

Nonsmooth Optimization Based Multiple Robust Controller Design Under Coupling Partitioned Uncertainty

Liu Jiaqi, X. Z. Gao, and K. Zenger

Abstract—This paper proposes a nonsmooth optimization technique to cope with the H_∞ synthesis problem under weak coupling uncertainty. For some kind of systems, a single controller cannot be found in terms of a given robust performance objective, whereas a multiple robust controller can satisfy the desired performances by introducing extra controllers and dividing the entire uncertainty into small partitioned sets. Such partitioned sets can be generated based on the small gain theorem and μ -analysis. In addition to solving the weak-coupled parametric uncertainty, the partition with decoupled algorithm can improve the design efficiency. However, the multiple robust controller design has to be transformed into a nonsmooth optimization problem. This paper uses the Clarke subdifferential and steepest descent method to acquire the gradient information of the nonsmooth function. The validity of our method is illustrated and verified by a practical controller design case-study for a satellite attitude control problem.

I. INTRODUCTION

IN this paper, a new technique is presented for the H_∞ synthesis problem in the case that a single robust controller cannot be found to satisfy the desired performances. The H_∞ synthesis at the presence of uncertainty is one of the main topics of modern control theory. In the formulation of any control problem, there is always a discrepancy between the actual plant dynamics and its mathematical model used for the controller design. These discrepancies usually come from external disturbances, unknown plant parameters, and unmodeled dynamics. Therefore, the controller has to be robust against those variations in the plant dynamics. Most existing research results on the robustness synthesis focus on the design of a single controller under the framework of a common single Lyapunov function and LMI formulations [1]. Using a common Lyapunov function for all performance indices leads to conservativeness and therefore prevents

robust control theory from further development limiting its applications to some extent [2].

To overcome the above drawbacks, a lot of work has been done based on the mixed H_∞/H_2 robust controller design [3], parameter dependent Lyapunov function [4], switched control and sliding model control [5]. Unfortunately, certain systems are impossible to control for the entire uncertainty with a single controller to achieve the desired performance. For example, in a controller design problem provided by Kajiwara, the system of floating platform with two thrusters cannot achieve robust control objectives, due to the actuator limitations [6]. Choi present a technique by using a number of controllers so as to guarantee robust performances [7][8]. This method partitions the uncertainty set into a series of disjoint subsets, and designs a robust controller for each subset. The system hereby improves the robustness of its performances, because each controller needs to be robust only in the corresponding subset rather than in the entire uncertainty set. The overall performance of the multiple controllers in the whole uncertainty set is evaluated as the maximum H_∞ norm in each subset. Thus, the overall performance is expected to be better than the H_∞ synthesis with only a single controller designed for the entire uncertainty. With the partitioned uncertainty idea, multiple robust controller design has to be transformed into another type of problem, in which a pair of subset and controller should be found for minimizing the H_∞ norm, and such H_∞ synthesis is a nonsmooth optimization problem [9].

This paper first provides an overview to the linear fraction transformation method, which can partition the uncertainty into subsets. A nonsmooth optimization technique that guarantees the desired numerical solution of the H_∞ synthesis problems with weak coupling parametric uncertainty is next proposed. The Clarke subdifferential and steepest descent method are used to obtain the gradient information of the nonsmooth function in case that the system is nonsmooth itself or driven by a nonsmooth control input. The weak coupling parametric uncertainty is decoupled by introducing a partition algorithm. Finally, the proposed multiple robust controller design is examined using a satellite attitude control problem

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with the consideration of multiple constraints, such as model uncertainty and disturbance torque. Numerical simulation results show the validity of our multiple robust controller strategy.

II. MULTIPLE ROBUST CONTROL SYNTHESIS AND UNCERTAINTY SEGMENTATION ALGORITHM

We first briefly explain the main principle of the multiple controller system synthesis. This method mainly focuses on the H_∞ performance case. However, the case of the H_2 norm or even mixed H_∞ / H_2 norm can be dealt with in an analogous manner. Therefore, each step of the segmentation algorithm on how to partition the uncertainty set into a series of disjoint smaller sets is discussed in detail. The control synthesis can be transformed into a nonsmooth optimization problem, and the method for solving this optimization problem will be presented here as well.

A. Controller Design

This paper considers a linear time-invariant generalized plant with real parametric uncertainty, in which the uncertainty set Δ and desired robust performance γ are given. Suppose that Δ consists of two parts, one of which is partitioned as Δ_{cell} , while the other is Δ_{unpart} . The partitioned Δ_{cell} is regarded as a key object to improve the robust performance. On the other hand, Δ_{unpart} is not measured or estimated, and it may not considerably affect the robust performance. If the interior of $\Delta_{cell} \subset \Delta$ is nonempty and closed, we suppose that:

$$\Delta_{cell} = \left[\begin{array}{c} \Delta_{cell}^1 \\ \vdots \\ \Delta_{cell}^q \end{array} \right] \cup \dots \quad (1)$$

$$\text{int}(\Delta_{cell}^i) \cap \dots = \emptyset \text{ for } i \neq j$$

In this paper, the uncertainty can be represented via the Linear Fraction Transformation (LFT) with respect to an uncertain parametric matrix. The LFT is a transformation, which can separate the certain and uncertain parts in the system by a state-space like model, and all the time-invariant uncertain systems can be transformed into the LFT models. The basic principle of using the LFT is referred to as ‘‘pulling out the Δ s’’ [10]. Pulling out the Δ_{cell}^i from uncertainty Δ and a general closed-loop feedback robust control system are shown in Fig. 1. Here, K_{cell}^i and Δ_{cell}^i are the controller and uncertainty in the i^{th} partitioned set, respectively. The multiple robust controller design can be formulated in an LFT

framework. $F_l(M, \Delta)$, $F_u(M, \Delta)$ are the lower and upper LFT with:

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

$$F_l(M, \Delta) = M_{11} + M_{12}\Delta(I - M_{22}\Delta)^{-1}M_{21} \quad (2)$$

$$F_u(M, \Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12} \quad (3)$$

Define the operator F in Fig. 1 with respect to Δ as:

$$F(K_{cell}^i, \Delta_{cell}^i) = F_l(F_u(P, \Delta_{cell}^i), K_{cell}^i) \quad (4)$$

The operator $F(K_{cell}^i, \Delta_{cell}^i)$ denotes simply the closed-loop transfer matrices of $w \mapsto z$ for the uncertainty Δ_{cell}^i and its corresponding controller K_{cell}^i . Any interconnection of the LFT is also an LFT, and, therefore, the operator $F(K_{cell}^i, \Delta_{cell}^i)$ can be regarded as the cascaded case of the so-called star product [9][10].

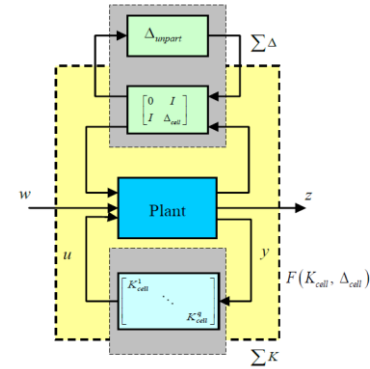


Fig. 1. Pulling the performance and feedback control channels from plant.

Theorem 2.1

Supposed that there are two uncertainty sets Δ_1 and Δ_2 , and K_1 and K_2 are the corresponding multiple robust controllers, such that

$$\|F(K_1, \Delta_1)\|_\infty < \|F(K_2, \Delta_2)\|_\infty \quad (5)$$

Consider the connected uncertainty sets Δ_1' and Δ_2' and suppose that for all $\Delta_r \in \Delta_2'$ and $\|\Delta_g\| \leq 1$, it holds:

$$\Delta_p = \begin{bmatrix} \Delta_r & \\ & \Delta_g \end{bmatrix}, \quad \|\Delta_p\|_\infty \|F(K_2, \Delta_2)\|_\infty < 1 \quad (6)$$

Then there exists a $\Delta_1' \supseteq \Delta_2'$, such that for all $\Delta_r \in \Delta_1'$ and, it holds:

$$\Delta_p = \begin{bmatrix} \Delta_r & \\ & \Delta_g \end{bmatrix}, \quad \|\Delta_p\|_\infty \|F(K_1, \Delta_1)\|_\infty < 1 \quad (7)$$

A detailed proof is given in [9]. The theorem guarantees that

the smaller $\|F(K_{cell}^i, \Delta_{cell}^i)\|_{\infty}$ can be found, the larger Δ_{cell}^i is partitioned in the segmentation algorithm. Hence, the multiple robust controller design is transformed into an optimization problem. In this optimization problem, we need to find a pair of subset uncertainty and controller for minimizing the H_{∞} norm of operator F and satisfying the desired robust performance γ :

$$(K_{cell}^i, \Delta_{cell}^i) = \arg \min_{\substack{K \in K_{multile} \\ \Delta \in \Delta_{cell}}} \{ \|F(K, \Delta)\|_{\infty} \} \quad (8)$$

$$\|T_{w,z}(K_{cell}^i, \Delta_{cell}^i)\|_{\infty} \leq \gamma$$

Where the closed loop transfer function from disturbance signal w to output signal z in subset Δ_{cell}^i is denoted by $T_{w,z}(cell)$. The formulated optimization problem is free of the Lyapunov variables, implying that it only needs moderate-size optimization algorithms even for the large systems. The segmentation algorithm on how to partition the uncertainty is next discussed.

B. Partition of Uncertainty

We present a segmentation algorithm for partitioning the entire uncertainty into a collection of small cells. Suppose that the uncertainty block Δ is given by:

$$\Delta = \begin{bmatrix} \Delta_{unpart} & & & & \\ & \Delta_{cell}^1 & & & \\ & & \ddots & & \\ & & & \Delta_{cell}^p & \\ & & & & \Delta_{cell}^p \end{bmatrix}, \Delta_{cell}^p = \begin{bmatrix} \delta_1^p & & & \\ & \ddots & & \\ & & & \delta_i^p \end{bmatrix} \quad (9)$$

Without loss of generality, we assume a tridimensional uncertainty set. Figure 2 shows how to divide the uncertainty into a series of disjoint subsets.

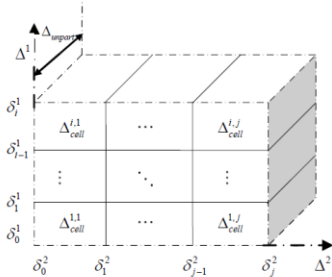


Fig. 2. Partitioning the uncertainty.

Our paper discusses the segmentation algorithm for partition as a standalone method. The integration of partitioning uncertainty into the H_{∞} synthesis and the performance of this algorithm will be addressed in Section IV. The segmentation algorithm consists of four stages, i.e., setting the initial uncertainty value and controller, local

uncertainty moving and local search (Fig. 3), partitioning the uncertainty into the subset Δ_{cell} , and removing the subset Δ_{cell} from the uncertainty (Fig. 4) according to the LFT mechanism. This iterative search procedure aims at fulfilling the partition requirements for a multiple robust controller design until some termination condition is satisfied.

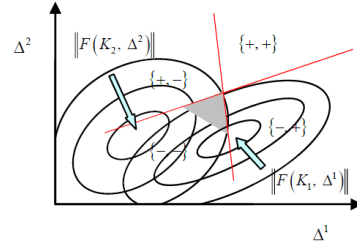


Fig. 3. Search area (shaded) for segmentation algorithm with two parameters and two objectives during initial stage.

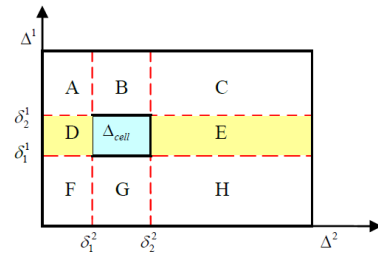


Fig. 4. Removing the partition Δ_{cell} from the uncertainty [9].

Algorithm 2.1

- 1) **Setting the initial uncertainty and controller**
Find an initial uncertainty set $\Delta_{initial}$ and a corresponding controller $K_{initial}$, which minimizes the performance of the given system.
- 2) **Local moving uncertainty and local search (Fig. 3)**
Theorem 2.1 shows that the algorithm can find the largest uncertainty subset Δ_{cell} around $\Delta_{initial}$, which meets the robust performance criterion. In this paper, the system has tridimensional uncertainty set, and the part to be partitioned is bi-dimension, as shown in Fig. 2. To partition uncertainty $\Delta_{part} = [\Delta^1, \Delta^2]$, there are two objectives $\|F(K_1, \Delta^1)\|$ and $\|F(K_2, \Delta^2)\|$, which need to be optimized. We describe the system under the circumstances of two optimization objectives, because most engineering problems have weak coupling parametric uncertainty. In Fig. 3, $\{-, -\}$ and $\{+, +\}$ denote the descent and ascent regions, respectively. The symbol $\{+, -\}$ indicates that in this direction, an improvement according to $\|F(K_1, \Delta^1)\|$

can be achieved while the values of $\|F(K_1, \Delta^2)\|$ will increase. The local search strategy is capable of both moving toward and along the Pareto set so as to obtain better performances for these two norms [11]. Along $\{-, -\}$, the local moving strategy d_{move} is used after N unsuccessful trials are made:

$$d_{move} = \frac{1}{N} \sum_{i=1}^N s_k \frac{\|\Delta^i\| - \|\Delta^0\|}{\|\Delta^i - \Delta^0\|} \quad (10)$$

where

$$s_k = \begin{cases} 1, & \text{if } \|F(K_i, \Delta^i)\| < \|F(K_0, \Delta^0)\| \\ -1, & \text{else} \end{cases} \quad (11)$$

3) Partitioning the uncertainty into the subset

Find the largest uncertainty subset Δ_{cell} in Step 2), and if the uncertainty Δ is convex, partition the uncertainty set by using the Cartesian cut cell method, as given in Fig. 3. As a matter of fact, cut cell meshes offer a potentially more automated and robust alternative to the boundary conforming method for complex and convex uncertainty [12]. In particular, partitioned cells shift the difficulty from geometry boundary conforming of the uncertainty to computational convex divisible polyhedron (high-order uncertainty). A three dimensional Cartesian cut cell method is used for arbitrarily shaped cells, and it treats all the cells as the convex polytypic uncertainty subsets [7].

4) Removing the subset from the uncertainty

Removing subset Δ_{cell} means that the uncertainty has to be redefined, and the segmentation algorithm may proceed in the rest part of Δ , as illustrated in Fig. 4. Dividing the uncertainty into several parts gives $\{A, B, \dots, H, \Delta_{cell}\} \subseteq \Delta$, and iterative Steps 1) to 3) result in the new uncertainty set Δ_{new} :

$$\begin{aligned} \Delta_{new} &= \{A, B, C\} \cup \{D\} \cup \{E\} \cup \{F, G, H\} = \Delta_{new}^a \\ &= \{A, D, F\} \cup \{B\} \cup \{G\} \cup \{C, E, H\} = \Delta_{new}^b \end{aligned}$$

This algorithm compares the value of $\|F(K, \Delta)\|$ in Δ_{new}^a and Δ_{new}^b until the uncertainty is partitioned into the desired situation. However, if the formed subsets are employed unsuccessfully after several iterations, they are also removed from the uncertainty.

III. NONSMOOTH OPTIMIZATION VIA CLARKE SUBDIFFERENTIAL

The main idea of the multiple robust controller design is to deal with the H_∞ synthesis by attacking the optimization problem in (8). As mentioned, the segmentation algorithm

partitions the uncertainty in a series of subset Δ^i_{cell} . Thus, a numerical nonsmooth method is used for optimizing the objective function $(K^i_{cell}, \Delta^i_{cell})$.

A. H_∞ Synthesis and Robust Stabilization

Given a Linear Time Invariant (LTI) system:

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix} \quad (12)$$

find a static output

$$u = \begin{bmatrix} K_{cell} \\ K_{unpart} \end{bmatrix} y \quad (13)$$

such that

$$\begin{aligned} K^i &= C_K^i (sI - A_K^i)^{-1} B_K^i + D_K^i \\ \alpha(K) &= \max \{ \text{Re}(\lambda) \} < 0 \end{aligned} \quad (14)$$

where K is stable, if and only if $\alpha(K) < 0$. $\alpha(K)$ is the spectral abscissa of matrix K , and λ is the eigenvalue of matrix K .

B. Objective Function and Optimization Algorithm

The objective function of the optimization problem in (8) is defined as:

$$f(K^i_{cell}, \Delta^i_{cell}) = \begin{cases} \arg \min_{\substack{K \in \text{stable} \\ \Delta \in \Delta_{cell}}} \{ -\|F(K, \Delta)\|_\infty^{-1} \}, & \alpha(K) < 0 \\ \|T_{w,z}(K_{cell}, \Delta_{cell})\|_\infty \leq \gamma, & \alpha(K) < 0 \end{cases} \quad (15)$$

This optimization problem is difficult to solve, due to the nonsmoothness of function f [13]. As we know that $\{K_{cell} | \alpha(K_{cell}) < 0\}$ is the set of stabilizing K . Finding a pair of $(K^i_{cell}, \Delta^i_{cell})$ that minimize $\|F(K, \Delta)\|_\infty$ is the same as finding $(K^i_{cell}, \Delta^i_{cell})$ that minimizes $-\|F(K, \Delta)\|_\infty^{-1}$. $\|F(K, \Delta)\|_\infty^{-1}$ has a positive correlation with uncertainty Δ , and $-\|F(K, \Delta)\|_\infty^{-1}$ converges to zero, if $\alpha(K)$ converges to zero. $\|F(K, \Delta)\|_\infty$ can only be evaluated at the stabilizing zone, and this makes the optimization problem in (8) even more difficult. Nevertheless, nonstabilizing $\{K_{cell} | \alpha(K_{cell}) > 0\}$ provides some information to be incorporated into f .

Our optimization algorithm for the problem in (15) has the following steps:

Algorithm 3.1

- 1) **Choose initial point** $K_{cell}^0 \cdot (K_{cell}^0, \Delta_{cell}^0)$ is used in Algorithm 2.1, if no other information is available for initialization.
- 2) **Find subgradient of f** . Use the Clarke subdifferentials of f , and apply a local search algorithm to the gradient descent. Search for a new solution $(K_{cell}^1, \Delta_{cell}^1)$ along with the direction d . The derivative of f with respect to d is defined as:

$$f^\circ(\Delta, \Delta) = \limsup_{t \rightarrow 0^+} \frac{f(K^{i+td}, \Delta^{i+td}) - f(K^i, \Delta^i)}{t} \quad (16)$$

- 3) **Moving**. Define $(K_{cell}^{i+1}, \Delta_{cell}^{i+1}) = (K_{cell}^i, \Delta_{cell}^i) + \Phi(K_{cell}^{i+td}, \Delta_{cell}^{i+td})$. Φ is an iterative search procedure that generate an improvement of the current iterate $(K_{cell}^i, \Delta_{cell}^i)$. If fixed subset Δ_{cell}^i moves to Δ_{cell}^{i+1} , $f(K_{cell}^{i+1}, \Delta_{cell}^{i+1}) < f(K_{cell}^i, \Delta_{cell}^i)$ is satisfied.
- 4) **Termination**. Apply algorithm 2.1 to (7) to obtain a new subset for Step 3). If for several iterations $f(K_{cell}^i, \Delta_{cell}^i) - T < f(K_{cell}^{i+1}, \Delta_{cell}^{i+1}) < f(K_{cell}^i, \Delta_{cell}^i)$, T is acquired, and the optimization algorithm is terminated.

C. Clarke Subdifferential and Gradient Descent

The operator F is defined in (4), which maps $(K_{cell}^i, \Delta_{cell}^i)$ into the space of the H_∞ norm. The function of $g(K, \Delta) = \|F(K, \Delta)\|_\infty$ is Clarke subdifferentiable at (K, Δ) with

$$\partial g(K, \Delta) = F'(K, \Delta)^* \left[\partial \|F(K, \Delta)\|_\infty \right] \quad (17)$$

where $\partial \| \cdot \|_\infty$ is the subdifferential of the H_∞ norm of the objective function, and $F'(K, \Delta)^*$ is the adjoin matrix of $F'(K, \Delta)$ [13].

The subdifferential of $g(K_{cell}^i, \Delta_{cell}^i)$ is obtained, and the subgradients are available by

$$\Theta_g = \|F\|_\infty^{-1} \text{Re}(G_{21}) F^H Q g Q^H G_{12}^T \quad (18)$$

where matrix G_{12} and G_{21} are defined [14]:

$$\begin{bmatrix} F & G_{12} \\ G_{21} & * \end{bmatrix} = \begin{bmatrix} P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} & P_{12}(I - KP_{22})^{-1} \\ (I - P_{22}K)^{-1}P_K & * \end{bmatrix} \quad (19)$$

$$= \begin{bmatrix} C(K) \\ C_2 \end{bmatrix} (sI - A(K))^{-1} \begin{bmatrix} B(K) & B_2 \end{bmatrix} + \begin{bmatrix} D(K) & D_{12} \\ D_{21} & * \end{bmatrix}$$

Here, there are:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, A(K) = A + B_2KC_2, B(K) = B_1 + B_2KD_{21}$$

where P is given in (4), and Q is orthonormal basis of eigenspace $F \cdot F^H$.

IV. NUMERICAL SIMULATIONS

This section demonstrates some numerical simulations for the attitude control system of a satellite with the consideration of both robust stabilization and H_∞ synthesis. The multiple robust controller technique is utilized to improve the robustness as well as dynamical performance. The European Space Agency (ESA) has developed a well-known method to cope with the satellite robust performance design [15]. The work is based on the development of an integrated modeling, analysis, and control framework, which incorporates the robustness analysis via the structured singular value, uncertainty modeling via the LFTs, and various robust control synthesis techniques. In this paper, the nonsmooth optimization techniques to solve the H_∞ synthesis problems under weak coupling parametric uncertainty are applied for the above attitude control system of the satellite.

The attitude dynamics and attitude kinematics of the satellite are defined as in [16]. The disturbing torque $T_d = [T_{dx} \ T_{dy} \ T_{dz}]$ (gravity gradient torque and solar light pressure torque) with $\pm 20\%$ disturbance is:

$$T_{dx} = T_{dz} = (2.5 + \sin w_0 t) \times 10^{-3} N \cdot m, T_{dy} = 0 \quad (20)$$

The initial states of the roll angle are $[\psi(0) \ \dot{\psi}(0) \ \ddot{\psi}(0)] = [0.8 \ -0.35]$. Note that our system only chooses the roll angle channel for the robust synthesis. The synthesis approach is applied on the multiple robust controller design

A series of continuous attitude control laws are given to ensure the system is always finite-time stable, thus guaranteeing the dynamic performance of the state variables, as depicted in Fig. 5 and 6.

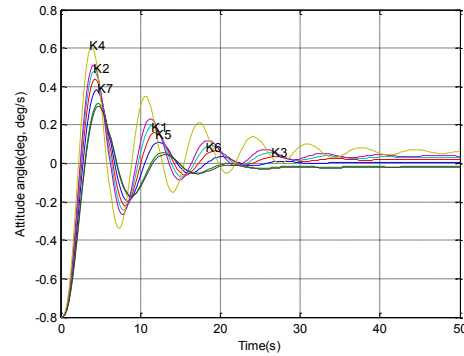


Fig. 5. Attitude angle curves (Pitch Channel I).

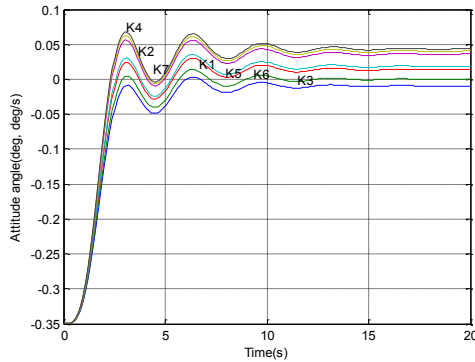


Fig. 6. Attitude angle velocity curves (Pitch Channel II).

In our simulation, the cases of the nominal system in (12) and (13) and the system with parameter uncertainty and external disturbing torque are considered. The simulation results show that the nominal system is finite-time stable and robust against uncertainties and disturbances based on the multiple designed controllers. Moreover, the gravity gradient torque and solar light pressure torque combat with the decoupling uncertainty using the Algorithm 2.1 and Algorithm 3.1. Actually, the control structure combines seven uncertainty subsets and the corresponding controllers. Therefore, this control problem is decomposed into seven lower-order subproblems, which are solved at different frequency bands.

V. CONCLUSIONS

In our paper, the H_∞ synthesis is first decomposed as a series of multiple robust controller designs by partitioning the uncertainty (model uncertainty, disturbance, and control effort constraint) into several subsets. The Clarke subdifferential and LFT techniques used offer powerful tools to deal with the nonsmooth problem. A local search incorporated optimization method is next employed for this nonsmooth optimization problem. An example of robust satellite attitude control demonstrates the effectiveness of the proposed multiple controller strategy.

ACKNOWLEDGMENTS

This research work was funded by the Academy of Finland under Grant 135225 and Finnish Funding Agency for Technology and Innovation (TEKES).

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