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Fractional Order PID Controller (FOPID)-Toolbox

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***Abstract*— This paper presents a fractional order PID controller (FOPID)-Toolbox to design robust fractional PID controllers achieving a desired crossover frequency and a desired phase margin. A novel approach based on nonsmooth optimization techniques is used. Two types of controllers are considered, the $(PID)^n$ and $PI^\alpha D^\beta$ controllers. The requirements to be fulfilled by the controller are expressed in terms of a desired open-loop response. Loop shaping configuration is used to synthesize the controller. To optimize the fractional orders an optimization algorithm based on the steepest descent method is used. Simulation results show the benefit of our method.**

I. INTRODUCTION

There is no doubt that the PID controller is one of the most used controller type in control-loops. The design and tuning of such a controller is well studied and still an active field of research, see [1]. A generalization of this type of controller is given by the fractional $PI^\alpha D^\beta$ controller and was introduced by [2], at first. Due to the additional fractional order α and β this controller, when well tuned, outperfoms the classical PID controller. In [3] a method to tune the $PI^\alpha D^\beta$ controller is presented. It is based on solving a set of nonlinear equations. In [4] and [5] a tuning rule for the PD^β controller with application to motion systems is given. Genetic algorithms are used in [6] to design a $PI^\alpha D^\beta$ controller.

Another class of fractional controllers is proposed by [7], namely $(PI)^n$ and $(PID)^n$ controllers. This class of controllers is more appropriate to ensure robustness of the closed-loop to static gain variations with a conventional $CRONE$ template. Based on output feedback techniques, the controller is derived using the Lyapunov stability condition expressed in terms of LMIs. The time domain constraints are introduced using the equality of moments between the closed-loop system and its fractional reference model.

Unlike in the PID controller case, the number of publications which deals with the design and tuning of the fractional PID controller is still small. Therefore, there is a need to explore new tuning methods. The goal of this work is to develop a systematic tool to optimize the parameters of the fractional order controller $(PID)^n$ and $PI^\alpha D^\beta$. Our approach is based on the recently developed nonsmooth optimization techniques by [8]. The requirements to be satisfied by the controller are expressed in terms of a desired open-loop response. The loop shaping approach is used to formulate the problem in the H_∞ framework.

This paper is organized as following, in section 2 we give the problem formulation for the $PI^\alpha D^\beta$ and the $(PID)^n$ controllers such as the solution of the related optimization problem. In section 3 we present the FOPID-Toolbox for Matlab. Numerical examples and simulation results are given in section 4.

II. PROBLEM FORMULATION

The approach proposed in this paper deals with the design of fractional PID controllers in the form $(PID)^n$ or $PI^\alpha D^\beta$. It is based on the work [8] in which the authors proposed an algorithm to solve the fixed structure H_∞ problem without using the Lyapunov stability in LMI form to avoid the related high number of decision variables. The motivation of this work is to extend this technique to cope with fractional order controllers.

A. Fractional $PI^\alpha D^\beta$ controller

The fractional controller in the form $PI^\alpha D^\beta$ is a generalization of the PID controller for fractional orders and was introduced by [2], at first. In this work an approach is proposed to optimize the parameters of this controller such that some requirements are fulfilled. For this purpose the feedback control-loop configuration, see Fig. 1, is considered. This configuration is well known as the loop shaping configuration [9], with $L_d(s)$ as the desired open-loop transfer function which includes the requirements to be fulfilled by the controller $K(s)$. The signal r denotes the reference signal, $e = r - y$ is the error signal, n_w and e_w are the exogenuous input and output, respectively. $G(s)$ is a SISO

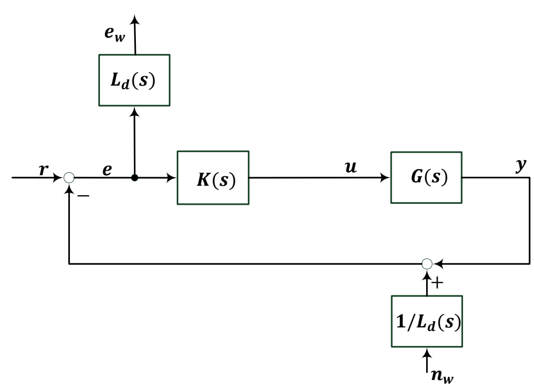

Fig. 1. Loop shaping configuration

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LTI-system with static gain variations. The transfer function from (r, n_w) to (y, e_w) is denoted by $T_{(r,n_w)\rightarrow(y,e_w)}(s)$.

We are interested in designing a fractional controller of the form

$$PI^{\alpha}D^{\beta} = K_P + \frac{K_I}{s^{\alpha}} + K_D s^{\beta} \quad (1)$$

achieving a given phase margin ϕ_m and a crossover frequency ω_c. The controller should also be robust in the presence of static gain variations which is given by

$$\left(\frac{\mathrm{d}\left(\arg(L\left(j\omega\right))\right)}{\mathrm{d}\omega}\right)_{\omega=\omega_c} = 0 \quad (2)$$

and means that the phase plot is flat around ω_c. These specifications are translated into a desired open-loop response

$$L_d(s) = \frac{\omega_f}{s^v}, \quad (3)$$

with

$$v = 2 - \frac{\phi_m}{90} \quad and \quad \omega_f = \omega_c^v\,. \quad (4)$$

Equation (3) defines a fractional integrator. To approximate the fractional order v in the whole frequency range, a high order transfer function is needed. For a band limited implementation of (3), the CRONE approximation method given in [10]

$$\prod_{i=1}^{N} \frac{1 + \frac{s}{\omega_i^{'}}}{1 + \frac{s}{\omega_i}}, \qquad \omega_i^{'}, \omega_i \quad \epsilon \mathbb{R} \quad (5)$$

with

$$\frac{\omega_i}{\omega_i^{'}} = \alpha, \quad \frac{\omega_{i+1}^{'}}{\omega_i^{'}} = \eta, \quad n = \frac{\log(\alpha)}{\log(\alpha\eta)} \quad (6)$$

is used. The order N should be choosen depending on the bandwidth in which the approximation is valid. A very important point to be mentioned here is that due to the inverse of $L_d(s)$, see Fig. 1, the CRONE approximation is a very suitable method to be used. Filter (5) is bi-proper with stable poles and zeros.

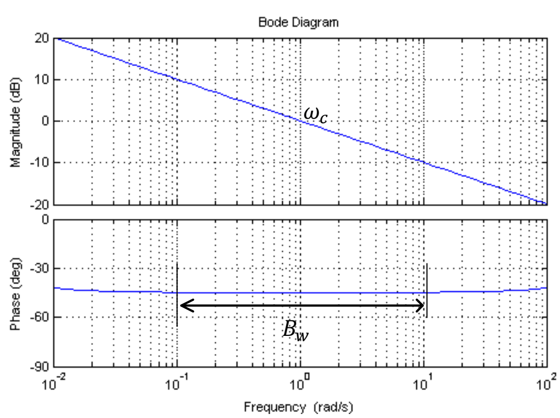

Fig. 2. Desired open-loop response

An example of a desired open-loop response using the fractional integrator

$$L_d(s) = \frac{1}{s^{1.5}} \quad (7)$$

is presented in Fig. 2. The approximation is valid in a specified bandwidth B_w around the crossover frequency $\omega_c = 1\ rad/s$. The phase margin for (7) is $\phi = 45^o$. The constant phase enforces the closed-loop to be robust against static gain variations. To express the phase margin specification in term of the overshoot of the related closed loop system see [11].

Before proceeding to the problem definition the following notation

$$\|G(s)\|_{\infty} := \max_{\omega} \bar{\sigma}(G(j\omega)) \quad (8)$$

is introduced to denote the H_{∞} norm of the transfer function $G(s)$. $\bar{\sigma}(G)$ is the maximal singular value of G. For SISO systems this norm is the maximum gain over all frequencies. In the MIMO case it is the peak value of the maximum singular value over all frequencies. With the help of this norm and using the configuration in Fig. 1 our optimization problem is formulated as follows

$$\min_{K\in\Omega} \quad \left\|T_{(r,n_w)\rightarrow(y,e_w)}(K)\right\|_{\infty} \quad (9)$$

with $K \in \Omega$ is a structural constraint on the controller. In our case this constraint is represented by fractional PID controllers in the form $PI^{\alpha}D^{\beta}$. Without the restriction $K \in \Omega$, problem (9) falls into the scope of convex optimization and can be solved efficiently. For example to solve (9) one can first define a generalized plant (10) consisting of the plant $G(s)$, the filter $L_d(s)$ and $1/(L_d(s))$

$$P : \left\{ \begin{array}{lcl} \dot{x} & = & A\ x + B_1\ \ w + B_2\ \ u \\ z & = & C_1\, x + D_{11}\, w + D_{12}\, u \\ e & = & C_2\, x + D_{21}\, w + D_{22}\, u \end{array} \right. \quad (10)$$

and a controller K

$$K : \left\{ \begin{array}{lcl} \dot{x}_K & = & A_K\ x_K + B_K\ e \\ u & = & C_K\ x_K + D_K\ e\ . \end{array} \right. \quad (11)$$

With the help of the bounded real lemma and the projection lemma [12], problem (9) can be transformed into the LMI form and then be solved efficiently using LMI-Solvers. The obtained controller is of full order which means that the size of A_k is equal to the size of A. As mentioned in [8], adding the constraint $K \in \Omega$ changes the whole situation. Problem (9) can not be converted into the LMI form or any other convex program. To solve this problem other algorithmic methodologies are required. The authors in [8] have proposed a new nonsmooth optimization technique to solve the H_{∞} problem under structural constraints on the controller. In the scope of this work, the set Ω consists of fractional PID controllers which includes an additional constraint, the fractional order.

Now considering the fractional controller (1) and using the approximation (5) for the fractional order α and β, this controller is equivalent to

$$\tilde{K}(s) = K_p + K_I F_I(s) + K_D F_D(s) \qquad (12)$$

with $F_I(s)$ and $F_D(s)$ used for the approximation of α and β, respectively. Substituting (12) in (9) and considering the case of a fixed value α_k and β_k problem (9) reduces to

$$\min_{\tilde{K}} \left\| T_{(r,n_w)\to(y,e_w)}(\tilde{K}, \alpha_k, \beta_k) \right\|_\infty . \qquad (13)$$

At this point we want to clarify the difference between (9) and (13). In (9) the fractional orders of the $PI^\alpha D^\beta$ controller are variables of the H_∞ minimization problem and so a posteriori known. Unlike in (13) they are known a priori. In the second case the optimal values of α and β are computed afterwards using an outer loop. Problem (13) can be seen as computing a static output feedback controller for the new plant $\tilde{G}(s)$ as shown in Fig. 3. This is a typical application of the method proposed in [8] which is implemented in the Matlab function **hinfstruct**.

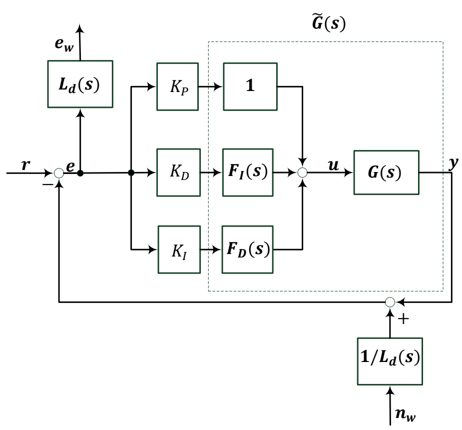

Fig. 3. Loop shaping configuration

The main idea in this work is to replace problem (9) by problem (13) and then optimize over the fractional orders α and β. Formally the considered optimization problem is expressed as follows

$$\min_{\alpha,\beta} \left(\min_{\tilde{K}} \left\| T_{(r,n_w)\to(y,e_w)}(\tilde{K}, \alpha, \beta) \right\|_\infty \right) \qquad (14)$$

and solved for α and β using the following algorithm

1) Initialize α_0 and β_0
2) Compute the gradient d_k with respect to α_k and β_k
3) If $\|d_k\|_2$ smaller then a value t STOP, else update α_k and β_k and go back to step (2).

To update α_k and β_k in step (3) we used the steepest descent algorithm with a line search. The computation of the gradient of the H_∞ norm in (14) with respect to α and β was performed numerically. The values of K_P, K_I and K_D are computed by substituting the obtained values of α_k and β_k in (13).

B. *Fractional $(PID)^n$ controller*

This kind of controller was presented in [7] and [13] and a method to tune the controller using the equality of moments between the closed-loop system and its fractional reference model is given. Contrary to [7] the fractional order n is also a tuning parameter to get an additional degree of freedom in designing the controller. The requirements to be fulfilled

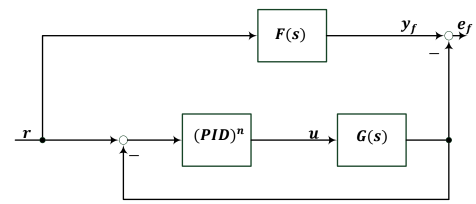

Fig. 4. Reference model configuration

by the $(PID)^n$ controller are the same used in the $PI^\alpha D^\beta$ controller case. To formulate the optimization problem, the configuration Fig. 4 is used. Formally speaking we want to solve the following problem

$$\min_{K\in\Omega} \left\| T_{r\to e_f}(K) \right\|_\infty \qquad (15)$$

with the set of fractional controllers Ω in the form

$$(PID)^n = \frac{1}{s^n}(K_P + \frac{K_I}{s} + K_D s) . \qquad (16)$$

To render the PID controller in (16) realizable, the derivative term $K_D s$ is replaced by $K_D \frac{s}{\tau s+1}$ and will be denoted with D_f. As mentioned in [7] it becomes necessary to define a new open-loop function given by

$$\tilde{L}(s) = \frac{L(s)}{\tau s + 1} . \qquad (17)$$

The filter describing the desired closed-loop response, see Fig. 4, becomes

$$F(s) = \frac{\tilde{L}(s)}{1 + \tilde{L}(s)} . \qquad (18)$$

To solve problem (15) the same idea as in the $PI^\alpha D^\beta$ controller case is adopted here. Using the order approximation (5) the controller (16) is equivalent to

$$\tilde{K}(s) = F_n(s)(K_P + \frac{K_I}{s} + K_D \frac{s}{\tau s + 1}), \qquad (19)$$

with $F_n(s)$ the integer approximation of the order n. Substituting (19) in (15) and considering the case of a known value n_k, problem (15) reduces to

$$\min_{\tilde{K}} \left\| T_{r\to e_f}(\tilde{K}, n_k) \right\|_\infty . \qquad (20)$$

Solving problem (20) can be seen as computing a PID controller for the augmented system $\tilde{G}(s) = G(s)F_n(s)$. The optimal value n_k that minimizes the H_∞ norm in (20) is computed using the approach proposed to optimize the $PI^\alpha D^\beta$ controller.

III. FOPID-TOOLBOX

As mentioned in [8], the problem of designing low order controllers is not convex. This means that the used algorithm does not necessarily lead to the global minimum. The authors in [8] deal with this difficulty by starting the algorithm with different initial sets of parameters and then pick up the one with the lowest value of the H_∞ norm. As our approach is based on this method, it is also a non convex one. Because of this fact we give the user the opportunity to analyse the relevant open and closed-loop plot of the resulting transfer function. Moreover with our FOPID-Toolbox the obtained controller parameters can be retuned if necessary. We also implemented several approximation methods for the fractional order (continuous and discrete). The main features of the FOPID-Toolbox are summarized as following

- Compute a fractional PID controller in the form $PI^\alpha D^\beta$ or $(PID)^n$ achieving robust performance in the presence of static gain variation
- Analyse the following plots
  - Open- and closed-loop Bode plot
  - Nyquist and Nichols plots
  - Step Response
- Tuning of the computed parameters
- Analyse and compare several approximation methods for the fractional orders

The discretization methods implemented in the toolbox are based on the work [14] in which the authors proposed about 28 methods. The FOPID-Toolbox contains seven methods to approximate the fractional orders α, β and n. For a detailled review about existing approximation methods we recommend the reader to view the work [14].

To compute a controller with the help of the FOPID-Toolbox, $Matlab\ Robust\ Control\ Toolbox$ (2011) or higher is required. The user has only to define the plant to be controlled, the desired phase margin and the desired crossover frequency. These specifications are translated into a desired open-loop response and then the related optimization problem is solved using the proposed approach.

IV. EXAMPLES

In this section we present several examples to show the benefit of using the FOPID-Toolbox to design fractional order PID controllers. We give also a comparison between the $(PID)^n$ and $PI^\alpha D^\beta$ controller.

A. Example 1

The plant for the first example is borrowed from the work [13], in which the authors considered the design of a $(PID_f)^n$ controller based on the method of moments. The plant (21) describes a time delay second order system

$$P(s) = \frac{G}{2s^2 + 3s + 1} e^{-0.2s}. \quad (21)$$

The parameter G is uncertain and varies in the region $[0.5\ 2]$. The goal is to optimize a $PI^\alpha D^\beta$ controller that fulfills the following requirements

- Phase margin $\phi_m = 51°$, approximately 24% overshoot
- Crossover frequency $\omega_c = 0.5\ rad/s$
- Flat phase at ω_c.

The FOPID-Toolbox translates these requirements into a desired response $L_d(s)$ using (3). Due to the time-delay of the plant, it is necessary to define a new desired response of the open-loop transfer function

$$\tilde{L}(s) = \frac{\omega_f}{s^v} e^{-Ts}. \quad (22)$$

To approximate the time-delay e^{-Ts}, the Pade method is used. After minimizing the objective function (9) in the parameters $(K_P, K_I, K_D, \alpha, \beta)$, we get the following controller

$$K(s) = 1.65 + \frac{1.9}{s^{1.14}} + 0.18\frac{s^{1.17}}{0.05s+1}. \quad (23)$$

The step response with the obtained controller is shown in Fig. 5. The overshoot is approximately constant for all values of G. The overshoot for the minimal and maximal value of G is 19% and 33% respectively. The performance of the $PI^\alpha D^\beta$ controller is good. For the seek of comparison we compute for the same plant with the same requirements a $(PID)^n$ controller. As in the previous case a new desired reference model is defined

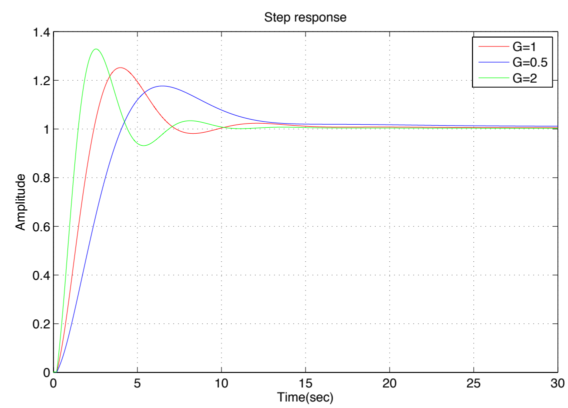

Fig. 5. Step response for different static gain variations ($PI^\alpha D^\beta$)

$$T(s) = \frac{\tilde{L}(s)}{1 + \tilde{L}(s)} \quad (24)$$

with

$$\tilde{L}(s) = \frac{\omega_f}{(\tau s + 1)s^v} e^{-0.2s}. \quad (25)$$

After minimizing (15) in the parameters (K_P, K_I, K_D, n), we get the following controller

$$K(s) = \frac{1}{s^{0.37}}(1.1 + \frac{0.78}{s} + 0.37\frac{s}{0.005s+1}). \quad (26)$$

The step response with the controller (26) is shown in Fig. 6. The overshoot is nearly constant for the nominal, minimal and maximal value of G. The robustness to gain variations is achieved with this controller. Comparing now the step response Fig. 5 and Fig. 6 it becomes clear that

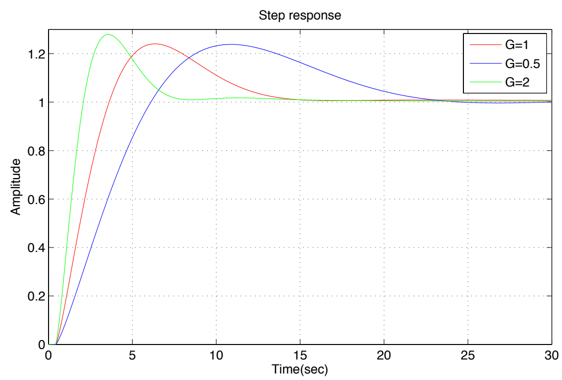

Fig. 6. Step response for different static gain variations $(PID_f)^n$

the $(PID_f)^n$ controller outperforms the $PI^\alpha D^\beta$ controller. With L_d denoting the desired open loop response Fig. 7 shows that the design requirements are not fully satisfied by the $PI^\alpha D^\beta$ controller. Fig. 8 shows that the $(PID)^n$

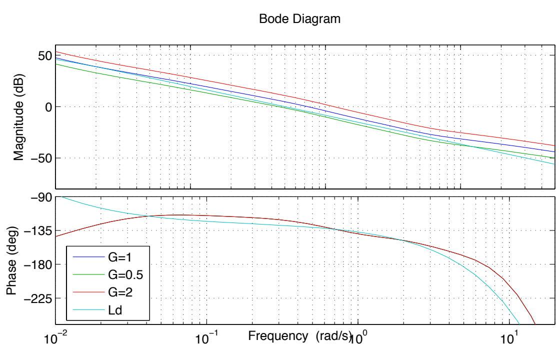

Fig. 7. Bode plot $PI^\alpha D^\beta$

controller provides a very good fit of L_d. Moreover, we give

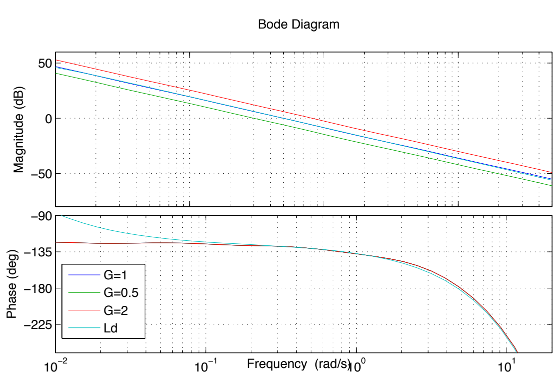

Fig. 8. Bode plot $(PID)^n$

also a comparison between the controllers (26) and (23) and the controllers in the work [13]. The results are shown in Table I. The FOPID-$(PID_f)^n$ controller gives a slightly better performance then the $(PID_f)^n$ controller in [13]. The performance of the FOPID-$PI^\alpha D^\beta$ controller is adequate

TABLE I
OVERSHOOTS IN (%)

G	0.5	1	2
FOPID-$(PID_f)^n$	24%	24.4%	28.9%
[13] $(PID_f)^n$	22%	24%	29.5%
FOPID-$(PI^\alpha D^\beta)$	19%	25%	33%
[13] (PID_f)	8.75%	24.5%	47.5%

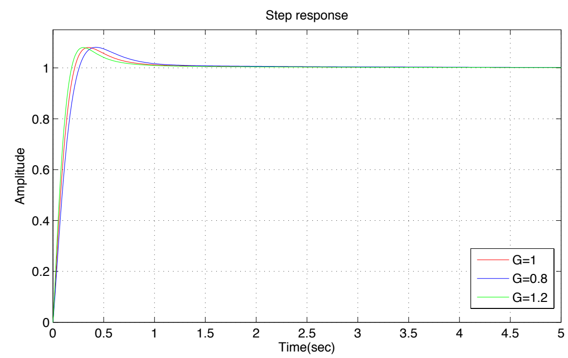

Fig. 9. Step response for different static gain variations $(PD)^n$

and the PID_f controller failed to ensure robustness in the presence of static gain variations.

B. Example 2

The plant for the second example is taken from the work [5], in which the authors designed a PD^α controller using a tuning rule based on a set of nonlinear equations. The following plant

$$P(s) = \frac{G}{s(0.4s+1)} \tag{27}$$

describes a simplified motion control system. The parameter G is uncertain with 0.8 as a minimal value and 1.2 as a maximal value. The controller should fulfill the following requirements

- Phase margin $\phi_m = 70°$
- Crossover frequency $\omega_c = 10\ rad/s$
- Flat phase at ω_c.

These requirements are translated using (4) into a desired open-loop response

$$L_d(s) = \frac{16.68}{s^{1.22}}. \tag{28}$$

After minimizing the objective function (9) in the parameters (K_P, K_D, β), we get the fractional controller

$$K(s) = \frac{1}{s^{0.21}}(16.36 + 6.42\frac{s}{0.005s+1}). \tag{29}$$

The step response with the controller (29) is presented in Fig. 9. The overshoot of the three step responses is constant. The robustness to static gain variations is achieved by this controller. The performance of this controller is approximately the same as with the PD^α in [5]. Both controllers satisfy the design requirements.

C. Example 3

With this example we want to show that our approach is also valid for higher order systems. The plant considered here is a fourth order model

$$P(s) = \frac{G}{(s+10)(s+2)(s+1)(s+0.5)} \quad (30)$$

with G an uncertain parameter varying in the region $[0.5\ 1.5]$. The requirements to be satisfied by the controller are

- Phase margin $\phi_m = 60$
- Crossover frequency $\omega_c = 0.5\ rad/s$
- Flat phase at ω_c.

We are interested in designing a $PI^\alpha D^\beta$ controller. After translating the requirements in the desired open-loop response

$$L_d(s) = \frac{0.39}{s^{1.33}} \quad (31)$$

the H_∞ norm (9) is minimized using the proposed method. The step response with the obtained controller

$$K(s) = 1.55 + \frac{0.74}{s^{1.0665}} + 1.34\frac{s^{0.93}}{0.001s+1} \quad (32)$$

is shown in Fig. 10. Clearly the controller ensures robustness for static gain variations. The overshoot of the three step responses is nearly constant. This is due to the flat phase around the crossover frequency $0.5\ rad/s$, see Fig. 11.

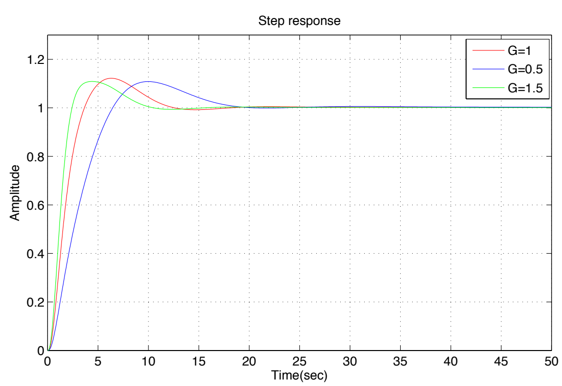

Fig. 10. Step response for different static gain variations ($PI^\alpha D^\beta$)

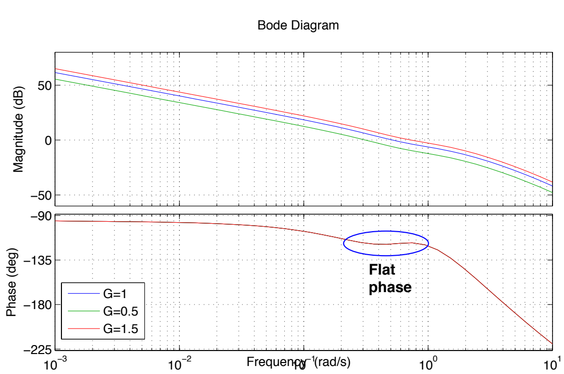

Fig. 11. Open-loop bode plot $PI^\alpha D^\beta$

V. CONCLUSIONS

In this work a new tuning method for fractional PID controllers in the form $PI^\alpha D^\beta$ or $(PID)^n$ is presented. It is based on the recently developed nonsmooth optimization techniques and a steepest descent algorithm. Moreover, based on our tuning method a FOPID-Toolbox for Matlab is presented. The user provides a desired phase margin ϕ_m and a crossover frequency ω_c. These specifications are then internally translated to a desired open-loop response. A fractional controller is computed that best fits this desired open-loop response in the frequency domain. Several numerical examples have shown that the proposed method provides a robust controller satisfying the prespecified requirements. Future works will be for example to extend the toolbox to cope with general uncertainty and to extend the results for MIMO systems. Another point will be to implement the controllers on a real plant and to explore automotive applications.

REFERENCES

[1] K. Åström and T. Hägglund, "The future of PID control," *Control Engineering Practice*, vol. 9, no. 11, pp. 1163–1175, 2001.

[2] I. Podlubny, "Fractional-order systems and $PI^\alpha D^\beta$-controllers," *IEEE Transactions on Automatic Control*, vol. 44, no. 1, pp. 208–214, 1999.

[3] C. Monje, B. Vinagre, V. Feliu, and Y. Chen, "Tuning and auto-tuning of fractional order controllers for industry applications," *Control Engineering Practice*, vol. 16, no. 7, pp. 798–812, 2008.

[4] H. Li, Y. Luo, and Y. Chen, "A fractional order proportional and derivative (FOPD) motion controller: Tuning rule and experiments," *IEEE Transactions on Control Systems Technology*, vol. 18, no. 2, pp. 516–520, 2010.

[5] Y. Luo and Y. Chen, "Fractional-order [proportional derivative] controller for robust motion control: Tuning procedure and validation," in *American Control Conference, 2009. ACC'09.* IEEE, 2009, pp. 1412–1417.

[6] S. Padhee, A. Gautam, Y. Singh, and G. Kaur, "A novel evolutionary tuning method for fractional order PID controller," *International Journal of soft computing and Engineering (IJSCE), Issue-3*, 2011.

[7] M. Tenoutit, N. Maamri, and J. Trigeassou, "An output feedback approach to the design of robust fractional PI and PID controllers," in *Proceedings of the 18th World Congress of the International Federation of Automatic Control (WC IFAC), Milan, Italy*, 2011.

[8] P. Apkarian and D. Noll, "Nonsmooth H_∞ synthesis," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 71–86, 2006.

[9] P. Gahinet and P. Apkarian, "Structured H_∞ synthesis in matlab," *Proc. IFAC, Milan, Italy*, 2011.

[10] P. Melchior, P. Lanusse, O. Cois, F. Dancla, and A. Oustaloup, "Crone toolbox for matlab: Fractional systems toolbox," in *Tutorial Workshop on" Fractional Calculus Applications in Automatic Control and Robotics", 41st IEEE CDC'02*, 2002, pp. 9–13.

[11] G. F. Franklin, J. D. Powell, and A. Emami-Naeini, *Feedback control of dynamic systems.* Prentice Hall, 2006.

[12] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H_∞ control," *International Journal of Robust and Nonlinear Control*, vol. 4, no. 4, pp. 421–448, 1994.

[13] M. Tenoutit, N. Maamri, and J. Trigeassou, "A time moments approach to the design of robust fractional PID controllers," in *Systems, Signals and Devices (SSD), 2011 8th International Multi-Conference on.* IEEE, 2011, pp. 1–7.

[14] D. Valério and J. da Costa, "Time-domain implementation of fractional order controllers," in *Control Theory and Applications, IEE Proceedings-*, vol. 152, no. 5. IET, 2005, pp. 539–552.

[15] A. Banos, J. Cervera, P. Lanusse, J. Sabatier, *et al.*, "Bode optimal loop shaping with crone compensators," in *Procedings of the 14th IEEE Mediterranean Electrotechnical Conference, MELECON 2008*, 2008.