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Energy and CO_2 Efficient Scheduling of Smart Home Appliances

Kin Cheong Sou, Mikael Kördel, Jonas Wu, Henrik Sandberg and Karl Henrik Johansson

Abstract— **A major goal of smart grid technology (e.g., smart meters) is to provide consumers with demand response signals such as electricity tariff and CO_2 footprint so that the consumers can consciously control their electricity consumption patterns. These demand response signals provide incentives for the consumers to help reduce peak energy demand by load balancing, as this is particularly relevant in a situation with high level of renewable energy penetration. However, the volume of information can be overwhelming for the consumers. Further, in some situation minimization of electricity bill and CO_2 emission can be conflicting goals and a trade-off analysis is required. To enable the consumers to participate in smart grid effort this paper proposes a decision aiding framework for optimal household appliances scheduling and trade-off analysis through Pareto frontier exploration. To compute the optimal schedules associated with Pareto optimal points, linear optimization problems with SOS2 (special ordered set of type 2) constraints are solved using CPLEX, in the case where the demand response signals are assumed to be piecewise constant. For arbitrary demand response signals, a corresponding dynamic programming solution is proposed. A numerical study demonstrates that in a realistic test case the Pareto frontier analysis can provide valuable information leading to schedules with drastically different electricity and CO_2 emission patterns. In addition, the case study verifies that the Pareto frontier can be computed in real-time in a realistic residential computing environment.**

I. Introduction

Electricity consumption varies between different hours of the day, between days of the week, and between seasons of the year, where the highest power demand in the Northern countries typically occurs when the outdoor temperature drops. In recent years, the power demand has reached new peak levels and created extra stress to balance demand and generation. Environmental and economical reasons will, in the near future, require distribution companies to consider more complex power balance scenarios based on the introduction of large scale renewable electricity generation, plug-in electrical vehicles (PEVs) and distributed electricity generation in residential areas. Intermittent renewable energy sources, such as wind, are dynamic by definition and will require additional balancing power to maintain quality of electrical supply to consumers. Additionally, an increasing number of PEVs will introduce high electricity consumption that is not always predictable. Both the wind power's dynamic contribution to electricity generation and the PEVs' random demand of electricity require a balancing force in the electricity grid.

Load balancing of urban electrical loads, such as residential/industrial electricity consumption, can be accomplished by minimizing the usage of non-renewable generation and scheduling controllable loads to times when renewable energy generation is high. Particular ways to engage the consumers in participating in load balancing is achieved through economic incentives such as time-varying electricity tariff (e.g., spot pricing [1], [2]), or CO_2 footprint [3] for environmentally concerned consumers (e.g., the Stockholm Royal Seaport project [4]).

References such as [5]–[8] have demonstrated the value of time-varying electricity tariff in the management of the power grid, especially in the reduction of peak power consumption; however, such load balancing is feasible only if the consumers are both able and willing to consider tariff information. For instance, it is unrealistic to expect most consumers to identify the most economical operation of their appliances in the presence of dynamic tariff prices and peak consumption penalties. Hence, an automatic decision support system is highly desirable, that either directly takes control of the appliance operation or provides simple advice to the consumers. This scheduling problem has been considered in the context of electricity bill minimization for a given electricity tariff, in both residential and industrial settings (e.g., [8]–[13]).

In general, electricity tariff is positively correlated with CO_2 footprint. This is, however, not always the case in certain countries including Sweden. During daytime Sweden utilizes its relatively clean energy sources such as hydro power plants and nuclear power plants. However, during nighttime Sweden imports relatively inexpensive but CO_2 intense energy from Denmark, Germany and Poland whose primary energy source is combustive fuel power plants [14]. See Fig. 1 for an illustration of the electricity tariff and CO_2 footprint. The situation in Fig. 1 presents a trade-off for the consumers who desire to simultaneously minimize their electricity bills and CO_2 emission. This trade-off can be studied through the Pareto frontier (i.e., the set of all Pareto optimal solutions). See Fig. 2 for an illustration and [15], [16] for more details.

A. Contributions of the Paper

This paper investigates the appliances scheduling problems whose solutions correspond to the Pareto optimal points in the electricity bill and CO_2 emission trade-off analysis.

Kin Cheong Sou is with the Department of Mathematical Sciences, Chalmers University of Technology and the University of Gothenburg, Sweden. The other authors are with the ACCESS Linnaeus Center and the Automatic Control Lab, the School of Electrical Engineering, KTH Royal Institute of Technology, Sweden. kincheong.sou@chalmers.se, {mkordel,ogwu,hsan,kallej}@kth.se
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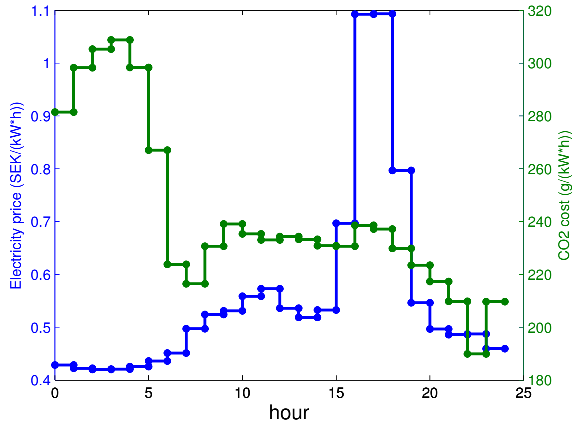

Fig. 1. Electricity tariff and CO_2 footprint in Sweden on January 5th, 2010. Tariff data is taken from Nordpool Spot (`www.nordpoolspot.com`). CO_2 footprint data is taken from [3]. The demand response signals are piecewise constant, with possible jumps at the beginning of each hour.

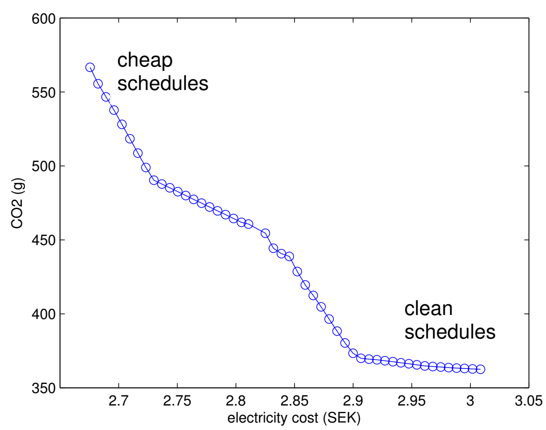

Fig. 2. Each point on the blue line and to its "northeast" corresponds to the electricity bill and CO_2 emission for a specific scheduling of the appliances. Points on the blue line are Pareto optimal. A point is Pareto optimal if there is no other point whose electricity bill and CO_2 emission are no worse, and better in at least one regard. The Pareto optimal points in the upper-left corners correspond to the cheap schedules, whereas the Pareto optimal points in the lower-right corners correspond to the clean schedules. Points to the "southwest" of a Pareto optimal point are not achievable, and points to the "northeast" of a Pareto optimal are not worth considering.

The formulation of the scheduling problem depends on the assumption on the demand response signals (i.e., electricity tariff and CO_2 intensity). Two cases are considered. In the first case, the demand response signals are assumed to be piecewise constant as specified by the Stockholm Royal Seaport project. It is demonstrated that the scheduling problem becomes a linear optimization problem with SOS2 constraints (SOS2 stands for special ordered set type 2 [17]). This formulation can be solved much more efficiently than the mixed integer programming formulation in [8]. In the second case the demand response signals are arbitrary. In this case the scheduling problem is a nonconvex nonlinear knapsack (or resource allocation) problem with precedence constraints. While solutions for special cases are known (e.g., nonconvex piecewise linear knapsack problem [18] and convex nonlinear resource allocation problem [19]), a complete treatment of the general problem considered in this paper is not yet known. In this paper a dynamic programming solution is provided for this scheduling problem with arbitrary demand response signals. In general, the dynamic programming problem is difficult to solve because of the arbitrariness of the signals. Appropriate discretization and quantization of relevant objects in the problem are introduced to ensure that the problem can be solved in practice.

B. Outline

In Section II the appliances scheduling problem is introduced. Then the procedure for Pareto frontier exploration for the electricity bill and CO_2 trade-off analysis is described. The corresponding optimal scheduling problems for finding the Pareto optimal points are defined. Section III derives the optimal scheduling problems under the assumption that demand response signals are piecewise constant. Section IV derives the optimal scheduling problems with arbitrary demand response signals. In Section V some case studies are presented, and the paper is concluded in Section VI.

II. Model of Scheduling Problem and Pareto Frontier Exploration

A. Demand Response Signals

The demand response signals considered in this paper are the 24-hour (i.e., day ahead) electricity tariff and CO_2 footprint, which are assumed known when the appliances scheduling problem is to be solved. The schedule determined according to the day ahead demand response signals can serve as the basis for further adjustments in real-time when updates become available. Two cases of these signals are considered. In the first case both the tariff and CO_2 footprint signals are piecewise constant with possible jumps at every hour [2], [3]. In the second case the demand response signals are arbitrary. See Fig. 1 for the signal values on January 5th, 2010 for the first case.

B. Appliances

In the proposed scheduling framework, the operation process of an appliance is divided into a set of sequential *energy phases*. An energy phase is a sub-task of the appliance operation, and it is uninterruptible. That is, once an energy phase starts, it must continue until it is finished. The time dependent power assignment to all energy phases is referred to as a power profile (for an appliance) [20]. See Fig. 3 for an illustration. In this paper, the models of the energy phases are further simplified: each energy phase requires a pre-specified amount of time to process and the power it can be assigned is *constant* and pre-specified. Therefore, the only decisions regarding the scheduling is when to start the energy phases. However, the assignment of the start time is not arbitrary since certain constraints must be observed. The energy phases are sequential since an appliance sub-task cannot begin until the previous sub-task is completed (e.g.,

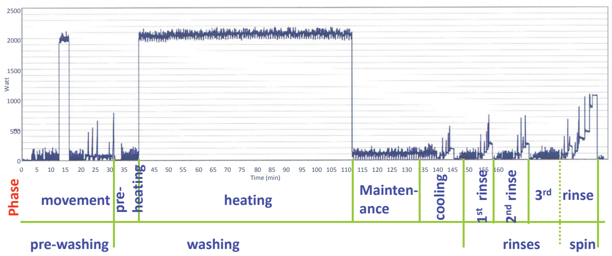

Fig. 3. The eight energy phases constituting the operation process of an example washing machine (movement, pre-heating, heating, etc.) [20].

the washing machine agitator cannot start until the basin is filled with water). In addition, there can be delays between the energy phases for an appliance (e.g., the washing machine agitator can delay starting after the basin is filled, but the delay cannot be longer than ten minutes). Furthermore, there can also be a temporal relationship between appliances (e.g., the dryer cannot start before the washing machine finishes). Nevertheless, in this paper the total power limit constraint is not considered since the scheduler cannot alter the assigned power levels of the appliances and it is assumed that it is safe to run multiple appliances at the same time.

C. User Preferences

The household users can optionally specify a preference that each appliance should be run between a corresponding time interval (e.g., the laundry must be completed by 17:00).

D. Pareto Frontier Exploration of Optimal Schedules

As discussed in Section I the trade-off between electricity bill and CO_2 emission minimization can be studied through the Pareto frontier. In the proposed scheduling framework the ε-constraint method [16], [21] is used to calculate the Pareto frontier. In the current context, this method contains two steps: In step one, the two endpoints of the Pareto frontier (cf., Fig. 2) are found by solving two separate optimization problems:

$$\begin{array}{ll} \text{minimize} & \text{electricity bill} \\ \text{subject to} & \text{constraints in Sections II-B an II-C.} \end{array} \tag{1}$$

and

$$\begin{array}{ll} \text{minimize} & CO_2 \text{ consumption} \\ \text{subject to} & \text{constraints in Sections II-B an II-C.} \end{array} \tag{2}$$

After step one, the ranges for possible electricity bill and CO_2 emission become known. For step two a griding of the range of electricity bill of interest is defined. Let ε_k denote possible values on the grid for $k = 1, 2, \ldots$. For each ε_k, the following problem is solved:

$$\begin{array}{ll} \text{minimize} & CO_2 \text{ consumption,} \\ \text{subject to} & \text{electricity bill less than } \varepsilon_k \\ & \text{constraints in Sections II-B and II-C.} \end{array} \tag{3}$$

After step two the Pareto frontier is obtained (Fig. 2 is obtained in this manner). Notice that a parallel approach can be taken, where the griding is on the CO_2 emission instead. It is merely a convention of the proposed framework that the electricity bill is gridded.

III. Optimal Scheduling with Piecewise-Constant Tariffs

In this section the precise formulation of (3) will be given, under the assumption that the electricity tariff and CO_2 footprint signals are piecewise constant. The formulations of (1) and (2) are similar to that of (3), and hence they are only briefly mentioned in the end of Section III-C.

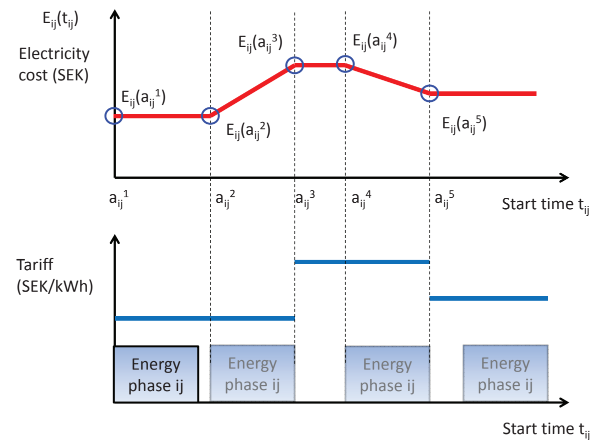

Fig. 4. Piecewise linear electricity cost function $E_{ij}(t_{ij})$ and its break points (circles in blue).

A. Decision Variables and Objective Function

Let N be the number of appliances which need to be scheduled, and these appliances are indexed by $i \in \{1, \ldots, N\}$. For appliance i, let n_i denote the number of energy phases associated with its operation. The energy phases are indexed by $j \in \{1, \ldots, n_i\}$ each for appliance. The decision regarding the appliances use is when the energy phases start. These start time decision variables are denoted by t_{ij} (for the energy phase j in appliance i). From the start time t_{ij} the energy phase continues to run until $t_{ij} + T_{ij}$, where T_{ij} is the pre-specified process time. During this operation, the power consumption is constant as assumed in Section II-B. The process of the energy phase induces electricity and CO_2 costs, which are functions of the start time t_{ij}. These cost functions are denoted by $E_{ij}(t_{ij})$ and $C_{ij}(t_{ij})$ respectively. In this subsection, because of the constant power consumption and the fact that the tariff and CO_2 footprint are piecewise constant, E_{ij} and C_{ij} are continuous piecewise linear functions (different for different energy phases). These piecewise linear cost functions are fully characterized by their "break points", which are points where the cost functions change slopes, plus the points with the earliest and latest time when the energy phase can be scheduled. The break points for E_{ij} are denoted as $\left(a_{ij}^k, E_{ij}\left(a_{ij}^k\right)\right)$ for the k^{th} break point, with energy phase j in appliance i. The break points (for

E_{ij}) can be computed with known tariff and appliances technical data. See Fig. 4 for an illustration. The CO_2 cost functions are defined similarly, and they are also described by the break points $\left(a_{ij}^k, C_{ij}\left(a_{ij}^k\right)\right)$. To take advantage well-studied optimization paradigms (i.e., linear optimization), a particular representation for the cost functions E_{ij} and C_{ij} is necessary. To begin, t_{ij} is described as a linear combination of the time-coordinates of the break points:

$$t_{ij} = \sum_{k=1}^{b_{ij}} \lambda_{ij}^k a_{ij}^k, \tag{4}$$

where b_{ij} is the number of break points for E_{ij} (same as that for the C_{ij} case), and λ_{ij}^k are auxiliary decision variables satisfying the following three sets of constraints:

$$\begin{aligned} &\lambda_{ij}^k \in [0,1], \quad \forall\, i,j,k, \\ &\sum_{k=1}^{b_{ij}} \lambda_{ij}^k = 1, \quad \forall\, i,j, \\ &\text{The tuple } (\lambda_{ij}^1, \ldots, \lambda_{ij}^{b_{ij}}) \text{ satisfies the SOS2 constraint [17],} \\ &\text{for all } i \text{ and } j. \end{aligned} \tag{5}$$

In (5), the SOS2 constraint requires that at most two variables in the tuple $(\lambda_{ij}^1, \ldots, \lambda_{ij}^{b_{ij}})$ can be nonzero, and in case there are two nonzero variables they must be consecutive. The SOS2 constraint can be either modeled using binary decision variables [22], or directly handled by an appropriate solver. Using the representation of t_{ij} in (4), the cost functions $E_{ij}(t_{ij})$ and $C_{ij}(t_{ij})$ can be described as

$$\begin{aligned} E_{ij}\big(t_{ij}(\lambda_{ij}^k)\big) &= \sum_{k=1}^{b_{ij}} \lambda_{ij}^k E_{ij}\left(a_{ij}^k\right), \\ C_{ij}\big(t_{ij}(\lambda_{ij}^k)\big) &= \sum_{k=1}^{b_{ij}} \lambda_{ij}^k C_{ij}\left(a_{ij}^k\right). \end{aligned} \tag{6}$$

With (6), the objective function and the first constraint in (3) can be described (by summing up the costs in (6) for all energy phases in all appliances).

B. Appliances and User Preference Constraints

The processing of the energy phases is subject to appliances and user preference constraints, as described in Sections II-B and II-C. Let nonnegative numbers $\underline{D}_{ij}$ and $\overline{D}_{ij}$ denote the minimum and maximum delays between energy phase j and its preceding one $(j-1)$ in appliance i. Then the sequential processing of the energy phases and the between-phase delay requirements can be jointly modeled as

$$t_{i(j-1)} + T_{i(j-1)} + \underline{D}_{ij} \le t_{ij} \le t_{i(j-1)} + T_{i(j-1)} + \overline{D}_{ij}, \quad \forall\, i, j \ge 2. \tag{7}$$

Also, for any pair (i_1, i_2) which requires a precedence relationship (e.g., i_1 being the washing machine and i_2 being the dryer), the following constraint is enforced:

$$t_{i_1 n_{i_1}} + T_{i_1 n_{i_1}} \le t_{i_2 1}, \tag{8}$$

where n_{i_1} denotes the number of energy phases for appliance i_1. Furthermore, the household users can specify a time interval during which an appliance should be run. For appliance i, the interval is specified by a lower bound T_{lb}^i before which the appliance cannot run, and a upper bound T_{ub}^i after which the appliance cannot run. The corresponding constraints are described as:

$$\begin{aligned} t_{i1} &\ge T_{\mathrm{lb}}^i, \quad \forall\, i \\ t_{in_i} + T_{in_i} &\le T_{\mathrm{ub}}^i, \quad \forall\, i. \end{aligned} \tag{9}$$

Constraints (9) are in fact always enforced. If the household users do not specify explicitly, T_{lb}^i would be equal to the earliest start time of the planning period. Similarly, T_{ub}^i would become the latest end time of the planning period.

C. Optimization Formulation with SOS2 Constraints

To sum up, the Pareto frontier exploration optimization problem in (3) can be modeled as:

$$\begin{aligned} \underset{t_{ij}, \lambda_{ij}^k}{\text{minimize}} \quad & \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{b_{ij}} \lambda_{ij}^k C_{ij}\left(a_{ij}^k\right) \\ \text{subject to} \quad & \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{b_{ij}} \lambda_{ij}^k E_{ij}\left(a_{ij}^k\right) < \varepsilon_k \\ & \text{constraints in (4), (5), (7), (8) and (9).} \end{aligned} \tag{10}$$

Again, because of the SOS2 constraint on λ_{ij}^k in (5) problem (10) should be solved with a solver which can handle SOS2 constraint or binary decision variables (e.g., CPLEX can handle both).

Finally, notice that the corresponding optimization problem for (2) has the same form as (10), except that the electricity bill budget constraint $\sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{b_{ij}} \lambda_{ij}^k E_{ij}\left(a_{ij}^k\right) < \varepsilon_k$ is removed. Further, the problem for (1) is similarly defined with the electricity cost replacing the CO_2 emission in the objective function.

IV. Optimal Scheduling with Arbitrary Tariffs

To solve the optimal scheduling problem with arbitrary tariffs a dynamic programming approach (e.g., [23], [24]) is proposed in this paper. In this section the start times for all energy phases for all appliances are labeled through a single index, namely $t_1, t_2, \ldots, t_{N_{ep}}$, where N_{ep} denotes the total number of all energy phases. Concerning an appliance the sequential energy phase processing constraint in (7) can be rewritten as

$$\underline{d}_n \le t_{n+1} - t_n \le \overline{d}_n \quad \forall\, n > 1, \tag{11}$$

where $\underline{d}_n$ and $\overline{d}_n$ are defined using $\underline{D}_{ij}$, $\overline{D}_{ij}$ and T_{ij} in (7). Setting $\underline{d}_n = -\infty$ means that the lower bound is not enforced. Similarly, setting $\overline{d}_n = \infty$ means that the upper bound is not enforced. (11) can model certain simple inter-appliance sequential relationship in (8). One such case is that the washing machine must be finished before the dryer can start, but there is no constraint on the dishwasher. The more complex situations can arise, for example when both the washing machine and the dishwasher need to finish before the dryer but there is no sequential relationship between the former two appliances. These more complex situations require a more complex formulation than what can be modeled by (11). Unfortunately they are out of the scope of this paper. The user time preference constraint in (9) can similarly be modeled as:

$$l_n \leq t_n \leq u_n \quad \forall\, n, \tag{12}$$

where l_n and u_n are defined through (9). Furthermore, for reasons to be explained shortly, the time axis of the planning period is *discretized*. That is,

$$t_n \in \{0, \Delta t, 2\Delta t, \ldots, (N_t - 1)\Delta t\}, \quad \forall\, n, \tag{13}$$

where N_t is given and $\Delta t = \frac{T_f}{N_t}$ with T_f being the last moment any energy phase can start during the entire planning period. N_t controls the granularity of the discretization of the time axis.

A. Dynamic Programming Formulation for (1) or (2)

The dynamic programming formulation for (1) or (2) is described first, since it is different from that for (3). Let $f_n(t_n)$ denote the cost (i.e., either electricity cost for (1) or CO_2 emission for (2)) for starting energy phase n at time t_n. By definition, the total cost for scheduling the appliances is $\sum_{k=1}^{N_{ep}} f_k(t_k)$. For any given n, let $J(t_n)$ denote the optimal partial cost for scheduling energy phases 1 through n in the following sense: in the calculation of $J(t_n)$ energy phase n is started at t_n, while energy phases 1 through $n-1$ are scheduled to minimize $\sum_{k=1}^{n-1} f_k(t_k)$ subjected to constraints (11) and (12). That is, for $n > 1$

$$J(t_n) = f_n(t_n) + \min_{\substack{t_1, t_2, \ldots, t_{n-1} \\ \text{constraint (11)} \\ \text{constraint (12)}}} \{f_1(t_1) + \ldots + f_{n-1}(t_{n-1})\}, \tag{14}$$

with the convention that $J(t_n) = \infty$ if the minimization above is infeasible, and for $n = 1$,

$$J(t_1) = f_1(t_1). \tag{15}$$

If $J(t_{N_{ep}})$ can be evaluated for all possible values of $t_{N_{ep}}$, then the cost of the optimal schedule for all energy phases can be obtained by minimizing $J(t_{N_{ep}})$ with respect to $t_{N_{ep}}$ subject to (12). By [23], [24], (14) can be computed through the following dynamic programming recursion:

$$J(t_n) = f_n(t_n) \quad + \min_{\substack{t_{n-1} \\ \text{constraint (11)} \\ \text{constraint (12)}}} J(t_{n-1}), \quad \forall\, n > 1, \tag{16}$$

where the boundary condition is given in (15). The argument of minimum in (16) can be stored while (16) is being evaluated. Hence, upon completion of the recursion the optimal schedule is also available. In general, the minimization in (16) is difficult to carry out because the cost function f_n is arbitrary. However, with assumption (13) this minimization, for each n, becomes a comparison of N_t scalars. As a result, the computation effort for evaluating $J(t_{N_{ep}})$ through the dynamic programming recursion (16) is $O(N_t N_{ep})$.

B. Dynamic Programming Formulation for (3)

The additional feature of (3) is the "electricity bill budget" constraint requiring that the total electricity cost for scheduling the appliances must be less than a given threshold ε_k. To handle this budget constraint, extra bookkeeping and added computation are needed in the dynamic programming formulation. The detail is as follows: For each $n = 1, \ldots, N_{ep}$, let $c_n(t_n)$ and $e_n(t_n)$ respectively denote the CO_2 emission and electricity cost when energy phase n is started at t_n. For this subsection, it is further assumed that the values of $e_n(t_n)$ are *quantized*. That is,

$$e_n(t_n) \in Q_E, \quad \forall\, n \text{ with } Q_E = \{0, \Delta e, 2\Delta e, \ldots, (N_e - 1)\Delta e\}, \tag{17}$$

and

$$\Delta e = \frac{E_{\max}}{N_e}, \quad E_{\max} = \max_n \big\{ \max_{t_n} \{e_n(t_n)\} \big\}, \quad N_e \text{ is given.} \tag{18}$$

In above, N_e controls the quantization level of the electricity cost e_n.

Similar to the formulation in Section IV-A, let t_n denote that start time of energy phase n. In addition, for $1 \leq n \leq N_{ep}$ let b_n denote the electricity bill budget left for running energy phase $n+1, n+2, \ldots, N_{ep}$. By convention, $b_{N_{ep}} \in Q_E$ represents the leftover budget after assigning all energy phases. In addition, $b_{n-1} = b_n + e_n{}'$ for all $n > 1$, for $e_n{}' \in Q_E$. This implies that b_n are quantized in the same way as e_n:

$$b_n \in Q_E, \quad \forall\, n. \tag{19}$$

Indeed, if (17) were not enforced, the set of all possible values of b_n would be very difficult to characterize.

Let $J_C(t_n, b_n)$ denote the optimal partial CO_2 emission, defined in a similar way as $J(t_n)$ in Section IV-A except with the additional electricity bill budget constraint. For $n > 1$,

$$J_C(t_n, b_n) = c_n(t_n) + \min_{\substack{t_1, t_2, \ldots, t_{n-1} \\ \text{constraint (11)} \\ \text{constraint (12)} \\ \sum_{i=1}^{n} e_i(t_i) < \varepsilon_k - b_n}} \Big\{ c_1(t_1) + \ldots + c_{n-1}(t_{n-1}) \Big\}, \tag{20}$$

with the convention that $J_C(t_n, b_n) = \infty$ if the minimization above is infeasible, and for $n = 1$,

$$J_C(t_1, b_1) = \begin{cases} c_1(t_1) & \text{if } e_1(t_1) < \varepsilon_k - b_1 \\ \infty & \text{if } e_1(t_1) \geq \varepsilon_k - b_1 \end{cases} \tag{21}$$

If $J_C\left(t_{N_{ep}}, b_{N_{ep}}\right)$ can be evaluated for all possible combinations of $\left(t_{N_{ep}}, b_{N_{ep}}\right)$ (i.e., $N_t \times N_e$ combinations in total because of (13) and (19)), then the minimum value of $J_C\left(t_{N_{ep}}, b_{N_{ep}}\right)$ is the optimal objective value of (3). The dynamic programming recursion to calculate (20) is

$$\begin{aligned} J_C(t_n, b_n) &= c_n(t_n) + \min_{\substack{t_{n-1}, b_{n-1} \\ \text{constraint (11)} \\ \text{constraint (12)} \\ b_{n-1} = b_n + e_n(t_n)}} J_C(t_{n-1}, b_{n-1}) \\ &= c_n(t_n) + \min_{\substack{t_{n-1} \\ \text{constraint (11)} \\ \text{constraint (12)}}} J_C(t_{n-1}, b_n + e_n(t_n)), \end{aligned} \tag{22}$$

where the boundary condition is given in (21). Similar to the case in Section IV-A the argument of minimum in (22) can be stored, and upon completion of the dynamic programming recursion in (22) the optimal scheduling for the appliances can be recovered. Again, because of (13) and (19) the minimization in (22) for each n is a comparison of $N_e \times N_t$ scalars. Therefore, the computation effort for evaluating $J_C\left(t_{N_{ep}}, b_{N_{ep}}\right)$ through (22) is $O(N_e N_t N_{ep})$.

Finally, notice that if the quantization assumption in (17) is not valid, then the procedure described in this subsection can still be used to obtain an approximate solution to (3). The violation of the electricity bill budget constraint and optimality is controlled through the quantization parameter N_e. The effect of the quantization in (17), as well as that of the time axis discretization in (13), will be evaluated in a numerical case study in Section V.

V. Numerical Case Study

The case study considers an apartment with three typical appliances (i.e., washing machine, dryer and dishwasher) as in [8]. The demand response signals are shown in Fig. 1, corresponding to the electricity tariff and CO_2 footprint in Sweden on January 5th, 2010. The specifications of the energy phases of the appliances are listed in Tables I, II and III respectively. The constant assigned power for each energy phase is obtained by dividing the required energy (column two in the tables) by the energy phase process time (column five in the tables). The washing machine is scheduled to finish working before the dryer can start. Furthermore, the user time-preference specifies that the washing machine and dryer must operate between 0:00 and 23:00, and the dishwasher must operate between 19:00 and 24:00. All computations in this case study were performed on a 32-bit machine with 2.4GHz processors and 2GB of RAM.

TABLE I

Dishwasher technical specifications [20]

Energy phase	Energy (Wh)	Min power (W)	Max power (W)	Process time (min)
pre-wash	16.0	6.47	140	14.9
wash	751.2	140.26	2117.8	32.1
1st rinse	17.3	10.28	132.4	10.1
drain	1.6	2.26	136.2	4.3
2nd rinse	572.3	187.3	2143	18.3
drain & dry	1.7	0.2	2.3	52.4

TABLE II

Washing machine technical specifications [20]

Energy phase	Energy (Wh)	Min power (W)	Max power (W)	Process time (min)
movement	118	27.231	2100	26
pre-heating	5.5	5	300	6.6
heating	2054.9	206.523	2200	59.7
maintenance	36.6	11.035	200	19.9
cooling	18	10.8	500	10
1st rinse	18	10.385	700	10.4
2nd rinse	17	9.903	700	10.3
3rd rinse	78	23.636	1170	19.8

TABLE III

Dryer technical specifications [20]

Energy phase	Energy (Wh)	Min power (W)	Max power (W)	Process time (min)
drying	2426.3	120.51	1454	120.8

A. Pareto Frontier Calculation via Linear Optimization with SOS2 Constraints

For this part of the case study the Pareto frontier is computed by solving (1), (2) and (3) which are specified to the linear optimization problem with SOS2 constraints in (10) and its variants. Fig. 5 shows the Pareto frontier with ten Pareto optimal points. The computation of the whole Pareto frontier took about 2 seconds. This is practical for real-time computation in a residential environment. The schedules corresponding to points A, B and C in Fig. 5 are shown in Fig. 6, 7 and 8 respectively. The results agree with intuition. Because of the user time preference constraint (usage between 19:00 and 24:00) the dishwasher is scheduled towards the end of the day. However, the schedules for the washing machine and dryer are subject to more drastic changes. For the cheap schedule in Figure 6 the two loads are scheduled during the earlier hours of the day because of the electricity is least expensive, even though it can be CO_2 costly. On the other hand, for the clean schedule in Figure 7 the loads are scheduled in the "valleys" of the CO_2 footprint curve. Finally, for the balanced schedule in Figure 8 the strategy seems to suggest the middle-ground, with the washing machine scheduled to the spot in the cheap case and the dryer scheduled to the spot in the clean case.

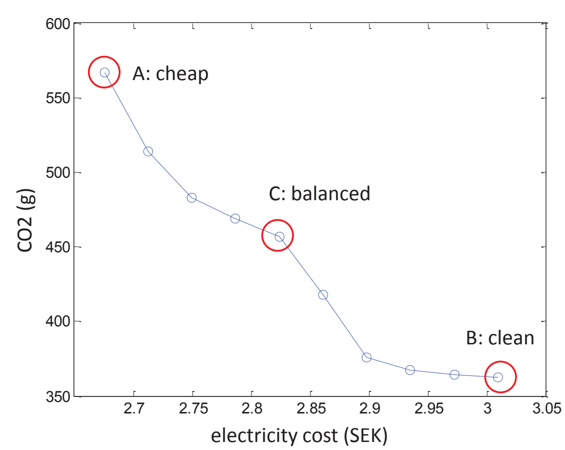

Fig. 5. Pareto frontier representing the electricity bill and CO_2 trade-off.

B. Pareto Frontier Calculation via Dynamic Programming

In this part of the case study the Pareto frontier is computed by solving (1), (2) and (3) using the proposed dynamic programming formulations in Section IV-A and Section IV-B. The demand response signals involved in this study are the same piecewise constant ones as depicted in Fig. 1, even though the dynamic programming formulations can handle the more general situation. Rather, the purpose of this study

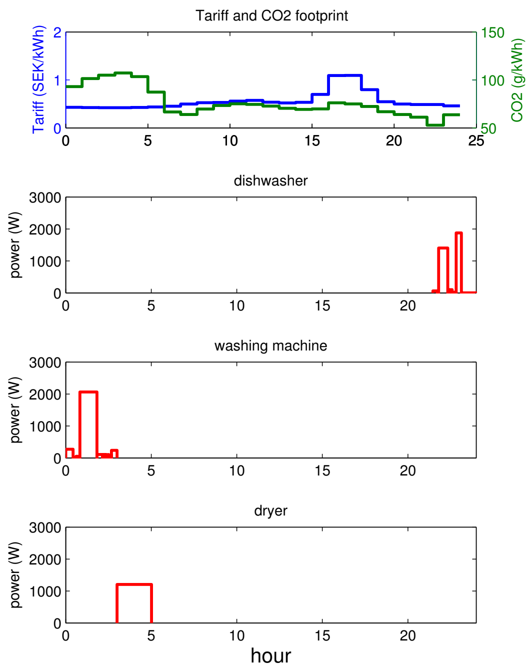

Fig. 6. Cost minimizing schedule corresponding to point A in Fig. 5.

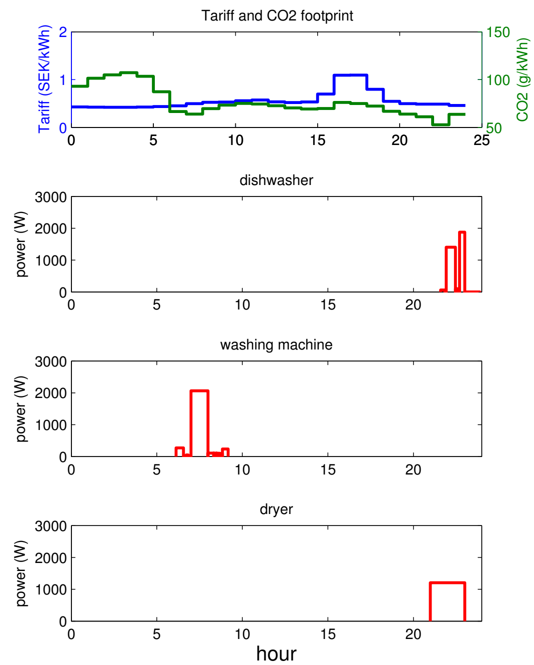

Fig. 7. CO_2 minimizing schedule corresponding to point B in Fig. 5.

is to evaluate the accuracy loss due to the discretization defined in (13) and the quantization in (17), where the granularity is controlled by N_t and N_e respectively. Fig. 9 shows the Pareto optimal points computed using dynamic programming with four different combinations of (N_t, N_e), as well as the exact Pareto frontier obtained by solving (10) and it variants. Fig. 9 indicates that, even with the approximation due to the discretization and quantization, the dynamic programming generated Pareto optimal points are reasonably accurate. In the least accurate case (i.e., $N_t = 300$ and $N_e = 300$), the average (over ten Pareto optimal points) relative error is less than 1%. Regarding computation time, obtaining ten Pareto optimal points requires 1 minute, 4.6 minutes, 3.5 minutes and 15.8 minutes, respectively for the case $(N_t, N_e) = (300, 300)$, $(N_t, N_e) = (300, 1000)$, $(N_t, N_e) = (1000, 300)$ and $(N_t, N_e) = (1000, 1000)$. While one minute of dynamic programming solving is much longer than the two-second solving of (10) and it variants. The increased computation effort is the price to pay for the generality of the dynamic programming formulation. Further refinements of the dynamic programming formulation can be studied to ensure that it is truly real-time implementable in a residential computing environment.

VI. Conclusion

In this paper an automatic decision framework to schedule smart home appliances to minimize electricity bill and CO_2 emission is considered. In a situation such as Sweden where the two objectives may conflict with each other, the Pareto frontier computed in this paper can provide valuable guideline for operating the appliances. In particular, with the Swedish demand response signals on January 5th, 2010 (a cold day), preferences over electricity bill minimization or over CO_2 emission minimization can lead to very different appliances schedules with drastically different consequences on electricity bill and CO_2 emission. This is demonstrated by Fig. 5 through Fig. 8. The figures also demonstrated that appropriate choices of the demand response signals will lead to appliances schedules which avoid the peak of electricity load during the day. By giving up certain degrees of freedom in the scheduling setup as compared to [8], a linear optimization problem with SOS2 constraints can be set up to compute the Pareto optimal points in a realistic setting in about 2 seconds using a standard laptop equipped with CPLEX solver. Further detail regarding the comparison of the computation times of the formulations in [8] and in Section III in this paper can be found in [25, Chapter 4.3]. For the more general case where the demand response signals are arbitrary, a dynamic programming based procedure to compute the Pareto frontier is possible as demonstrated. However, more investigations are needed to make the dynamic programming solution implementable in real-time and to allow it to handle the case with arbitrary precedence relationship among the operations of the appliances.

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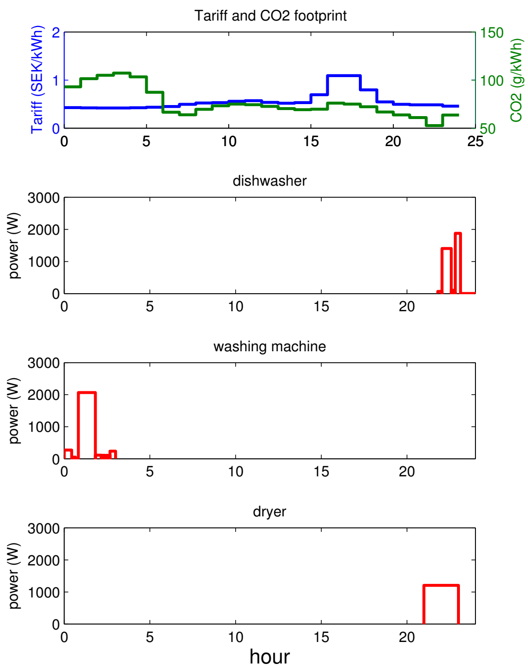

Fig. 8. Balanced schedule corresponding to point C in Fig. 5.

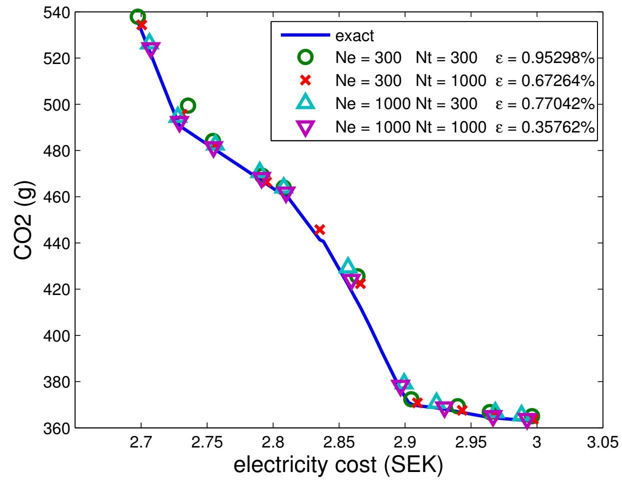

Fig. 9. Pareto frontiers computed exactly and by various dynamic programming formulations with different combinations of (N_t,N_e). N_t is the number of discretization points of the time axis as defined in (13). N_e is the number of quantization levels of the electricity tariff as defined in (17). ε is the average relative error in the CO_2 cost.

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