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A Distributed Fault Detection Filtering Approach for a Class of Interconnected Input-Output Nonlinear Systems

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Abstract— **This paper develops a filtering approach for distributed fault detection of a class of interconnected input-output nonlinear systems with modeling uncertainties, disturbances and measurement noise. A distributed fault detection filtering scheme and the corresponding adaptive thresholds are designed based on filtering certain signals so that the effect of the measurement noise and disturbances is attenuated, which facilitates less conservative thresholds and enhanced robustness. Further analysis leads to a quantitative characterization of the class of detectable faults and simulation results are used to illustrate the proposed distributed fault diagnosis filtering approach.**

I. Introduction

Recent technological advancements in communications and real-time computing have facilitated the advancement of distributed approaches for monitoring and control of large-scale complex systems. The safe and reliable operation of such systems through the early detection of any "small" faults before they become serious failures is a crucial component of the overall system performance and sustainability.

Most approaches for fault detection and accommodation that have been developed so far are based on a centralized architecture, where information about the state of the system are gathered and processed centrally [1], [2]. Recent advances in communications and distributed sensing signified the advancement of these schemes towards distributed and hierarchical fault diagnosis methods [3]–[9]. An important issue that is often overseen is the presence of measurement noise and disturbances. In most real world applications the presence of measurement noise and modeling uncertainty may influence significantly the performance of fault detection schemes by causing, for example, false alarms.

Recently Ferrari *et al.* [7] investigated the problem of distributed fault detection and isolation for large-scale, discrete-time, nonlinear, uncertain systems with measurement uncertainty, considering full state measurement, where the monolithic system is decomposed into smaller interconnecting subsystems and in Boem *et al.* [8] the proposed methodology was presented in the case of continuous-time systems. Other research dealing with the fault detection problem for interconnected continuous-time nonlinear systems with modeling uncertainty and in the presence of measurement noise and disturbances was conducted by Keliris *et al.* [9], [10], where a general class of filters is embedded into the design of the residual and threshold signals in a way that takes advantage of the filtering noise suppression properties. The work in [9] includes a rigorous detectability analysis providing results regarding the magnitude of the detected faults, an upper bound on the detection time and the relation of the detection time with respect to the order and pole locations of the filters used, under the assumption of full state measurement.

In this work, a distributed fault detection filtering scheme is presented for a class of input-output interconnected continuous-time nonlinear systems with modeling uncertainty and in the presence of measurement noise and disturbances. A key novelty of the proposed scheme is the design of suitable residual and threshold signals utilizing a certain class of filters in a way that takes advantage their noise suppression properties. Therefore, the noise and disturbance effects in a certain frequency range is diminished allowing for less conservative thresholds to be derived guaranteeing no false alarms.

The paper is organized as follows: in Section II, a problem formulation for distributed fault detection of a class of input-output nonlinear dynamical systems with modeling uncertainties, measurement noise and disturbances is presented. In Section III the design of the distributed fault detection scheme based on a filtering approach is presented in detail and in Section IV the fault detectability condition that characterizes the class of faults detectable by the proposed scheme is derived. In Section V a simulation example illustrates the concepts presented. Finally, Section VI provides some concluding remarks.

II. Problem formulation

We consider a large-scale distributed nonlinear dynamic system comprised of N subsystems Σ_I, $I \in \{1, ..., N\}$, where each subsystem is described by:

$$\Sigma_I : \begin{cases} \dot{x}_I(t) = A_I x_I(t) + f_I(C_I x_I(t), \bar{C}_I \bar{x}_I(t), u_I(t)) \\ \qquad + \eta_I(x_I(t), \bar{x}_I(t), u_I(t), t) + \zeta_I(t) \\ \qquad + \beta_I(t - T_0)\phi_I(x(t), u_I(t)) & (1) \\ y_I(t) = C_I x_I(t) + \xi_I(t). & (2) \end{cases}$$

where $x_I \in \mathbb{R}^{n_I}$, $u_I \in \mathbb{R}^{m_I}$ and $y_I \in \mathbb{R}^{p_I}$ are the state, input and measured output vectors of the I-th subsystem respectively and $x \triangleq \left[x_1^\top, x_2^\top, \ldots, x_N^\top\right]^\top \in \mathbb{R}^n$ is the state vector of the overall system. The vectors $\bar{x}_I \in \mathbb{R}^{\bar{n}_I}$ and $\bar{C}_I \bar{x}_I \in \mathbb{R}^{\bar{p}_I}$ denote the state variables and the corresponding output variables respectively of neighboring subsystems that affect the I-th subsystem. The matrix $A_I \in \mathbb{R}^{n_I} \times \mathbb{R}^{n_I}$ and

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the function $f_I : \mathbb{R}^{p_I} \times \mathbb{R}^{\bar{p}_I} \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{n_I}$ characterize the known nominal function dynamics and the matrix $C_I \in \mathbb{R}^{p_I} \times \mathbb{R}^{n_I}$ is the known nominal output matrix of the I-th subsystem. Note that the function f_I contains also the known part of the interconnection function between the I-th and its neighboring subsystems. The vector function $\eta_I : \mathbb{R}^{n_I} \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^{m_I} \times \mathbb{R}^{+} \mapsto \mathbb{R}^{n_I}$ denotes the modeling uncertainty associated with the nominal dynamics, $\zeta_I(t) \in \mathbb{R}^{n_I}$ is the unknown disturbance associated with the I-th subsystem and $\xi_I \in \mathbb{R}^{p_I}$ represents the measurement noise. The term $\beta_I(t-T_0)\phi_I(x, u_I)$ characterizes the fault function dynamics affecting the I-th subsystem including its time evolution. More specifically, the term $\phi_I : \mathbb{R}^{n} \times \mathbb{R}^{m_I} \mapsto \mathbb{R}^{n_I}$ represents the unknown fault function and the term $\beta_I(t-T_0) : \mathbb{R} \mapsto \mathbb{R}^{+}$ models the time evolution of the fault, where T_0 is the unknown time of the fault occurrence. Note that the fault function ϕ_I may depend on the global state variable vector x and not only on the local state vector x_I. From a practical perspective, this allows faults that are functions of the overall state vector, not only states that are available to the I-th subsystem. The fault time profile $\beta_I(t-T_0)$ can be used to model both abrupt and incipient faults using a decaying exponential type function:

$$\beta_I(t-T_0) \triangleq \begin{cases} 0 & \text{, if } t < T_0 \\ 1 - e^{-b_I(t-T_0)} & \text{, if } t \geq T_0, \end{cases} \tag{3}$$

where $b_I > 0$ is typically an unknown parameter representing the time evolution of the fault. Large values of b_I indicate abrupt faults, whereas smaller values of b_I indicate slowly developing faults (incipient faults).

The objective is to design and analyze a distributed fault detection scheme, where to each subsystem Σ_I, we associate a local fault detection agent $\mathcal{F}_I$, which receives local measurements y_I and partial information from neighboring fault detection agents $\mathcal{F}_J$. It is assumed that there exist feedback controllers for selecting u_I such that some desired control objectives are achieved. In this paper, we do not deal explicitly with the control design problem, but instead we focus on the fault detection issue in the case of partial state measurement and in the presence of faults ϕ_I, modeling uncertainties η_I, disturbances ζ_I and measurement noise ξ_I. Particular emphasis is placed on addressing issues related to measurement noise and disturbances, which may affect significantly the performance of the fault detection scheme.

The following assumptions are used throughout the paper:

Assumption 1: For each subsystem Σ_I, $I \in \{1, ..., N\}$ the pair (A_I, C_I) is observable.

Assumption 2: For each subsystem Σ_I, $I \in \{1, ..., N\}$ the local state variables $x_I(t)$ and the local inputs $u_I(t)$ are uniformly bounded before and after the occurrence of a fault (well-posedness).

Assumption 3: The modeling uncertainty η_I in each subsystem is an unstructured and unknown nonlinear function of x_I, $\bar{x}_I$, u_I and t but bounded by a *known* positive functional $\bar{\eta}_I$; i.e.,

$$\|\eta_I(x_I, \bar{x}_I, u_I, t)\| \leq \bar{\eta}_I(y_I, \bar{y}_I, u_I), \tag{4}$$

for all $t \geq 0$ and for all $(x_I, \bar{x}_I, u_I) \in \mathcal{D}_I$, where $\bar{y}_I \subset \mathbb{R}^{\bar{p}_I}$ is the noisy counterpart of $\bar{C}_I \bar{x}_I$, i.e. $\bar{y}_I = \bar{C}_I \bar{x}_I + \bar{\xi}_I$, $\bar{\xi}_I \subset \mathbb{R}^{\bar{p}_I}$ and $\bar{\eta}_I(y_I, \bar{y}_I, u_I) \geq 0$ is a known bounding function in some compact region of interest $\mathcal{D}_I = \mathcal{D}_{x_I} \times \mathcal{D}_{\bar{x}_I} \times \mathcal{D}_{u_I} \subset \mathbb{R}^{n_I} \times \mathbb{R}^{\bar{n}_I} \times \mathbb{R}^{m_I}$.

Assumption 3 characterizes the class of modeling uncertainties being considered. In practice, the system can be modeled more accurately in certain regions of the state space. Therefore, the fact the bound $\bar{\eta}_I$ is a function of y_I, $\bar{y}_I$ and u_I provides more flexibility by allowing the designer to take into consideration any prior knowledge of the system.

To dampen the effect of measurement noise $\xi_I(t)$ and disturbances $\zeta_I(t)$, each measured variable $y_I^{(j)}$ (j-th component of y_I) is filtered by $H_p(s)$, where $H_p(s)$ is a strictly proper, asymptotically stable transfer function of the form:

$$H_p(s) = \frac{d_{p-1}s^{p-1} + d_{p-2}s^{p-2} + \ldots + d_0}{s^p + c_{p-1}s^{p-1} + \ldots + c_1 s + c_0}. \tag{5}$$

The filter $H_p(s)$ is asymptotically stable and hence BIBO stable. Therefore, for bounded measurement noise $\xi_I(t)$ and bounded disturbance $\zeta_I(t)$, the filtered measurement noise $\epsilon_{\xi_I}(t) \triangleq H_p(s)[\xi_I(t)]$ and filtered disturbance $\epsilon_{\zeta_I}(t) \triangleq H_p(s)[\zeta_I(t)]$ are bounded as follows:

$$\|\epsilon_{\xi_I}(t)\| \leq \bar{\epsilon}_{\xi_I}(t), \tag{6}$$

$$\|\epsilon_{\zeta_I}(t)\| \leq \bar{\epsilon}_{\zeta_I}(t), \tag{7}$$

where $\bar{\epsilon}_{\xi_I}(t)$, $\bar{\epsilon}_{\zeta_I}(t)$ are computable bounding functions. It is important to note that filtering the output measurements is crucial to the proposed distributed fault detection scheme because it dampens the effect of measurement noise and, as will be shown later on in the paper, it causes the derived detection threshold to be less conservative. A key challenge is the design of a distributed fault detection scheme within the filtering framework that reduces the effect of measurement noise and disturbances, and at the same time maintains the desired stability and robustness properties.

III. Distributed Fault Detection

For each subsystem Σ_I, we design a local estimation model, based on the known components of (1) under healthy mode of operation:

$$\begin{aligned} \dot{\hat{x}}_I(t) = A_I \hat{x}_I(t) + f_I(y_I(t), \bar{y}_I(t), u_I(t)) \\ + L_I(y_I(t) - \hat{y}_I(t)) \end{aligned} \tag{8}$$

$$\hat{y}_I(t) = C_I \hat{x}_I(t), \tag{9}$$

where the gain matrix L_I is computed so that $(A_I - L_I C_I)$ is Hurwitz (see Assumption 1). For simplicity, the initial conditions of the observer are set to zero; i.e., $\hat{x}_I(0) = 0$. Note that the measured output $y_I(t)$ is multiplied by the observer gain L_I (to be designed), therefore the noise term $\xi_I(t)$ is also multiplied, thus resulting in conservative detection threshold. Through the use of filtering, as will be shown, the term $L_I \xi_I(t)$ becomes $L_I \epsilon_{\xi_I}(t)$, thus allowing for less conservative detection threshold to be obtained.

In this work, the residual signal $r_I(t)$ to be used for fault detection in each subsystem Σ_I is given by

$$r_I(t) = H_p(s)\left[y_I(t) - \hat{y}_I(t)\right], \tag{10}$$

where $H_p(s)$ is given by (5). The detection decision of a fault in the overall system is made when $|r_I^{(j)}(t)| > \bar{r}_I^{(j)}(t)$ for at least one component $j = 1, 2, \ldots, p_I$, in any local subsystem Σ_I, where $\bar{r}_I^{(j)}(t)$ is the detection threshold, to be designed later on.

In the following, we will use the filtered state variable $x_{I,f}$ defined as:

$$x_{I,f}(t) \triangleq H_p(s)\left[x_I(t)\right], \tag{11}$$

and the filtered state estimate $\hat{x}_{I,f}$ defined as:

$$\hat{x}_{I,f}(t) \triangleq H_p(s)\left[\hat{x}_I(t)\right]. \tag{12}$$

Let $h_p(t)$ be the impulse response associated with $H_p(s)$; i.e. $h_p(t) \triangleq \mathcal{L}^{-1}\left[H_p(s)\right]$. Then (11) can be written as

$$x_{I,f}(t) = \int_0^t h_p(\tau) x_I(t-\tau)\,\mathrm{d}\tau. \tag{13}$$

By taking the derivative of $x_{I,f}(t)$ in (13) and using Leibniz integral rule, we obtain

$$\dot{x}_{I,f}(t) = H_p(s)\left[\dot{x}_I(t)\right] + h_p(t) x_I(0). \tag{14}$$

Similarly, by taking the derivative of $\hat{x}_{I,f}(t)$ we obtain

$$\dot{\hat{x}}_{I,f}(t) = H_p(s)\left[\dot{\hat{x}}_I(t)\right] + h_p(t)\hat{x}_I(0). \tag{15}$$

Let us consider the time interval $[0, T_0)$, prior to the occurrence of any fault. In this time interval, using (1), the filtered state dynamics (14) satisfy:

$$\begin{aligned}\dot{x}_{I,f}(t) &= H_p(s)\big[A_I x_I(t) + f_I\big(C_I x_I(t), \bar{C}_I \bar{x}_I(t), u_I(t)\big) \\ &\qquad + \eta_I(x_I, \bar{x}_I, u_I, t) + \zeta_I(t)\big] + h_p(t) x_I(0) \\ &= A_I x_{I,f}(t) + H_p(s)\left[f_I\big(C_I x_I(t), \bar{C}_I \bar{x}_I(t), u_I(t)\big)\right] \\ &\quad + H_p(s)\left[\eta_I(x_I, \bar{x}_I, u_I, t)\right] + \epsilon_{\zeta_I}(t) + h_p(t) x_I(0).\end{aligned} \tag{16}$$

Similarly, using (8) and $\hat{x}_I(0) = 0$, the filtered state estimate dynamics (15) satisfy:

$$\begin{aligned}\dot{\hat{x}}_{I,f}(t) &= A_I \hat{x}_{I,f}(t) + H_p(s)\big[f_I\big(y_I(t), \bar{y}_I(t), u_I(t)\big)\big] \\ &\quad + L_I\big(y_{I,f}(t) - \hat{y}_{I,f}(t)\big),\end{aligned} \tag{17}$$

where $y_{I,f}(t) \triangleq H_p(s)[y_I(t)]$ and $\hat{y}_{I,f}(t) \triangleq H_p(s)[\hat{y}_I(t)]$.

Using (16) and (17), the filtered estimation error $\tilde{x}_{I,f}(t) \triangleq x_{I,f}(t) - \hat{x}_{I,f}(t)$ satisfies

$$\dot{\tilde{x}}_{I,f}(t) = A_{I,0}\tilde{x}_{I,f}(t) + \chi_I(t), \tag{18}$$

where

$$A_{I,0} \triangleq A_I - L_I C_I, \tag{19}$$

$$\begin{aligned}\chi_I(t) &\triangleq H_p(s)\left[\eta_I(x_I, \bar{x}_I, u_I, t)\right] - L_I \epsilon_{\xi_I}(t) \\ &\quad + \epsilon_{\zeta_I}(t) + \epsilon_{\Delta_I}(t) + h_p(t) x_I(0),\end{aligned} \tag{20}$$

$$\epsilon_{\Delta_I}(t) \triangleq H_p(s)\left[\Delta f_I(t)\right], \tag{21}$$

$$\begin{aligned}\Delta f_I(t) &\triangleq f_I\big(C_I x_I(t), \bar{C}_I \bar{x}_I(t), u_I(t)\big) \\ &\quad - f_I\big(y_I(t), \bar{y}_I(t), u_I(t)\big).\end{aligned} \tag{22}$$

The solution of (18) is

$$\tilde{x}_{I,f}(t) = e^{A_{I,0}t}\tilde{x}_{I,f}(0) + \int_0^t e^{A_{I,0}(t-\tau)}\chi_I(\tau)\,\mathrm{d}\tau. \tag{23}$$

Assumption 4: The filtered function mismatch term $\epsilon_{\Delta_I}(t)$ is bounded by a computable bounding function $\bar{\epsilon}_{\Delta_I}(t)$; i.e., $\|\epsilon_{\Delta_I}(t)\| \le \bar{\epsilon}_{\Delta_I}(t)$ for all $t \ge 0$.

Assumption 4 is based on the observation that filtering dampens the error effect of measurement noise present in the function mismatch term $\Delta f_I(t)$. A suitable selection of the bound $\bar{\epsilon}_{\Delta_I}$ can be made through the use of simulations.

Using (2) and (9) the residual (10) satisfies

$$\begin{aligned}r_I(t) &= C_I H_p(s)\left[x_I(t) - \hat{x}_I(t)\right] + H_p(s)\left[\xi_I(t)\right] \\ &= C_I \tilde{x}_{I,f}(t) + \epsilon_{\xi_I}(t).\end{aligned} \tag{24}$$

Using (23) and noting that $\tilde{x}_{I,f}(0) = 0$ due to the filters' zero initial condition, (24) becomes

$$r_I(t) = \int_0^t C_I e^{A_{I,0}(t-\tau)}\chi_I(\tau)\,\mathrm{d}\tau + \epsilon_{\xi_I}(t). \tag{25}$$

Using the triangle inequality, the j-th element of $r_I(t)$, i.e. $r_I^{(j)}(t)$ satisfies

$$\begin{aligned}|r_I^{(j)}(t)| &\le \left|\int_0^t C_I^{(j)} e^{A_{I,0}(t-\tau)}\chi_I(\tau)\,\mathrm{d}\tau\right| + |\epsilon_{\xi_I}^{(j)}(t)| \\ &\le \int_0^t \|C_I^{(j)} e^{A_{I,0}(t-\tau)}\|\|\chi_I(\tau)\|\,\mathrm{d}\tau + |\epsilon_{\xi_I}^{(j)}(t)|,\end{aligned} \tag{26}$$

where $C_I^{(j)}$ indicates the j-th row of the matrix C_I. Note that since $A_{I,0}$ is Hurwitz, there exist positive constants $\kappa_I^{(j)}$, $\lambda_I^{(j)}$ such that $\|C_I^{(j)} e^{A_{I,0}t}\| \le \kappa_I^{(j)} e^{-\lambda_I^{(j)}t}$. In addition, using the fact that $|\epsilon_{\xi_I}^{(j)}(t)| \le \|\epsilon_{\xi_I}(t)\| \le \bar{\epsilon}_{\xi_I}(t)$, (26) becomes

$$|r_I^{(j)}(t)| \le \int_0^t \kappa_I^{(j)} e^{-\lambda_I^{(j)}(t-\tau)}\|\chi_I(\tau)\|\,\mathrm{d}\tau + \bar{\epsilon}_{\xi_I}(t). \tag{27}$$

Now, consider the term $\|\chi_I(t)\|$ which satisfies

$$\begin{aligned}\|\chi_I(t)\| &\le \|H_p(s)\left[\eta_I(x_I, \bar{x}_I, u_I, t)\right]\| + \|L_I \epsilon_{\xi_I}(t)\| \\ &\quad + \|\epsilon_{\zeta_I}(t)\| + \|\epsilon_{\Delta_I}(t)\| + \|h_p(t) x_I(0)\| \\ &\le \int_0^t |h_p(t-\tau)|\|\eta_I(x_I(\tau), \bar{x}_I(\tau), u_I(\tau), \tau)\|\,\mathrm{d}\tau \\ &\quad + \|L_I \epsilon_{\xi_I}(t)\| + \|\epsilon_{\zeta_I}(t)\| + \|\epsilon_{\Delta_I}(t)\| \\ &\quad + |h_p(t)|\|x_I(0)\|.\end{aligned} \tag{28}$$

According to (28), a suitable bounding function (i.e., $\|\chi_I(t)\| \le \bar{\chi}_I(t)$) is designed as

$$\begin{aligned}\bar{\chi}_I(t) \triangleq\ & \bar{H}_p(s)\left[\bar{\eta}_I(y_I(t), \bar{y}_I(t), u_I(t))\right] + \|L_I\|\bar{\epsilon}_{\xi_I}(t) \\ & + \bar{\epsilon}_{\zeta_I}(t) + \bar{\epsilon}_{\Delta_I}(t) + |h_p(t)|\bar{x}_{I,d},\end{aligned} \tag{29}$$

where $\bar{x}_{I,d}$ is a bounding estimate of $x_I(0)$ such that $\|x_I(0)\| \le \bar{x}_{I,d}$ and $\bar{H}_p(s)$ is a transfer function with impulse response $\bar{h}_p(t) \ge |h_p(t)|$ that will be defined below using the following well-known fact (i.e., see [11]) which states that the impulse response $h_p(t)$ of a strictly proper and asymptotically stable transfer function $H_p(s)$ decays exponentially, i.e. $|h_p(t)| \le \mu_I e^{-\nu_I t}$ for some $\mu_I > 0$,

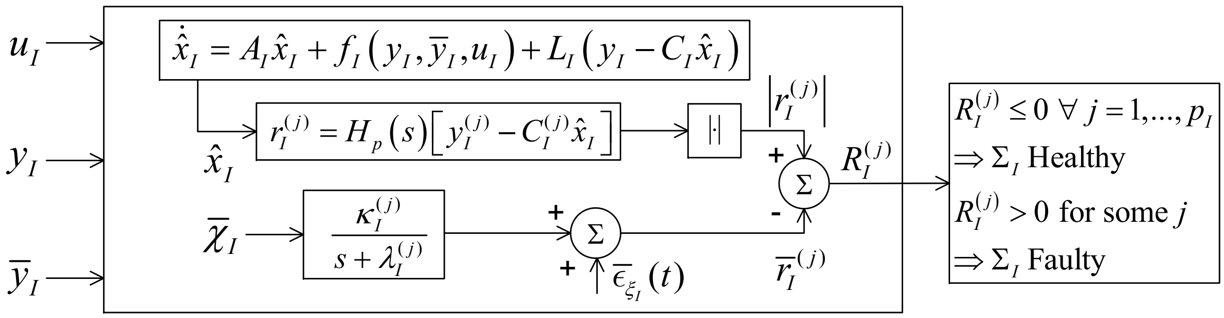


Fig. 1: Local Filtered Fault Detection Scheme.

$\nu_I > 0$ for all $t > 0$. Therefore, $\bar{h}_p(t)$ can be selected as $\bar{h}_p(t) = \mu_I e^{-\nu_I t}$ and therefore $\bar{H}_p(s) = \frac{\mu_I}{s+\nu_I}$. In addition, note that $|h_p(t)|$ is exponentially decaying, thus a conservative estimate $\bar{x}_{I,d}$ affects the detection threshold only during the initial transient period. Moreover, note that if $h_p(t)$ is non-negative; i.e. $h_p(t) \geq 0$, for all $t \geq 0$, then the calculation of $\bar{H}_p(s)$ can be omitted since $|h_p(t-\tau)| = h_p(t-\tau)$ (see (28)). Necessary and sufficient conditions for non-negative impulse response for a specific class of filters are given in [12].

So, by using (27), (28) and (29), we obtain $|r_I^{(j)}(t)| \leq \bar{r}_I^{(j)}(t)$, where the detection threshold $\bar{r}_I^{(j)}(t)$ is given by

$$\bar{r}_I^{(j)}(t) \triangleq \int_0^t \kappa_I^{(j)} e^{-\lambda_I^{(j)}(t-\tau)} \bar{\chi}_I(\tau)\,\mathrm{d}\tau + \bar{\epsilon}_{\xi_I}(t) \tag{30}$$

and can be implemented using simple linear filtering techniques as $\bar{r}_I^{(j)}(t) = \frac{\kappa_I^{(j)}}{s+\lambda_I^{(j)}}\left[\bar{\chi}_I(t)\right] + \bar{\epsilon}_{\xi_I}(t)$.

The findings of the preceding analysis are summarized in the following lemma.

Lemma 1: Consider a distributed system made of N subsystems Σ_I given by (1), (2). In the absence of any faults, the residual signals $r_I^{(j)}(t)$ given by (10), where the signals $\hat{y}_I^{(j)}$ are given by (8) and (9), are bounded by the detection thresholds $\bar{r}_I^{(j)}(t)$, given by (30), thus guaranteeing no false alarms.

Figure 1 illustrates the implementation of the local filtered fault detection scheme for the I-th subsystem resulting from equations (5), (8), (9), (10), (29) and (30).

In general, the distributed fault detection scheme is constituted by N local filtered fault detection modules, one for each subsystem Σ_I. Each subsystem is monitored by a local fault detection module which requires the input and output measurements of the subsystem that is monitoring and also the measurements of all interconnecting subsystems that are influencing the subsystem that the specific module is monitoring. Therefore, there is the need of a communication infrastructure enabling the information exchange between the local fault detection modules depending on their interconnections.

IV. Fault detectability

The design and analysis in the previous section was based on devising suitable thresholds $\bar{r}_I^{(j)}(t)$ such that the absolute values of the residual signals $r_I^{(j)}(t)$ are bounded by $\bar{r}_I^{(j)}(t)$ in the absence of any fault. In this section a detectability condition of the aforementioned fault detection scheme is presented. This condition constitutes a theoretical result that provides a quantitative characterization of the class of faults detectable by the proposed scheme.

Theorem 1: Consider the nonlinear interconnected system (1), (2) with the distributed fault detection scheme described in (5), (8), (9), (10), (29) and (30). A fault in the I-th subsystem occurring at $t = T_0$ is detectable if the filtered fault function $\phi_{I,f}\big(x(t), u_I(t), t\big) \triangleq H_p(s)[\beta_I(t - T_0)\phi_I(x(t), u_I(t))]$ satisfies the following inequality for some $j = 1, 2, \ldots, p_I$:

$$\left|\int_{T_0}^t C_I^{(j)} e^{A_{I,0}(t-\tau)} \phi_{I,f}\big(x(\tau), u_I(\tau), \tau\big)\,\mathrm{d}\tau\right| > 2\bar{r}_I^{(j)}(t).$$

Proof: In the presence of a fault that occurs at $t = T_0$, equation (23) becomes:

$$\begin{aligned}\tilde{x}_{I,f}(t) &= e^{A_{I,0}t}\tilde{x}_{I,f}(0)\\ &\quad + \int_0^t e^{A_{I,0}(t-\tau)}\big(\chi_I(\tau) + \phi_{I,f}\big(x(\tau), u_I(\tau), \tau\big)\big)\,\mathrm{d}\tau\end{aligned}$$

and the residual from (25) becomes

$$\begin{aligned}r_I(t) &= \int_0^t C_I e^{A_{I,0}(t-\tau)}\big(\chi_I(\tau) + \phi_{I,f}\big(x(\tau), u_I(\tau), \tau\big)\big)\,\mathrm{d}\tau\\ &\quad + \epsilon_{\xi_I}(t).\end{aligned}$$

Note that the term $C_I e^{A_{I,0}t}\tilde{x}_{I,f}(0)$ is omitted since $\tilde{x}_{I,f}(0) = 0$ due to the filters' zero initial conditions. By using the triangle inequality, the j-th element of $r_I(t)$ for $t > T_0$ satisfies:

$$\begin{aligned}|r_I^{(j)}(t)| \geq &- \left|\int_0^t C_I^{(j)} e^{A_{I,0}(t-\tau)}\chi_I(\tau)\,\mathrm{d}\tau\right| - |\epsilon_{\xi_I}^{(j)}(t)|\\ &+ \left|\int_{T_0}^t C_I^{(j)} e^{A_{I,0}(t-\tau)}\phi_{I,f}\big(x(\tau), u_I(\tau), \tau\big)\,\mathrm{d}\tau\right|.\end{aligned} \tag{31}$$

Note that the lower limit of the last integral in (31) is T_0 due to (3). Following a similar procedure as in the derivation of the detection threshold (30), equation (31) becomes

$$\begin{aligned}|r_I^{(j)}(t)| \geq &- \bar{r}_I^{(j)}(t)\\ &+ \left|\int_{T_0}^t C_I^{(j)} e^{A_{I,0}(t-\tau)}\phi_{I,f}\big(x(\tau), u_I(\tau), \tau\big)\,\mathrm{d}\tau\right|.\end{aligned}$$

For fault detection, the inequality $|r_I^{(j)}(t)| > \bar{r}_I^{(j)}(t)$ must hold for some $j = 1, 2, \ldots, p_I$, so the final fault detectability condition given in Theorem 1 is obtained. ■

The above fault detectability theorem provides a measure of the type of faults that can be detected with the proposed distributed fault detection scheme. Clearly, the fault functions $\phi_I(x, u_I)$ are typically unknown and therefore this condition cannot be checked apriori. However, it provides useful intuition about the type of faults that are detectable.

V. Simulation Results

In this section, we consider a numerical example to illustrate some of the concepts developed in this paper. The example is based on a system of two interconnected one-link manipulators with revolute joints actuated by a DC motor, where the elasticity of the joint can be modeled by linear tensional spring [13]. The state variable $x_I^{(1)}$ represents the motor position, $x_I^{(2)}$ the motor velocity, $x_I^{(3)}$ the link position and $x_I^{(4)}$ the link velocity. The system dynamics of the two subsystems $I = 1, 2$, with $x_I = \left[x_I^{(1)} x_I^{(2)} x_I^{(3)} x_I^{(4)}\right]$, are given by

$$\dot{x}_I = A_I x_I + f_I + \zeta_I + \eta_I \tag{32}$$

$$y_I = C_I x_I + \xi_I \tag{33}$$

where for the first subsystem $I = 1$:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}, \eta_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1sin(x_1^{(3)}) \end{bmatrix},$$

$$f_1 = \begin{bmatrix} 0 \\ 43.2u \\ 0 \\ -3.33sin(x_1^{(3)}) + 2sin(x_2^{(4)}) \end{bmatrix}.$$

For the second subsystem $I = 2$, we have:

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -24.3 & -0.625 & 24.3 & 0 \\ 0 & 0 & 0 & 1 \\ 9.75 & 0 & -9.75 & 0 \end{bmatrix}, \eta_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1sin(x_2^{(3)}) \end{bmatrix},$$

$$f_2 = \begin{bmatrix} 0 \\ 21.6u \\ 0 \\ -1.665sin(x_2^{(3)}) + sin(x_1^{(4)}) \end{bmatrix}$$

and the matrix C_I is given in both subsystems $I = 1, 2$ by $C_I = [\ 0\ 1\ 0\ 0\ ;\ 0\ 0\ 1\ 0\ ;\ 0\ 0\ 0\ 1\ ]$. For simplicity, the input u_I for both subsystems is a sinusoid of magnitude 1 and frequency 1 Hz. In this simple example, we consider an abrupt multiplicative actuator fault in subsystem 1 where the input changes to $u_1 = (1 + \theta_1)\bar{u}_1$ where $\bar{u}_1$ is the nominal input in the non-fault case and $\theta_1 \in [-1, 0]$ is the parameter characterizing the magnitude of the fault. The fault occurs at $T_0 = 2$ sec with a magnitude $\theta_1 = -0.25$.

Two FDI modules are implemented and each one monitors its associated subsystem. For every measurable state variable, the residual signal is generated according to (10) and the corresponding detection threshold using (30), (29). The estimator $\hat{x}_1(t)$ is implemented according to (8) and the gain matrix L_1 is designed so that $(A_1 - L_1 C_1)$ is Hurwitz. More specifically, L_1 is given by

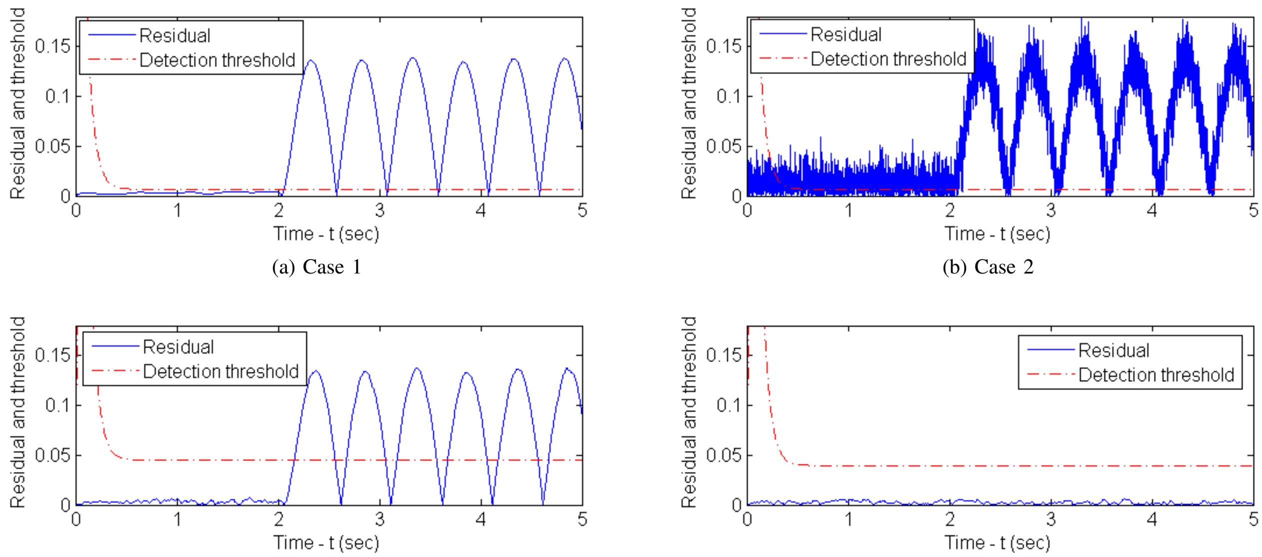
$$L_1 = \begin{bmatrix} -9.3178 & 0 & 5.0575 \\ 44.6928 & 48.6000 & -10.0984 \\ 0 & 15.0000 & 1.0000 \\ -8.4207 & -19.5000 & 29.0572 \end{bmatrix}.$$

Similarly, the estimator $\hat{x}_2(t)$ is computed in the second FDI module, where the gain matrix L_2 is designed so that $(A_2 - L_2 C_2)$ is Hurwitz.

In "Case 1" we consider the ideal case where no measurement noise or disturbances are present. For this case, we omit the use of filtering and therefore the residual and threshold signals are given by modifying accordingly (10), (29) and (30). More specifically, the residual is given by $r_I(t) = y_I(t) - \hat{y}_I(t)$ and the threshold by $\bar{r}_I^{(j)}(t) = \frac{\kappa_I^{(j)}}{s+\lambda_I^{(j)}} [\bar{\eta}_I(y_I(t), \bar{y}_I(t), u_I(t))] + \kappa_I^{(j)} e^{-\lambda_I^{(j)} t} \bar{x}_{I,d}$. For all the cases that will be demonstrated we consider that the bound on the modeling uncertainty is $\bar{\eta}_1 = 0.1$ and the bound on the system initial conditions is $\bar{x}_{1,d} = 1$. In addition the simulation results concern the subsystem $I = 1$ and more specifically the residual $r_1^{(4)}$ and threshold $\bar{r}_1^{(4)}$. Figure 2a shows the residual and threshold signals in "Case 1", where it can be seen that the fault is successfully detected with no false alarms prior to the fault occurrence.

Next, in "Case 2" the noise and disturbance terms are added to demonstrate their effect in the detectability scheme presented in "Case 1". The measurement noise is of Gaussian type with mean $\mu_\xi = 0$, variance $\sigma_\xi^2 = 0.00025$ and max magnitude 0.05. The disturbance is also considered Gaussian with mean $\mu_\zeta = 0.01$, variance $\sigma_\zeta^2 = 0.0025$ and max magnitude 0.09. More specifically, in "Case 2" we use the same residual and threshold signals as in "Case 1", but in the presence of measurement noise and disturbances. Figure 2b demonstrates the results in "Case 2" where it can be seen that the residual exceeds significantly the threshold prior to the fault occurrence, causing false alarms.

Finally, in "Case 3" we demonstrate the effectiveness of the proposed fault filtering approach where a low-pass filter with transfer function $H_p(s) = \frac{50^2}{(s+50)^2}$ is used for the generation of the residual and threshold signals. The bounds that are used for the detection threshold generation are $\bar{\epsilon}_{\xi_I} = 0.006$, $\bar{\epsilon}_{\zeta_I} = 0.04$ and $\bar{\epsilon}_{\Delta_I} = 0.01$ for both FDI modules. Note that the transfer function $H_p(s)$ has a non-negative impulse response $h_p(t)$ and therefore the calculation of $\bar{h}_p(t)$ is not needed since $h_p(t)$ can be used. Figure 2c shows the residual $r_1^{(4)}$ and detection threshold $\bar{r}_1^{(4)}$ where it is clearly seen that the fault is successfully detected at around $t = 2.15$ sec without any false alarms prior to the fault occurrence. Figure 2d shows the results for the subsystem $I = 2$ and more specifically the residual $r_2^{(4)}$ and threshold



(a) Case 1

(b) Case 2

(c) Case 3a

(d) Case 3b

Fig. 2: Residual signal and fault detection threshold for Cases 1-3, under different assumptions.

$\bar{r}_2^{(4)}$. The filter that is used has the same transfer function as before. The simulation results show that the fault effects are not propagated to the second subsystem and the residual does not exceed its threshold, thus the fault in subsystem 1 is not detected in subsystem 2. Therefore, partial fault isolation by identifying the subsystem that the fault has occurred is also achieved.

VI. Conclusion

In this paper a distributed fault detection filtering approach for a class of input-output interconnected, continuous-time, nonlinear systems with modeling uncertainties, measurement noise and disturbances is presented. A fault detectability condition is obtained that characterizes quantitatively the class of faults that can be detected using the proposed approach. The main contribution of this paper is the novel use of filters in a distributed framework that takes advantage of their noise dampening characteristics for the derivation of tight residual and threshold signals. Intuitively, filtering allows dampening of the noise in a certain frequency range, thus facilitating the design of less conservative thresholds, which enhances fault detectability. The implementation of the scheme relies on a distributed framework, where each subsystem is monitored by a local fault detection module which requires the input and output measurements of the subsystem that is monitoring and also the measurements of all interconnecting subsystems that are influencing the subsystem that the specific module is monitoring. Future research efforts will be devoted to the extension of the filtering fault detection approach to include a greater class of input-output nonlinear systems and to develop a comprehensive distributed fault isolation methodology.

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