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Distributed Control of Swarm Motions as Continua using Homogeneous Maps and Agent Triangulation

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Abstract— **In this paper, presented is an approach for the control of swarm motions that treats a swarm as a deformable body or a continuum. By considering a special class of motion maps between a current configuration prescribed at a given time t and a desired configuration prescribed at a subsequent time t+Δt called a homogeneous map two strategies are proposed for the control of a swarm or a multi-agent system (MAS). In the first strategy, MAS motion control is achieved with no communication among agents in the idealized case where the required map from any t to t+Δt may be pre-determined. It is the case when a desired swarm task can be scripted ahead of time with some certainty. The second control strategy is based on three leaders prescribing the motion map for a desired swarm task which is then propagated to the followers via a local inter-agent communication protocol. The proposed communication protocol exploits some special features of homogeneous maps. It achieves the desired MAS motion control with three leaders and minimum inter-agent communications. Simulation results validate the effectiveness of the proposed strategies.**

I. INTRODUCTION

Some common approaches to formation control of MAS include leader-follower, virtual structures, behavioral, and potential functions [1, 2]. More recently PDE based methods that use boundary control ideas for the formation control of MAS have been proposed [3-7]. Many of the latter are based on the solution properties of the 1D heat diffusion equation [8-11]. In essence these PDE methods treat motion control of a MAS evolving in R^n as n 1D motion control problems, where each agent is modeled as a 1D first or second order linear PDE [3-7]. The motion control problem then is equivalent to the state of the PDE models representing the position of agents and the corresponding family of equilibrium solutions of the PDEs prescribing the desired formation patterns of the MAS. One drawback of these PDE based methods is the slow rate of convergence of the MAS to its desired formation. It is a manifestation of the control at the boundary which requires leaders to be at the boundary of the MAS. In addition, obtaining a desired MAS formation using PDE based approaches requires the solution of a PDE with spatially varying parameters, that may be difficult to solve analytically.

In this paper, two control strategies for motion control of MAS which are in some sense close to the PDE formalism are presented. They are based on the properties of homogenous maps that are central to the study of the time evolution of continua or motion of deformable bodies. as The first uses a predefined motion map for a MAS and requires no inter-agent communications for its evolution. It is attained by properly selecting the Jacobian defining the desired deformation of the MAS and a rigid body translation vector such that all motion constraints are satisfied. The second strategy employs three non-aligned agents as the leaders who prescribe the required motion map from any *t* to *t*+Δ*t* to achieve the desired MAS formation. In this case the rest of the follower agents acquires the required map through communication among agents. In particular, the followers learn the leader prescribed motion map through communication with three non-aligned local agents by preserving certain distance ratios with respect to the positions of the local adjacent agents. Consequently, 2D motion of a MAS is achieved with three possible leaders and limited inter-agent communications. In addition, unlike the PDE based methods the strategy proposed here provides for a faster rate of convergence to the desired configuration as exact desired formation of the MAS can be obtained by followers through local communications with some neighbor agents.

The paper consists of four sections with this section I being the introduction. Developed in section II are the basic homogeneous kinematic maps for MAS motions as deformable bodies and the two proposed control strategies, which are followed by some simulation results in section III. Conclusions are in section IV.

II. PROBLEM STATEMENT

A. Basic Kinematics

Let the MAS consist of N agents moving in a plane. Let $\boldsymbol{R}_i = X_i\boldsymbol{e}_x + Y_i\boldsymbol{e}_y$ be the initial position of agent i in a reference configuration Ω_0, and $\boldsymbol{r}_i(t) = x_i(t)\boldsymbol{e}_x + y_i(t)\boldsymbol{e}_y$ $(i = 1,2,\dots,N)$ be the position of agent i in the current configuration Ω_t, at any time $t > 0$. Suppose the mapping from Ω_0, to Ω_t that maps each point in the continuum Ω_0 to

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Ω_t is given by $\boldsymbol{r}(t) = \boldsymbol{f}(\boldsymbol{R}, t)$, where $\boldsymbol{r}(t)$ denotes the current location of the point that occupied position $\boldsymbol{R}$ in the reference configuration (see Fig. 1). We assume that this mapping and its inverse are one-to-one and continuously differentiable in its arguments. The inverse will exist throughout Ω_t provided the Jacobian of the mapping is non-vanishing at every point in Ω_0: $Q(\boldsymbol{R}, t) = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{R}}$ is non-singular. In fact from basic continuum mechanics it follows that $|Q(\boldsymbol{R}, t)| > 0$ for all $\boldsymbol{R}$, t.

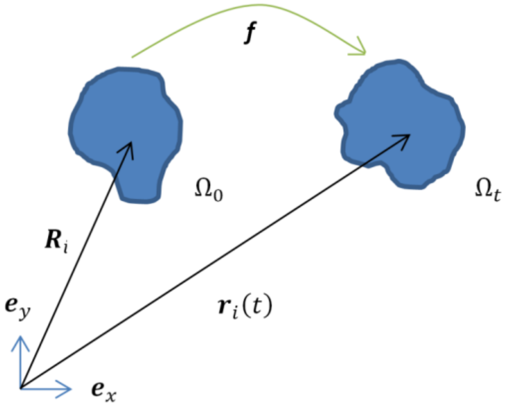

Fig. 1. Schematic of continuum based mapping

While the above description is valid for any general motion, we will focus on motions of the form

$$\boldsymbol{r}_i(t) = Q(t)\boldsymbol{R}_i + \boldsymbol{D}(t). \tag{1}$$

where Q and $\boldsymbol{D}$ are constants or functions of time and do not depend on the material point. These special motions are called *homogenous deformations*. We note that $Q(t)$ characterizes any material deformation and $\boldsymbol{D}(t)$ denotes a rigid body translation. Equation (1) forms the basis of the two control strategies for swarm motions developed in the paper. In the first each agent is assumed to know what Q and $\boldsymbol{D}$ are at the outset and in the second Q and $\boldsymbol{D}$ are only known to or are determined by three leaders.

B. Motion Control under no Inter-Agent Communication

For this case, the motion map is pre-defined and every agent is assumed to know the Jacobian $Q(t)$ and the rigid body displacement vector $\boldsymbol{D}(t)$ that prescribes the MAS motion from Ω_0 to Ω_t. Since the Jacobian matrix is positive definite, it is automatically guaranteed that no inter agent collision occurs. In order to compute $Q(t)$ and $\boldsymbol{D}(t)$, all that is needed are positions of three vertex agents such that they are initially non-aligned and remain non-aligned for all time $t \geq t_0$. Then, by knowing the initial positions $\boldsymbol{R}_i = (X_i, Y_i), i = 1,2,3$ and transient positions $\boldsymbol{r}_i(t) = (x_i, y_i), i = 1,2,3$ of the three vertices of the corresponding triangular domains the Q and $\boldsymbol{D}$ entries can be determined by simple matrix algebra as follows:

$$\begin{bmatrix} Q_{11}(t) \\ Q_{12}(t) \\ Q_{21}(t) \\ Q_{22}(t) \\ D_{11}(t) \\ D_{21}(t) \end{bmatrix} = \begin{bmatrix} X_1 & Y_1 & 0 & 0 & 1 & 0 \\ X_2 & Y_2 & 0 & 0 & 1 & 0 \\ X_3 & Y_3 & 0 & 0 & 1 & 0 \\ 0 & 0 & X_1 & Y_1 & 0 & 1 \\ 0 & 0 & X_2 & Y_2 & 0 & 1 \\ 0 & 0 & X_3 & Y_3 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}. \tag{2}$$

Remark 2: An important limitation of homogenous transformation of MAS with zero communication is that it only applies to predefined motions. For example, in case of an abrupt change of the motion plan or appearance of an unpredictable obstacle in the motion field, the defined Jacobian matrix $Q(t)$ and the rigid body displacement vector $\boldsymbol{D}(t)$, must be updated to a different $\boldsymbol{Q}'(t)$ and $\boldsymbol{D}'(t)$ as quickly as possible. This is handled by developing a local communication protocol among the agents.

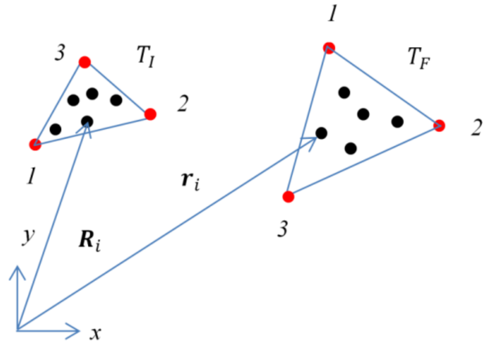

Fig. 2. Schematic of homogenous transformation

C. Motion Control of a MAS with inter-agent communication

Suppose that a follower agent i initially located at (X_i, Y_i) communicates with adjacent agents i_1, i_2, and i_3, which are located at $\left(X_{i_1}, Y_{i_1}\right)$, $\left(X_{i_2}, Y_{i_2}\right)$, and $\left(X_{i_3}, Y_{i_3}\right)$ at time $t = 0$, and define the ratios termed ***communication weights*** of the follower agent i with respect to three adjacent agents i_1, i_2, and i_3 as follows:

$$w_{i,i_1} = \frac{(X_{i_3}-X_{i_2})(Y_i-Y_{i_2})-(Y_{i_3}-Y_{i_2})(X_i-X_{i_2})}{(X_{i_3}-X_{i_2})(Y_{i_1}-Y_{i_2})-(Y_{i_3}-Y_{i_2})(X_{i_1}-X_{i_2})}, \tag{3}$$

$$w_{i,i_2} = \frac{(X_{i_1}-X_{i_3})(Y_i-Y_{i_3})-(Y_{i_1}-Y_{i_3})(X_i-X_{i_3})}{(X_{i_1}-X_{i_3})(Y_{i_2}-Y_{i_3})-(Y_{i_1}-Y_{i_3})(X_{i_2}-X_{i_3})}, \tag{4}$$

$$w_{i,i_3} = \frac{(X_{i_2}-X_{i_1})(Y_i-Y_{i_1})-(Y_{i_2}-Y_{i_1})(X_i-X_{i_1})}{(X_{i_2}-X_{i_1})(Y_{i_3}-Y_{i_1})-(Y_{i_2}-Y_{i_1})(X_{i_3}-X_{i_1})}, \tag{5}$$

where

$$w_{i,i_1} + w_{i,i_2} + w_{i,i_3} = 1. \tag{6}$$

Sign of weight ratios: Based on the location of agent i, the weights w_{i,i_1}, w_{i,i_2}, and w_{i,i_3} can be positive, zero, or negative. In Fig. 3, the x-y plane is split to seven sub-regions where in each sub-region, the sign of weights don't change. We note that if agent i is located inside the triangle with the adjacent agents i_1, i_2, and i_3 as vertices then all the weights remain positive. Moreover, it is impossible that all weight ratios are simultaneously negative since it violates condition (6).

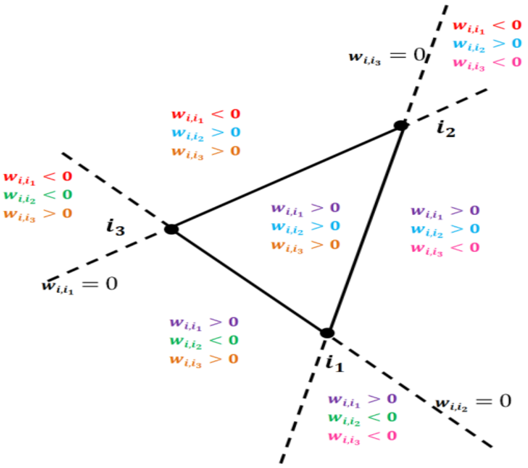

Fig. 3. Sign of weights of communication depending on positions of agents i, i_1, i_2, and i_3

Maintaining the Homogenous Map: If agents i, i_1, i_2, and i_3 can be forced to move in such a way that the weights (3-5), remain constant during the motion of the MAS, then these four agents satisfy conditions for a homogenous map with the position of the agent i given by:

$$\boldsymbol{r}_{i_d} = w_{i,i_1}\boldsymbol{r}_{i_1} + w_{i,i_2}\boldsymbol{r}_{i_2} + w_{i,i_3}\boldsymbol{r}_{i_3}, \tag{7}$$

where w_{i,i_1}, w_{i,i_2}, and w_{i,i_3} are constant and $\boldsymbol{r}_{i_d}$ is the desired position of agent i.

Now, suppose that agent 1, 2, and 3 are leaders that move independently (such that they are always non-aligned) with the desire to propagate the motion map to their follower agents, (agents 4, 5, …, N) which are initially embedded in the leading triangle that is defined as the triangle formed by the three leaders. Each follower agent i (i = 4, 5, …, N) communicates only to three local agents i_1, i_2, and i_3 with weights of communication w_{i,i_1}, w_{i,i_2}, and w_{i,i_3}, respectively, and always tries to reach $\boldsymbol{r}_{i_d}$(See eqn. (7).). Thus a communication topology as shown in Fig. 4 is introduced to acquire the motion map.

In this scenario although the desired position of any follower agent does not follow what is given by the homogeneous map prescribed by the leaders the initial configuration of the MAS will indeed be what was intended with respect to the configuration at the time the leaders changed the map. This fact is established in Theorem 2 below.

Let G be the communication graph corresponding to the local inter-agent communication topology of Fig. 4. Then, G is the union of the sub-graph Ω and its boundary, $\partial\Omega$. The follower agents all belong to Ω and the leaders belong to the boundary $\partial\Omega$. An edge connecting any two follower nodes (Shown by purple lines in Fig. 4) represents bi-directional communication between the two nodes, where they are not necessarily the same in both directions. Furthermore, the edges that connect a leader to a follower agent (Shown by green arrows in Fig. 4 where they terminate on the followers) show the directed communication from a leader to a follower.

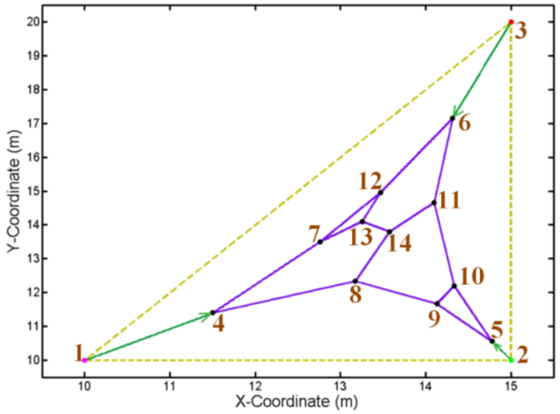

Fig. 4. A schematic of the graph showing local inter-agent communication

Kinematics of the Motion: The desired position $\boldsymbol{r}_{i_d}$ of any follower agent i, which moves under the specified communication protocol can be restated in the following general form:

$$\boldsymbol{r}_{i_d} = \sum_{\substack{j=1 \\ i\sim j}}^{N} w_{i,j}\boldsymbol{r}_j, \tag{8}$$

where the symbol "~" denotes adjacency between two agents i and j, $i = 4,5,\ldots,N$ is the index specifying a follower agent and $w_{i,j}s$ are constant weights of communication of agent i with its adjacent agents, which are assigned based on initial position of the agents, according to eqns. (3-5).

Equation (8) can be rewritten as

$$\boldsymbol{r}_{i_d} = \sum_{\substack{j=4 \\ i\sim j}}^{N} w_{i,j}\boldsymbol{r}_j + \sum_{\substack{j=1 \\ i\sim j}}^{3} w_{i,j}\boldsymbol{r}_j. \tag{9}$$

The first term on the right hand side of eqn. (9) corresponds to those followers that only communicate with three other follower agents and the second term on the right hand side of equation (9) is associated with those followers that communicate directly with one leader and two followers.

For simplicity we model any follower agent i (for $i = 4,5,\dots,n$) with simple kinematics:

$$\dot{\boldsymbol{r}}_i = \boldsymbol{v}_i, \tag{10}$$

where $\boldsymbol{v}_i$ is control velocity input. The objective is to assign $\boldsymbol{v}_i$ such that the distance between desired position $\boldsymbol{r}_{i_d}$ and the actual position $\boldsymbol{r}_i$ of the follower agent i is kept as small as possible for all time t with the error going to zero soon after the leaders settle or reach the final configuration. One possible strategy is to choose $\boldsymbol{v}_i$ as

$$\boldsymbol{v}_i = g\left(\boldsymbol{r}_{i_d} - \boldsymbol{r}_i\right), \tag{11}$$

where the velocity vector of the agent i is continuously updated such that it is in the direction of the vector from its current position to the desired position as shown in Fig. 5 In (11) g is a positive control parameter which can be used to adjust the rate of convergence of the position error.

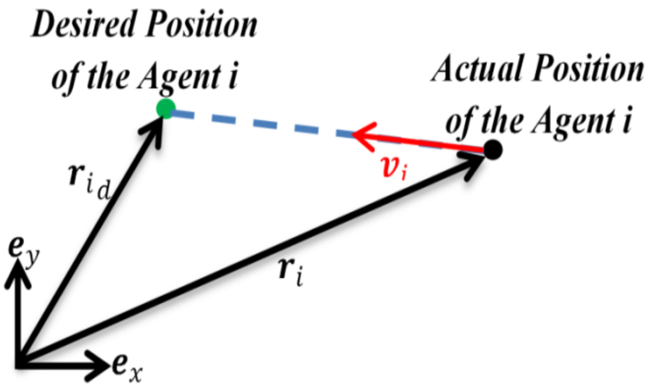

Fig. 5. Geometry of the velocity control input

Substituting for $\boldsymbol{r}_{i_d}$ from eqn. (9) in (11) together with (10) leads to

$$\dot{\boldsymbol{r}}_i = -g\left(\boldsymbol{r}_i - \sum_{\substack{j=4 \\ i\sim j}}^{N} w_{i,j}\boldsymbol{r}_j\right) + g\sum_{\substack{j=1 \\ i\sim j}}^{3} w_{i,j}\boldsymbol{r}_j. \tag{12}$$

Equation (12) is the $(i-3)-th$ row of the following $N-3$ dimensional state space dynamical system:

$$\dot{\boldsymbol{Z}} = g(A\boldsymbol{Z} + B\boldsymbol{U}), \tag{13}$$

where $\boldsymbol{Z} = [\boldsymbol{r}_4 \ \cdots \ \boldsymbol{r}_N]^T$, $\boldsymbol{U} = [\boldsymbol{r}_1 \ \ \boldsymbol{r}_2 \ \ \boldsymbol{r}_3]^T$, A is $(N-3)$ x $(N-3)$ matrix, and B is $(N-3)$ x 3 matrix.

Weight Matrix W: Equation (13) can also be written in the following form:

$$\dot{\boldsymbol{Z}} = gW\begin{bmatrix}\boldsymbol{U}\\ \boldsymbol{Z}\end{bmatrix}, \tag{14}$$

where W is a $(N-3)$x N matrix which can be partitioned as follows:

$$W = [B_{(N-3)\times 3} \ \vdots \ A_{(N-3)\times(N-3)}]. \tag{15}$$

The weight matrix W has following three useful properties:

1- It is row-stochastic (sum of each row of W is zero.). Furthermore, every row of W has four non-zero elements.
2- All diagonal elements of partition A of the matrix W are equal to -1.
3- A is not necessarily symmetric, however, if $A_{ij} = 0$, then, $A_{ji} = 0$, and if $A_{ij} \neq 0$, then, $A_{ji} \neq 0$.

Now we develop (i) a sufficient condition for matrix A to be negative definite so that we can guarantee convergence of the MAS to the desired formation, and (ii) show that the final formation of the MAS satisfies the homogenous map that starts with the initial configuration of the MAS with the final configuration defined by the leaders. These are given in the two theorems that follow.

Theorem 1: *If the graph corresponding to local inter-agent communication of a MAS is assigned based on the initial configuration of the agents such that any agent i is embedded inside the triangle formed by three vertex agents that are adjacent to agent i, then the matrix* $A_{(N-3)\times(N-3)}$ *is negative definite.*

Proof:

Note that matrix A can be written as$-(I-F)$ where I is the identity and F is an irreducible non-negative matrix. We know from Fig. 3 and eqns. (3-5) that when any agent i is located inside the triangle formed by three adjacent agents of i, then all communication weights are positive and less than one. Since, the sum of every row of the weight matrix W is zero and the matrix A is obtained by deleting the first three columns of W, it follows that the sum of three rows of A is negative while the sum of the rest of the rows of A are zero. Consequently, by invoking the Perron-Frobenius theorem [14], it can be concluded that the spectrum $\rho(F)$ of F, is less than 1, and that $-A$ is non-singular M-matrix and A is negative definite. ■

From theorem 1, we know that the state vector $\boldsymbol{Z}$ of eqn. (13) converges asymptotically to the equilibrium state vector $\boldsymbol{Z}_s$ given by:

$$\boldsymbol{Z}_s = -A^{-1}B\boldsymbol{U}. \tag{16}$$

The only thing that remains to be verified is whether or not the communication weights of the followers with respect to the leader agents, in the final configuration, are the same as the initial weight ratios of the followers with respect to the leader agents. If the answer is yes then the final formation of the MAS is the same as that prescribed by the homogeneous map defined by the leaders which assumed the initial configuration of the agents as its starting or the reference configuration.

Theorem 2: *If a MAS motion occurs under fixed communication weights with (i) any follower agent solely communicating with three adjacent agents, (ii) time invariant weights of communication are chosen by each agent based on the initial configuration of the agents, and (iii) the communication graph is designed properly such that any follower agent is embedded inside the triangle with its three chosen adjacent agents as the vertices. Then, the final formation of the MAS is the same as the one resulting from the homogenous map that assumes the initial configuration. Equivalently, the initial and final weights of communication of the follower agents with respect to leaders are the same.*

Proof:

Since every follower agent communicates with three local agents, any row of the $(N-3)\times N$ matrix $W=[B_{(N-3)\times 3} \ \vdots \ A_{(N-3)\times(N-3)}]$ has four non-zero elements where all diagonal elements of partition A are *-1*. Because the weights are calculated from the initial configuration of the MAS, and in the initial formation, any follower is chosen to be inside the triangle with three adjacent vertex agents, it follows that all non-zero off-diagonal elements of partition A and the non-zero elements of B are positive. Using the fact that A is negative definite from Theorem 1 and row operations on W it can be shown to be equivalent to the partitioned $W\sim S=[S_1=A^{-1}B \ \vdots \ S_2=-I]$. Now it can be shown that S is row stochastic which follows from the fact that W is row stochastic. Therefore, the sum of every row of partition $S_1=-A^{-1}B$ is equal to 1. In other words, the elements of any row i of S_1, $S_{1_{i1}}$, $S_{1_{i2}}$, and $S_{1_{i3}}$, can be viewed as weights of communication of the follower agent i with the leaders (agents 1, 2 and 3), respectively, as the vertices of the embedding triangle when the MAS has reached the equilibrium state in the final configuration. When the equilibrium state or the final locations of the leaders are reached the final position vectors of the leaders follow from eqn. (14) where $\boldsymbol{Z_s}=-A^{-1}BU$. Since, we assume that the leaders come to rest at the final configuration it therefore follows that $-A^{-1}B$ for the initial and final configuration must be the same. This in turn implies that the weights of communication of the follower agents with the leaders as the vertices are the same in both the initial and final configurations. Therefore, it can be concluded that the final configuration of the MAS is nothing but the one prescribed by the homogenous map that is computed on the basis of the initial configuration. ■

III. SIMULATION RESULTS

We present two scenarios for simulations where in the first a homogenous map is utilized with zero inter-agent communication. In the second a motion plan is prescribed by three leaders and is propagated to the followers through the inter-agent communication protocol developed in the paper. We consider a MAS consisting of 14 agents (3 leaders and 11 followers) with leaders 1, 2, and 3 denoted by magenta, green and red which are initially located at $(10,10)$, $(15,10)$, and $(15,20)$, respectively. All the followers are assumed to be embedded in the triangle defined by the leaders as vertices. The goal is to move the followers so that they are finally bounded by a triangle whose vertices are the final positions of the leaders, located at $(30,20)$, $(40,15)$, and $(35,25)$.

The follower initial positions are as listed in Table 1.

Table 1 Position of the followers in the initial configuration

Agent	*x*	*y*
4	**11.50**	**11.40**
5	**14.78**	**10.56**
6	**14.32**	**17.16**
7	**12.76**	**13.50**
8	**13.17**	**12.34**
9	**14.13**	**11.67**
10	**14.33**	**12.20**
11	**14.10**	**14.66**
12	**13.47**	**14.96**
13	**13.25**	**14.10**
14	**13.58**	**13.80**

Figure 6 shows the initial and final configuration of the MAS. The followers are embedded by the triangle with its vertices at the initial positions of the leaders. The goal is to see that the followers are finally embedded in the triangle with the vertices at the final positions of the leaders located at $(30,20)$, $(40,15)$, and $(35,25)$. Note that the final locations of the follower agents are not pre-specified and the agents in black in the final configuration should be simply assumed to be inside the final triangle.

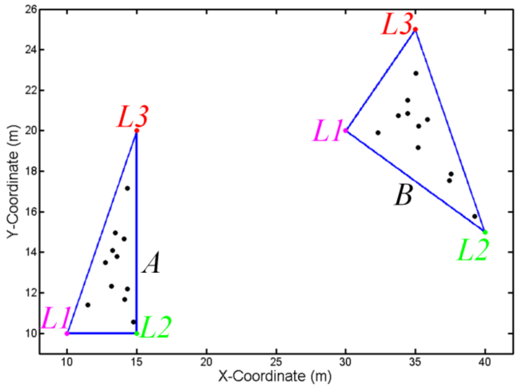

Fig. 6. Initial and final (desired) configuration of the MAS

For both case studies, the travel time for the motion of the leaders are taken as $20s$ with their motion pre-specified by a polynomial function of time such that velocity and acceleration of the leaders are both zero at the initial and final time. The resulting three leader trajectories are shown in Fig. 7.

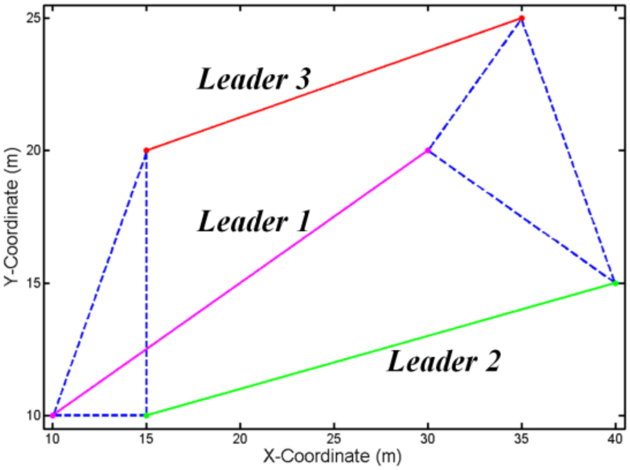

Fig. 7. Leader trajectories

A. Scenario I: MAS motion control with no communication

Using relation (2), together with the initial positions of the leaders the Jacobian matrix $Q(t)$ and the rigid body displacement vector $\boldsymbol{D}(t)$ defining the homogeneous map is computed and each follower agent is assumed to have knowledge of Q and $\boldsymbol{D}$.

Figure 8 shows the evolution of the MAS under zero communication at five intermediate times t = 0s, t = 5s, $t = 10s$, $t = 15s$ and $t = 20s$ shown as black, blue, green, red, and magenta spots, respectively.

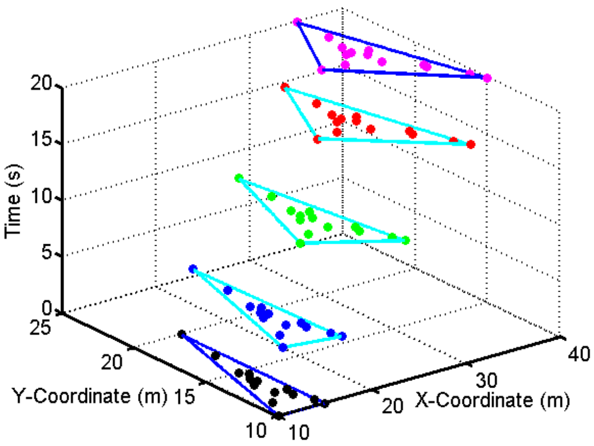

Fig. 8. Configuration of the MAS at time $t = 0s$, $t = 5s$, $t = 10s$, $t = 15s$ and $t = 20s$

B. Scenario II: : Decentralized Control of Motion of the MAS

As alluded to earlier, when the leaders' motion is not pre-specified the followers do not have access to the required homogeneous map defined by mapping, $Q(t)$, and rigid body displacement vector *D(t)*. The follower agents will acquire the homogenous map prescribed by the leaders through local inter agent communications. The inter-agent communication graph of the MAS for scenario 2 is shown in Fig. 4. The weights of communication of the follower agents that are calculated based on their initial positions, are all positive, and give a negative definite matrix A and a 11x3 matrix B according to eqns. (10) and (11). Weights of communications are listed in Table 2.

Table 2 Weights of Communications of follower agents

	Weights of Communication
F4	$w_{4,1} = 0.5, w_{4,7} = 0.2$ $w_{4,8} = 0.3$
F5	$w_{5,2} = 0.71, w_{5,9} = 0.15$ $w_{5,10} = 0.14$
F6	$w_{6,3} = 0.45, w_{6,11} = 0.25$ $w_{6,12} = 0.3$
F7	$w_{7,4} = 0.32, w_{7,12} = 0.30$ $w_{7,13} = 0.38$
F8	$w_{8,4} = 0.29, w_{8,9} = 0.36$ $w_{8,14} = 0.35$
F9	$w_{9,5} = 0.35, w_{9,8} = 0.31$ $w_{9,10} = 0.34$
F10	$w_{10,5} = 0.33, w_{10,9} = 0.37$ $w_{10,11} = 0.40$
F11	$w_{11,6} = 0.4, w_{11,10} = 0.3$ $w_{11,14} = 0.3$
F12	$w_{12,6} = 0.34, w_{12,7} = 0.29$ $w_{12,13} = 0.37$
F13	$w_{13,7} = 0.35, w_{13,12} = 0.35$ $w_{13,14} = 0.30$
F14	$w_{14,8} = 0.30, w_{14,11} = 0.41$ $w_{14,13} = 0.29$

The leader trajectories for this case are the same as in scenario *i,* but are not known to the followers. Therefore, the follower agents communicate with local agents as per the communication graph shown in Fig. 4 with weights of communication determined according to those given in eqns. (3-5).

Threshold of the Control Parameter: The control parameter g (See eqn. (11)) needs to be chosen appropriately to guarantee that none of the follower agents fall outside the moving triangle with leaders' positions as vertices at any time t. For this purpose, weights of communication of any follower agent with three leaders ($w_{i,1}, w_{i,2}$, and $w_{i,3}$ for $= 4,5,\ldots,14$) must be greater than zero at any time $t > 0$. For this case study, the control gain $g = 15$ which is greater than the minimum of 9.488 is chosen to perform the simulations. It is noted that $g_{min} = 9.488$ guarantees none of the follower agents leave the leading triangle. We note that $w_{i,1}(t), w_{i,2}(t)$, and $w_{i,3}(t)$ can be obtained by using eqns. (3-5), based on transient configuration of the MAS at time *t,* where indices i_1, i_2, and i_3 represent the leader agents *1, 2, and 3,* respectively.

Evolution of the MAS motion is depicted in Fig. 9 with its intermediate configurations at five sample times $t = 0s$, $t = 5s$, $t = 10s$, $t = 20s$ and $t = 30s$.

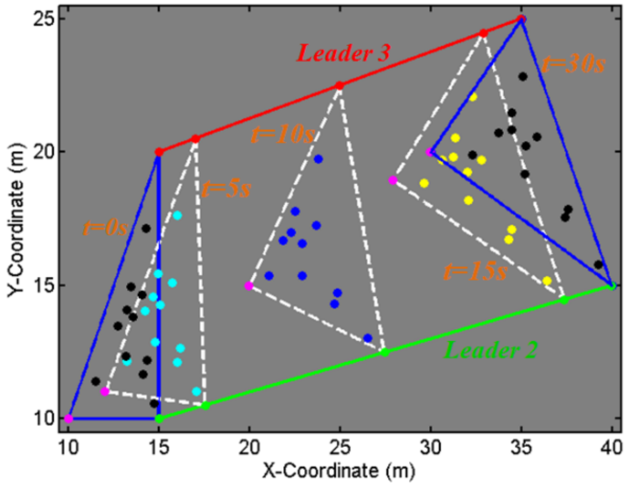

Fig. 9. Configurations of the MAS at times $t=0s, t=5s$, $t=10s, t=15s, t=20s$, and $t=30s$

Furthermore, x and y coordinates of position of follower agent 13 versus time are illustrated in Fig. 10 by blue and red, respectively. Also, x and y coordinate of state of homogenous transformation of follower agent 13 are depicted by hashed green and black, respectively, in order to illustrate the extent of deviation from homogenous transformation during transition.

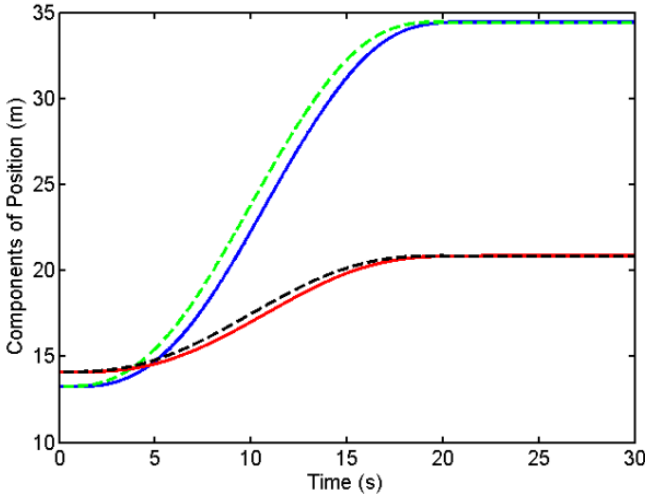

Fig. 10. x and y coordinates of position of follower agent 13; x and y coordinate of state of homogenous transformation of follower agent 13

IV. CONCLUSION

A continuum based approach for controlling the motion of a MAS, which moves in a plane, with the some communication was studied and two MAS motion control strategies: one based on zero inter agent communications and the other based on local inter agent communications were developed. It is easy to guarantee that no inter-agent collisions occur and is quite transparent in handling obstacles. The design is reduced to a consideration of leader motion paths. The approach compares very favorably in terms of convergence rates and the required inter-agent communications when compared to boundary control ideas using PDE methods.

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