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Learning control in spatial coordinates for the path-following of autonomous vehicles

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***Abstract*— On the basis of a kinematic third order nonlinear model of an autonomous vehicle, a space-learning control is designed for the tracking of planar curves whose uncertain curvature is L-periodic in the curvilinear abscissa. The behaviour of a human driver, who repetitively learns the correct action from the past experience, is mathematically reproduced. Simulation results demonstrate the effectiveness of the presented approach.**

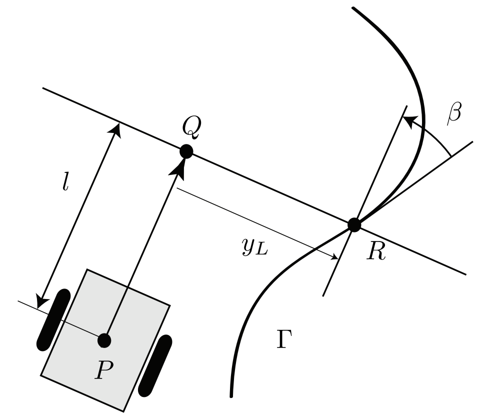
I. Introduction

Repetitive learning control techniques[1] rely on the consideration that human beings are able to effectively improve task executions when trials are repeated (see [5], [16], [33] for recent theoretical results and [3], [4], [23], [21] for significant applications[2]). In contrast to general non-learning ones, repetitive learning controls use, in a repetitive scenario, the richness of information which error signals possess from previous executions ([2], [7]). Repetitive learning controls thus iteratively extract from the past the sufficient experience to improve the closed loop performances and to guarantee the output tracking.

In this paper we present a learning control for autonomous vehicles moving at constant speed $v > 0$. In particular we consider the nonholonomic mobile vehicle model

$$\begin{aligned} \dot{x} &= v\cos(\theta), \quad x(0) = x_0 \\ \dot{y} &= v\sin(\theta), \quad y(0) = y_0 \\ \dot{\theta} &= \omega, \quad \theta(0) = \theta_0, \end{aligned} \tag{1}$$

where $(x, y) \in \mathbb{R}^2$ and $\theta \in S^1$ identify position and orientation of the vehicle in the plane, while x_0, y_0, θ_0 are the initial conditions. Let Γ be the image of the function $\gamma(\cdot) : (-nL, +\infty) \to \mathbb{R}^2$ ($n \in \mathbb{N}^+$, $L \in \mathbb{R}^+$) with $\|\gamma'(s)\| = 1$. Consider a point Q placed in front of the vehicle, on its symmetry axis, at a distance l from the vehicle position $P = [x, y]^T$. Let R be the intersection of the normal to the vehicle direction passing at Q with the curve Γ. Let y_L be the length of the segment that joins R and Q, with a positive sign if R is on the right of the vehicle symmetry axis. Furthermore, define the angle $\beta = \theta - \arg\gamma'(s)$ (see Figure 1 for all the aforementioned quantities). In the following,



$\tau(\theta)$ and $\nu(\theta)$ denote the vectors $[\cos(\theta), \sin(\theta)]^T$, $\nu(\theta) = [-\sin(\theta), \cos(\theta)]^T$, respectively. In other words, $\nu(\theta)$ is the unit vector parallel to the vehicle symmetry axis and $\nu(\theta)$ is its orthogonal. The scalar curvature of Γ is given by $\kappa(s) = \frac{d \arg\gamma'(s)}{ds}$.

A. Problem formulation

We address the problem of designing a space-learning control in order to guarantee the tracking - by the autonomous vehicle - of the planar curve Γ whose curvature $\kappa(s)$ is L-periodic in the curvilinear abscissa s. The design is based on the measurement (provided by an artificial vision system) of the lateral offset y_L at a certain preview distance l. The problem addressed in this paper is precisely the following: *on the basis of (y_L, s) and of (v, l, L), design an infinite-memory feedback learning control ω which guarantees asymptotic regulation to zero of the output y_L for any uncertain $\kappa(s)$ (periodic of period L) in a suitable compact set*. Remark that differently from other approaches to path-following (see for instance [9] and [10]) function $\kappa(s)$ is not known in advance by the controller.

Remark 1.1: If we interpret the planar curve as an ideal path guaranteeing periodic obstacle avoidance, then solving the problem addressed in this paper means guaranteeing safe, reliable, not driver-dependent collision avoidance maneuvers such as the ones involved in car tests with predetermined cone placements at a known constant distance L on the road[3]. With this respect, it is clear that when the vehicle moves exactly along the reference curve with $l = 0$, the

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[1]We adopt in this paper the classical distinction between iterative and repetitive learning controls (see [34] and [2]): in iterative learning control the initial conditions of the system are typically set to the same value at each repetition while in repetitive control the initial conditions of the system on each trial are set equal to the final conditions of the previous repetition.

[2]See also [15], [29] and [28], [27], [30], [31], [32] for theoretical developments and applications of adaptive learning controls.

[3]Recall that shortly after that the Mercedes-Benz A-class was introduced in 1997 and when the Swedish magazine "Teknikens Vaerld" gave one A-class in the hands of the car tester Robert Collin, it was an unfortunate surprise that he flipped it on its top during a routine test to avoid a moose at just 60 km/h.

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set of maneuvers could be actually considered time-periodic of known constant period $T = L/v$. The time-periodicity is however destroyed as soon as the preview distance l as well as the distance y_L between the vehicle and the ideal curve become non-zero. What is actually periodic of constant period L is in fact only the curvature $\kappa(s)$ which satisfies

$$\kappa(s-L) = \kappa(s).$$

No constant time-periods could be found if we look at $\kappa(s(t))$ as a time-varying disturbance. To this purpose, let the positive real T satisfying

$$\int_0^T \dot{s}(\tau)\mathrm{d}\tau = L$$

be the first time elapse when the arc length[4] from $t = 0$ is L. By interpreting the curvilinear abscissa s as a function of time we can only write

$$\kappa(s(t) - L) = \kappa(s(t)), \quad \forall\, t \geq T$$

which shows that, for general non-constant $\dot{s}(t)$, the space-periodicity does not imply the time-periodicity (with time-periodicity denoting "periodicity of constant period"). A time-varying "period" $T_t(t)$ (sufficiently smooth according to the inverse function theorem ([11])) should be in fact considered: it would satisfy

$$s(t) - L = s(t - T_t(t)), \quad \forall\, t \geq T$$

that is

$$\int_0^t \dot{s}(\tau)\mathrm{d}\tau - L = \int_0^{t-T_t(t)} \dot{s}(\tau)\mathrm{d}\tau, \quad \forall\, t \geq T$$

which would be equivalent to (see [1])

$$\int_{t-T_t(t)}^t \dot{s}(\tau)\mathrm{d}\tau = L, \quad \forall\, t \geq T.$$

For such a $T_t(t)$ only, we can write for any $t \geq T$

$$\kappa(s(t)) = \kappa(s(t) - L) = \kappa(s(t - T_t(t))).$$

The main difficulty addressed in this paper is thus the requirement of mathematically stating and solving the corresponding space-learning problem (instead of solving the time-learning one) in perfect analogy with human drivers' behaviours. The specific theoretical contribution of this paper is demonstrated by considering that, in the related paper [1] in which a space learning control is designed on the basis of the previously defined $T_t(t)$ (with notation P_t), the time-varying $T_t(t)$ is only considered for simulations and experiments while a constant period is implicitly assumed in deriving crucial equations[5].

[4]It is referred in [1] to as the first trajectory cycle.

[5]See to this purpose equations (18) and (19) in [1].

II. Control Design

A. Change of coordinates and time-reparameterization

To overcome the difficulty presented in the previous section, system (1) is rewritten using the arc-length coordinate s as independent variable. After a change of coordinates, the curvature $\kappa(s)$ acts as a L-periodic disturbance in the system dynamics.

Note that the output y_L is not always well defined. We will implicitly assume, in all future computations, a scenario in which every variable exhibits small deviations from the corresponding ideal, reference repetitive behaviour (guaranteeing the existence of the point R and leading to $\dot{s}(\cdot) \geq c_s > 0$). This scenario will be guaranteed to survive by suitably restricting the system initial conditions and by suitably choosing the control parameters at the end of the control design procedure.

According to the previously introduced notation, the output y_L satisfies the following equality

$$P + l\tau(\theta) - y_L\nu(\theta) = \gamma(s) \tag{2}$$

so that taking the time derivative of (2) leads to

$$v\tau(\theta) + l\nu(\theta)\omega - \dot{y}_L\nu(\theta) + y_L\tau(\theta)\omega = \gamma'(s)\dot{s}.$$

Taking the scalar product of (2) by $\tau(\theta)$ and using the equality $\langle\tau(\theta), \gamma'(s)\rangle = \cos(\beta)$ we obtain

$$\dot{s} = \frac{v + \omega y_L}{\cos(\beta)}. \tag{3}$$

On the other hand, taking the scalar product of (2) by $\nu(\theta)$ and using the fact that $\langle\nu(\theta), \gamma'(s)\rangle = -\sin(\beta)$ we write

$$\dot{y}_L = l\omega + \sin(\beta)\dot{s}. \tag{4}$$

Finally, computing the time derivative of β we obtain

$$\dot{\beta} = \omega - \kappa(s)\dot{s}. \tag{5}$$

Equations (3)-(5) constitute a system description in different coordinates. The last equation however shows that the dynamics for β is directly influenced by the control input ω. In order to force ω to appear in the y_L-dynamics only, we define the variable

$$\eta = l\beta - y_L$$

whose dynamics is

$$\dot{\eta} = -\dot{s}\left[l\kappa(s) + \sin(\beta)\right].$$

As we shall see, the change of coordinates from (x, y, θ) to (y_L, η, s) will be used to our advantage for the control design.

We now define the control input

$$\omega(t) = \frac{v v_c(t)}{\cos(\beta(t)) - y_L(t)v_c(t)}, \tag{6}$$

that is

$$\omega(t) = \dot{s}(t)v_c(t),$$

where $v_c(t)$ is yet to be designed. For the generic time variable $h(t)$ with h taking its definition among the elements of the set $\{y_L, \eta, \omega, v_c\}$ define

$$\bar{h}(s) \doteq h(g(s))\,,$$

where $g(\cdot) : \mathcal{R}(s(t)) \to \mathbb{R}_0^+$ [$\mathcal{R}(s(t))$ is the range of the function $s(t)$] exists according to the inverse function theorem (see [11]). Therefore, the dynamics for $\bar{y}_L(s)$ and $\eta(s)$ can be written as

$$\begin{aligned} \bar{y}'_L(s) &= \frac{\dot{y}_L(t)}{\dot{s}(t)} \\ \bar{\eta}'(s) &= \frac{\dot{\eta}(t)}{\dot{s}(t)}\,, \end{aligned} \tag{7}$$

or, equivalently,

$$\begin{aligned} \bar{y}'_L(s) &= l\bar{v}_c(s) + \sin\left(\frac{\bar{\eta}(s)+\bar{y}_L(s)}{l}\right) \\ \bar{\eta}'(s) &= -l\kappa(s) - \sin\left(\frac{\bar{\eta}(s)+\bar{y}_L(s)}{l}\right). \end{aligned}$$

The control input $\omega(t)$, when explicitly expressed as a function of the curvilinear abscissa s, becomes

$$\bar{\omega}(s) = \frac{v\bar{v}_c(s)}{\cos\left(\frac{\bar{\eta}(s)+\bar{y}_L(s)}{l}\right) - \bar{y}_L(s)\bar{v}_c(s)} = \dot{s}(g(s))\bar{v}_c(s).$$

The advantage of using the above time reparameterization is that (7) now describes a system evolving in s (instead of t) with a forcing disturbance signal $\kappa(s)$ that is s-periodic.

Note that the change of variables presented above requires the denominators appearing in (3) and (6) to be bounded away from zero. This will be guaranteed by the controller design, that will keep the angle β and the output y_L sufficiently small.

B. Definition of the reference signal

We are now able to define[6] the reference signal $\bar{\eta}_*(s)$ for $\bar{\eta}(s)$ and the reference input $\bar{v}_{c*}(s)$ satisfying

$$\begin{aligned} 0 &= l\bar{v}_{c*}(s) + \sin\left(\frac{\bar{\eta}_*(s)}{l}\right) \\ \bar{\eta}'_*(s) &= -l\kappa(s) - \sin\left(\frac{\bar{\eta}_*(s)}{l}\right). \end{aligned} \tag{8}$$

For a suitable initial condition $\bar{\eta}_*(-nL)$, $\bar{\eta}_*(s)$ and $\bar{v}_{c*}(s)$ are s-periodic of period L, that is

$$\begin{aligned} \bar{\eta}_*(s+L) &= \bar{\eta}_*(s) \\ \bar{v}_{c*}(s+L) &= \bar{v}_{c*}(s) \end{aligned}$$

for all $s \geq -nL$. This can be proved by defining the error

$$\xi(s) = \bar{\eta}_*(s+L) - \bar{\eta}_*(s),$$

[6]Note that this is not describing a reference virtual vehicle moving along the curve since s is inherited by the controlled vehicle behaviour and the corresponding s_* is defined by $\dot{s}^* = \dot{s}$. In other words, a path following (and not trajectory tracking) problem is here solved: the goal is only to steer the vehicle to reach and follow a geometric path with specific constraints - such as time reparameterization for the vehicle movement along the path - being of secondary importance (see [12], [17], [18], [19], [20] and references therein).

by considering the Lyapunov function

$$V_\xi = \frac{1}{2}\xi^2(s)$$

and its s-derivative along the trajectories of the $\xi(s)$- dynamics[7] and by applying the Brouwer fixed point theorem ([6]) to the continuous function $T(\cdot)$ which associates to the initial condition $\bar{\eta}_*(-nL) = q$ the solution of the second equation in (8) evaluated at $s = (n-1)L$. The set

$$\Pi_\eta = [-l\arcsin(p), l\arcsin(p)]$$

for $l|\kappa(s)| < p < 1$ is in fact $[-nL, +\infty)$-invariant for the second equation in (8) since $f(\bar{\eta}_*, s) \doteq -l\kappa(s) - \sin(\bar{\eta}_*/l)$ is such that $f(l\arcsin(p), s) < 0$ and $f(-l\arcsin(p), s) > 0$.

C. Design of the learning controller and stability analysis

We define the tracking error

$$\tilde{\eta}(s) = \bar{\eta}(s) - \bar{\eta}_*(s)$$

so that the error system, according to (8), is

$$\begin{aligned} \bar{y}'_L(s) &= \bar{y}'_L(s) - 0 = l\left[\bar{v}_c(s) - \bar{v}_{c*}(s)\right] \\ &\quad + \sin\left(\frac{\bar{\eta}(s)+\bar{y}_L(s)}{l}\right) - \sin\left(\frac{\bar{\eta}_*(s)}{l}\right) \\ \tilde{\eta}'(s) &= -\sin\left(\frac{\bar{\eta}(s)+\bar{y}_L(s)}{l}\right) + \sin\left(\frac{\bar{\eta}_*(s)}{l}\right) \end{aligned}$$

where $\bar{v}_{c*}$ is defined by the first equation in (8). Let: the reals k_P, k_I, M_v be suitable positive control parameters; $\mathrm{sat}_{M_v}(\cdot) : \mathbb{R} \to [-M_v, M_v]$ be a continuous odd increasing function satisfying $\mathrm{sat}_{M_v}(q) = q$ for any $q \in (0, M_v]$ and $\mathrm{sat}_{M_v}(q) = M_v$ for any $q > M_v$; $\varphi_x(\cdot) : \mathbb{R}^+ \cup \{0\} \to [0,1]$ $(x > 0)$ be a continuous increasing function for $t \in [0, x]$ with $\varphi_x(0) = 0$ and $\varphi_x(t) = 1$ for any $t \geq x$; α_i, $1 \leq i \leq n$, be positive design parameters satisfying $\sum_{i=1}^n \alpha_i = 1$, $\alpha_i \geq 0$, $1 \leq i \leq n$; j_* be equal to $\min\{j : \alpha_j > 0\}$; b be equal to $\alpha_1 + 2\alpha_2 + \ldots + n\alpha_n$. Stability arguments similar to those used in [5], [16] and [33] (with s in place of t and with error nonlinearities appearing in the dynamics of $(\bar{y}_L, \bar{\eta})$ to play the only role of complicating all the derivations) can be now used to prove that the P-type learning control

$$\begin{aligned} \bar{v}_c(s) &= -k_P\bar{y}_L(s) + \hat{\bar{v}}_{c*}(s) \\ \hat{\bar{v}}_{c*}(s) &= \alpha_1\mathrm{sat}_{M_v}\left(\hat{\bar{v}}_{c*}(s-L)\right) + \alpha_2\mathrm{sat}_{M_v}\left(\hat{\bar{v}}_{c*}(s-2L)\right) \\ &\quad + \ldots + \alpha_n\mathrm{sat}_{M_v}\left(\hat{\bar{v}}_{c*}(s-nL)\right) - k_I L b \varphi_{j_* L}(s)\bar{y}_L(s) \\ \hat{\bar{v}}_{c*}(r) &= 0, \quad \forall\ r \leq 0 \end{aligned} \tag{9}$$

is able to guarantee (with a proper choice of the control gains) the asymptotic output tracking

$$\begin{aligned} \lim_{s\to+\infty} \bar{y}_L(s) &= 0 \\ \lim_{s\to+\infty} \tilde{\eta}(s) &= 0. \end{aligned} \tag{10}$$

[7]Use the mean value theorem ([11]).

On the other hand, the previous asymptotic equations imply the weak convergence to zero of $\hat{\bar{v}}_c(s) - \bar{v}_{c*}(s)$ in the space $\mathcal{L}^2[0, L]$ ([6], [14]). We can in fact write

$$\begin{aligned}\bar{y}_L(s) - \bar{y}_L(s-a) &= \int_{s-a}^{s} \bar{y}'_L(\tau)\mathrm{d}\tau \\ &= -l\int_{s-a}^{s} [\bar{v}_{c*}(\tau) - \hat{\bar{v}}_{c*}(\tau)]\mathrm{d}\tau + r(s)\end{aligned}$$

for any $a \in (0, L]$ with $\lim_{s\to+\infty} r(s) = 0$, which leads to ($m \in \mathbb{N}^+$, χ_i is the characteristic function on the set $\Sigma_i \subset [0, L]$, $a_i \in \mathbb{R}^+$)

$$\lim_{s\to+\infty} \int_{s-L}^{s} [\bar{v}_{c*}(\tau) - \hat{\bar{v}}_{c*}(\tau)] \sum_{i=1}^{n} a_i\chi_i \mathrm{d}\tau = 0.$$

The previous results hold for any sufficiently small $|\bar{y}_L(0)|$, $|\tilde{\eta}(0)|$ and sufficiently high k_P guaranteeing for all $s \geq 0$ ($\Theta \in [0, 1]$)

$$\begin{aligned}&\cos\left(\frac{\bar{\eta}(s) + \bar{y}_L(s)}{l}\right) \geq c_\beta > 0 \\ &\cos\left(\bar{\eta}_*(s)\right) \geq c_{\eta*} > 0 \\ &\cos\left((1-\Theta)\left[\frac{\bar{\eta}(s) + \bar{y}_L(s)}{l}\right] + \Theta\bar{\eta}_*(s)\right) \geq c_\eta > 0 \\ &\cos\left(\frac{\bar{\eta}(s) + \bar{y}_L(s)}{l}\right) - \bar{y}_L(s)\bar{v}_c(s) \geq c_v > 0.\end{aligned}$$

The proof is based on the use (see [16] and [33]) of the Lyapunov functions $V_y(s) = 1/2\bar{y}_L^2(s)$, $V_\eta(s) = 1/2\tilde{\eta}^2(s)$ satisfying for all $s \geq 0$

$$\begin{aligned}V_y(s) &\leq V_y(0) + \frac{a_1}{k_P^2} \\ V_\eta(s) &\leq V_\eta(0) + a_2 V_y(s)\end{aligned}$$

and of the function

$$\mathcal{V}(s) = V_y(s) + V_\eta(s) + l\mathcal{P}(s) \tag{11}$$

satisfying for any $s \geq j_* L$

$$\mathcal{V}'(s) \leq -a_3 V_y(s) - a_4 V_\eta(s)$$

with a_1, a_2 (not depending on k_P), a_3, a_4 suitable positive reals and

$$\begin{aligned}\mathcal{P}(s) = (2bk_I L)^{-1}\Bigg\{ &\overbrace{(\alpha_1 + \ldots + \alpha_n)}^{=1} \int_{s-L}^{s} g_v(\tau)\mathrm{d}\tau \\ &+ (\alpha_2 + \ldots + \alpha_n)\int_{s-2L}^{s-L} g_v(\tau)\mathrm{d}\tau \\ &+ \ldots + \alpha_n \int_{s-nL}^{s-(n-1)L} g_v(\tau)\mathrm{d}\tau \Bigg\} \\ g_v(s) = \left[\bar{v}_{c*}(s) - \mathrm{sat}_{M_v}\left(\hat{\bar{v}}_{c*}(s)\right)\right]^2&.\end{aligned}$$

According to [16] and [33], the learning steering control (9) can be interpreted as a generalized PI control in spatial coordinates: the P action is the same one while the integral action is generalized to the periodic framework by the learning estimation scheme in (9) which is apparently designed on the basis of the past executions. The learning thus becomes a bridge between knowledge and experience, i.e. insufficient knowledge can be effectively bridged by learning through repetitive practice. Furthermore, the above learning estimation scheme in (9) uses not only the most recent previous information but relies on a weighted sum of the information stored in the n previous executions ([8]) as typically happens for humans driving along repetitive paths. The weights α_i, $1 \leq i \leq n$, constitute extra degrees of freedom in the control design which may be chosen in order to improve the closed loop performances (see recent advances in [13], [22], [24], [25], [26]). The constraint

$$\sum_{i=1}^{n} \alpha_i = 1$$

finally guarantees that the sensitivity function $S(s)$ is zero for $s = kj2\pi/L$, $k = 0, 1, 2, \ldots$ in accordance with the typical property, achieved by correctly-initialized learning controls, of reproducing periodic disturbances of period L (see [22], [24], [25] for related results).

Remark 2.1: The second asymptotic equation in (10), according to the definition $\eta = l\beta - y_L$, implies

$$\lim_{s\to+\infty} \left[\bar{\beta}(s) - \bar{\beta}_*(s)\right] = 0$$

where, according to (5), $\bar{\beta}_*(s)$ satisfies the differential equation

$$\bar{\beta}'_*(s) = \bar{v}_{c*} - \kappa(s) = -\frac{1}{l}\sin\left(\bar{\beta}_*(s)\right) - \kappa(s).$$

In the special case of constant curvature $\kappa(s) \equiv c_k$, the last asymptotic equation becomes

$$\lim_{s\to+\infty} \left[\bar{\beta}(s) - \arcsin(lc_k)\right] = 0$$

showing that $\bar{\beta}(s)$ reasonably converges to a constant for $s \to +\infty$.

Remark 2.2: When expressed with respect to the time t, the proposed control becomes

$$\begin{aligned}\omega(t) &= \dot{s}(t)v_c(t) = \dot{s}(t)\left[-k_P y_L(t) + \hat{v}_{c*}(t)\right] \\ \hat{v}_{c*}(t) &= \alpha_1 \mathrm{sat}_{M_v}\left(\hat{v}_{c*}(t - T_t(t))\right) \\ &\quad + \alpha_2 \mathrm{sat}_{M_v}\left(\hat{v}_{c*}(t - 2T_t(t))\right) + \ldots \\ &\quad + \alpha_n \mathrm{sat}_{M_v}\left(\hat{v}_{c*}(t - nT_t(t))\right) - k_I Lb\varphi_T(t)y_L(t) \\ \hat{v}_{c*}(r) &= 0, \quad \forall\, r \leq 0\end{aligned} \tag{12}$$

which reduces to the classical time-learning control when $\dot{s}(t)$ is constant (recall, with this regard, the related comments in the Introduction[8]).

[8]The "time-varying period" $T_t(t)$ satisfies

$$\int_{t-T_t(t)}^{t} \dot{s}(\tau)\mathrm{d}\tau = L, \quad \forall\, t \geq T.$$

When $\dot{s}$ is constant, T_t is in fact constant and identically equal to $L/\dot{s}$.

III. Simulation results

We have applied in simulation the learning method presented in Section II to the following two cases. In the first case, the curve Γ is given by $\gamma([0, 2\pi])$ with

$$\gamma(s) = (10 + \sin(8s))[\sin(s), -\cos(s)]^T,$$

reparameterized according to the arc-length. The curve is represented in Figure 1, the length of the curve is $L \simeq 71.94$ m and the curvature is periodic with period $L/8$. We have implemented in simulation the control law (12) with $n = 1$, $k_P = 0.5$, $k_I L = 30$ and $\varphi_x = \min\{x, 1\}$. The vehicle speed is constant and is given by $v = 1$. Figures 1 and 2, in which the reference and actual vehicle trajectories are reported along with the lateral offset $y_L(t)$, illustrate the effectiveness of the proposed control design and analysis.

In the second case we have considered the curve Γ given by $\gamma([0, 2\pi])$ with

$$\gamma(s) = [2s, -\sin(s)]^T,$$

reparameterized according to the arc-length. Even though this second curve, that is represented in Figure 1, is not closed, its curvature is anyway periodic with period $L \simeq 13.32$ m. We have implemented in simulation the control law (9) with $n = 1$, $k_P = 0.2$, $k_I L = 40$ and $\varphi_x = \min\{x, 1\}$. Also in this simulation, the vehicle speed is constant and is given by $v = 1$. The results are reported in Figures 3 and 4: satisfactory tracking is achieved also in this case.

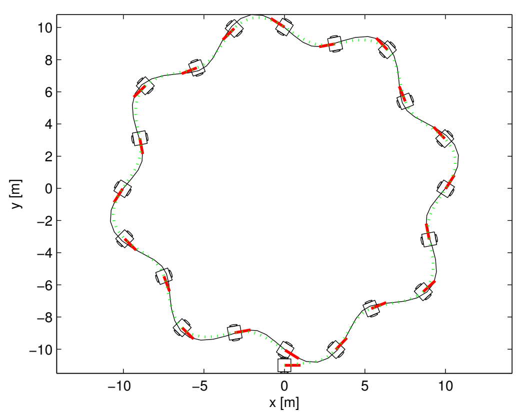

Fig. 1. The reference curve Γ (black line), the vehicle path (green dotted line) and the segment joining points P and Q (red segment) for the first simulation.

Conclusions

A space-learning control is designed for tracking of planar curves whose uncertain curvature is L-periodic in the curvilinear abscissa. The behaviour of a human driver who repetitively learns the correct action from the past experience is mathematically reproduced. Simulation results demonstrate the effectiveness of the presented control design and analysis. Further research efforts will be devoted to the experimental validation of the proposed approach.

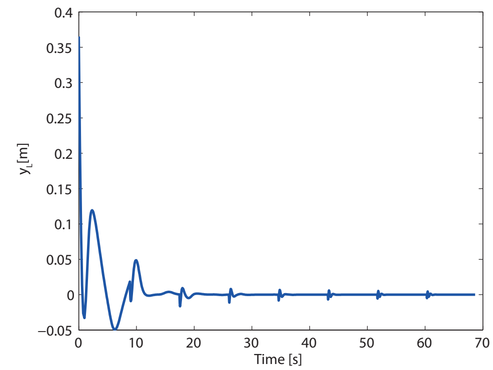

Fig. 2. The lateral offset $y_L(t)$ for the first simulation.

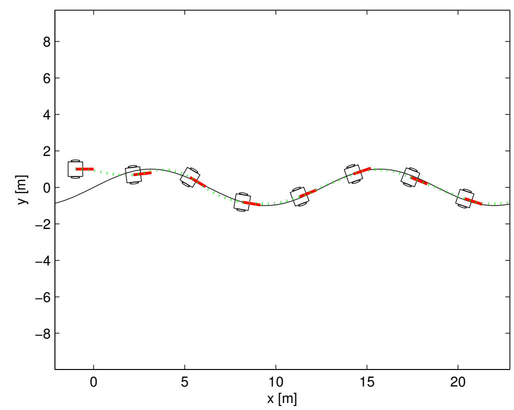

Fig. 3. The reference curve Γ (black line), the vehicle path (green dotted line) and the segment joining points P and Q (red segment) for the second simulation.

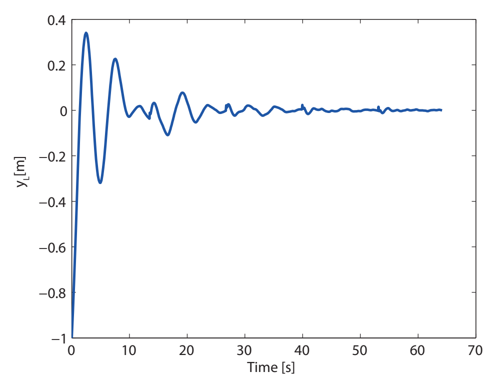

Fig. 4. The lateral offset $y_L(t)$ for the second simulation.

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