2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Fractional order control of the injection system in a CNG engine*

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Abstract— **Common rail injection systems of compressed natural gas (CNG) engines require robust control, because of large variations in their parameters and working conditions and because of their highly nonlinear behavior. This paper shows that a noninteger (fractional) order PI (or FOPI) controller, designed by a loop-shaping approach, improves robustness to parameter variations and guarantees an optimal performance. Efficacy of the FOC is tested by simulation based on a CNG linearized model and on the approximation of the FOC by a rational transfer function.**

KeyWords: Fractional-order proportional-integral controllers, loop-shaping, approximation of irrational operators, compressed natural gas engines, injection systems.

I. INTRODUCTION

In a compressed natural gas (CNG) common rail injection system, pressure control is necessary for accurately metering the air-fuel mixture that strongly affects combustion efficiency, and thus consumptions, performance, and pollutant emissions. However, the control task is difficult because the working conditions vary in large extent due to the gas compressibility. Hence, to obtain robustness to parametric variations and model uncertainties, this paper proposes noninteger (fractional) order controllers (FOC). A FOC is designed as follows.

Firstly, a reliable and accurate state-space nonlinear model with two state variables is linearized at different equilibrium points, depending on the working conditions set by the driver power request, engine speed and load. The linearized models yield transfer functions taking into account the dominant dynamics, i.e. the filling/empting of the common rail volume that is larger than ones of the other components in the injection system. Only one input is considered as driving command. Then, a transfer function is used to derive a control law for regulating the injection pressure. The structure of the transfer function is characterized by the static gain, a time constant, and a dead time. This is the most common process model used in papers on classical controller tuning.

As for the tuning of the FOC, a recently developed loop-shaping approach is applied to set the parameters of a fractional order PI controller. To synthesize, a first tuning criterion seeks an optimal feedback control system, where "the absolute value of the return difference is at least one at all frequencies" [1]. A second criterion aims at obtaining robust stability to gain and parameter variations in the loop, by keeping a nearly constant phase margin in a sufficiently wide frequency range around the gain cross-over frequency, that in turn is strictly dependent on the desired closed-loop bandwidth.

The tuning criteria provide relatively simple formulas that directly relate performance specifications to the FOC parameters. The formulas are new, even if they share the same criteria underlying the tuning of FOC for systems with different models. Simulation experiments confirm the simplicity and efficacy of the method in satisfying performance and robustness requirements. The paper is organized as follows. Section II introduces the CNG system. After a brief background on fractional calculus and fractional order control in Section III, the design method is presented in Section IV. Then, Section V illustrates how to approximate fractional operators for realization and Section VI shows simulation results. Finally, Section VII draws some conclusions.

II. THE COMPRESSED NATURAL GAS SYSTEM

The main parts of the common rail injection system for CNG engines are (see [2], [3] for details): a fuel tank storing high pressure gas, a mechanical pressure reducer, including a main chamber and a control chamber, a solenoid valve, and a fuel metering system, consisting of a common rail and four electro-injectors (Fig. 1). The fuel from the pressure reducer directly flows towards the rail, and the solenoid valve regulates the intake flow in a secondary circuit including the control chamber. The injection flow only depends on the rail pressure, which is almost equal to the main chamber pressure, and on the injection timings, that are precisely driven by an Electronic Control Unit (ECU). Then the flow control acts on the solenoid valve, as described in the following. The ECU sets the injectors opening time intervals t_j, depending on the engine speed and load. The whole injection cycle takes place in a 720° interval, with a 180° delay between each injection command.

The variable inflow section of the pressure reducer is varied by the axial displacement of a spherical shutter coupled with a moving piston. Gas pressure in the main chamber acts on the piston lower surface by pushing it up, and gas pressure in the control chamber pushes it down and causes the shutter opening. When the solenoid valve is energized, the fuel enters the control chamber and builds up the pressure on the upper surface of the piston. Then, the piston is pushed down with the shutter, and more fuel enters into the main chamber, where the pressure increases. On the contrary, if the solenoid valve is not energized, the pressure on the upper side of the piston decreases. Thus the piston raises and the main chamber shutter closes under the action

*This work was supported by the Italian Ministry of University and Research (MIUR) under project *"Non integer order systems in modeling and control"*, grant no. 2009F4NZJP.

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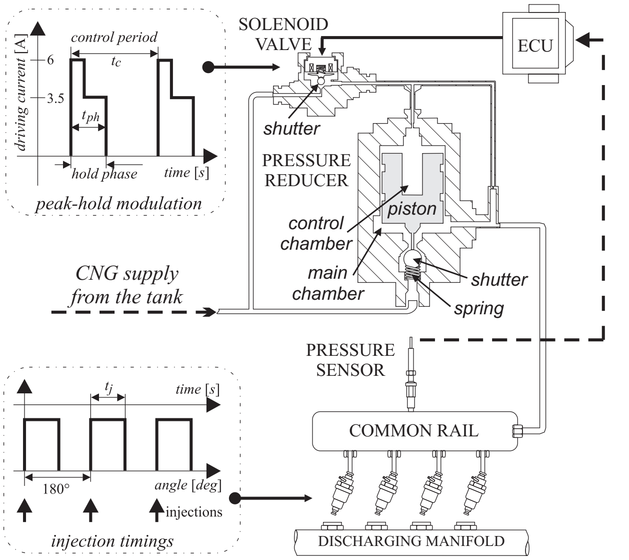

Fig. 1. Block scheme of the common rail CNG injection system

of a preloaded spring. The solenoid valve driving current is applied for a t_{ph} time interval, and it consists of a peak phase followed by a variable duration hold phase. Now let t_c be the valve control period, i.e. the time interval between two leading edges of the current signal. Then, varying the duty cycle of the driving current in a control period, namely the $t_{ph}\backslash t_c$ ratio, makes the valve to open and close in turn, so as to control the chamber pressure (Fig. 1).

A. The CNG Injection System Model

A second order state space nonlinear model describes the pressure dynamics in the injection system [3]. It considers two control volumes having a uniform, time varying, pressure distribution: the regulator control chamber and the rail circuit, respectively. The tank pressure acts as an input rather than a state variable, because its measure is always available on board as it is related to the fuel supply and it is approximatively constant within a large time interval. Furthermore, by assumption, the injection pressure equals the one of the rail, so that we don't detail the model of electro-injectors, but include them in the rail circuit as control electronic valves. Finally, assuming a constant temperature in the whole system, the dynamics can be completely defined by the pressure variations in the control chamber and the rail circuit. The control chamber, the rail, and the tank pressures are denoted by x_1, x_2, and p_{tk}, respectively. Moreover, u_1 and u_2 are the signals driving the solenoid valve and the injectors, respectively. With this notations the CNG injection system model is [3]:

$$\begin{cases} \dot{x}_1(t) = c_{11}p_{tk}(t)u_1(t) - c_{12}\sqrt{x_2(t)[x_1(t)-x_2(t)]} \\ \dot{x}_2(t) = c_{21}p_{tk}(t)[c_{24}x_1(t) - c_{25}x_2(t) - c_{26}p_{tk}(t) - c_{27}] \\ \qquad\qquad - c_{22}x_2(t)u_2(t) + c_{23}\sqrt{x_2(t)[x_1(t)-x_2(t)]} \end{cases} \tag{1}$$

where c_{ii} are constant coefficients. The model is appropriate for control, because the tank and rail pressures allow us to approximately but significantly describe the complex dynamics of the nonlinear system in several distinct equilibrium points. Moreover, the rail pressure is the main controlled variable to achieve the desired performance in CNG injection [3].

Now, we linearize the equations (1) at different equilibrium points, so that each tuning of the controller parameters refers to a current working point. This simplification is justifiable because the control action has to keep the pressure close to a reference value, in dependence of the working conditions set by the driver power request, speed and load. Linearization with respect to $\bar{\mathbf{x}}$ and $\bar{\mathbf{u}}$ yields: $\delta\dot{\mathbf{x}}(t) = \mathbf{A}\delta\mathbf{x}(t) + \mathbf{B}\delta\mathbf{u}(t)$, where $\delta\mathbf{x}(t) = \mathbf{x}(t) - \bar{\mathbf{x}}$ and $\delta\mathbf{u}(t) = \mathbf{u}(t) - \bar{\mathbf{u}}$. Choosing $\delta x_2(t)$ as output gives $\delta y(t) = \mathbf{C}\delta\mathbf{x}(t)$, where $\mathbf{C} = [0\ 1]$. See [3] for further details on the elements $a_{i,j}$ and $b_{i,k}$ of $\mathbf{A}$ and $\mathbf{B}$.

To compute $\mathbf{A}$ and $\mathbf{B}$, equilibrium pressures can be determined from (1). However, since the valve and injectors driving signals, $\bar{u}_1$ and $\bar{u}_2$, result from modulation of discrete values, they cannot be used directly into the equations. Then, we consider their mean values within an injection cycle instead of the instantaneous values. In particular, $\bar{u}_1$ is the driving current duty cycle, while $\bar{u}_2 = 4 \cdot t_j \cdot \upsilon/120$, i.e. the ratio between the injection time interval t_j and the duration of whole injection cycle, multiplied by the number (4) of injectors and the engine speed υ in rpm. Under this assumption, the linear model coefficients are determined by the following steps. First, simulation and experimental tests point out that the tank pressure is the main variable affecting controller performances, thus allowing to set different main families of sub-models. Hence, to inject the proper fuel amount corresponding to the actual working conditions, we draw the appropriate values of the opening time interval t_j (as a function of engine load and rail pressure x_2) and frequency $4 \cdot t_j \cdot \upsilon/120$ (as a function of engine speed). Furthermore, we obtain u_2 from a look-up table. Finally, after determining the equilibrium conditions by fixing p_{tk}, x_2 and u_2, we compute u_1 and x_1 to obtain the linear model.

The state space linear model straightforwardly yields a 1×2 transfer-function matrix, whose elements give the relations between δy and the inputs δu_1 and δu_2, respectively. We derive a family of transfer functions suitable for the controller design by considering δu_1 as the control signal. With some abuse of notation, we drop the symbol δ, use u instead of u_1, and obtain:

$$y(s) = \frac{a_{21}b_{11}}{s^2 - (a_{11}+a_{22})s + a_{11}a_{22} - a_{12}a_{21}} u(s) \tag{2}$$

where s is the Laplace variable. Transfer function in (2) has the real poles $\{s_1, s_2\}$ on the left-half s-plane. Namely, for every value of the variable system parameters, it always holds that $a_{11}, a_{22} < 0$ and $a_{12}, a_{21} > 0$; moreover, the discriminant of characteristic equation can be put in the form $\Delta = (a_{11} - a_{22})^2 + 4a_{12}a_{21} > 0$) (see [3]). If we leave apart the working point, then one pole, say s_1, has a dominant effect on the transient response. Thus, if we neglect the smaller time constant and introduce a time delay τ for the pressure propagation from the main chamber to the common rail, then we derive a family of first order plus time delay models:

$$y(s) = G(s) \cdot u(s) = \frac{Ke^{-\tau s}}{1+Ts} u(s) \tag{3}$$

where $K = a_{21} \cdot b_{11}/(s_1 \cdot s_2)$ and $T = -1/s_1$. The delay τ can be experimentally determined. The linearized model is sufficiently accurate to represent the nonlinear CNG injection system, as it was tested by simulation in different working points. Moreover, we may compare different controllers that can be designed for first order plus time delay systems.

Since the system may have several equilibrium points, the parameters in (3) are subject to variations. Then, different values of K, T, and τ can be obtained and used as reference to design the FOC. Then, a gain scheduling approach should be adopted to adjust the controller to variations in the CNG system parameters. However, this approach is beyond the scope of this paper, and it is omitted for sake of space.

III. BACKGROUND ON FRACTIONAL CALCULUS AND FRACTIONAL CONTROL

After the pioneering studies in fractional calculus of some scientists of the past centuries (Leibniz, Heaviside, etc.), there have been many attempts to define and exploit noninteger (fractional) integrals and derivatives. The popular Riemann-Liouville definition of fractional derivative of order α of a function $f(t)$ with respect to time t and starting point a is:

$${}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^{(n)}}{dt^{(n)}} \left\{ \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \right\} \tag{4}$$

where $\alpha \in \mathbb{R}^+$ is the non-integer order of differentiation, with $n \in \mathbb{N}$ and $n-1 < \alpha < n$, and where $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ is the Euler gamma function. The Grünwald-Letnikov formulation is often used for discrete-time implementation:

$$\begin{aligned} {}_aD_t^{\alpha} f(t) &= \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{i=0}^{[\frac{t-a}{h}]} (-1)^i \binom{\alpha}{i} f(t-ih) \\ &= \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{i=0}^{[\frac{t-a}{h}]} (-1)^i \frac{\Gamma(\alpha+1)}{\Gamma(i+1)\Gamma(\alpha-i+1)} f(t-ih) \end{aligned} \tag{5}$$

where $[x]$ gives the integer part of x and h is the step time increment. However, we here consider the Caputo's definition:

$${}_0D_t^{\alpha} f(t) = \frac{1}{\Gamma([\alpha]-\alpha)} \int_0^t \frac{f^{([\alpha])}(\tau)}{(t-\tau)^{\alpha+1-[\alpha]}} d\tau \tag{6}$$

where $[\alpha] = n$ is the smallest integer such that $\alpha < [\alpha]$ and $f^{([\alpha])}$ denotes the standard derivative of integer order $[\alpha]$. This choice is because the commonly used initial conditions appear in the Laplace transform

$$\mathscr{L}\left\{ {}_0D_t^{\alpha} f(t) \right\} = s^{\alpha} F(s) - \Sigma_{k=0}^{n-1} f^{(k)}(0)\, s^{\alpha-k-1} \tag{7}$$

whereas the Laplace transform of the Riemann-Liouville definition depends on initial conditions that do not have obvious physical interpretation. In (7), for $f^{(k)}(0) = 0$, $k = 0, 1, \ldots, (n-1)$, it holds: $\mathscr{L}\left\{ {}_0D_t^{\alpha} f(t) \right\} = s^{\alpha} F(s)$, then some difficulties arise because s^{ν} is an infinite-dimensional irrational operator. More specifically, we need to approximate s^{α} to realize FOC based on this kind of differintegral operator. Section V will focus on this problem.

Fractional calculus has recently given raise to many applications in different fields of science and engineering. As regards control systems engineering, the origin can probably be dated back to the seminal works by Bode [4], Manabe [5], and Tustin [6]. More recently, the CRONE (Commande Robuste d'Ordre Non Entiér) control approach was widely applied [7]. Noninteger (fractional) derivatives and integrals allow us to generalize PID controllers and define Fractional Order PID (FOPID) controllers or $PI^{\lambda}D^{\mu}$ controllers, where λ and μ are the noninteger orders of integration and differentiation, respectively [8].

FOC may overcome PID or more complex integer order controllers because they may guarantee increased closed-loop performance and enhanced robustness to gain and parameter variations, both when the controlled plant is modeled as integer order system and especially when the plant is described as noninteger order system, in its turn [9], [10]. Therefore, if one thinks about all industrial loops in which PID are dominant, it is easy to imagine the potential impact and success of FOC. Recent results justify the efforts to exploit these potentialities [11], [12], but there is no established and widely accepted method for designing or tuning FOC. Therefore, our contribution is in the line of research for systematic design procedures and easy tuning rules, and it is, in general, conceived to tune FOC for plants modeled by a gain, a time constant, and a dead time.

IV. DESIGN OF THE FRACTIONAL ORDER CONTROLLER

The FOC, namely a FOPI controller, is designed by modifying and adopting a recently proposed loop-shaping procedure [13] that was inspired by the classical Symmetrical Optimum method for electro-mechanical system. The same ideas were also tested to control processes with time delays [14]. Here we refer to an unitary feedback control system, neglecting the sensor dynamics so that the plant is a first-order system plus a time delay. Hence the model of an injection system in a CNG engine is:

$$G_p(s) = \frac{K}{1+Ts} e^{-\tau s} \tag{8}$$

where K, T, and τ are the gain, the time constant, and the deadtime (see section II). Moreover, the FOPI controller is defined by an irrational integral operator depending on the noninteger (fractional) order $0 < \nu < 1$:

$$G_c(s) = K_P + \frac{K_I}{s^{\nu}} = \frac{K_I}{s^{\nu}}(1 + T_I s^{\nu}) \tag{9}$$

with $T_I = \frac{K_P}{K_I}$. Then, three design parameters are available, K_I, T_I and ν. Then, the open-loop transfer function is:

$$G(s) = G_c(s)\, G_p(s) = \frac{K K_I (1 + T_I s^{\nu})}{s^{\nu}(1+Ts)} e^{-\tau s} \tag{10}$$

and the open-loop frequency response becomes:

$$G(j\omega)=\frac{K K_I\left[1+T_I\,(j\omega)^{\nu}\right]}{(j\omega)^{\nu}\,(1+j\omega T)}\,e^{-j\omega\tau}. \tag{11}$$

Putting $\theta=\frac{\pi}{2}\nu$, the (11) yields:

$$G(j\omega)=\frac{K K_I\left\{1+T_I\,\omega^{\nu}\left[\cos(\theta)+j\sin(\theta)\right]\right\}}{\omega^{\nu}\left[\cos(\theta)+j\sin(\theta)\right](1+j\omega T)}\,e^{-j\omega\tau}. \tag{12}$$

Now, using a nondimensional frequency $u=\omega T$ leads to:

$$G(ju)=\frac{K K_I T^{\nu}\left\{1+T_I\,(\frac{u}{T})^{\nu}\left[\cos(\theta)+j\sin(\theta)\right]\right\}}{u^{\nu}\left[\cos(\theta)+j\sin(\theta)\right](1+ju)}\,e^{-j\frac{u\tau}{T}} \tag{13}$$

with

$$|G(ju)|=\frac{K K_I T^{\nu}}{u^{\nu}}\sqrt{\frac{1+2T_I\,(\frac{u}{T})^{\nu}\cos(\theta)+T_I^2\,(\frac{u}{T})^{2\nu}}{1+u^2}} \tag{14}$$

and

$$\angle G(ju)=\arctan\left(\frac{T_I\,(\frac{u}{T})^{\nu}\sin(\theta)}{1+T_I\,(\frac{u}{T})^{\nu}\cos(\theta)}\right)-\arctan(u)-\theta-\frac{u\tau}{T} \tag{15}$$

If we consider the closed-loop transfer function $F(ju)=\frac{1}{1+G^{-1}(ju)}$, the requirement $|F(ju)|\equiv 1$ would imply an optimal feedback system, i.e. a perfect input-output tracking, that is achieved if $|G(ju)|\gg 1$ holds for all frequencies. The requirement can only be approximated in a limited but significant bandwidth u_B. Moreover, to avoid instability determined by high loop gains, we shape $|G(ju)|$ so that it is high at low frequencies and rolls-off at high frequencies [15]. The appropriate loop-shaping in the range around the cross-over frequency is important to guarantee robust stability and specified performance despite parameter changes in the injection system. To this aim, the fractional integrator is very useful because it provides a magnitude plot with a fractional slope of -20ν dB/decade and allows a nearly "flat" phase diagram of $G(ju)$ in a sufficiently wide range around the cross-over, then a nearly constant phase margin in this range.

Once u_B is fixed, a good estimation of the cross-over is given by $u_c\in[\frac{u_B}{1.7},\frac{u_B}{1.3}]$ [16]. For example, we may choose $u_c=\frac{u_B}{1.5}$. Then, the phase margin $PM=\pi+\angle G(ju_c)$ is obtained:

$$\begin{aligned}PM&=\arctan\left(\frac{T_I\,(\frac{u_c}{T})^{\nu}\sin(\theta)}{1+T_I\,(\frac{u_c}{T})^{\nu}\cos(\theta)}\right)-\arctan(u_c)-\frac{u_c\tau}{T}+\pi-\theta\\&=\varphi_1(u_c)-\varphi_2(u_c)-\frac{u_c\tau}{T}+\pi-\theta\end{aligned} \tag{16}$$

where $\varphi_1(u_c)$ and $\varphi_2(u_c)$ are the first two arguments in (16). Now, the design idea is to choose T_I such that $\varphi_1(u_c)-\varphi_2(u_c)-u_c\frac{\tau}{T}=-\pi/2$ in order to obtain $PM=\pi/2-\theta$. Hence easy algebra involving trigonometric functions gives:

$$T_I=\frac{\left[u_c-\cot(\frac{u_c\tau}{T})\right]T^{\nu}}{u_c^{\nu}\left\{\sin(\theta)-u_c\cos(\theta)+\left[\cos(\theta)+u_c\sin(\theta)\right]\cot(\frac{u_c\tau}{T})\right\}}. \tag{17}$$

In this way, we obtain a direct design relation between the phase margin and the noninteger (fractional) order ν if we have a specification PM_s on the phase margin in u_c:

$$PM_s=\pi/2-\theta=0.5\,(1-\nu)\,\pi \tag{18}$$

To tune K_I, we remind that definition of cross-over implies that $|G^{-1}(ju_c)|^2=1$. This condition yields:

$$K_I=\frac{u_c^{\nu}}{K T^{\nu}}\sqrt{\frac{1+u_c^2}{1+2T_I\,(\frac{u_c}{T})^{\nu}\cos(\theta)+T_I^2\,(\frac{u_c}{T})^{2\nu}}} \tag{19}$$

The tuning formulas (17) and (19) require a proper choice of ν and u_B, to obtain $T_I>0$. Moreover, high values of u_B reduce the rise time in the response of the closed-loop system. However, lower values of u_B shift u_c toward a more centered position in the flat region of the phase characteristic. Therefore, a trade-off is necessary. As an example, if we refer to the injection system in section II, we choose $u_B=2\pi$ to associate the bandwidth to the plant time constant and we fix some values of the fractional order ν.

V. APPROXIMATION OF FRACTIONAL ORDER OPERATORS

The irrational operator s^{ν} is the main unit of the FOPI controller. For realization purpose it requires an approximation by a rational transfer function that must be characterized by stable poles and minimum-phase zeros. Namely, right half plane (RHP) zeros and RHP poles in the open-loop transfer function severely reduce the benefits of feedback. RHP zeros put a limit on the maximum achievable bandwidth of the feedback controlled systems [17], [18]. RHP poles imply constrained large gain bandwidth, that generally leads a highly amplified sensor noise at the input to the plant [17], [18].

One of the earliest approximation methods is the Oustaloup's CRONE approach [7]. Other approaches approximate s^{ν} via truncated continued fractions and rational function interpolation (see [19], [20] for a short review) or signal processing techniques [21]. Many methods lead to rational functions with simple zeros interlacing simple poles on the negative real axis of the s-plane [22], [23].

Here we refer to an efficient approximation of s^{ν}, with $0<\nu<1$, that guarantees the interlacing between stable poles and minimum-phase zeros (Maione (2008)). The method has the advantage that closed formulas allow the computation of the convergent. To synthesize, the approximation is given by a rational transfer function:

$$s^{\nu}\approx\frac{\alpha_{N0}(\nu)s^N+\alpha_{N1}(\nu)s^{N-1}+\ldots+\alpha_{NN}(\nu)}{\beta_{N0}(\nu)s^N+\beta_{N1}(\nu)s^{N-1}+\ldots+\beta_{NN}(\nu)} \tag{20}$$

where $N\geq 1$, $\alpha_{Nj}(\nu)=\beta_{N,N-j}(\nu)$ for $j=0,\ldots,N$ and the generic coefficient is determined by the closed formula:

$$\alpha_{Nj}(\nu)=(-1)^j\binom{N}{j}(\nu+j+1)_{(N-j)}(\nu-N)_{(j)} \tag{21}$$

in which the so-called Pochammer functions are $(\nu+j+1)_{(N-j)}=(\nu+j+1)(\nu+j+2)\ldots(\nu+N)$ and $(\nu-N)_{(j)}=(\nu-N)(\nu-N+1)\ldots(\nu-N+j-1)$, with $(\nu+N+1)_{(0)}=(\nu-N)_{(0)}=1$. Approximation errors are reduced even if the number N of zero-pole pairs is small [22].

Note that if $1\leq\nu<2$, we may consider $s^{\nu}=s\,s^{\nu-1}$ and then approximate $s^{\nu-1}$ with the prevously recalled method. Moreover, if the FOPI includes a noninteger integrator $s^{-\nu}$, with $0 < \nu < 1$, we may approximate $s^{1-\nu}$ by considering that $s^{-\nu} = s^{-1}s^{1-\nu}$, so that we exploit the integer order integrator s^{-1} to compensate step disturbances at steady state.

We remark that the interlacing between simple zeros and poles was analyzed in details and proven in [24], [23] for two different classes of approximations based on CF. Moreover, the same techniques can be applied for efficient digital implementation of fractional operators that show more robustness to truncation and round-off errors due to finite word length [25], [22].

VI. SIMULATION RESULTS

To evaluate the tracking and disturbance rejection capabilities of the FOC, we test it on the nonlinear state space model implemented in the Matlab/Simulink simulation environment. Each working condition of the injection system (see [3] for details) provides a different triple (K, T, τ), then an associated controller. All the coefficients in (1) and the consequent values of (K, T, τ) can be recovered from [3].

All the tested controllers are tuned according to the design steps of Section IV by taking as reference the linearized models at the starting equilibrium point. More in details, the value of u_B is determined according to the time constant each linearized model provides. $N = 5$ is used to approximate the fractional integrator in the tuned FOPI controllers. Significant results are discussed in the following. Note that it is very important to limit the overshoot in the rail pressure because too much fuel alters the stoichiometric air-fuel ratio so as to increase consumptions and emissions.

In the first test, a step variation in the reference input is applied, while keeping constant the injectors exciting time interval, the engine speed, and the tank pressure. This mode results in a constant injectors driving command. Fig. 2(a) shows the simulated responses of the system controlled by a FOC characterized by $\nu = 0.3, 0.4, 0.5, 0.6$, when a step reference variation occurs from 5 to 5.5 bar, with a constant 2500 rpm engine speed, a 3 ms injectors exciting time interval, and a constant 50 bar tank pressure. A low pass filter has been applied to the rail pressure signal, because of an oscillating behavior due to the solenoid valve dynamics in the control period and to the injections within a cycle. A fast step response of the FOC controlled system occurs with a limited overshoot, that increases with ν. The rising request of a quick pressure imposes a large variation of duty cycle, that however always keeps within the maximum allowed range. The same figure further shows the performance obtained in analogous working conditions by replacing the FOC with a classical PI controller. The PI controller is tuned according to the open-loop Ziegler-Nichols rule because it is the most widely used method in industry for controlling fuel injection. In this case, even if a quite prompt response is obtained with a reduced rise time, the overshoot is much higher, that corresponds to an inaccurate metering of the injected fuel. The settling time is instead of the same magnitude than that obtained by the FOC. A set-point filter could smooth the response both for the PI and FOPI controlled systems, but this would make the control structure more complex.

Another test refers to a higher magnitude of the step variation in the reference pressure (see Fig. 2(b)). Similar considerations hold true. However, this test shows the robustness of the controllers because the tuned parameters are not optimized for the new varied working conditions. Also in this case, the FOC outperform the PI controller.

To evaluate disturbance rejection by the FOC, another test keeps constant the tank pressure and rail pressure reference, while varying engine speed and load, and thus the injection timings. Fig. 2(c) depicts the model output for a constant 50 bar tank pressure, a 5 bar rail pressure reference, and a varying injectors driving signal. Simulation starts from a steady-state condition corresponding to a 2500 rpm engine speed and 3 ms injectors exciting time interval within the injection cycle. At time 2.5 s, a 2500 rpm speed step is applied, and the injection time interval is raised to 8 ms. Because of the increased amount of injected fuel, the current duty cycle is no longer able to maintain the reference rail pressure so that the rail pressure decreases. Fig. 2(c) shows a prompt response of the FOC, which is able to take back the rail pressure to the reference value. The response by the PI controller shows much higher oscillations, then the same previously cited problems for fuel regulation.

Comparing the results to those provided by other controllers [3], the FOPI controller guarantees much lower rise times and similar settling times. Moreover, it is less complex than the generalized predictive controller designed in [3].

VII. CONCLUSIONS

This paper proposes a fractional order control for regulating the pressure in the injection system of a compressed natural gas engine. The FOPI controller is designed by applying a loop-shaping approach for optimizing the feedback control system and for obtaining high robustness to plant parameter variations.

The importance of the method lies in the easy reproducibility and capability to conjugate performance and robustness specifications. Namely, the tuning formulas strictly and easily relate the requirements to the FOC parameters. Finally, the approach can be extended to include a gain scheduling for adapting the controller to varying parameters according to different working conditions. Future work will further investigate the method both by simulation and real experiments.

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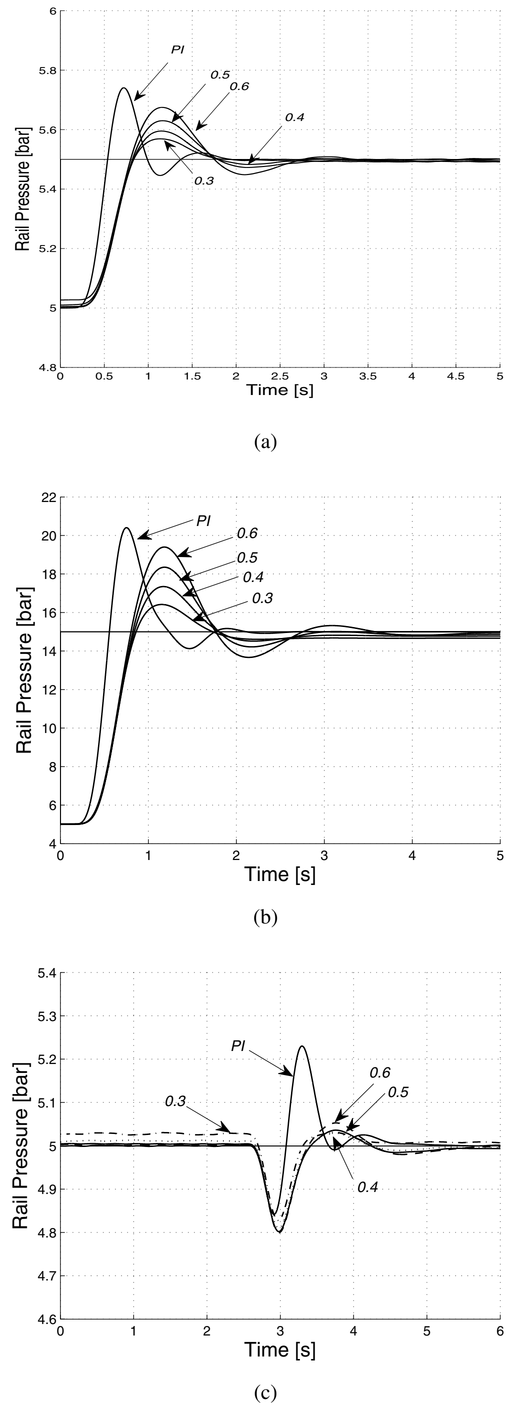

Fig. 2. System response for: (a) 0.5 bar step variation in reference pressure; (b) 10 bar step variation in reference pressure; (c) load disturbances in engine speed and injectors exciting time interval.