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A Comparison of Fractional Smith Predictors

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Abstract **: A new class of fractional Smith predictors is proposed in this paper, associating Internal Model Control for the design of time delay controllers to the CRONE template in order to improve robustness to static gain variations. Numerical simulations illustrate intrinsic and structural robust performances of the fractional PI and PID predictors. Comparisons of the two controllers show that the Smith PID presents the greatest ability to reject gain variations and system structure uncertainty.**

Keywords **: Fractional PID, Time delay systems, Internal model control, Smith predictor, Robust performances.**

I. Introduction

PID control and the tuning of PID controllers remain an active research field. Fractional calculus [11,12,14] has certainly contributed to this intense activity, thanks to the fractional PID controller introduced by Podlubny [13] with its generalized integral and derivative actions. Since this initial paper, a lot of publications have been devoted to the tuning of the $PI^{\alpha} D^{\beta}$ controller [7,8,20]. Time delay systems can be controlled by classical PID regulators, but it is well known that for increased performance, a specific controller is required, which is called the Smith predictor [16]. Because the time delay cannot be cancelled, it has to be included in the closed loop reference model, according to Internal Model Control (IMC) theory [9]. So, the equivalent feedback controller corresponds to the Smith predictor, which can be interpreted as a generalized PID controller for time delay systems. Quite naturally, combining this classical approach to fractional integral and derivatives actions, the fractional Smith predictor has been introduced [2,17].

Since the seminal works of Bode [1], Manabe [5] and Oustaloup [10], it has been demonstrated that the main interest of fractional controllers is to improve robustness of the closed loop to plant uncertainties. Robustness to static gain variations was the main feature of these early fractional controllers. Based on the CRONE open loop reference model, a new class of fractional PI and PID controllers has been defined [4]. Using IMC principle [9], it is easily generalized to fractional PI and PID Smith controllers. In this paper, we propose to compare the performances of these two controllers in terms of robust performances. Two types of robustness are investigated: robustness to static gain variations and analysis of its limitations caused by the time delay, and robustness to the real structure of the plant, which is generally more complex than that of the model used for parameter tuning.

The paper is organized as follows. Section II is a reminder of fractional PI and PID controllers based on the CRONE model. Using IMC theory, fractional Smith controllers are defined in section III. Finally, based on numerical simulations, a comparison of the robust performances of these controllers is presented in section IV.

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II. Fractional PI and PID Controllers

A. Introduction

The objective is to design a controller $C(s)$, according to the conventional feedback loop of figure n°1, where $H(s)$ is the plant. Our design objective is that the closed loop system behaves as a given reference model $H_{ref}(s)$:

$$H_{CL}(s)=\frac{C(s)H(s)}{1+C(s)H(s)}=H_{ref}(s) \tag{1}$$

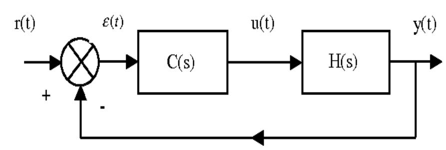

Figure n°1: Conventional feedback control loop

B. Fractional PI controller

Oustaloup has defined an ideal open loop CRONE template [10]:

$$H_{OL}(s)=\frac{\omega_{ref}}{s^{n+1}} \quad \text{with } 0<n<1 \tag{2}$$

in order to get rid of static gain variations. The reference closed loop transfer function corresponding to this CRONE template is:

$$H_{ref}(s)=\frac{1}{1+\dfrac{s^{n+1}}{\omega_{ref}}}=\frac{1}{1+\tau_{ref}\, s^{n+1}} \tag{3}$$

Consider the first order system:

$$H(s)=\frac{G}{1+\tau s} \tag{4}$$

whose static gain G is subject to variations. Because our design objective is $H_{CL}(s)=H_{ref}(s)$, it is straightforward to get the corresponding controller:

$$C(s)=\frac{1+\tau s}{G\tau_{ref}s^{n+1}} \tag{5}$$

which we can write as:

$$C(s)=\frac{1}{s^n}(\frac{1}{T_i s}+K_p) \tag{6}$$

with

$$T_i=G\tau_{ref} \text{ and } K_p=\frac{\tau}{G\tau_{ref}} \tag{7}$$

$C(s)$ is a classical PI controller associated to a fractional integrator $1/s^n$. We define it as the (PI)^n controller: it is the generalization of the PI controller to the fractional case.

Remark: Thanks to this fractional controller, the open loop transfer function $C(s)H(s)$ matches perfectly the CRONE template (2), (i.e. the curve representing $C(j\omega)H(j\omega)$ in the Nichols chart is exactly vertical [10]), so there is perfect rejection of static gain variations.

C. Fractional PID controller

Let

$$H(s)=\frac{G}{1+a_1 s+a_2 s^2} \tag{8}$$

be the process to be controlled, where G is subject to variations. Robustness to static gain variations is provided by the following open loop CRONE template [4]:

$$H_{OL}(s)=\frac{\frac{\omega_0}{2m}}{s^{n+1}(1+\frac{s}{2m\omega_0})} \tag{9}$$

where $0<n<1$ provides robustness and ω_0 and m are tuning parameters.

The corresponding closed loop reference model is:

$$H_{ref}(s)=\frac{1}{1+\frac{2m}{\omega_0}s^{n+1}+\frac{s^{n+2}}{\omega_0^{\ 2}}} \tag{10}$$

$H_{ref}(s)$ is a generalized second order reference model, where n, ω_0 and m characterize the time response and the damping of the closed loop system. A fractional PID controller (PID$_f$)^n corresponding to the transfer function [4]:

$$C(s)=\frac{1}{s^n}(\frac{1}{T_i s}+K_p+\frac{T_d s}{1+\tau_f s}) \tag{11}$$

applied to $H(s)$ insures exactly the open loop template $H_{OL}(s)$ and the closed loop reference model $H_{ref}(s)$. The tuning of the parameters corresponds to:

$$T_i=\frac{2mG}{\omega_0} \qquad \tau_f=\frac{1}{2m\omega_0}$$
$$K_p=\frac{a_1-\tau_f}{T_i} \qquad T_d=\frac{a_2}{T_i}-K_p\tau_f \tag{12}$$

Let us remind that any PID controller has to include a filtering τ_f of the derivative action $T_d\ s$ in the design of the controller. This filtering action is necessary for a realistic implementation of the controller and to reduce noise sensitivity of the control input $u(t)$.

On the other hand, the presence of $\tau_f=1/2m\omega_0$ in the open loop transfer function (9) prevents the curve $H_{OL}(j\omega)$ (in the Nichols chart) to be exactly vertical, so there is no longer perfect rejection of static gain variations. This remark applies also to the fractional Smith predictors.

III. FRACTIONAL SMITH PREDICTORS

A. Internal Model Control

IMC [9] refers to the closed loop structure illustrated on figure n°2.

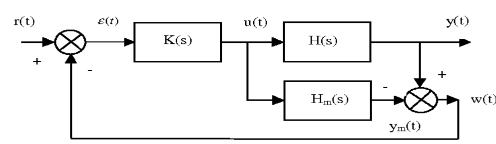

Figure n° 2 : Internal Model Control loop

$H(s)$ is the system, $H_m(s)$ is the model of the system: it represents the user's knowledge concerning the true system. $K(s)$ is the IMC controller:

Let $H_{ref}(s)$ be the reference model of the closed loop. For the synthesis of $K(s)$, we consider the nominal case, i.e. we assume that $H_m(s)$ represents exactly $H(s)$. If moreover we assume that there is no disturbance in the system, we get:

$y(t)=y_m(t)$ thus $w(t)=0$, i.e. the system operates in open loop. So we can write:

$$H(s)K(s)=H_m(s)K(s)=H_{ref}(s) \tag{13}$$

or equivalently:

$$K(s)=H_m(s)^{-1}H_{ref}(s) \tag{14}$$

which is the relation defining $K(s)$.

Because $w(t)=0$, the system operates in open loop, so notice that there is no stability problem in the nominal case, provided that $H(s)$ and $K(s)$ are stable transfer functions.

Indeed, there is generally a mismatch between $H(s)$ and $H_m(s)$, so the closed loop transfer function is:

$$H_{CL}(s)=\frac{K(s)\,H_m(s)}{1+K(s)\,(H(s)-H_m(s))} \tag{15}$$

which means that the modeling error $H(s)-H_m(s)$ is the cause of closed loop instability.
There is exact equivalence between IMC and conventional feedback [9], corresponding to the equivalence equation:

$$C(s)=\frac{K(s)}{1-K(s)H_m(s)}=\frac{K(s)}{1-H_{ref}(s)} \tag{16}$$

This equivalence allows implementation of IMC using a conventional feedback loop; this approach is used by the Smith predictor.

B. PI Smith controller

Let $\quad H(s)=H_{1,T}(s)=\dfrac{G\,e^{-T\,s}}{1+\tau\,s} \qquad (17)$

be the process to be controlled.
It is well known that it is impossible to cancel the time delay T : so it remains necessarily in the closed loop response. We have to include this time delay in the closed loop reference model according to the IMC principle [9]. Moreover, because our objective is to get rid of static gain variations, we have to use the closed loop CRONE model (3). So the resulting reference IMC model is:

$$H_{ref}(s)=\frac{e^{-T\,s}}{1+\tau_{ref}\,s^{n+1}} \tag{18}$$

Then, using the equivalence between IMC and conventional feedback, it is straightforward to demonstrate that the controller $C(s)$ corresponds to:

$$C(s)=\frac{1+\tau\,s}{\tau_{ref}\,s^{n+1}}\;\frac{1}{1+\dfrac{1-e^{-Ts}}{\tau_{ref}\,s^{n+1}}} \tag{19}$$

A transformation of this equation, inspired by the Smith predictor technique [16], shows that $C(s)$ corresponds to the structure represented figure n°3:

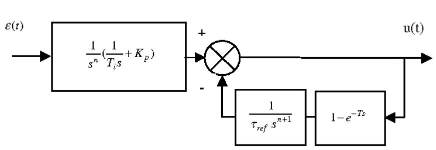

Figure n°3: Fractional PI Smith controller

This controller is composed of a fractional PI controller whose parameter tunings are given by (7), associated to a closed loop term, analogous to that of the Smith predictor, including the time delay $e^{-T\,s}$ and a second fractional integrator $\dfrac{1}{s^{n+1}}$.

Notice that if we impose $n=0$, the previous equations define the classical integer order Smith PI predictor [16].

C. PID Smith controller

Let $\quad H(s)=H_{2,T}(s)=\dfrac{G\,e^{-T\,s}}{1+a_1\,s+a_2\,s^2} \qquad (20)$

be the process to be controlled.
Using the same approach as previously, we have to use the generalized CRONE closed loop reference model, so:

$$H_{ref}(s)=\frac{e^{-T\,s}}{1+\dfrac{2m}{\omega_0}s^{n+1}+\dfrac{s^{n+2}}{\omega_0^{\,2}}} \tag{21}$$

Then, using IMC methodology, it is straightforward to demonstrate that the controller $C(s)$ corresponds to:

$$C(s)=\frac{1+a_1 s+a_2 s^2}{\dfrac{2mG}{\omega_0}s^{n+1}(1+\dfrac{s}{2m\omega_0})}\;\frac{1}{1+\dfrac{1-e^{-Ts}}{\dfrac{2m}{\omega_0}s^{n+1}(1+\dfrac{s}{2m\omega_0})}} \tag{22}$$

This controller corresponds to the structure represented figure n°4:

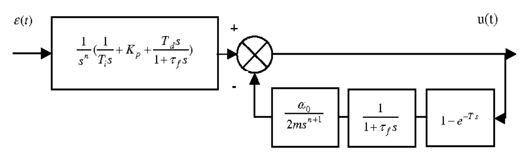

Figure n°4: Fractional PID Smith controller

$C(s)$ is composed of a fractional PID controller, whose parameter tunings are given by (12), associated to a closed loop term, including the time delay $e^{-T\,s}$ and a second fractional integrator $\dfrac{1}{s^{n+1}}$.

Obviously, this closed loop term increases the complexity of the Smith controller. On the other hand, it guarantees unconditional stability of the closed loop in the nominal case, i.e. if $H_m(s)=H_{2,T}(s)=H(s)$.

IV. A Comparative Analysis of Smith Predictors

A. Problem Statement

Robustness of fractional controllers depends on several factors. First of all, because they have been designed with the CRONE template, it is necessary to test their ability to reject static gain variations, i.e. gain variations associated to changes of the plant operating point. Secondly, because the system model used for the design is an approximation of the true plant model, it is necessary to investigate the influence of this mismatch on robustness. So, we propose two types of tests.

The first one concerns intrinsic robustness, i.e. robustness to gain variations when there is no mismatch between the model $H_m(s)$ and the true system $H(s)$.

The second one concerns structural robustness, i.e. robustness to gain variations when there is a mismatch between the system and its model.

Assume that the true system is characterized by:

$$H(s)=e^{-\Delta_H s}\,G(s) \qquad (23)$$

where Δ_H is a true time delay and $G(s)$ represents the dynamical part of the system.

Practically, we have to consider two models:

$H_{1,T}(s)$ for the PI Smith controller

$H_{2,T}(s)$ for the PID Smith controller

There exist many techniques to derive these models: there are elementary (graphical) techniques (like Strejc, Broïda, …) and techniques based on system identification. In our approach, we have derived these two models using output error identification [3,15] which is based on non linear optimization [6].

Notice that $T=\Delta_H$ only if $H_m(s)=H(s)$. Because of mismatch ($H_m(s)\neq H(s)$) , we get :

$$T=\Delta_H+\Delta_G \qquad (24)$$

where Δ_G is an equivalent time delay corresponding to $G(s)$.

B. Intrinsic Robustness

B.1 PI Smith predictor

We consider

$$H(s)=H_m(s)=H_{1,T}(s)=\frac{G\,e^{-Ts}}{1+\tau s} \qquad (25)$$

with $G_{nom}=1$ $\tau=1s$ and $T=0.1s$ $T=0.5s$ $T=1s$.

G varies in the interval :

$$\frac{G_{nom}}{2}<G<2G_{nom} \qquad (26)$$

This test interval has been used for all the other controllers.

Because we expect a constant overshoot (of the closed loop response) when G varies, we characterize robustness with the value $D_{\%}$ of the achieved overshoot.

Each controller has been designed with $\tau_{ref}=0.5s$ and $n=0.4$.

The results are presented in Table 1 for the three values of the time delay.

G	2	1.5	1	0.666	0.5
T = 0.1s	28.5	25	21.7	19.5	18.3
T = 0.5s	76.3	47	21.7	11.2	8.2
T = 1s	unstable	70.8	21.7	4.7	0

Table n° 1 : Fractional PI controller , $H(s)=H_{1,T}(s)$

As expected, robustness is correctly achieved only for small time delays (i.e. $T=0.1s$) and there is a degradation for large time delays (notice instability for $G=2$ $T=1s$).

B. 2 PID Smith predictor

We consider :

$$H(s)=H_m(s)=H_{2,T}(s)=\frac{G\,e^{-Ts}}{(1+\tau_1 s)(1+\tau_2 s)} \qquad (27)$$

with $G_{nom}=1$ $\tau_1=1s$ $\tau_2=2s$ $T=0.25s$; $0.5s$; $1s$

As previously, robustness depends on the time delay, but also on the closed loop time response. So, we have used two values for this time responses, referred as slow and fast. They are obtained with the following tuning parameters:

Slow : $n=0.35$ $m=2.5$ $\omega_0=1.25\,rd/s$

Fast : $n=0.35$ $m=2.5$ $\omega_0=5\,rd/s$

The respective values of the closed loop overshoot are presented in tables 2 and 3.

G	2	1.5	1	0.666	0.5
T=0.25s	25.4	22.9	20.8	19.5	19
T=0.5s	30	25	20.8	18.4	17.4
T=1s	42.5	30.4	20.8	16.2	14.3

Table n°2: Fractional PID controller, $H(s)=H_{2,T}(s)$, *slow response*

G	2	1.5	1	0.666	0.5
T=0.25s	32.4	25.3	19.4	16.1	14.6
T=0.5s	51.2	33.5	19.4	13.1	10.9
T=1s	87.5	51.5	19.4	8.3	6

Table n°3: Fractional PID controller, $H(s)=H_{2,T}(s)$, *fast response*

Because the classical Smith predictor corresponds to $n=0$, we have also compared the fractional predictor to its integer order equivalent for the two responses:

Slow : $n=0$ $m=0.52$ $\omega_0=0.45\,rd/s$

Fast : $n=0$ $m=0.5$ $\omega_0=1.2\,rd/s$

The corresponding values of the closed loop overshoot are presented in tables 4 and 5.

G	2	1.5	1	0.666	0.5
T=0.25s	38	29.5	19.2	11.2	7.5
T=0.5s	43.8	32.4	19.2	9.9	6.3
T=1s	56.8	38.5	19.2	7.5	4.5

Table n°4: Conventional Smith PID controller, $H(s)=H_{2,T}(s)$, *slow response*

G	2	1.5	1	0.666	0.5
T=0.25s	49.6	35.3	19.3	8	3.5
T=0.5s	67.4	43.7	19.3	4.8	1.9
T=1s	99.3	59.3	19.3	1	1

Table n°5: Conventional Smith PID controller, $H(s)=H_{2,T}(s)$ *, fast response*

Firstly, comparison with conventional Smith predictor shows that the fractional controller exhibits a better robustness in all situations, thanks to the CRONE template. However, as previously, this robustness is excellent only for small values of T and a slow response; it is degraded for large values of T and faster responses.

C. Structural robustness

C. 1 PI Smith controller

We consider

$$H_m(s)=\frac{G\,e^{-Ts}}{1+\tau s}=H_{1,T}(s) \qquad (28)$$

and

$$H(s)=\frac{G\,e^{-\Delta_H s}}{(1+\tau_1 s)(1+\tau_2 s)} \qquad (29)$$

with $G_{nom}=1$ $\tau_1=1s$ $\tau_2=2s$ $\Delta_H=0.25s;\ 0.5s\ ;\ 1s$

$H_m(s)$ has been estimated with output error technique. Estimates are listed in table n°6.

Δ_H	G	τ	T	J
0.25s	1.003	2.44	0.893	0.686
0.5s	1.003	2.44	1.149	0.686
1s	1.003	2.44	1.646	0.686

Table n°6 Output error estimation of $H_{1,T}(s)$

We can notice that $H_m(s)$ is a rough approximation of $H(s)$, as indicated by the large value of quadratic criterion J.

Notice that $T=\Delta_H+\Delta_G$, according to (24), with $\Delta_G\approx 0.65\,s$.

Then, we have tested (see table n°7) the closed loop robustness to G variations for the three values of Δ_H, using the previous slow case. Notice that it has been necessary to modify n for each value of Δ_H, with $\tau_{ref}=3s$.

Thus:

$\Delta_H=0.25s$ $n=0.24$; $\Delta_H=0.5s$ $n=0.22$; $\Delta_H=1s$ $n=0.18$.

G	2	1.5	1	0.666	0.5
$\Delta_H=0.25s$	35.8	28.5	21.1	17.2	16.2
$\Delta_H=0.5s$	40.5	30.7	21.2	16.6	15.6
$\Delta_H=1s$	48.9	33.9	20.5	15.0	14.1

Table n° 7: PI Smith predictor applied to the time delayed second order system

Indeed, we can compare these results to those of the PID Smith predictor, slow case. We notice immediately that the Smith PID exhibits better robust performance than the Smith PI.

So, we can conclude: though the two Smith predictors have good robust intrinsic robustness, the PI Smith predictor is not able to ensure the same performance level as the PID predictor if there is a structural mismatch between $H_m(s)$ and $H(s)$. Indeed, this is explained by the presence of the derivative action of the PID controller and by the important mismatch indicated by the quadratic criterion.

C. 2 PID Smith controller

We consider now :

$$H_m(s)=\frac{G\,e^{-Ts}}{(1+a_1 s+a_2 s^2)}=H_{2,T}(s) \qquad (30)$$

and

$$H(s)=\frac{G\,e^{-\Delta_H s}}{(1+\tau_1 s)(1+\tau_2 s)(1+\tau_3 s)} \qquad (31)$$

with: $G_{nom}=1$ $\tau_1=1s$ $\tau_2=2s$ $\tau_3=3s$ $\Delta_H=0.2s;\ 1s\ ;\ 5s$

As previously, $H_m(s)$ has been estimated with the output error technique, see table n°8, with the same excitation.

Δ_H	G	a_1	a_2	T	J
0.2s	0.997	5.34	7.61	0.867	0.066
1s	0.996	5.33	7.61	1.666	0.065
5s	0.997	5.35	7.75	5.641	0.040

Table n°8: Output error estimation of $H_{2,T}(s)$

Notice now the low value of quadratic criterion J, indicating a good match between $H_m(s)$ and $H(s)$.

Again, we notice that $T=\Delta_H+\Delta_G$, with $\Delta_G\approx 0.65\,s$

Then, we have tested robustness to gain variations, for two values of the time response:

Slow : $n=0.33$ $m=2.5$ $\omega_0=0.4\,rd/s$

Fast : $n=0.35$ $m=2.5$ $\omega_0=0.8\,rd/s$

We notice good robustness to G variations for the different values of Δ_H (see Tables n° 9 and 10, figures n° 9 and 10) with the slow response, whereas there is an important degradation (and even instability) for $\Delta_H=5s$ with the fast response.

G	2	1.5	1	0.666	0.5
$\Delta_H=0.2s$	28	24.4	21.8	20.7	20.5
$\Delta_H=1s$	35.4	27.6	21.8	19.2	18.2
$\Delta_H=5s$	81.8	49.3	21.8	12.4	10

Table n°9: PID Smith predictor applied to the time delayed third order system, slow response

G	2	1.5	1	0.666	0.5
$\Delta_H = 0.2s$	34.3	27.2	21.3	18.5	17.5
$\Delta_H = 1s$	48.8	33.8	21.3	15.9	14
$\Delta_H = 5s$	unstable	67	21.3	6.5	0

Table n°10 : PID Smith predictor applied to the time delayed third order system, fast response

However, the PID Smith controller exhibits a better ability than the PI one to reject structural mismatch because it presents more tuning parameters and moreover because $H_{2,T}(s)$ model has more flexibility than $H_{1,T}(s)$ to reduce structural mismatch.

D. Conclusion

Fractional and integer order Smith predictors exhibit equivalent performances in the nominal case ($G = G_{nom}$) and with no mismatch between the system and its model: this means that they are theoretically able to provide perfect control with large time delays. Practically, due to their high sensitivity to gain uncertainty for large time delays, the real interest of conventional Smith predictors is limited by the risk of closed loop instability.

As exhibited by previous results, fractional controllers provide more security to the user thanks to their static gain robustness, i.e. their ability to reject static gain uncertainty.

Robustness to structural uncertainty is an essential feature for all systems, but particularly for time delay systems. The fractional PID Smith controller is certainly the one which exhibits the greatest ability to reject static gain and structural uncertainties.

V. CONCLUSION

A new class of fractional Smith predictors has been proposed in this paper: it associates Internal Model Control principle for the design of time delay controllers to the CRONE template in order to improve robustness to static gain variations. A particular feature of these controllers is their tuning simplicity: in fact, it relies essentially on system identification using $H_{1,T}(s)$ or $H_{2,T}(s)$ models. Numerical simulations have illustrated intrinsic and structural robust performances of the fractional PI and PID predictors. However, comparisons have shown that the Smith PID controller presents the greatest ability to reject gain variations and system structure uncertainty. So, an interesting application of the PID predictor is the robust control of a large class of uncertain plants in association with system identification [18,19].

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