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Soft sensor based dynamic flow rate estimation in low speed radial pumps

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Abstract— **Today, poorly controlled low power centrifugal pumps are responsible for a great fraction of the overall pump related energy consumption in the European Union [1]. It is expected that the energy efficiency can be improved considerably with appropriate automatic flow rate control. Automatic flow rate controllers are not popular among practitioners, however, since their installation and tuning is an additional effort, and since they require flow rate sensors, which result in an increased unit cost. We present a dynamic flow rate estimation method which makes automatic control without traditional sensors possible. The setup of the estimation method and all required measurements can be carried out by the manufacturer. As a result, the additional effort for the customer is kept to a minimum.**

I. Introduction

Pumps are used in a wide range of industrial and service applications. Approximately 16% of the overall consumption of electrical energy in the service sector in the European Union [1] is consumed by pumps.

This consumption is mainly caused by a very large number of relatively low power pumping units up to about 30 kW. A closer analysis reveals that the energy efficiency of these units could be improved considerably by automatic variable speed control [2]. In fact, many pumps of this type operate at fixed speed and are manually throttled to the desired flow rate. Since flow rate measurement equipment, its installation, and its continuos maintenance [3] incur additional cost, and since setting up a flow rate controller is an additional effort, automatic flow rate control is often not considered to be an option. In this paper we summarize and compare existing approaches to flow rate estimation in variable speed centrifugal pumps without traditional flow rate sensors. We present a new hybrid estimation method that provides sufficiently accurate estimation results even under transient operating conditions. Furthermore, the proposed method implicitly calculates a dynamic model of the pumping unit which serves as reference model for a flow rate controller synthesis.

II. Notation and Preliminaries

Let q and $[q_{\min}, q_{\max}]$ denote the flow rate (measured in m^3/s, though we use m^3/h in all figures for ease of interpretation) and its operating range, respectively. Correspondingly, denote the pump outlet pressure (Pa) by $p_e \in [p_{e,\min}, p_{e,\max}]$, the pump differential pressure (Pa) by $\Delta p \in [\Delta p_{\min}, \Delta p_{\max}]$, the rotational speed (1/min) by $n \in [n_{\min}, n_{\max}]$, the pump power (W) by $P \in [P_{\min}, P_{\max}]$, and the effective stator current (A) in the electrical drive of the pump by $i \in [i_{\min}, i_{\max}]$. The differential pressure is often substituted by the *pump head* $h = \Delta p/\rho g = [h_{\min}, h_{\max}]$, which is usually measured in m, where the bounds on Δp and h are related in the obvious way. The pumped fluid is assumed to be water under standard conditions for simplicity.

Pumps are commonly characterized by several input-output relationships ("curves" or "characteristics"). The *q-h-curve* describes the pump head h as a function of the flow rate q, where the rotational speed n is assumed to be constant. The *q-h-n-diagram* is a collection of q-h-curves for various n. The *q-P-curve* and the *q-i-curve* describe the pump power P and the effective stator current i, respectively, as a function of the flow rate q for constant rotational speed n. The relation between power P and effective stator current i is described by the ***P-i-curve***. The hydraulic system into which the pump is embedded is commonly described by the *process-curve*, which relates the pump outlet pressure p_e to the flow rate q and the static pressure height h_{stat} (see Fig. 3 for an example).

III. Model-based flow rate estimation methods

Existing methods for flow rate estimation belong to one of two categories. The *process-based strategy* uses a pump outlet pressure measurement to estimate the flow rate through the process, where a process-curve serves as a very simple static process model. On the other hand, *pump-based strategies* use either a differential pressure measurement or online torque and speed measurements provided by a frequency converter.

A. Process based flow rate estimation

If a pump outlet pressure measurement is available (cf. Fig. 1), the characteristic process-curve (1) can be used to estimate the flow rate [4]. The process-curve always consists

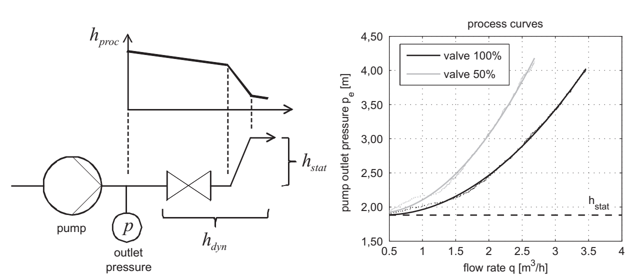

Fig. 1: Sample process with measured h_{proc} at two different valve openings

of a static and a dynamic part

$$h_{\text{proc}}(q) = h_{\text{stat}} + h_{\text{dyn}}(q), \tag{1}$$

where the static part h_{stat} depends mainly on the process layout and the dynamic part models the friction losses. Since

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$h_{\mathrm{dyn}}(q)$ usually is monotonic (cp. Fig. 1), the process-curve can be uniquely inverted and therefore be used to infer q if p_e is measured.

The field of application of the process-curve based estimation method is significantly limited by the fact that a constant process-curve is required. This renders a process curve useless in, for example, water distribution networks, where varying water consumption results in changes of the network structure, which in turn cause the static head and the friction losses to change (see Fig. **??** for an illustration of the error caused by mismatches between the process-curve and the actual process). The need of an in-situ measurement of the process-curve, which requires customer action, may be seen as an additional drawback, though fully automatic algorithms for these measurements exist [4].

B. Pump based flow rate estimation

In contrast to process-based estimation, a pump-based estimation is independent of the specific application, it can be implemented by the manufacturer prior to delivery, and therefore does not require any customer action. There exist several approaches. We summarize those aspects that are relevant to the new approach described in Sect. IV.

1) q-h-curve flow rate estimation method: If the pump differential pressure Δp can be measured, the flow rate q can be estimated from the q-h-curve or q-h-n-diagram of the pump. For typical low speed radial pumps, the q-h-curve has a parabolic shape [5] and therefore can be fitted to a function of the form

$$h(q) = \frac{\Delta p(q)}{\rho g} = C_{p0} + C_{p1} q + C_{p2} q^2. \quad (2)$$

Variable speeds n can be accounted for with the well known hydraulic affinity laws [5]

$$h = h_0 \left(\frac{n}{n_0}\right)^2 \frac{\eta_h}{\eta_{h,0}},\ q = q_0 \left(\frac{n}{n_0}\right) \frac{\eta_q}{\eta_{q,0}} \quad (3)$$

with h_0, q_0, η_0 denoting differential pressure, flow rate and the corresponding efficiency factors at nominal rotational speed n_0. Extending (2) by (3) while assuming constant efficiency factors leads to a function that describes the q-h-n-diagram

$$h(q,n) = C_{p0} \left(\frac{n}{n_0}\right)^2 + C_{p1} q \left(\frac{n}{n_0}\right) + C_{p2} q^2, \quad (4)$$

where we use the same symbol h in (2) and (4) for simplicity.

Most low speed radial pumps show unstable q-h-curves with partially non-negative gradient (type "F" instability [5], see Fig. 2 for an example). Consequently, q cannot be determined uniquely for given h and n. In this case, flow rate estimation is limited to the stable part of the q-h-curve or has to be used in combination with other methods. In addition, the estimation performance is poor in the low flow rate region due to the low sensitivity dh/dq, i.e. in the flat part of the q-h-curve. Admittedly, the unstable region is often avoided in practice for other reasons, for example, because it results in a high stress on the pump components. The small size pumps of interest here, however, can be operated at $q = 0$ without

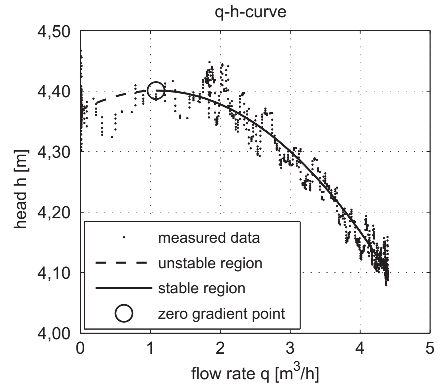

Fig. 2: q-h-curve of a typical low speed radial centrifugal pump at nominal operating speed

problems. Finally, it must be noted that the assumption of a constant efficiency factor η in (3) is only valid in a margin of about 20% around the operating point [6] (see Fig. 10 for an illustration).

2) q-P-curve flow rate estimation method: The vast majority of pumps is driven by squirrel cage induction motors [2], which can be equipped with frequency converters for variable speed control. Two control strategies exist: scalar based control which uses a fixed voltage/frequency relation (U/f-curve), and model based control algorithms [11], where the latter can further be subdivided into direct torque control (DTC) and flux oriented control (FOC). DTC and FOC are standard in modern frequency converters and undergo continuous optimization [12]. Both methods can control and estimate motor torque and speed even in transient operating conditions [9], [10]. The estimated power $P = torque \cdot speed$ can be used to estimate the flow rate q if a combined q-P-curve of pump and motor is available to describe the relation between flow rate q and pump shaft power. The pump shaft power P_s typically consists of a hydraulic and a mechanical part

$$P_s(q,n) = P_{\mathrm{hyd}}(q,n) + P_{\mathrm{mech}}(n), \quad (5)$$

where P_{mech} depends on various factors such as the friction in bearings and seals. The hydraulic power P_{hyd} can be calculated from (4) according to

$$P_{\mathrm{hyd}}(q,n) = q \cdot h = C_{p0} q \left(\frac{n}{n_0}\right)^2 + C_{p1} q^2 \left(\frac{n}{n_0}\right) + C_{p2} q^3. \quad (6)$$

The relation between P_{mech} and n is more complex and depends on the actual design of the shaft, bearings and seals. While a precise first principles modelling may be attractive, a set of defined measurements can lead to a reliable description that does not suffer from the restrictions induced by the affinity laws mentioned in Sect. III-B.1 [7].

3) q-i-curve flow rate estimation method: A flow rate estimation can be based on a measurement of the effective stator current i. This measurement is provided by most frequency converters even when operating in static U/f mode (which is common in pumping applications), i.e. even when no torque and speed estimation is available. The relation between the

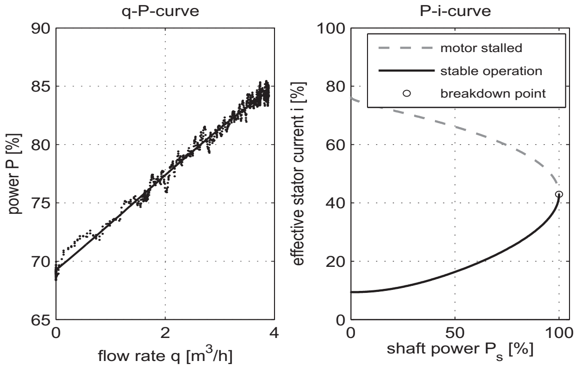

Fig. 3: q-P-curve of a low speed radial centrifugal pump at nominal operating speed and model-based P-i-curve of a standard induction motor

effective stator current i and flow rate q does not only depend on the pump characteristics but also on the motor and the settings of the frequency converter. Motor parameters, such as stator and rotor resistances and inductivities, are usually not disclosed by manufacturers. Consequently, first principles modelling is difficult. An example for a representative motor is shown in Fig. 3. Since the P-i-curve is invertible over the whole region of stable motor operation, P_s can be inferred from a measurement of i. The flow rate can be estimated with the q-P-curve subsequently. If changes in rotational speed have to be accounted for, frequency converter settings have to be known, since the U/f-curve affects the relation between rotational speed and the effective motor current. Varying motor temperatures may also affect the effective stator current. Since an internal motor temperature measurement (e.g. PT100) is available in most applications (typically because over temperature protection is required), it can be used to adjust the stator current measurement according to the motor temperature.

4) Hybrid flow rate estimation methods: Hybrid methods have been devised in an attempt to mitigate some of the problems mentioned in the preceding sections. Ahonen et al. [4] show that a process-curve can be determined with the pump. The authors of [8] present a method that is based on the quotient $P/\Delta p$ as a function of q, which is shown to provide more accurate estimations than those based on q-Δp- or q-P-curves. Because the quotient is independent of fluid density, the method is robust with respect to varying fluid parameters. To overcome the limited accuracy of the hydraulic affinity laws, Kernan et al. [7] present a method that measures power versus rotational speed with closed discharge valve to calculate a calibration curve. This calibration curve is used to adjust the q-P-curve according to the actual rotational speed. The approach implicitly accounts for the influence of a variable efficiency factor. It results in an accurate flow rate estimation over a wide rotational speed range.

IV. Boundary Curve Flow Rate Estimation Method

In this section we present a new method for the online estimation of the flow rate q through centrifugal pumps. It is our goal to cover all relations between frequency converter, motor and pump automatically over the entire hydraulic operating region $q \in [q_{\min}, q_{\max}]$ of the pump. We recall that the method needs to be simple to be accepted, which, unfortunately, rules out many first principles models. Our approach is similar to the one presented in [7], where a single calibration curve is used to adjust the q-P-curve of the pump. Essentially, our approach involves *two* such calibration curves for two limiting cases, resulting in an increased steady state performance. Moreover, our method addresses problems that occur during transient operation and considerably improves transient estimation performance.

We assume a q-i-curve $q : [i_{\min}, i_{\max}] \to [q_{\min}, q_{\max}]$ and its inverse, which we denote by q^{-1}, are available in the remainder of the section. We use the normalized inverse, $q_{\text{norm}}^{-1} : [0,1] \to [0,1]$, to simplify expressions. The normalized inverse maps normalized flow rates $q_{\text{norm}} = (q - q_{\min})/(q_{\max} - q_{\min})$ to normalized stator currents $i_{\text{norm}} = (i - i_{\min})/(i_{\max} - i_{\min})$. We furthermore assume that the frequency converter output voltage corresponds to a predefined U/f-curve.

The static model proposed here relies on the *stator current envelope* that requires to measure the *lower* and *upper boundary curve*.

Lower boundary curve measurement

The lower boundary curve is obtained by measuring the effective stator current as a function of the rotational speed while the discharge valve is closed. Since the flow rate q is forced to be zero, the hydraulic power P_{hyd} attains its minimum. Therefore, the effective stator current is measured at its lowest possible value i_0 for every rotational speed n. We refer to the underlying function

$$i^{\text{low}} : [n_{\min}, n_{\max}] \to [i_{\min}, i_{\max}]$$

as the *lower boundary curve*. It is approximated by fitting a polynomial

$$i^{\text{low}}(n) = \sum_{j=0}^{d_{i^{\text{low}}}} c_{i^{\text{low}},j} \cdot n^j \qquad (7)$$

to the measured data, where the degree $d_{i^{\text{low}}}$ is case dependent. In section V we show that lower boundary curve measurement can also be used to determine a dynamic model of the pumping unit.

Upper boundary curve measurement

The second limiting case is covered by measuring the stator current i as a function of the rotational speed n for maximum flow rate[1], $q \overset{!}{=} q_{\max}$. We refer to the underlying function

$$i^{\text{high}} : [n_{\min}, n_{\max}] \to [i_{\min}, i_{\max}]$$

as the *upper boundary curve*, and assume that a polynomial of appropriate degree can be fitted to the measurements as described above. These measurements can also be used to

[1]The maximum flow rate $q_{\max}$ defines the upper normal operation margin as intended by the manufacturer. A violation of this margin may lead to cavitation and/or overload. Flow rate estimation is therefore not provided outside the normal operation region but can be used to indicate a violation.

determine an approximation of the maximum flow rate as a function of the rotational speed n, which we denote by

$$q^{\text{high}} : [n_{\min}, n_{\max}] \to [q_{\min}, q_{\max}]$$

and refer to as the *maximum flow rate curve*.

Stator current envelope

As long as the pump is operated within the allowed intervals of $n \in [n_{\min}, n_{\max}]$ and $q \in [q_{\min}, q_{\max}]$ the stator current $i(q, n)$ is bounded according to $i^{\text{low}}(n) \leq i(n) \leq i^{\text{high}}(n)$ by construction of i^{low} and i^{high}. In this sense, i^{low} and i^{high} form the stator current envelope. Values that do not lie within the envelope indicate that the pump left its normal operating region. An example for the stator current envelope is shown in Fig. 8.

The estimated flow rate q_{est} can be calculated from

$$q_{\text{est}}(n, i) = q^{\text{high}}(n) \cdot q_{\text{norm}}^{-1}(x(n, i)), \tag{8}$$

where

$$x(n, i) = \frac{i - i^{\text{low}}(n)}{i^{\text{high}}(n) - i^{\text{low}}(n)}. \tag{9}$$

Notice that (9) involves a linear interpolation between the limiting curves i^{low} and i^{high}.

We briefly note that the boundary curve method is not restricted to a specific measured variable (i.e. the effective stator current). Furthermore, it can be extended to a hybrid method similar to those discussed in Sect. III-B.4. This is necessary, for example, if q_{norm} cannot be inverted uniquely. Figure 7 illustrates a hybrid method that combines stator current and differential pressure measurements.

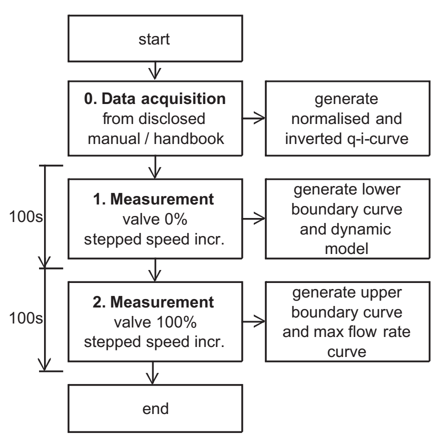

Fig. 4: Measurement sequence to set up the boundary curve method

V. Improving performance during transient operation

All flow rate estimation methods presented in the preceding sections work well during steady state operation. During transient operation, especially during rotational speed changes, large deviations can occur between the estimated and actual flow rates. An illustration is shown in Fig. 11. In the static estimation case, n and i are input signals to equation (9). Clearly, different dynamics of the input signals lead to a miscalculation of the position $x(n, i)$ during transient operation. In our case the speed signal n inherits much faster dynamics than the effective stator current measurement i, since n is a setpoint signal from the process control system (PLC) (cf. Fig. 7). To address this problem, we use a first order differential equation with an additional delay to model the pump and motor inertia as well as the lag and delay of the effective stator current measurement device

$$T_L \frac{di(t)}{dt} + i(t) = K(n, q) n(t - T_D), \tag{10}$$

where $K(n, q)$ denotes a nonlinear function that covers the steady state relation between n, q and i. Since first principles modelling of K is difficult (i.e. due to unknown parameters), we use the boundary curve method presented in section IV to describe the steady state behaviour. Using (10) with $K = 1$ and substituting i with n_{adj} leads to a speed signal n_{adj} that inherits the same dynamic behaviour as the measured effective stator current in relation to rotational speed changes.

$$G_{\text{dyn}}(s) = \frac{n}{n_{\text{adj}}} = \frac{1}{T_L s + 1} e^{-T_D s} \tag{11}$$

Therefore, using n_{adj} and i in (9) and (8) results in an exact calculation of q_{est} during transient operation. The time constants T_L and T_D can be determined from step responses obtained during the measurement of the lower boundary curve as follows (see Fig. 5 for an illustration). Therefore, no additional measurement is needed for the

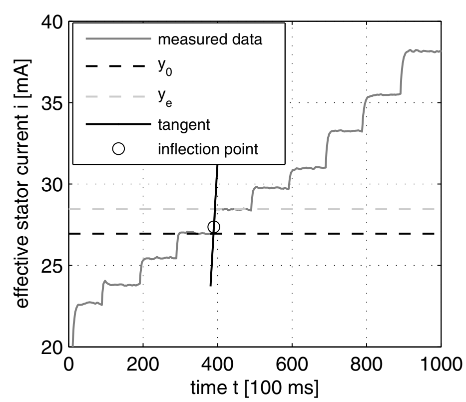

Fig. 5: Speed step responses of the effective stator current measurement, see (12) for the resulting dynamic reference system

setup of the dynamic estimation. We note that the dynamic model does not cover fluid inertia since it does not affect the dynamic estimation. However, fluid inertia may have to be considered if the dynamic model should be used for a controller synthesis.

VI. Performance evaluation

We apply the proposed approach to the test setup sketched in Fig. 6. All components are standard industrial components. The pumping unit consists of a low speed radial pump (KSB Etanorm G32-125.1), a standard 0.55kW squirrel cage induction motor and a frequency converter (KSB "Pump-Drive"). The rotational speed bounds for this setup read

$n_{\min} = 800\,\mathrm{min}^{-1}$ and $n_{\max} = 1500\,\mathrm{min}^{-1}$, the maximum possible head is $h_{\max} = 4.3\,\mathrm{m}$ and the maximum possible flow rate is $q_{\max} = 12\mathrm{m}^3/\mathrm{h}$. The sensing equipment

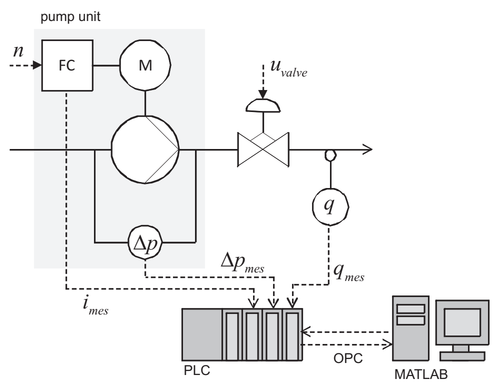

Fig. 6: Setup of the test stand used for measurements and performance evaluation

consists of a pump mounted differential pressure device, an ultrasonic reference flow rate sensor, a motor temperature sensor, and the frequency converter output, which is configured to measure the effective stator current. Measured data are logged by a PLC system and connected via OPC to a MATLAB interface, which carries out the estimation calculations, processes measurements, and sends setpoints to the system. An overview of the estimation scheme is given in Fig. 7. Measurements are carried out as described in Fig. 4.

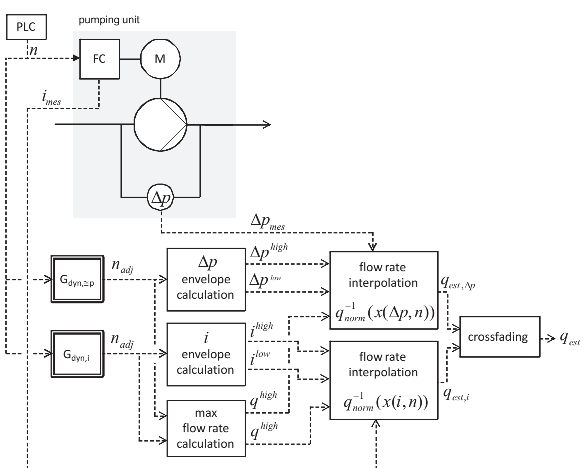

Fig. 7: Boundary curve method using stator current and differential pressure

The resulting boundary curves and the normalised i-q-curve are depicted in Fig. 8.

A. Steady state performance

We compare four methods for the estimation of q during steady state operation, which we refer to as method 1–4 for short: The process-curve based method as described in [4] (method 1, cf. Sect. III-A), a method based on the affinity laws (method 2, cf. Sect. III-B.1), the single calibration curve method as described in [7] (method 3, cf. Sect. III-B.4), and the boundary curve method proposed in the present paper (method 4, cf. Sects. IV and V). All four methods are tested at four different rotational speeds with discharge valve openings of 25%, 50%, 75% and 100%. Figure 9 lists the relative estimation error $e_{q,rel} = \Delta q/q_{\mathrm{BEP}}$, where $e_{q,rel} = \Delta q/q_{\mathrm{BEP}}$ with $\Delta q = |q - q_{\mathrm{est}}|$ and $q_{\mathrm{BEP}} = 8m^3/h$. The symbols q, q_{est} and q_{BEP} refer to the actual and measured flow rates, and the flow rate at the best efficiency point, respectively.

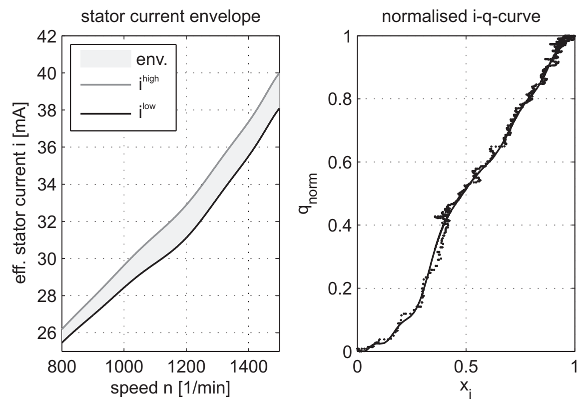

Fig. 8: Stator current envelope and normalized inverse q-i-curve q^{-1}_{norm} of the example test setup

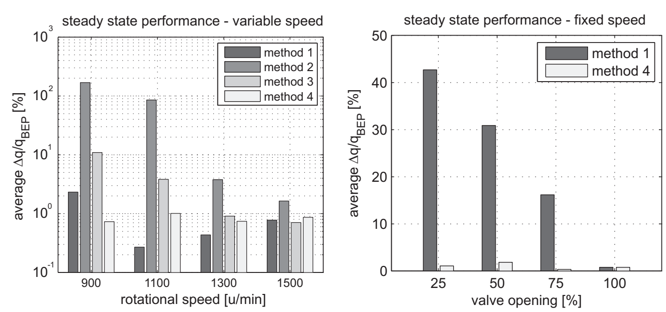

Fig. 9: Left: Relative estimation error during steady state operation at different rotational speeds for methods 1–4; Right: Relative estimation error during steady state operation with fixed speed at different discharge valve openings

These results indicate that the estimation based on the affinity laws (method 2) can only be used reliably for a fixed rotational speed or in a narrow range. This low performance is based on the limited accuracy of the affinity laws, especially when a fixed efficiency factor η is assumed. Incorporating a variable η improves the estimation performance as shown in Fig. 10, though it requires a function that relates η to the rotational speed n.

In comparison to the process and the boundary curve methods (methods 1 and 4, respectively), the single calibration curve method (method 3) gives higher estimation errors for low rotational speeds. This results, because the q-i-curve gradient varies only slightly with n, while the overall shape of the curve remains nearly constant. Information about the q-i-curve gradient is contained in the distance between i^{low} and i^{high}, therefore the boundary curve method is able to adjust the gradient, leading to improved estimation results in the low speed region.

The process curve method also shows high steady state

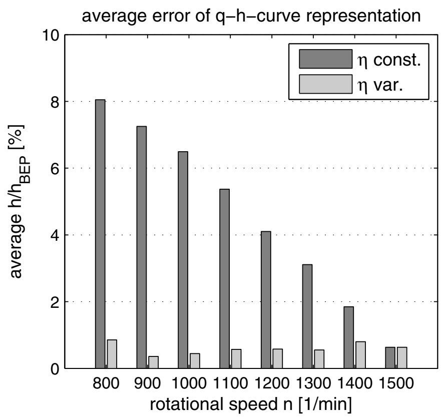

Fig. 10: Average error of the q-h-curve representation of a low speed centrifugal pump with constant and variable efficiency factor η

estimation quality which may not hold in practice since the calculation of the friction factor needs the actual flow rate. The results shown in the left diagram in Fig. 9 were generated using a precise flow rate measurement[2] for the calculation of the friction factor and not as proposed in [4] by using a possibly less precise q-P-curve based flow rate estimation. Additionally, using the process curve method for processes with variable friction factor leads to high estimation errors as shown the right diagram in Fig. 9.

B. Transient performance

The dynamic estimation model described in section V can be used to significantly improve transient performance. The automatic calculation of the dynamic reference model from the effective stator current measurement shown in Fig. 5 yields

$$G_{\mathrm{dyn,i}}(s) = \frac{1}{4.44s + 1} e^{-8.78s} \tag{12}$$

This dynamic model is used to calculate the adjusted speed signal n_{adj} which is used in (8) and (9) together with i to calculate the estimated flow rate q_{est}. Figure 11 shows a sample time series to illustrate the improved transient performance. It is obvious from Fig. 11 that the static

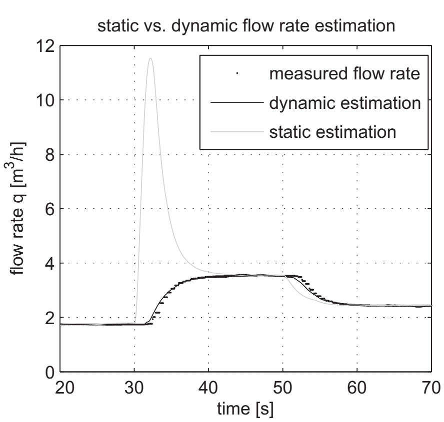

Fig. 11: Estimation quality during transient operation

[2]In most applications a reference flow rate measurement will not be available, since this would obviate the need for a flow rate estimation.

estimation leads to problems when used for continuous flow rate control purposes. The estimation based on the dynamic model shows nearly the same dynamics as the actual flow rate measurement. Moreover, the dynamic model simplifies the flow rate controller synthesis.

VII. Conclusion

We proposed an approach to estimating the flow rate through centrifugal pumps in the absence of a flow rate sensor. It was our main goal to achieve precise estimations even during transient operation with as little as possible implementation effort for the customer. We compared our approach to existing ones and demonstrated that significant improvements in transient performance can be achieved with little additional effort compared to established state-of-the-art methods.

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References

[1] A.T. de Almeida, P. Fonseca, H. Falkner and P. Bertoldi *Market transformation of energy-efficient motor technologies in the EU* Energy Policy, vol. 31, 2003.
[2] F. J. T. E. Ferreira, J. A. C. Fong, and A. T. de Almeida, *Ecoanalysis of variable-speed drives for flow regulation in pumping systems* Energy Policy, vol. 31, 2003.
[3] R.C. Baker *Flow Measurement Handbook* Cambridge University Press, 2005.
[4] T. Ahonen, J. Tamminen, J. Ahola and J. Kestilä, *Frequency-Converter-Based Hybrid Estimation Method for the Centrifugal Pump Operational State* IEEE Trans. on Industr. Electr., vol. 59, NO. 12, December 2012.
[5] J.F. Gülich, *Centrifugal Pumps (in German)*, Heidelberg, 2010.
[6] P. Muszynski *Impeller pumps: relating η and n* World Pumps, vol. 2010, NO. 7, July 2010.
[7] D.J. Kernan *Method for determining pump flow without the use of traditional sensors* U.S. Patent, 7,945,411, May 2011.
[8] KSB corp. *Flow Rate Measurement (in German)* German Patent DE10359726A1
[9] T.G. Habetler, F. Profumo, M. Pastorelli and L.M. Tolbert, *Direct Torque Control of Induction Machines Using Space Vector Modulation* IEEE Trans. on Ind. Appl., vol. 28, NO. 5, October 1992.
[10] D. Casadei, F. Profumo, G. Serra and A. Tani, *FOC and DTC: Two Viable Schemes for Induction Motors Torque Control* IEEE Trans. on Power Electr., vol. 17, NO. 5, September 2002.
[11] G.S. Buja and M.P. Kazmierkowski *Direct Torque Control of PWM Inverter-Fed AC Motors - A Survey* IEEE Trans. on Power Electr., vol. 17, NO. 5, September 2002.
[12] T. Geyer, G. Papafotiou and M. Morari *Model Predictive Direct Torque Control - Part I: Concept, Algorithm and Analysis* IEEE Trans. on Industr. Electr., vol. 56, NO. 6, June 2009.