems

$$(\Sigma_1) \left\{ \begin{array}{l} \mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{C}_i\dot{\mathbf{q}}_i = \boldsymbol{\tau}_i + \boldsymbol{\tau}_{ext,i}, \\ \boldsymbol{\tau}_i = \sum_{j \in N_i} \mu_{ij} \left(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\right) - \boldsymbol{\tau}_{ext,i} + \overline{\boldsymbol{\tau}}_i, \\ \mu_{ij} = |\mathbf{q}_i(0) - \mathbf{q}_j(0)|, \quad j \in N_i, \\ \overline{\boldsymbol{\tau}}_i = -\mathbf{K}_i\mathbf{w}_i, \;\; \mathbf{K}_i = \mathbf{K}_i^T > 0, \\ \mathbf{w}_i(t) = \dot{\mathbf{q}}_i(t) - \sum_{j \in N_i} \mu_{ij}\mathbf{q}_{ij}(t), \end{array} \right.$$

and the μ_i in (2) as

$$\mu_i = \sum_{j \in N_i} |\mathbf{q}_i(0) - \mathbf{q}_j(0)|, \quad i = 1, \dots, N. \tag{9}$$

Remark 2: The above can easily be adapted for the special case of node-dependent modification of the synchronizing signal (2) with $\mu_i = \mu_{ij}$, $j \in N_i$. For example, the control torque from (4) is now given by

$$\boldsymbol{\tau}_i = \mu_i \sum_{j \in N_i} \left(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\right) - \boldsymbol{\tau}_{ext,i} + \overline{\boldsymbol{\tau}}_i$$

with $\overline{\boldsymbol{\tau}}_i = -\mathbf{K}_i\mathbf{v}_i$. Finally, the synchronizing signals in (2) written in vector form are

$$\dot{\mathbf{q}} = -(\mathcal{M} \otimes \mathbf{I}_3)\mathbf{q} + \mathbf{I}_{3\times N} \otimes \mathbf{v}$$

where $\mathcal{M}$ denotes the graph Laplacian weighted by the weights μ_i and defined via

$$\mathcal{M} = \mathcal{D}_\mu - \mathcal{A}_\mu, \qquad [\mathcal{A}_\mu]_{ij} = \mu_i[\mathcal{A}]_{ij}, \;\; j \in N_i$$
$$[\mathcal{D}_\mu]_{ii} = \sum_{j \in N_i} \mu_i, \qquad [\mathcal{D}_\mu]_{ij} = 0, \;\; \forall j \neq i.$$

A compact form similar to (7) can easily be written with the expression for the synchronizing signal $\mathbf{w}$ replaced by $\mathbf{v}$ and the modified Laplacian $\mathcal{N}$ replaced by $\mathcal{M}$.

IV. Adaptation of synchronization penalty terms

The proposed control and adaptation schemes is considered here; it builds on the work in [3] and attempts to adapt the consensus gains relative to some idealized fixed gain.

A. Adaptation of consensus weights: node-dependent case

To aid with the Lyapunov-based stability arguments, we define the synchronization signal

$$\mathbf{x}_i(t) \triangleq \dot{\mathbf{q}}_i(t) - \lambda_i^* \sum_{j \in N_i} \mathbf{q}_{ij}(t) \tag{10}$$

where $\lambda_i^* > 0$, $i = 1 \dots, N$ are the fixed synchronization gains. These gains are known and user-defined, and can be chosen according to (9). The above case includes the special case of uniform gains $\lambda_i^* = \lambda^*$.

The time derivative of (10) is given by

$$\dot{\mathbf{x}}_i(t) \triangleq \ddot{\mathbf{q}}_i(t) - \lambda_i^* \sum_{j \in N_i} \dot{\mathbf{q}}_{ij}(t). \tag{11}$$

The control law in this case is an extension to (4) and now includes *adaptation of the consensus weights*

$$\boldsymbol{\tau}_i = \lambda_i(t) \sum_{j \in N_i} \Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \boldsymbol{\tau}_{ext,i} + \overline{\boldsymbol{\tau}}_i,$$

where $\lambda_i(t)$, $i = 1, \dots, N$ denote the adaptive consensus gains. The above control law is re-written in order to include terms related to $\mathbf{x}_i$ and its derivative in (11) as follows

$$\begin{aligned} \boldsymbol{\tau}_i = \lambda_i^* \sum_{j \in N_i} \Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \boldsymbol{\tau}_{ext,i} + \overline{\boldsymbol{\tau}}_i \\ + \widetilde{\lambda}_i(t) \sum_{j \in N_i} \Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big), \end{aligned} \tag{12}$$

where $\widetilde{\lambda}_i(t) \triangleq \lambda_i(t) - \lambda_i^*$, $i = 1, \dots, N$ is the parameter error. When the above torque is applied to the system (1), it results in the closed loop system

$$\mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{C}_i\dot{\mathbf{q}}_i = \lambda_i^* \sum_{j\in N_i}\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) + \widetilde{\lambda}_i(t)\sum_{j\in N_i}\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) + \overline{\boldsymbol{\tau}}_i.$$

Using the definition of the synchronization signal (10) and its time derivative (11), the closed-loop system is (*cf.* (5))

$$\mathbf{M}_i\dot{\mathbf{x}}_i + \mathbf{C}_i\mathbf{x}_i = \widetilde{\lambda}_i(t)\sum_{j\in N_i}\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) + \overline{\boldsymbol{\tau}}_i. \quad (13)$$

The adaptation of the gains $\lambda_i(t)$, generated via an application of Lyapunov redesign methods to a Lyapunov function [12], is given by

$$\dot{\lambda}_i(t) = \dot{\widetilde{\lambda}}_i(t) = -\gamma_i\mathbf{x}_i^T\sum_{j\in N_i}\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big),$$

where $\gamma_i > 0$ are the adaptive gains [12] and whose role is to speed up adaptation of $\lambda_i(t)$. It should be noted that the above adaptive law is feasible since the fixed gains λ_i^* are known.

A possible modification which takes the form of a diffusion term, takes advantage of the knowledge of the value of λ_i^* to produce

$$\dot{\widetilde{\lambda}}_i(t) = -\gamma_i\mathbf{x}_i^T\sum_{j\in N_i}\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \widetilde{\lambda}_i(t). \quad (14)$$

The following lemma examines the stability of the closed-loop system (13) using the proposed control law (12) and adaptive law (14).

Lemma 1 (node-dependent gain): Consider the closed loop system

$$(\Sigma_2)\begin{cases} \mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{C}_i\dot{\mathbf{q}}_i = \boldsymbol{\tau}_i + \boldsymbol{\tau}_{ext,i}, \\ \boldsymbol{\tau}_i = \lambda_i(t)\sum_{j\in N_i}\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \boldsymbol{\tau}_{ext,i} + \overline{\boldsymbol{\tau}}_i, \\ \dot{\lambda}_i(t) = -\gamma_i\mathbf{x}_i^T\sum_{j\in N_i}\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \widetilde{\lambda}_i(t), \\ \overline{\boldsymbol{\tau}}_i = -\mathbf{K}_i\mathbf{x}_i, \quad \mathbf{K}_i = \mathbf{K}_i^T > 0, \\ \mathbf{x}_i(t) = \dot{\mathbf{q}}_i(t) - \lambda_i^*\sum_{j\in N_i}\mathbf{q}_{ij}(t), \end{cases}$$

which uses the proposed adaptation of the consensus weights. Then all signals are bounded and the attitude synchronization is achieved.

Proof. The proof is given in Appendix I.

Remark 3: The adaptive law (13) is made feasible due to the knowledge of the fixed gains $\lambda_i^* > 0$, which are design parameters. They can be chosen using the guidelines given by (9) in Section III. The convergence of $\widetilde{\lambda}_i(t)$ to zero is exponential, yielding $\lim_{t\to\infty}\lambda_i(t) = \lambda_i^*$. While the convergence is exponential, the choice of the time-variation of $\widetilde{\lambda}_i(t)$ through the adaptation (14) is made in order to enhance the transient response of the synchronization signals. Even in the event that the diffusion term is not included in (14), one can still obtain $|\mathbf{q}_i - \mathbf{q}_j| \to 0$ but without the knowledge of an explicit convergence limit of $\lambda_i(t)$.

B. Adaptation of consensus weights: edge-dependent case

In this case, the synchronization signal is defined (*cf.* (3))

$$\mathbf{y}_i(t) = \dot{\mathbf{q}}_i(t) - \sum_{j\in N_i}\lambda_{ij}^*\mathbf{q}_{ij}(t)$$

with a corresponding derivative given by

$$\dot{\mathbf{y}}_i(t) = \ddot{\mathbf{q}}_i(t) - \sum_{j\in N_i}\lambda_{ij}^*\dot{\mathbf{q}}_{ij}(t).$$

The control law is similarly given by

$$\boldsymbol{\tau}_i = \sum_{j\in N_i}\lambda_{ij}^*\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \boldsymbol{\tau}_{ext,i} + \overline{\boldsymbol{\tau}}_i + \sum_{j\in N_i}\widetilde{\lambda}_{ij}(t)\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big),$$

with the closed loop system becoming

$$\mathbf{M}_i\dot{\mathbf{y}}_i + \mathbf{C}_i\mathbf{y}_i = \sum_{j\in N_i}\widetilde{\lambda}_{ij}(t)\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) + \overline{\boldsymbol{\tau}}_i.$$

The edge-dependent gain adaptation is given by

$$\dot{\widetilde{\lambda}}_{ij}(t) = -\gamma_{ij}\mathbf{y}_i^T\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \widetilde{\lambda}_{ij}(t).$$

The stability and convergence properties of the closed-loop system are similar to the node-dependent case and are simply stated.

Lemma 2 (edge-dependent gain): Consider the closed loop system

$$(\Sigma_3)\begin{cases} \mathbf{M}_i\ddot{\mathbf{q}}_i + \mathbf{C}_i\dot{\mathbf{q}}_i = \boldsymbol{\tau}_i + \boldsymbol{\tau}_{ext,i}, \\ \boldsymbol{\tau}_i = \sum_{j\in N_i}\lambda_{ij}(t)\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \boldsymbol{\tau}_{ext,i} + \overline{\boldsymbol{\tau}}_i, \\ \dot{\lambda}_{ij}(t) = -\gamma_{ij}\mathbf{y}_i^T\Big(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}\Big) - \widetilde{\lambda}_{ij}(t), \\ \overline{\boldsymbol{\tau}}_i = -\mathbf{K}_i\mathbf{y}_i, \quad \mathbf{K}_i = \mathbf{K}_i^T > 0, \\ \mathbf{y}_i(t) = \dot{\mathbf{q}}_i(t) - \sum_{j\in N_i}\lambda_{ij}^*\mathbf{q}_{ij}(t), \end{cases}$$

which uses the proposed adaptation of the consensus weights. Then all signals are bounded and the attitude synchronization is achieved.

Proof. The proof is similar to the one for Lemma 1 and is therefore omitted due to space limitations.

V. Numerical Studies

We consider $N = 4$ spacecraft with a communication topology described via a directed graph depicted in Figure 1. For simplicity, we used the system parameters considered in [3]. The inertia matrices of the four spacecraft were chosen as $J_1 = \mathrm{diag}(17, 12, 9)$, $J_2 = \mathrm{diag}(14, 13, 10)$, $J_3 = \mathrm{diag}(20, 10, 9)$ and $J_4 = \mathrm{diag}(15, 9, 16)$. For simplicity, the external disturbances are assumed zero.

In the first part of the numerical studies, examine the the effects of a node-dependent gain as summarized in (9).

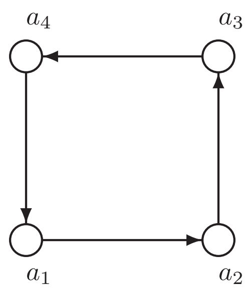

Fig. 1. Communication topology (directed graph on 4 vertices).

In the second study we implement the adaptation of the synchronizing signals via the gain $\lambda_i(t)$ as presented in (14).

The connectivity of the 4 systems is represented by the directed graph of Figure 1 with the corresponding graph Laplacian given by

$$\mathcal{L}(\mathcal{G}) = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

An appropriate measure for the synchronization, that is independent of the graph topology is the deviation-from-the-mean, defined via

$$\delta_i(t) = \mathbf{q}_i(t) - \frac{1}{N}\sum_{j=1}^{N} \mathbf{q}_j(t).$$

The cumulative measure is taken as the Euclidean norm of the aggregate deviations-from-the-mean

$$\|\Delta(t)\| = \sqrt{\sum_{i=1}^{N} |\delta_i(t)|^2}$$

A. *Effect of node-dependent synchronization gains*

Using the following initial conditions

$$\mathbf{q}_1 = \begin{bmatrix} 3.0 & 2.0 & 1.0 \end{bmatrix}^T, \mathbf{q}_2 = \begin{bmatrix} 3.2 & 2.4 & 1.9 \end{bmatrix}^T,$$
$$\mathbf{q}_3 = \begin{bmatrix} 2.7 & 1.2 & 2.9 \end{bmatrix}^T, \mathbf{q}_4 = \begin{bmatrix} 5.1 & 4.2 & 4.0 \end{bmatrix}^T,$$

the gains μ_i were chosen as

$$\begin{aligned} \mu_1 &= |\mathbf{q}_1(0) - \mathbf{q}_4(0)| = 4.2720, \\ \mu_2 &= |\mathbf{q}_2(0) - \mathbf{q}_1(0)| = 1.0050, \\ \mu_3 &= |\mathbf{q}_3(0) - \mathbf{q}_2(0)| = 1.6401, \\ \mu_4 &= |\mathbf{q}_4(0) - \mathbf{q}_3(0)| = 3.9962. \end{aligned}$$

The cumulative deviation from the mean using the above variable weights is depicted in Figure 2. Additionally, it includes the case of uniformly fixed weights (or gains) $\mu_i = 0.2$, $i = 1, \ldots, 4$.

One can easily observe the effects of a node-dependent gain versus a uniform gain on the measure of agreement. While further optimization on the choice of the gains μ_i is warranted and is the topic of a forthcoming publication by the authors, the numerical studies reveal that when the gains are chosen in proportion to the initial mismatch between the spacecraft states, an improvement over the gain that is uniformly chosen for all spacecraft is observed. The $L_2(0, 10)$ norm of $\|\Delta(t)\|$ presented in Table I, further supports the improvement of node-dependent gain versus a uniform gain in $\Delta(t)$.

case	$L_2(0, 10)$ norm of $\|\Delta(t)\|$
μ_i given by (9)	1.85371
uniform μ	5.14726

TABLE I
$L_2(0, 10)$ NORM OF $\|\Delta(t)\|$.

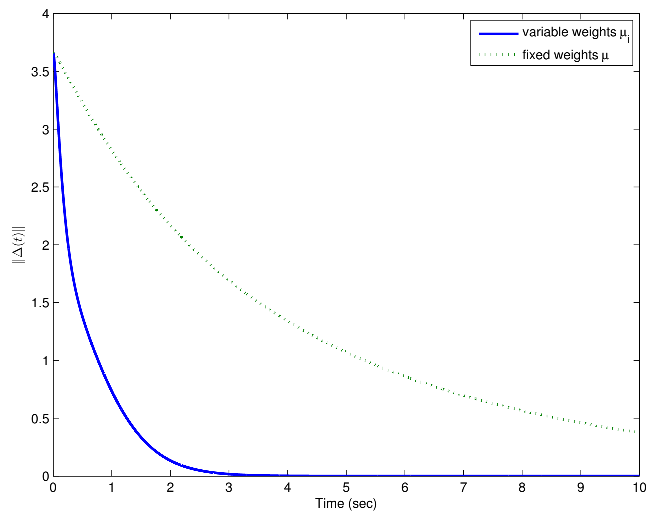

Fig. 2. Evolution of the norm of the deviation-from-the-mean $\Delta(t)$.

B. *Adaptation of synchronization signal*

We consider the adaptation of the synchronizing gain $\lambda_i(t)$ via (14). We compare the cumulative deviation-from-the-mean $\Delta(t)$ for the time-varying gains of the node-dependent case with the fixed case $\lambda_i = 5$, $i = 1, \ldots, 4$. The deviation is depicted in Figure 3. The use of adaptive gains resulted in an improvement of the transient response of $\Delta(t)$. The $L_2(0, 10)$ norm of $\|\Delta(t)\|$ is also presented in Table II, further pointing to the transient improvement of $\Delta(t)$ when gain adaptation is implemented.

case	$L_2(0, 10)$ norm of $\|\Delta(t)\|$
adaptive	1.29409
fixed	1.66425

TABLE II
$L_2(0, 10)$ NORM OF $\|\Delta(t)\|$.

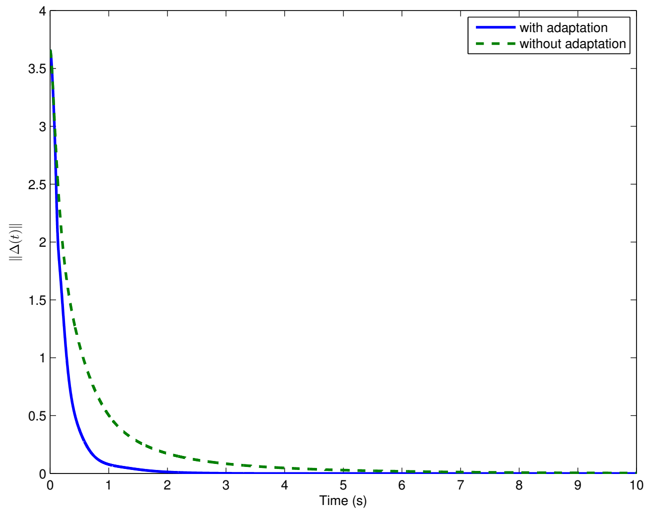

Fig. 3. Evolution of the norm of the deviation-from-the-mean $\Delta(t)$.

VI. Conclusions

This work considered two modifications in the control architecture for spacecraft attitude synchronization. The first one examined the use of different consensus gains enforcing consensus by penalizing the distance between the states of different spacecraft. The second one provided an adaptation of the consensus gains in order to improve the transient response of the deviation-from-the-mean consensus measure. The choice of such a measure to examine the agreement of the spacecraft is independent of the communication topology making it the ideal choice for examining the success of synchronization controllers.

A natural extension of the above case is the optimization of the consensus gains μ_{ij} in (7). In this case, the cost functional should penalize both the deviation from the mean and rotational kinetic energy and must provide a balance between fast convergence of $\|\Delta(t)\|$ to zero while maintaining acceptable values of the kinetic energy. Such an optimization will provide the optimal μ_{ij} via

$$\mu_{ij} = \arg\min \int_0^{t_f} \left(\|\Delta(\tau)\|^2 + \sum_{i=1}^N \boldsymbol{\omega}_i^T(\tau)\mathbf{J}_i\boldsymbol{\omega}_i(\tau)\right) d\tau$$

subject to the dynamics (7). The above can also be used as the choice of the λ_i^* in (10) for the adaptive case. Both extensions are currently being examined by the authors and will appear in a forthcoming publication.

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Appendix I
Proof of Lemma 1

Proof: We consider the Lyapunov-like function

$$V = \sum_{i=1}^N V_i(\mathbf{x}_i, \widetilde{\lambda}_i) = \sum_{i=1}^N \left(\frac{1}{2}\mathbf{x}_i^T\mathbf{M}_i\mathbf{x}_i + \frac{1}{2\gamma_i}\widetilde{\lambda}_i^2\right)$$

where $\gamma_i > 0$ denote the adaptive gains. The derivative of V_i along the trajectories of (13) is given by

$$\begin{aligned}
\dot{V}_i(\mathbf{x}_i, \widetilde{\lambda}_i) &= \frac{1}{2}\mathbf{x}_i^T\dot{\mathbf{M}}_i\mathbf{x}_i + \mathbf{x}_i^T\mathbf{M}_i\dot{\mathbf{x}}_i + \widetilde{\lambda}_i\frac{\dot{\widetilde{\lambda}}_i}{\gamma_i} \\
&= \frac{1}{2}\mathbf{x}_i^T\dot{\mathbf{M}}_i\mathbf{x}_i + \widetilde{\lambda}_i\frac{\dot{\widetilde{\lambda}}_i}{\gamma_i} + \mathbf{x}_i^T\overline{\boldsymbol{\tau}}_i \\
&\qquad + \mathbf{x}_i^T\Big(-\mathbf{C}_i\mathbf{x}_i + \widetilde{\lambda}_i\sum_{j\in N_i}(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_{ij}\mathbf{q}_i)\Big) \\
&= \frac{1}{2}\mathbf{x}_i^T\Big(\dot{\mathbf{M}}_i - 2\mathbf{C}_i\Big)\mathbf{x}_i + \mathbf{x}_i^T\overline{\boldsymbol{\tau}}_i \\
&\qquad + \widetilde{\lambda}_i\Big(\mathbf{x}_i^T\sum_{j\in N_i}(\mathbf{M}_i\dot{\mathbf{q}}_{ij} + \mathbf{C}_i\mathbf{q}_{ij}) + \frac{\dot{\widetilde{\lambda}}_i}{\gamma_i}\Big).
\end{aligned}$$

Using the adaptive law (14) and $\overline{\boldsymbol{\tau}}_i = -\mathbf{K}_i\mathbf{x}_i$ along with the property of skew symmetry of $\dot{\mathbf{M}}_i - 2\mathbf{C}_i$, one arrives at

$$\sum_{i=1}^N \dot{V}_i(\mathbf{x}_i, \widetilde{\lambda}_i) = -\sum_{i=1}^N \left(\mathbf{x}_i^T\mathbf{K}_i\mathbf{x}_i + \frac{\widetilde{\lambda}_i^2}{\gamma_i}\right) < 0.$$

As a consequence, one has $|\mathbf{x}_i| \to 0$ as $t \to \infty$ and with the modified adaptation (14), one also has $\lambda_i(t) \to \lambda_i^*$ as $t \to \infty$. Examining the definition of the synchronizing signal and implicitly assuming that the graph contains a spanning tree, one can then follow the arguments in [3] to conclude

$$\lim_{t\to\infty} |\mathbf{q}_i(t) - \mathbf{q}_j(t)| = 0, \text{ and } \lim_{t\to\infty} |\boldsymbol{\omega}_i(t)| = 0.$$

■