2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

A statistically robust payment sharing mechanism for an aggregate of renewable energy producers

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***Abstract*— Variability of supply is a fundamental difficulty associated with renewable resources in the electricity market. One way of mitigating this difficulty is to aggregate a diverse collection of resources in order to exploit the negative correlations that may exist among them. We consider an aggregation scheme where individual renewable energy producers offer day-ahead contracts to an aggregate manager which in turn participates in a two stage electricity market. The net payment received by the aggregate manager from the market has to be fairly distributed among the participants in the aggregate. Since the actual power supplied by the aggregate is random, the net payment it receives is also random. The problem of sharing this random payment is complicated by the fact that different participants may have different statistical models for the payment because they have different statistical models for their and other producers' net generation. We propose a simple payment sharing mechanism that is independent of the statistical models of the participants. We show that our payment sharing mechanism ensures that individual producers are better off in the aggregate than on their own. Further, under certain conditions, aggregation provides the social benefit of increasing the amount of renewable energy available in the day-ahead market.**

I. INTRODUCTION

The goal of reducing carbon emissions has led to a global drive to increase the utilization of renewable energy resources such as wind and solar energy in the electricity market. However, unlike the traditional sources of electricity such as coal and natural gas, renewable resources are uncontrollable and unpredictable. Natural variations in wind speed and insolation result in a time-varying supply of electric power from renewable resources. Moreover, these variations are difficult to forecast. The variability in the supply of renewable power is one of the main challenges of integrating renewable resources into the electricity market [1], [2].

One way of mitigating variability is to aggregate a diverse collection of renewable resources in order to exploit the negative correlations that may exist among different renewable resources. For example, aggregating wind power resources over a large geographic area can reduce the overall variability of wind power [3]. The aggregate of renewable resources can offer a joint contract to the system operator and share the payments received from the system operator. The key question in such aggregates is the following: how should the payment from the system operator be fairly distributed among the participants in the aggregate?

Since the actual power supplied by the aggregate is random, the payment it receives from the system operator is also random. If all the renewable energy producers in the aggregate have a common statistical model for the random amount of power they can jointly provide, then a coalition game theory based approach for aggregation can be used [4], [5]. It is shown in [4] that a coalition game among wind power producers connected to a single bus has a non-empty core and hence a coalition stabilizing mechanism for payment sharing necessarily exists. This payment sharing mechanism divides the coalition's expected payments among its members.

The formation of a stable coalition is predicated on the ability of the renewable energy producers to agree upon a common statistical model of the power they can jointly provide. Coalition formation may be disrupted not because fair ways of distributing expected payments don't exist but because the producers cannot agree on the statistical model to be used to compute these fair distributions. Further, since the renewable power statistics are changing from one delivery period to another, coalition formation would require continuous exchange of credible information among the producers to update the common statistical model and continuous adjustments in the payment sharing mechanism based on the time-varying statistical model.

In order to circumvent the above problems, we propose a simple mechanism of sharing realized payments (not expected payments) that does not rely on any statistical model of the renewable power. Thus, the aggregate can operate without requiring the producers to agree upon a common statistical model. Further, the proposed mechanism does not require modifications from one delivery period to another.

Once the aggregate's payment sharing mechanism has been specified, each producer decides a power contract to offer to the aggregate. The contract specifies a constant power level to be delivered/consumed over a specified time in the future. The payoff of each producer in the aggregate depends on the contracts offered by all the producers in the aggregate. Thus, producers are involved in a game with the offered contracts as the actions of this game. We study the equilibrium properties of the game induced by our proposed mechanism. We show that at any Nash equilibrium of the game, each producer is better off in the aggregate than outside it. Further, under some conditions, aggregation incen-

This work was supported in part by EPRI and CERTS under sub-award 09-206, NSF under Grants EECS-0925337 and 1129001, Idaho National Labs, and Robert Bosch LLC through its Bosch Energy Research Network funding program.

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tivizes each producer to offer a larger amount of renewable power in the day-ahead market. The increased availability of renewable energy in the day-ahead market would reduce the amount of non-renewable energy scheduled in the day-ahead market.

A. Notation

Bold letters denote vectors. For a vector $\mathbf{E} = (E^1, \ldots, E^n)$, $\mathbf{E}^{-i}$ denotes $(E^1, \ldots, E^{i-1}, E^{i+1}, \ldots, E^n)$. $x^+ := \max\{x, 0\}$.

B. Sign convention

For any renewable energy producer,

(i) A positive value of payment represents money paid to the producer, a negative value represents money paid by the producer.

(ii) A positive value of power represents power flowing from the producer to the system, a negative value represents power flowing to the producer.

II. The Model

A. Renewable Energy Producers

We consider a collection of n renewable energy producers connected to a common bus in the power network. The renewable energy producers are indexed by $i \in \mathcal{N} = \{1, 2, \ldots, n\}$. The renewable energy producers can be:

(a) *Producers coupled with loads:* These are renewable energy producers whose primary role is to supply power to the loads they are coupled with but they can sell their excess power to the system operator. At times, they may also buy power from the system operator if their production is expected to be less than their load's demand. Examples include wind power generators coupled with loads [6] and loads with distributed photo-voltaic generation.

(b) *Uncoupled producers:* These are renewable energy producers such as wind farms, remote solar power plants that can sell all of their produced power to the system operator.

Let w_t^i be the net generation available to the system operator from the ith producer at time t. For an uncoupled producer, $w_t^i \geq 0$, is simply the renewable power produced. For renewable producers coupled with loads, w_t^i is the difference between the renewable power produced at time t and the power consumed by the loads. Note that for producers coupled with loads w_t^i may be positive or negative. A negative w_t^i means that the system operator is supplying power to the ith producer. The power available to the system operator from the ith producer in a time interval $[t_0, t_1]$ is a stochastic process $w_t^i, t_0 \leq t \leq t_1$.

B. Market Structure

We consider the market structure where each producer can offer power contracts in the day-ahead market. A contract specifies a constant power level to be delivered or consumed during a specified time period. A positive value of contracted power means that power is delivered whereas a negative value of contracted power means power is consumed. The price of power in the day-ahead market is p. Deviations from contracted power result in ex-post payments that depend on ex-post imbalance prices: q, for negative deviations (shortfalls), and λ for positive deviations (surpluses)[1]. The net payment received by producer i if it offered a contract to deliver/consume a constant power of C^i during the time-interval $[t_0, t_1]$ is

$$pC^i(t^1 - t^0) + \int_{t_0}^{t_1} \lambda[w_t^i - C^i]^+ dt - \int_{t_0}^{t_1} q[C^i - w_t^i]^+ dt, \quad (1)$$

where w_t^i is the actual power delivered/consumed by producer i at time t. The first term in (1) is positive if $C^i > 0$, that is, if the ith producer offers a contract to *deliver* power; the first term is negative if $C^i < 0$, that is, if the ith producer offers a contract to *consume* power.

We assume that: (a) the day-ahead price p is fixed and known and (b) the imbalance prices are random variables with the shortfall price $q \geq 0$. The surplus price λ can be positive or negative, however, we assume that the inequality $\lambda \leq q$ is always true. We refer the reader to [7] for a discussion of one possible way by which the imbalance prices are determined. In the model in [7], $0 < \lambda \leq q$. A positive λ means that production in excess of the contracted value or consumption short of the contracted value are paid for at a price λ by the system operator. A negative value of λ means that production in excess of the contracted value or consumption short of the contracted value are penalized.

C. Aggregation Model

Our goal is to find a scheme for aggregating the renewable energy producers in order to exploit the negative correlations that may exist among them. We assume the presence of an aggregate manager to facilitate the aggregation process. The aggregate manager can accept day-ahead power contracts from individual renewable energy producers and offer an aggregate contract to the system operator in the day-ahead market. The power delivered by the aggregate manager to the system operator is sum of powers from individual producers. The net payment received by the aggregate manager from the system operator if it offered a contract to deliver/consume a constant power of $C^{agg} := \sum_{i \in \mathcal{N}} C^i$ during the time-interval $[t_0, t_1]$ is

$$pC^{agg}(t^1 - t^0) + \int_{t_0}^{t_1} \lambda \left[w_t^{agg} - C^{agg}\right]^+ dt - \int_{t_0}^{t_1} q \left[C^{agg} - w_t^{agg}\right]^+ dt, \quad (2)$$

where $w_t^{agg} := \sum_{i \in \mathcal{N}} w_t^i$ is the actual power delivered/consumed. The aggregate manager is responsible for distributing the payment received from the system operator among the producers.

[1] A surplus can either be production in excess of the contracted value or consumption less than the contracted value. Similarly, a shortfall can either be production less than the contracted value of consumption in excess of contracted value.

D. Informational Assumptions

We assume that:

(i) The aggregate manager has *no statistical information* about the renewable power and about the imbalance prices. The distribution of payments by the aggregate manager depends on the *actual realizations* of the power delivered and the imbalance prices.

(ii) At the time of offering day ahead contracts, each producer has *private statistical information* about all the random variables involved, namely, the powers that can be delivered in the time interval $[t_0, t_1]$ by all the producers as well as the imbalance prices. The statistical information available to different producers may be different. Producer i's statistical information can be any valid probability distribution on renewable power process $\mathbf{w}_t = (w_t^1, w_t^2, \dots, w_t^n)$ and the imbalance prices (q, λ).

III. Payment Distribution in the Aggregate

If producer i offers a contract to deliver a power level C^i to the aggregate manager, it is paid according to the day-ahead market price p. At the time of delivery, the producer is paid an imbalance payment (which may be positive or negative). The imbalance payment for producer i at a time-instant $t \in [t_0, t_1]$ is a function of the imbalance prices q, λ and the vector of individual deviations of all participants in the aggregate. We define the vector of deviations at time t as

$$\mathbf{E}_t := \mathbf{w}_t - \mathbf{C},$$

where $\mathbf{w}_t = (w_t^1, w_t^2, \dots, w_t^n)$ and $\mathbf{C} = (C^1, C^2, \dots, C^n)$. The ith element E_t^i of $\mathbf{E}_t$ represents the excess production from producer i. The net payment received by producer i if it offered a power contract C^i to the aggregate manager is given by

$$pC^i(t^1 - t^0) + \int_{t_0}^{t_1} D^i(\mathbf{E}_t, q, \lambda) dt, \tag{3}$$

where $D^i(\mathbf{E}_t, q, \lambda)$ specifies the imbalance payments. We refer to the collection of functions $D^i(\cdot), i = 1, 2, \dots, n$, as the *Aggregate's Payment Sharing Mechanism*.

A. Desirable Properties of the Aggregate's Payment Sharing Mechanism

We would like the aggregate's payment sharing mechanism to satisfy the following desirable properties:

P1. *Ex-post Individual Rationality:* If producer i does not participate in the aggregate but directly offers a contract C^i to the system operator, its payment is given by (1) while its payment from the aggregate manager is given by (3). We would like to ensure that for every realization of the renewable power process and the imbalance prices, producer i's payment from the aggregate is at least as large as the payment it could have obtained through an individual contract with the system operator. That is, we require that

$$D^i(\mathbf{E}_t, q, \lambda) \geq \lambda[w_t^i - C^i]^+ - q[C^i - w_t^i]^+ \tag{4}$$

P2. *Budget Balance:* The aggregate manager offers the sum of individual contracts, $\sum_i C^i$, in the day-ahead market and it delivers a total power of $\sum_i w_t^i$ at time t. We require that the sum of payments made by the aggregate manager to all producers is equal to the payment received by the aggregate manager from the system operator. Therefore, we require that

$$\sum_i D^i(\mathbf{E}_t, q, \lambda) = \lambda \left[w_t^{agg} - C^{agg}\right]^+ - q\left[C^{agg} - w_t^{agg}\right]^+ \tag{5}$$

P3. *Fairness:* Finally, we require that whenever $(w_t^i - C^i) = (w_t^j - C^j)$ then $D^i(\mathbf{E}_t, q, \lambda) = D^j(\mathbf{E}_t, q, \lambda)$.

B. Proposed Mechanism

For ease of exposition, we present the cases where $\lambda \geq 0$ and $\lambda < 0$ separately.

Case A: $\lambda < 0$ This corresponds to the situation where both shortfalls and surpluses are penalized. We first define the sets of producers with shortfalls and surpluses as:

$$\mathcal{P}_t := \{i \in \mathcal{N} | E_t^i \geq 0\}, \quad \mathcal{Q}_t := \{i \in \mathcal{N} | E_t^i < 0\}.$$

Further, we define $E_t^p := \sum_{i \in \mathcal{P}_t} E_t^i, \quad E_t^q := \sum_{i \in \mathcal{Q}_t} E_t^i$. We now specify the functions $D^i(\mathbf{E}_t, q, \lambda)$ as follows:

1) *No net deviation:* If $\sum_i E_t^i = 0$, then $D^i(\mathbf{E}_t, q, \lambda) = 0$, for all $i \in \mathcal{N}$.

2) *Net shortfall:* If $\sum_i E_t^i < 0$, then let a to be the unique solution of the equation

$$\sum_{i \in \mathcal{Q}_t} \min(a, |E_t^i|) = E_t^p. \tag{6}$$

Then,

$$D^i(\mathbf{E}_t, q, \lambda) = 0, \quad i \in \mathcal{P}_t$$
$$D^j(\mathbf{E}_t, q, \lambda) = -q(|E_t^j| - \min(a, |E_t^j|)), \quad j \in \mathcal{Q}_t \tag{7}$$

The main idea behind the above imbalance payment functions is the following: the net positive excess at time t, E_t^p, should be used to compensate for the shortfalls of producers in $\mathcal{Q}_t$. Each term in the summation in (6) represents the amount of producer i's shortfall being compensated. The amount of shortfall compensation producer i receives cannot be more than its shortfall (hence the $\min(a, |E_t^i|)$) and the total shortfall compensated must equal the net positive excess E_t^p. (See Figure 1).

3) *Net excess:* If $\sum_i E_t^i > 0$, then let b to be the unique solution of the equation

$$\sum_{i \in \mathcal{P}_t} \min(b, E_t^i) = |E_t^q|. \tag{8}$$

Then,

$$D^i(\mathbf{E}_t, q, \lambda) = \lambda(E_t^i - \min(b, E_t^i)), \quad i \in \mathcal{P}_t$$
$$D^j(\mathbf{E}_t, q, \lambda) = 0, \quad j \in \mathcal{Q}_t \tag{9}$$

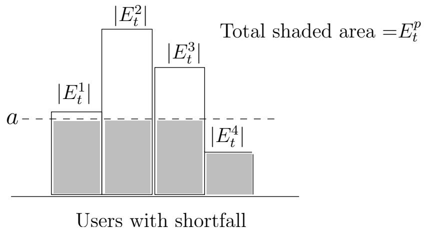

Fig. 1. a satisfying (6).

The above imbalance payments are analogous to the payments in case of net shortfall with the roles of sets $\mathcal{P}_t$ and $\mathcal{Q}_t$ reversed.

Case B: $\lambda \geq 0$ This corresponds to the situation where producers are paid for surpluses. Recall that we assumed $\lambda \leq q$. We now specify the functions $D^i(\mathbf{E}_t, q, \lambda)$ as follows:

1) *No net deviation:* If $\sum_i E_t^i = 0$, then

$$D^i(\mathbf{E}_t, q, \lambda) = \lambda E_t^i, \quad i \in \mathcal{P}_t$$
$$D^j(\mathbf{E}_t, q, \lambda) = \lambda E_t^j, \quad j \in \mathcal{Q}_t$$

2) *Net shortfall:* If $\sum_i E_t^i < 0$, then let c to be the unique solution of the equation

$$\sum_{i \in \mathcal{Q}} \min(c, |E_t^i|) = E_t^p. \tag{10}$$

Then,

$$D^i(\mathbf{E}_t, q, \lambda) = \lambda E_t^i, \quad i \in \mathcal{P}_t$$
$$D^j(\mathbf{E}_t, q, \lambda) = -q(|E_t^j| - \min(c, |E_t^j|)) - \lambda \min(c, |E_t^j|), \quad j \in \mathcal{Q}_t \tag{11}$$

The main idea behind the above imbalance payment functions is the following: the net positive excess at time t, E_t^p, should be used to compensate for the shortfalls of producers in $\mathcal{Q}_t$. However, each producer receiving a compensation for its shortfall must pay for it at the price λ.

3) *Net excess:* If $\sum_i E_t^i > 0$, then

$$D^i(\mathbf{E}_t, q, \lambda) = \lambda E_t^i, \quad i \in \mathcal{P}_t$$
$$D^j(\mathbf{E}_t, q, \lambda) = \lambda E_t^j, \quad j \in \mathcal{Q}_t$$

Theorem 1: The proposed aggregate's payment mechanism satisfies properties P1, P2 and P3.

Proof: See Appendix A. ■

The following lemma will be used later in the analysis.

Lemma 1: For any q and λ such that $q \geq 0, q \geq \lambda$, the following statements are true:

1) $D^i(\mathbf{E}_t, q, \lambda)$ is a continuous function of $\mathbf{E}_t$.
2) For any fixed $\mathbf{E}_t^{-i}$, $D^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ is a piecewise linear and concave function of E_t^i.

Proof: See Appendix B. ■

IV. Contract Game

Given the aggregate's payment mechanism of Section III-B, we now investigate the contracts offered by producers to the aggregate manager. Each producer would like to offer a contract C^i to the aggregate manager that maximizes its expected payoff. This expectation is with respect to producer i's *private statistical information* about the renewable power $\mathbf{w}_t$ and the imbalance prices q and λ. Because the payments from the aggregate depend on the contracts from all the producers, the producers are involved in a game with the offered contract levels as the actions of this game. We assume that producer i can offer a contract $C^i \in [-m^i, M^i]$, $m^i, M^i \geq 0$. (Recall that negative contracts represent power consumption.) If the n producers offer contract levels $\mathbf{C} = (C_1, C_2, \ldots, C_n)$ to the aggregate manager, then producer i's expected payoff is

$$\Pi^i(\mathbf{C}) = pC^i(t^1 - t^0) + \int_{t_0}^{t_1} \mathbb{E}^i[D^i(\mathbf{w}_t - \mathbf{C}, q, \lambda)]dt, \tag{12}$$

where $\mathbb{E}^i$ denotes that the expectation is with respect to producer i's private statistical model on $(\mathbf{w}_t, q, \lambda)$.

Before investigating the game induced by aggregation, we briefly review the problem of choosing optimal contract if producer i directly offered a contract to the system operator. This problem has been investigated in detail in [8]. If producer i offers a contract C^i to the system operator, its expected payoff is

$$J^i(C^i) := pC^i(t^1 - t^0) + \int_{t_0}^{t_1} \mathbb{E}^i[\lambda(w_t^i - C^i)^+ - q(C^i - w_t^i)^+]dt \tag{13}$$

Let S^i be the contract that maximizes the expected payoff given by (13). (In case the maximizer is not unique, we define S^i to be the smallest maximizer.)

Lemma 2: Irrespective of contracts offered by other producers, producer i can guarantee itself an expected payoff at least as large as $J^i(S^i)$.

Proof: This is simply a consequence of ex-post individual rationality (Property P1). Offering the contract S^i to the aggregate gives a better payoff to producer i than offering the same contract directly to the system operator. Therefore, by offering S^i to the aggregate, producer i can guarantee itself an expected payoff no smaller than $J^i(S^i)$. ■

If the n producers engage in some negotiation/message exchange process to decide the contract levels they offer to the aggregate manager, then the contract levels $\mathbf{C}^* = C^{1*}, C^{2*}, \ldots, C^{n*})$ they converge to must satisfy the Nash equilibrium conditions, that is, for all $i \in \mathcal{N}$,

$$\Pi^i(\mathbf{C}^*) \geq \Pi^i(C^i, \mathbf{C}^{*,-i}), \forall C^i \in [-m^i, M^i] \tag{14}$$

The following lemma ensures the existence of equilibrium contract levels.

Theorem 2: The contract game has a Nash equilibrium in pure strategies. That is, there exists $\mathbf{C}^* = (C^{1*}, C^{2*}, \ldots, C^{n*})$ that satisfies (14) for all $i \in \mathcal{N}$.

Proof: To prove the existence of a Nash equilibrium in pure strategies, it suffices to show that for any i, $\Pi^i(\cdot)$ is a continuous function and for any $\mathbf{C}^{-i}$, $\Pi^i(C^i, \mathbf{C}^{-i})$ is concave in C^i [9, Chapter 4]. Lemma 1 implies that $D^i(\mathbf{w}_t - \mathbf{C}, q, \lambda)$ is a continuous function of $\mathbf{C}$. Therefore, $\Pi^i(\mathbf{C})$ is a continuous function of $\mathbf{C}$ as well. Lemma 1 also implies that, for any given $\mathbf{C}^{-i}$, $D^i(w_t^i - C^i, \mathbf{w}_t^{-i} - \mathbf{C}^{-i}, q, \lambda)$ is a concave function of C^i. Since averaging over $(\mathbf{w}_t, q, \lambda)$ and integrating over time preserves concavity, $\Pi^i(C^i, \mathbf{C}^{-i})$ is concave in C^i. Therefore, a Nash equilibrium in pure strategies is guaranteed to exist. ■

Lemma 3: At any Nash equilibrium of the contract game, producer i gets an expected payoff no smaller than $J^i(S^i)$.

Proof: This result follows directly from Lemma 2. ■

Lemma 3 ensures that aggregation gives larger expected payoffs to each producer than the payoffs without aggregation. The following lemma shows that when surpluses are not penalized (that is, when $\lambda \geq 0$), aggregation also has a social benefit as it incentivizes each producer to offer a larger amount of renewable power in the day-ahead market. The increased availability of renewable energy in the day-ahead market would reduce the amount of non-renewable energy scheduled in the day-ahead market.

Lemma 4: If $0 \leq \lambda \leq q$, then at any Nash equilibrium $\mathbf{C}^*$, the contract C^{i*} offered by producer i to the aggregate is at least as large as producer i's optimal contract S^i without aggregation.

Proof: See Appendix C ■

If producer i's optimal contract without aggregation, S^i, is positive, Lemma 4 proves that producer i will offer to deliver more than S^i amount of power to the aggregate manager. On the other hand, if S^i is negative, that is if producer i would have bought power in the day-ahead market without aggregation, Lemma 4 ensures that producer i will either ask for less power from the aggregate manager (if $S^i \leq C^{i*} < 0$) or actually sell power (if $S^i < 0 \leq C^{i*}$) to the aggregate manger in the day-ahead contract. In either case, aggregation would reduce the amount of non-renewable power scheduled in the day-ahead market.

Remark 1: The argument in Appendix C actually proves that for $0 \leq \lambda \leq q$, contract levels less than S^i are strictly dominated, that is, irrespective of other producers' contract, producer i should never offer a contract less than S^i.

V. Conclusion

We considered the problem of sharing payments in an aggregate of renewable energy producers connected to a common bus in a power network. We proposed a payment sharing mechanism that does not require the participants to find a common statistical model for net generation available from all producers. The proposed mechanism is individually rational, budget-balanced and fair. Thus, the mechanism incentivizes aggregation and ensures larger expected payments for individual producers than what they could have obtained on their own. Further, under certain conditions, it incentivizes individual producers to offer larger contracts in the day-ahead market than what they would have offered without aggregation. This increases the amount of renewable energy available in the day-ahead market and reduces the amount of non-renewable that needs to be scheduled.

The strongest assumption in our analysis is that all producers face the same day-ahead and imbalance prices. In general, producers connected to different buses would face different locational marginal prices. Extending our mechanism to a scenario with locational prices would be an important next step for this work. In this paper, we restricted ourselves to a simple two stage market. In a multi-stage market, producers may be able to take advantage of improved forecasts of their net generation to change their contract levels (see, for example, [10]). Finding a multi-stage payment mechanism and studying the accompanying multi-stage game among producers would be an important direction for future work.

Appendix

A. Proof of Theorem 1

Property P1: For any $i \in \mathcal{P}_t$ (that is, for i with $E_t^i = w_t^i - C^i \geq 0$), it is easy to show that the mechanism proposed in Section III-B always satisfies $D^i(\mathbf{E}_t, q, \lambda) \geq \lambda E_t^i$. Similarly, for any $j \in \mathcal{Q}_t$ (that is, for i with $E_t^j = w_t^j - C^j < 0$), we can show that the proposed mechanism always satisfies $D^i(\mathbf{E}_t, q, \lambda) \geq -q|E_t^i|$, where we make use of the fact that $\lambda \geq q$.

Property P2: We want to establish that

$$\sum_i D^i(\mathbf{E}_t, q, \lambda) = \lambda \left[\sum_i w_t^i - \sum_i C^i \right]^+ - q \left[\sum_i C^i - \sum_i w_t^i \right]^+ \quad (15)$$

Consider first the case when $\lambda < 0$. If $\sum_i E_t^i = \sum_i w_t^i - \sum_i C^i = 0$, then $D^i(\mathbf{E}_t, q, \lambda) = 0$ for all i and (15) is satisfied. If $\sum_i E_t^i = \sum_i (w_t^i - C^i) < 0$, then for all $i \in \mathcal{P}_t$, $D^i(\mathbf{E}_t, q, \lambda) = 0$, whereas for all $j \in \mathcal{Q}_t$, $D_t^j(\mathbf{E}_t, q, \lambda) = -q(|E_t^j| - \min(a, |E_t^j|))$, where a satisfies (6). Therefore,

$$\begin{aligned} \sum_i D^i(\mathbf{E}, q, \lambda) &= -q \sum_{i \in \mathcal{Q}_t} (|E_t^i| - \min(a, |E_t^i|)) \\ &= -q \left(\sum_{i \in \mathcal{Q}_t} |E_t^i| - E_t^p \right) \quad (16) \\ &= -q(-E_t^q - E_t^p) = -q \left(\sum_i (-E_t^i) \right) \\ &= -q \left(\sum_i C^i - \sum w_t^i \right) \quad (17) \end{aligned}$$

where we used (6) in (16). The right hand side of (17) is the same as the right hand side of (15) when $\sum_i (w_t^i - C^i) < 0$, thus (15) holds in this case. The argument for the situation where $\sum_i E_t^i = \sum_i (w_t^i - C^i) > 0$ proceeds in the same manner.

Now, consider the case when $\lambda \geq 0$. If $\sum_i E_t^i = \sum_i (w_t^i - C^i) \geq 0$, $D^i(\mathbf{E}_t, q, \lambda) = \lambda E_t^i$ and hence

$$\sum_i D^i(\mathbf{E}_t, q, \lambda) = \lambda \sum_i E_t^i = \lambda \left(\sum_i w_t^i - \sum_i C^i \right), \tag{18}$$

which establishes (15). Finally, if $\sum_i E_t^i = \sum_i (w_t^i - C^i) < 0$, using (11), we can write the sum of $D^i(\mathbf{E}, q, \lambda)$ as

$$\sum_i D^i(\mathbf{E}, q, \lambda) = \lambda \sum_{i \in \mathcal{P}_t} E_t^i - q \sum_{j \in \mathcal{Q}_t} (|E_t^j| - \min(c, |E_t^j|)) - \lambda \sum_{j \in \mathcal{Q}_t} \min(c, |E_t^j|), \tag{19}$$

where c satisfies (10). Because of (10), the first and the last summation in the above equation cancel out and we get

$$\begin{aligned} &- q \left(\sum_{j \in \mathcal{Q}_t} |E_t^j| - \sum_{j \in \mathcal{Q}_t} \min(c, |E_t^j|) \right) \\ &= -q \left(\sum_{j \in \mathcal{Q}_t} |E_t^j| - E_t^p \right) \qquad (20) \\ &= -q(-E_t^q - E_t^p) = -q \left(\sum_i (-E_t^i) \right) \\ &= -q \left(\sum_i C^i - \sum w_t^i \right) \qquad (21) \end{aligned}$$

where we used (10) in (20).The right hand side of (21) is the same as the right hand side of (15) when $\sum_i (w_t^i - C^i) < 0$, thus (15) holds.

Property P3: The symmetric nature of the functions D^i implies that Property P3 is satisfied.

B. Proof of Lemma 1

Before proving the lemma, we establish the following claim which will be used later.

Claim 1: Given m positive numbers, $\theta^1, \theta^2, \ldots, \theta^m$ and a positive number Δ, let $x \geq 0$ and $x > \Delta - \sum_{j=1}^m \theta^j$. Define $f(x)$ to be the unique solution of the equation

$$\min(y, x) + \sum_{j=1}^m \min(y, \theta^j) = \Delta. \tag{22}$$

Then, there exists a threshold h such that

$$\min(f(x), x) = \begin{cases} x & \text{if } x < h, \\ h & \text{if } x \geq h \end{cases} \tag{23}$$

Proof: Consider any x such that $\min(f(x), x) = f(x)$. Then for any $x' > x$, it is straightforward to verify that $f(x)$ satisfies (22) when the first term $\min(y, x)$ is replaced with $\min(y, x')$. Therefore, $f(x') = f(x)$ and $\min(f(x'), x') = f(x')$. This implies that (23) holds with the threshold h defined to be the smallest x such that $\min(f(x), x) = f(x)$. ■

We now turn to the proof of Lemma 1.

Case A: $\lambda < 0$ Consider any fixed $\mathbf{E}_t^{-i}, q, \lambda$. We can write the function $D^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ as follows:

1) *No net deviation:* If $x + \sum_{j \neq i} E_t^j = 0$, then $D^i(x, \mathbf{E}_t^{-i}, q, \lambda) = 0$.
2) *Net shortfall:* If $x + \sum_{j \neq i} E_t^j < 0$, then

$$D^i(x, \mathbf{E}_t^{-i}, q, \lambda) = \begin{cases} -q(|x| - \min(a, |x|)) \text{if } x < 0 \\ 0 \quad \text{if } x \geq 0 \end{cases} \tag{24}$$

where a satisfies

$$\min(a, x) + \sum_{j \in \mathcal{Q}_t, j \neq i} \min(a, |E_t^j|) = E_t^p.$$

Using Claim 1, (24) can be written as

$$D_t^i(x, \mathbf{E}_t^{-i}, q, \lambda) = \begin{cases} -q(|x| - h^a)) & \text{if } x \leq -h^a, \\ 0 & \text{if } -h^a < x < 0 \ , \\ 0 & \text{if } x \geq 0 \end{cases} \tag{25}$$

for some $h^a \geq 0$.
3) *Net excess:* If $x + \sum_{j \neq i} E_t^j > 0$, then

$$D_t^i(x, \mathbf{E}_t^{-i}, q, \lambda) = \begin{cases} \lambda(x - \min(b, x))) & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{26}$$

where b satisfies

$$\min(b, x) + \sum_{j \in \mathcal{P}_t, j \neq i} \min(a, E_t^j) = |E_t^q|.$$

Using Claim 1, (26) can be written as

$$D_t^i(x, \mathbf{E}_t^{-i}, q, \lambda) = \begin{cases} \lambda(x - h^b) & \text{if } x \geq h^b, \\ 0 & \text{if } 0 \leq x < h^b \ , \\ 0 & \text{if } x < 0 \end{cases} \tag{27}$$

for some $h^b \geq 0$.

Combining (25) and (27), $D_t^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ can be graphically represented as shown in Figure 2. Thus, $D_t^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ is continuous, piecewise linear and concave.

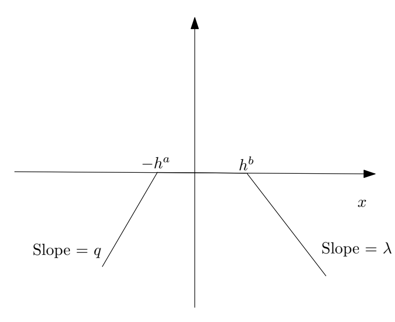

Fig. 2. $D^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ for $\lambda < 0$.

Case B: $0 \leq \lambda \leq q$ Consider any fixed $\mathbf{E}_t^{-i}, q, \lambda$. We can write the function $D^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ as follows:

1) *Net shortfall:* If $x + \sum_{j \neq i} E_t^j < 0$, then

$$D_t^i(x, \mathbf{E}_t^{-i}, q, \lambda) = \begin{cases} -q(|x| - \min(c, |x|)) - \lambda \min(c, |x|) & \text{if } x < 0 \\ \lambda x & \text{if } x \geq 0 \end{cases} \tag{28}$$

where c satisfies

$$\min(c, x) + \sum_{j \in \mathcal{Q}_t, j \neq i} \min(c, |E_t^j|) = E_t^p.$$

Using Claim 1, (28) can be written as

$$D_t^i(x, \mathbf{E}_t^{-i}, q, \lambda) = \begin{cases} -q(|x| - h^c) - \lambda h^c \text{ if } x \leq -h^c, \\ \lambda x \text{ if } -h^c < x < 0 \ , \\ \lambda x \text{ if } x \geq 0 \end{cases} \tag{29}$$

for some $h^c \geq 0$.

2) *Net excess or no deviation:* If $x + \sum_{j \neq i} E_t^j \geq 0$, then

$$D^i(x, \mathbf{E}_t^{-i}, q, \lambda) = \lambda x. \tag{30}$$

Combining (29) and (30), $D_t^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ can be graphically represented as shown in Figure 3. Thus, $D_t^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ is continuous, piecewise linear and concave.

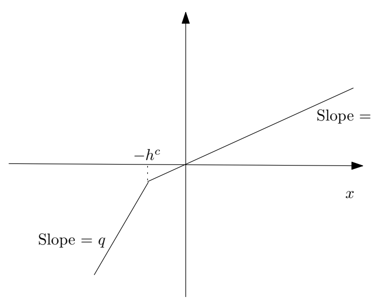

Fig. 3. $D^i(\cdot, \mathbf{E}_t^{-i}, q, \lambda)$ for $\lambda \geq 0$.

C. Proof of Lemma 4

Without aggregation, producer i's optimal contract S^i is obtained by maximizing $J^i(C^i)$ defined in (13). Differentiating $J^i(C^i)$ with respect to C^i, we get

$$\frac{dJ^i(C^i)}{dC^i} = p(t^1 - t^0) + \int_{t^0}^{t^1} \left[\mathbb{E}^i[-\lambda \mathbb{1}_{\{w_t^i > C^i\}}] - \mathbb{E}^i[q \mathbb{1}_{\{w_t^i < C^i\}}]\right] dt, \tag{31}$$

Because $\lambda \leq q$ the derivative is non-increasing in C^i. The optimal contract S^i is the smallest value of C^i such that

$$\int_{t^0}^{t^1} \left[\mathbb{E}^i[q \mathbb{1}_{\{w_t^i < C^i\}}] + \mathbb{E}^i[\lambda \mathbb{1}_{\{w_t^i > C^i\}}]\right] dt = p(t^1 - t^0) \tag{32}$$

Let $\mathbf{C}^*$ be a Nash equilibrium. Then, C^{i*} maximizes $\Pi^i(\cdot, \mathbf{C}^{*,-i})$. Differentiating $\Pi^i(\cdot, \mathbf{C}^{*,-i})$ with respect to C^i, we get

$$\frac{d\Pi^i(C^i, \mathbf{C}^{*,-i})}{dC^i} = p(t^1 - t^0) + \int_{t^0}^{t^1} \mathbb{E}^i \left[\frac{dD^i(w_t^i - C^i, \mathbf{w}_t^{-i} - \mathbf{C}_t, q, \lambda)}{dC^i}\right] dt \tag{33}$$

Recall that $D^i(E_t^i, \mathbf{E}_t^{-i}, q, \lambda)$ is concave in E_t^i; therefore $D^i(w_t^i - C^i, \mathbf{w}_t^{-i} - \mathbf{C}_t, q, \lambda)$ is concave in C^i and hence the derivative in (33) is non-increasing in C^i. Thus, C^{i*} is no less than the smallest C^i satisfying

$$\int_{t^0}^{t^1} \mathbb{E}^i \left[-\frac{dD^i(w_t^i - C^i, \mathbf{w}_t^{-i} - \mathbf{C}_t, q, \lambda)}{dC^i}\right] dt = p(t^1 - t^0) \tag{34}$$

Equivalently, C^{i*} is no less than the smallest C^i satisfying

$$\int_{t^0}^{t^1} \mathbb{E}^i \left[\left. \frac{dD^i(x, \mathbf{w}_t^{-i} - \mathbf{C}_t, q, \lambda)}{dx} \right|_{x = w_t^i - C^i} \right] dt = p(t^1 - t^0) \tag{35}$$

From figure 3 in the proof of Lemma 1, it is easy to verify that

$$\left. \frac{dD^i(x, \mathbf{w}_t^{-i} - \mathbf{C}_t, q, \lambda)}{dx} \right|_{x = w_t^i - C^i} \leq \lambda \mathbb{1}_{\{w_t^i \geq C^i\}} + q \mathbb{1}_{\{w_t^i < C^i\}}$$

whenever the derivative on left hand side exists. Thus, the left hand side of (35) is upper bounded by $\int_{t^0}^{t^1} \mathbb{E}^i[\lambda \mathbb{1}_{\{w_t^i \geq C^i\}} + q \mathbb{1}_{\{w_t^i < C^i\}}] dt$. Since S^i is the smallest value of C^i for which the above integral equals $p(t^1 - t^0)$, it follows that for any $C^i < S^i$, the left hand side of (35) cannot be equal to $p(t^1 - t^0)$. Therefore, $C^{i*} \geq S^i$.

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