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Reduced-order two-parameter pLPV controller for the rejection of nonstationary harmonically related multisine disturbances

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***Abstract*—A reduced-order gain-scheduling controller based on two time-varying parameters for the rejection of harmonic disturbances with time-varying frequencies is presented. The frequencies are harmonically related and assumed to be known. The control design results in a discrete-time controller with an order which is two times the number of frequency components of the multisine disturbance. Two gain-scheduling parameters are used independently of the number of harmonic components and a triangle is considered as the polytope, therefore the controller is obtained by interpolation between three controllers calculated for the vertices of the polytope. The resulting controller structure is therefore very simple. Experimental results on an active vibration control test bed are used to validate the controller.**

I. Introduction and Motivation

For the rejection of multisine disturbances with time-varying frequencies, where the frequencies are assumed to be known, various approaches for gain-scheduled feedback controllers have been proposed [1-19]. This specific control problem often occurs in active control of noise and vibration (ANC/AVC) in applications where rotating machinery operates at varying speed. The proposed feedback control approaches constitute alternatives to the commonly used adaptive (usually feedforward) filters based on the filtered-x least-mean-square algorithm [20]. Especially approaches based on methods for linear parameter-varying (LPV) systems [5-19] offer the advantage that stability of the closed-loop system is guaranteed for (usually arbitrarily fast) frequency variations. Thus “exact” stability results are available, whereas only “approximate” stability results seem to be available for the adaptive filtering approaches [20, 21].

In the feedback approaches [1-19], the controllers contain a model of the disturbance. This is necessary for the controllers to have high gain at the frequencies present in the disturbance to be rejected and is referred to as the internal model principle [22]. Various approaches can be chosen to ensure that the controller contains a model of the disturbance: The plant can be augmented with a disturbance model and a disturbance observer can be used to estimate the states of the overall system. The states of the disturbance model and plant are then used in a state-feedback control law [1-7]. Alternatively, the plant can be augmented with an output filter and the states of this output filter and the plant states (estimated by an observer) are used in state feedback [7-9]. Both of these approaches result in observer-based state-feedback controllers. The disturbance model can also be used as a weighting function to build a generalized plant and a general norm-optimal output-feedback controller can be designed [10-19]. The disturbance model is either modeled as a polytopic linear parameter-varying (pLPV) model or as an LPV model in linear fractional transformation (LPV-LFT) form. A detailed description of these control approaches is given in [7] for observer-based state-feedback approaches and [16] for output-feedback controllers. These approaches suffer from two drawbacks. First, the order of the resulting controller is the order of the plant plus the order of the disturbance model (which is, as discussed below, twice the number of frequency components that should be rejected) plus the order of additional weighting functions (if used). Thus, the controller could be of very high order. Second, in the discrete-time approaches, the disturbance model is transformed into the required pLPV or LPV-LFT form by introducing one independent gain-scheduling parameter for each frequency. This is a severe disadvantage in the commonly occurring case of harmonically related frequencies (multiples of a fundamental frequency).

Both problems are addressed in this paper. First, an idea suggested in [17, 18] is used to reduce the number of gain-scheduling parameters to two (independently of the number of harmonically related frequencies). A further simplification is achieved by considering a triangle as the polytope instead of the rectangle used in [17, 18]. Second, an approach is presented that results in a controller with an order of $2N_{\mathrm{h}}$, where N_{h} is the number of frequency components in the multisine disturbance. This is actually the minimum order required for the asymptotic rejection of the specified disturbance as it is the order of the disturbance model. Combining this with the parameter reduction approach and using a triangle in the parameter space results in a very simple controller structure: In each sampling instant, the resulting controller is calculated by interpolating between three controllers calculated for the vertices of the triangle. The whole approach is based on linear matrix inequalities (LMIs).

The remainder of this paper is organized as follows. In Sec. II, the controller and the overall closed-loop structure are presented. A method to obtain static output-feedback gains is reviewed and extended to pLPV systems in Sec. III. In Sec. IV, the disturbance model in pLPV form and the approximation used for the reduction to two parameters is discussed. The control design is explained in Sec. V and the controller is validated experimentally in Sec. VI. Possible extensions and future work are described in Sec. VII. Conclusions are given in Sec. VIII.

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II. Controller and Closed-Loop Structure

As mentioned above, for asymptotic disturbance rejection the controller has to include a model of the disturbance. One straightforward method to achieve this is to augment the plant with a disturbance model in form of an output filter. Then, an observer-based full-order state-feedback controller with additional dynamics prescribed through the output filter can be designed. For the rejection of multisine disturbances with time-varying frequencies, the method can be used with time-varying output filter dynamics. This approach is used e.g. in [9, 7]. However, the resulting controller is of high order (order of the plant plus order of the output filter).

If the focus is on disturbance rejection only, a controller that just contains the dynamics of the disturbance in the output filter would suffice, which is the design approach pursued in this paper. This structure is shown in Fig. 1. The plant G_p is described by the state-space model

$$\boldsymbol{x}_\mathrm{p}(k+1)=\boldsymbol{A}_\mathrm{p}\boldsymbol{x}_\mathrm{p}(k)+\boldsymbol{B}_\mathrm{p}u(k)+\boldsymbol{B}_\mathrm{p}y_\mathrm{d}(k), \tag{1}$$

$$y_\mathrm{p}(k)=\boldsymbol{C}_\mathrm{p}\boldsymbol{x}_\mathrm{p}(k), \tag{2}$$

where $\boldsymbol{x}_\mathrm{p}$ is the plant state vector, y_p is the plant output, u is the plant input and y_d is a disturbance that enters at the plant input. The dynamics of the output filter are given as

$$\boldsymbol{x}_\mathrm{M}(k+1)=\boldsymbol{A}_\mathrm{M}\boldsymbol{x}_\mathrm{M}(k)+\boldsymbol{B}_\mathrm{M}y_\mathrm{p}(k) \tag{3}$$

with state vector $\boldsymbol{x}_\mathrm{M}$. The resulting dynamic output-feedback controller is then given by the output filter and a controller gain $\boldsymbol{K}$ as

$$\boldsymbol{x}_\mathrm{M}(k+1)=\boldsymbol{A}_\mathrm{M}\boldsymbol{x}_\mathrm{M}(k)+\boldsymbol{B}_\mathrm{M}y_\mathrm{p}(k), \tag{4}$$

$$u_\mathrm{p}(k)=-\boldsymbol{K}\boldsymbol{x}_\mathrm{M}(k). \tag{5}$$

For the design of the output-feedback gain, the overall system consisting of plant and output filter, given by

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}_u u(k)+\boldsymbol{B}_w y_\mathrm{d}(k), \tag{6}$$

$$\boldsymbol{y}(k)=\boldsymbol{C}_y\boldsymbol{x}(k), \tag{7}$$

with

$$\boldsymbol{x}(k)=\begin{bmatrix}\boldsymbol{x}_\mathrm{p}(k)\\ \boldsymbol{x}_\mathrm{M}(k)\end{bmatrix},\ \boldsymbol{y}(k)=\boldsymbol{x}_\mathrm{M}(k), \tag{8}$$

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{A}_\mathrm{p} & \boldsymbol{0}\\ \boldsymbol{B}_\mathrm{M}\boldsymbol{C}_\mathrm{p} & \boldsymbol{A}_\mathrm{M}\end{bmatrix},\ \boldsymbol{B}_u=\boldsymbol{B}_w=\begin{bmatrix}\boldsymbol{B}_\mathrm{p}\\ \boldsymbol{0}\end{bmatrix},\ \boldsymbol{C}_y=\begin{bmatrix}\boldsymbol{0} & \mathbf{I}\end{bmatrix}, \tag{9}$$

is considered. The design problem is then to find a gain matrix $\boldsymbol{K}$ for output feedback of the form

$$u(k)=-\boldsymbol{K}\boldsymbol{y}(k), \tag{10}$$

that rejects the disturbance y_d. This is equivalent to partial state feedback of $\boldsymbol{x}_\mathrm{M}$ only. For the rejection of harmonic disturbances with time-varying frequencies, an output filter with time-varying dynamics is used. This usually requires a time-varying feedback gain. Thus, in the controller structure shown in Fig. 1 and (10), the matrices $\boldsymbol{A}_\mathrm{M}$ and $\boldsymbol{K}$ become time-varying. One approach to compute the time-varying gain matrix is to use quadratic stability theory for pLPV systems, if the output filter can be expressed as a pLPV system. Then, the gain matrix $\boldsymbol{K}$ can be obtained from the interpolation of gain matrices for vertex systems.

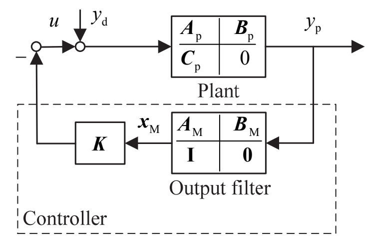

Figure 1. Closed-loop structure.

III. Gain-Scheduled H_∞ Suboptimal Output Feedback

As pointed out in Sec. II, the controller design problem no longer corresponds to full state feedback but rather to output feedback, which means that standard LMI techniques cannot be used since the problem is no longer convex. However, a design method for an H_∞ suboptimal static output-feedback controller has recently been proposed in [23]. In this section, this method is recapped and then extended to pLPV gain-scheduling controller design.

For a design approach based on H_∞ performance, a linear time-invariant (LTI) discrete-time system in its general form as in (6) and (7) is provided with an additional performance output $\boldsymbol{q}$, such that

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}_w w(k)+\boldsymbol{B}_u u(k), \tag{11}$$

$$\boldsymbol{q}(k)=\boldsymbol{C}_q\boldsymbol{x}(k)+\boldsymbol{D}_{qu}u(k), \tag{12}$$

$$\boldsymbol{y}(k)=\boldsymbol{C}_y\boldsymbol{x}(k), \tag{13}$$

while the input w is considered as performance input. The objective is to compute a static output-feedback gain $\boldsymbol{K}$, such that the closed-loop system

$$\boldsymbol{x}(k+1)=\boldsymbol{A}_\mathrm{cl}\boldsymbol{x}(k)+\boldsymbol{B}_w w(k), \tag{14}$$

$$\boldsymbol{q}(k)=\boldsymbol{C}_\mathrm{cl}\boldsymbol{x}(k), \tag{15}$$

$$\boldsymbol{y}(k)=\boldsymbol{C}_y\boldsymbol{x}(k) \tag{16}$$

with feedback control law as in (10) and

$$\boldsymbol{A}_\mathrm{cl}=\boldsymbol{A}-\boldsymbol{B}_u\boldsymbol{K}\boldsymbol{C}_y,\ \boldsymbol{C}_\mathrm{cl}=\boldsymbol{C}_q-\boldsymbol{D}_{qu}\boldsymbol{K}\boldsymbol{C}_y, \tag{17}$$

is stable and the H_∞-norm $\|T_{qw}(z)\|_\infty$ of the transfer function from $\boldsymbol{w}$ to $\boldsymbol{q}$, given by

$$T_{qw}(z)=\boldsymbol{C}_\mathrm{cl}(z\mathbf{I}-\boldsymbol{A}_\mathrm{cl})^{-1}\boldsymbol{B}_w, \tag{18}$$

is minimized. The design procedure is based on the Discrete-Time Bounded Real Lemma. Defining

$$\boldsymbol{Q}=\mathrm{null}(\boldsymbol{C}_y),\ \boldsymbol{R}=\boldsymbol{C}_y^\mathrm{T}(\boldsymbol{C}_y\boldsymbol{C}_y^\mathrm{T})^{-1}, \tag{19}$$

$$\boldsymbol{X}=\boldsymbol{Q}\boldsymbol{X}_Q\boldsymbol{Q}^\mathrm{T}+\boldsymbol{R}\boldsymbol{X}_R\boldsymbol{R}^\mathrm{T},\ \boldsymbol{Y}=\boldsymbol{Y}_R\boldsymbol{R}^\mathrm{T}, \tag{20}$$

the LMI $\mathfrak{M}(\boldsymbol{X}_Q,\boldsymbol{X}_R,\boldsymbol{Y}_R,\gamma)>\boldsymbol{0}$, with $\mathfrak{M}(\boldsymbol{X}_Q,\boldsymbol{X}_R,\boldsymbol{Y}_R,\gamma)$ as

$$\left[\begin{array}{c|c|c|c}\boldsymbol{Q}\boldsymbol{X}_Q\boldsymbol{Q}^\mathrm{T}+\boldsymbol{R}\boldsymbol{X}_R\boldsymbol{R}^\mathrm{T} & * & * & *\\ \boldsymbol{0} & \gamma\mathbf{I} & * & *\\ \boldsymbol{A}(\boldsymbol{Q}\boldsymbol{X}_Q\boldsymbol{Q}^\mathrm{T}+\boldsymbol{R}\boldsymbol{X}_R\boldsymbol{R}^\mathrm{T})-\boldsymbol{B}_u\boldsymbol{Y}_R\boldsymbol{R}^\mathrm{T} & \boldsymbol{B}_w & \boldsymbol{Q}\boldsymbol{X}_Q\boldsymbol{Q}^\mathrm{T}+\boldsymbol{R}\boldsymbol{X}_R\boldsymbol{R}^\mathrm{T} & *\\ \boldsymbol{C}_q(\boldsymbol{Q}\boldsymbol{X}_Q\boldsymbol{Q}^\mathrm{T}+\boldsymbol{R}\boldsymbol{X}_R\boldsymbol{R}^\mathrm{T})-\boldsymbol{D}_{qu}\boldsymbol{Y}_R\boldsymbol{R}^\mathrm{T} & \boldsymbol{0} & \boldsymbol{0} & \gamma\mathbf{I}\end{array}\right], \tag{21}$$

is solved for $\boldsymbol{X}_Q$, $\boldsymbol{X}_R$ and $\boldsymbol{Y}_R$ while minimizing the scalar

$\gamma > 0$. If solutions can be found, it holds that with

$$\boldsymbol{K} = \boldsymbol{Y}_R \boldsymbol{X}_R^{-1}, \tag{22}$$

the closed-loop system (14)-(16) is stable and $\|T_{qw}(z)\|_\infty < \gamma$. For a detailed derivation and proof, see Theorem 2 in [23]. There it is incorrectly stated that the associated H_∞-norm $\|T_{qw}(z)\|_\infty$ is equal to γ , which is actually not the case due to the conservativeness of the approach introduced by the prescribed structure of $\boldsymbol{X}$ in (20).

The controller obtained above can be extended directly to a pLPV gain-scheduling approach, if (11)-(13) is a pLPV system where only the system matrix $\boldsymbol{A}$ depends affinely on a parameter vector $\boldsymbol{\theta}(k) = \begin{bmatrix}\theta_1(k) & \cdots & \theta_N(k)\end{bmatrix}^{\mathrm{T}}$, that is

$$\boldsymbol{A}(\boldsymbol{\theta}(k)) = \boldsymbol{\mathcal{A}}_0 + \theta_1(k)\boldsymbol{\mathcal{A}}_1 + \ldots + \theta_N(k)\boldsymbol{\mathcal{A}}_N \tag{23}$$

with some constant matrices $\boldsymbol{\mathcal{A}}_i$, $i = 1,\ldots,N$. The parameter vector $\boldsymbol{\theta}(k)$ varies in a convex polytope $\boldsymbol{\Theta}$ with M vertices $\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_M$ in $\mathbb{R}^N$. A point $\boldsymbol{\theta}(k) \in \boldsymbol{\Theta}$ can then be written as a convex combination of vertices, i.e. there exist coordinates $\boldsymbol{\lambda}(k) = \begin{bmatrix}\lambda_1(k) & \cdots & \lambda_M(k)\end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^M$ with

$$\lambda_j(k) \geq 0\,,\ \sum_{j=1}^{M}\lambda_j(k) = 1 \text{ and } \boldsymbol{\theta}(k) = \sum_{j=1}^{M}\lambda_j(k)\,\boldsymbol{\theta}_j\,. \tag{24}$$

Then, M linear time-invariant vertex systems in the form of (11)-(13) can be defined with system matrices $\boldsymbol{A}_{\theta,j} = \boldsymbol{A}(\boldsymbol{\theta}_j)$ for $j = 1,\ldots,M$. It then holds that

$$\boldsymbol{A}(\boldsymbol{\theta}(k)) = \boldsymbol{A}(\boldsymbol{\lambda}(k)) = \lambda_1(k)\boldsymbol{A}_{\theta,1} + \ldots + \lambda_M(k)\boldsymbol{A}_{\theta,M}\,. \tag{25}$$

The following quadratic stability result can now be exploited for controller design. Assuming that the output-feedback gain $\boldsymbol{K}(k)$ has the structure

$$\boldsymbol{K}(k) = \sum_{j=1}^{M}\lambda_j(k)\boldsymbol{K}_{\theta,j}, \tag{26}$$

the closed-loop system (14)-(16) becomes a pLPV system with

$$\boldsymbol{A}_{\mathrm{cl}}(\boldsymbol{\lambda}(k)) = \sum_{j=1}^{M}\lambda_j(k)(\boldsymbol{A}_{\theta,j} - \boldsymbol{B}_u\boldsymbol{K}_{\theta,j}\boldsymbol{C}_y), \tag{27}$$

$$\boldsymbol{C}_{\mathrm{cl}}(\boldsymbol{\lambda}(k)) = \sum_{j=1}^{M}\lambda_j(k)(\boldsymbol{C}_q - \boldsymbol{D}_{qu}\boldsymbol{K}_{\theta,j}\boldsymbol{C}_y). \tag{28}$$

The closed-loop pLPV system then is *quadratically stable* if and only if there exists a matrix $\boldsymbol{X} = \boldsymbol{X}^{\mathrm{T}} > \boldsymbol{0}$ such that

$$\begin{bmatrix} \boldsymbol{X} & * \\ (\boldsymbol{A}_{\theta,j} - \boldsymbol{B}_u\boldsymbol{K}_{\theta,j}\boldsymbol{C}_y)^{\mathrm{T}}\boldsymbol{X} & \boldsymbol{X} \end{bmatrix} > \boldsymbol{0} \tag{29}$$

for all $j = 1,\ldots,M$ [24].

It can be shown that quadratic stability of (14)-(16) is implied by the existence of solutions to the LMI given in (21). This leads to a design method for a reduced-order pLPV gain-scheduling controller with bounded induced $\mathfrak{L}_2$-gain. First, the set of M LMIs $\mathfrak{M}(\boldsymbol{X}_Q, \boldsymbol{X}_R, \boldsymbol{Y}_{R,\theta,j}, \gamma) > \boldsymbol{0}$, $j = 1,\ldots,M$, is solved for the $M+2$ matrix variables $\boldsymbol{X}_Q$, $\boldsymbol{X}_R$, and $\boldsymbol{Y}_{R,\theta,j}$, $j = 1,\ldots,M$. If solutions can be found, the controller gains $\boldsymbol{K}_{\theta,j}$ are determined for every vertex of the parameter polytope via

$$\boldsymbol{K}_{\theta,j} = \boldsymbol{Y}_{R,\theta,j}\boldsymbol{X}_R^{-1}. \tag{30}$$

The current controller gain at instant k is then determined online via interpolation through (26). Due to the results presented before, the closed-loop system is quadratically stable even for arbitrarily fast changes of the parameters [24]. Additionally, the induced $\mathfrak{L}_2$-gain from input signal $w(k)$ to performance output $\boldsymbol{q}(k)$ of the pLPV system considered is bounded by γ .

IV. Disturbance Modeling

For the design of a controller with the structure described in Sec. II based on the method of Sec. III, a pLPV model for the multisine disturbance with time-varying frequencies is required. A single harmonic disturbance

$$y_{\mathrm{d},i}(k) = \alpha\sin\varphi_i(k) + \beta\cos\varphi_i(k) \tag{31}$$

can be modeled as the output of the state-space system

$$\boldsymbol{x}_{\mathrm{d},i}(k+1) = \boldsymbol{A}_{\mathrm{d},i}(k)\boldsymbol{x}_{\mathrm{d},i}(k), \tag{32}$$

$$y_{\mathrm{d},i}(k) = \boldsymbol{C}_{\mathrm{d},i}\boldsymbol{x}_{\mathrm{d},i}(k), \tag{33}$$

with

$$\boldsymbol{A}_{\mathrm{d},i}(k) = \begin{bmatrix} \cos\Omega_i(k) & \sin\Omega_i(k) \\ -\sin\Omega_i(k) & \cos\Omega_i(k) \end{bmatrix},\ \boldsymbol{C}_{\mathrm{d},i} = \begin{bmatrix}1 & 1\end{bmatrix}, \tag{34}$$

where the (instantaneous) normalized discrete-time phase variation $\Omega_i(k)$ is given through

$$\varphi_i(k+1) = \varphi_i(k) + \Omega_i(k) = \varphi_i(k) + 2\pi f_i(k)T_{\mathrm{s}}, \tag{35}$$

where $f_i(k)$ is the frequency and T_{s} is the sampling time. The amplitudes α and β depend on the initial conditions.

Using the frequency $\Omega_i(k)$ as the time-varying parameter would not result in a pLPV or LPV-LFT system. A simple solution would be to use $\cos\Omega_i(k)$ and $\sin\Omega_i(k)$ as independent time-varying parameters. This, however, would result in a conservative approach, as the parameters are actually not independent. Also, two time-varying parameters would be used for one single frequency. To only use one parameter per frequency, the state-space model with

$$\boldsymbol{A}_{\mathrm{d},i}(k) = \begin{bmatrix} 0 & 1 \\ -r^2 & 2r\cos\Omega_i(k) \end{bmatrix},\boldsymbol{C}_{\mathrm{d},i} = \begin{bmatrix}1 & 0\end{bmatrix}, \tag{36}$$

has been used in [1, 2, 6, 7, 9, 13-19]. Here, $r < 1$ is a real number close to 1 which is included for numerical reasons. As pointed out in [6], this is not a correct representation of the signal $y_{\mathrm{d},i}$ in (31), as the model does not account for frequency variations. Nevertheless, the controller based on this disturbance model achieved excellent disturbance rejection even for very fast frequency variations.

A multisine disturbance with N_{h} frequency components can then be described by the overall disturbance model

$$\boldsymbol{x}_{\mathrm{d}}(k+1) = \boldsymbol{A}_{\mathrm{d}}(k)\boldsymbol{x}_{\mathrm{d}}(k), \tag{37}$$

$$y_{\mathrm{d}}(k) = \boldsymbol{C}_{\mathrm{d}}\boldsymbol{x}_{\mathrm{d}}(k), \tag{38}$$

$$\boldsymbol{A}_{\mathrm{d}}(k) = \begin{bmatrix} \boldsymbol{A}_{\mathrm{d},1}(k) & & \\ & \ddots & \\ & & \boldsymbol{A}_{\mathrm{d},N_{\mathrm{h}}}(k) \end{bmatrix},\boldsymbol{C}_{\mathrm{d}} = \begin{bmatrix}\boldsymbol{C}_{\mathrm{d},1} & \cdots & \boldsymbol{C}_{\mathrm{d},N_{\mathrm{h}}}\end{bmatrix}, \tag{39}$$

and matrices $\boldsymbol{A}_{\mathrm{d},i}$ and $\boldsymbol{C}_{\mathrm{d},i}$, $i=1,\ldots,N_{\mathrm{h}}$, given by (36). In this disturbance model, one parameter is used per frequency.

In most pLPV approaches, a hyper-rectangle is used as a polytope to cover all possible parameter combinations in the given parameter ranges. For the problem considered here, this means that all frequencies are allowed to vary independently of each other. The same holds for the LPV-LFT approaches based on the above disturbance model. In many applications, though, the individual frequency components are harmonically related. Then, using one independent parameter per frequency makes the approach very conservative, because frequency combinations are included in the model that do not occur in reality. Also, the computational effort increases with the number of time-varying parameters. For example, in the pLPV-approaches [6, 7, 9, 16], the actual controller in each sampling step is obtained by interpolating between $2^{N_{\mathrm{h}}}$ vertex controllers.

A useful idea has been proposed in [17, 18], where the cosine function $\cos\Omega$ is approximated as

$$\cos\Omega \approx a_0 + a_1\Omega^2 + a_2\Omega^4 + \ldots + a_{N_{\mathrm{p}}}\Omega^{2N_{\mathrm{p}}}. \tag{40}$$

For harmonically related frequencies $f, h_1 f, \ldots, h_{N_{\mathrm{h}}-1} f$, individual approximations

$$\cos h_i\,\Omega \approx a_{i,0} + a_{i,1}\Omega^2 + a_{i,2}\Omega^4 + \ldots + a_{i,N_{\mathrm{p}}}\Omega^{2N_{\mathrm{p}}} \tag{41}$$

for $i=1,\ldots,N_{\mathrm{h}}-1$ are used. The coefficients $a_{i,0},\ldots,a_{i,N_{\mathrm{p}}}$ can be obtained through a least squares fit within the range of the frequency variation. Usually, an excellent approximation is obtained with only three coefficients per frequency. Then, the parameters

$$\theta_1(k) = \Omega^2(k) = (2\pi f(k) T_{\mathrm{s}})^2, \tag{42}$$

$$\theta_2(k) = \Omega^4(k) = (2\pi f(k) T_{\mathrm{s}})^4 \tag{43}$$

can be used as time-varying parameters for the pLPV model. With this approach, only two parameters are used, independently of the number of harmonically related frequency components N_{h}.

In [17, 18], a pLPV approach was considered for controller design and a rectangle was used as the polytope. The resulting controller at each sampling instant is then an interpolation of four controllers. However, using a rectangle as a polytope does not utilize the relationship $\theta_2(k)=\theta_1^2(k)$ that describes all admissible parameter combinations. It can be used to enclose all possible parameter combinations by a triangle rather than a rectangle. It is easy to show that for $\theta_{1,\min} \le \theta_1(k) \le \theta_{1,\max}$ the curve in $\mathbb{R}^2$ described by $\theta_2(k)=\theta_1^2(k)$ is enclosed by the triangle $\boldsymbol{\Theta}\subset\mathbb{R}^2$ with the three corners

$$\boldsymbol{\theta}_1 = \begin{bmatrix}\theta_{1,\min} & \theta_{1,\min}^2\end{bmatrix}^{\mathrm{T}}, \boldsymbol{\theta}_2 = \begin{bmatrix}\dfrac{\theta_{1,\max}+\theta_{1,\min}}{2} & \theta_{1,\max}\theta_{1,\min}\end{bmatrix}^{\mathrm{T}}, \tag{44}$$
$$\boldsymbol{\theta}_3 = \begin{bmatrix}\theta_{1,\max} & \theta_{1,\max}^2\end{bmatrix}^{\mathrm{T}}.$$

To conclude, for a multisine disturbance with N_{h} frequency components, a disturbance model given by (37)-(39) is used with $\boldsymbol{C}_{\mathrm{d},i}$ as given in (36) and

$$\boldsymbol{A}_{\mathrm{d},i}(k) = \begin{bmatrix}0 & 1\\ -1 & 2a_{i,0}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 2a_{i,1}\end{bmatrix}\theta_1(k) + \begin{bmatrix}0 & 0\\ 0 & 2a_{i,2}\end{bmatrix}\theta_2(k), \tag{45}$$

where $\theta_1(k)$ and $\theta_2(k)$ are given by (42) and (43), respectively, and the coefficients $a_{i,0}$, $a_{i,1}$ and $a_{i,2}$ are obtained from the approximation given in (41). For a fundamental frequency varying between $f_{\min}$ and $f_{\max}$, the parameter bounds

$$\theta_{1,\min} = (2\pi f_{\min} T_{\mathrm{s}})^2,\ \theta_{1,\max} = (2\pi f_{\max} T_{\mathrm{s}})^2 \tag{46}$$

are calculated. The disturbance model is then a pLPV model with the polytope given as the triangle specified by the three corners given in (44).

V. Controller Design

The pLPV disturbance model described in the previous section is now used in the output filter for the design of the controller shown in Fig. 1. Thus

$$\boldsymbol{A}_{\mathrm{M}}(k) = \boldsymbol{A}_{\mathrm{M}}(\boldsymbol{\theta}(k)) = \boldsymbol{A}_{\mathrm{d}}(\boldsymbol{\theta}(k)). \tag{47}$$

The input matrix of the output filter is chosen as

$$\boldsymbol{B}_{\mathrm{M}} = \begin{bmatrix}0 & 1 & 0 & 1 & \cdots & 0 & 1\end{bmatrix}^{\mathrm{T}}. \tag{48}$$

As mentioned above, the output filter

$$\boldsymbol{x}_{\mathrm{M}}(k+1) = \boldsymbol{A}_{\mathrm{M}}(\boldsymbol{\theta}(k))\boldsymbol{x}_{\mathrm{M}}(k) + \boldsymbol{B}_{\mathrm{M}} y_{\mathrm{p}}(k) \tag{49}$$

is now in pLPV form. Building the overall model as given in (6)-(9), it follows that also the overall model is a pLPV system and therefore the design and gain-scheduling method described in Sec. III can be applied. In order to achieve disturbance rejection through the H_∞-performance design, the artificial performance input has to be the unknown disturbance input, that is

$$w(k) = y_{\mathrm{d}}(k),\ \boldsymbol{B}_w = \boldsymbol{B}_u, \tag{50}$$

and the performance output $\boldsymbol{q}(k)$ is defined as in (12) with

$$\boldsymbol{C}_q = \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ W_y \boldsymbol{C}_{\mathrm{p}} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{W}_{x_{\mathrm{M}}}\end{bmatrix},\ \boldsymbol{D}_{qu} = \begin{bmatrix}W_u\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}, \tag{51}$$

with constant weights W_y, W_u and $\boldsymbol{W}_{x_{\mathrm{M}}}$ for the control signal, the plant output and the states of the error filter, respectively, as shown in Fig. 2. It is crucial to include the states of the output filter, since in open loop the transfer function from $w(k)$ to $\boldsymbol{W}_{x_{\mathrm{M}}}\boldsymbol{x}_{\mathrm{M}}(k)$ then has high gain at the specified frequencies.

Following the pLPV design approach presented in Sec. III, for each vertex of the triangle $\boldsymbol{\Theta}$, the system matrix is determined by $\boldsymbol{A}_{\mathrm{M},j} = \boldsymbol{A}_{\mathrm{M}}(\boldsymbol{\theta}_j)$ via

$$\boldsymbol{A}(\boldsymbol{\theta}_j) = \begin{bmatrix}\boldsymbol{A}_{\mathrm{p}} & \mathbf{0}\\ \boldsymbol{B}_{\mathrm{M}}\boldsymbol{C}_{\mathrm{p}} & \boldsymbol{A}_{\mathrm{M},j}\end{bmatrix},\ j=1,2,3. \tag{52}$$

The design method gives a controller gain $\boldsymbol{K}_j$ for every vertex. The resulting controller including output filter and gain matrix is then given by

$$\boldsymbol{x}_{\mathrm{M}}(k+1) = \boldsymbol{A}_{\mathrm{M}}(k)\boldsymbol{x}_{\mathrm{M}}(k) + \boldsymbol{B}_{\mathrm{M}} y_{\mathrm{p}}(k), \tag{53}$$

$$u(k) = -\boldsymbol{K}(k)\boldsymbol{x}_{\mathrm{M}}(k) \tag{54}$$

with

$$\boldsymbol{A}_{\mathrm{M}}(k) = \lambda_1(k)\boldsymbol{A}_{\mathrm{M},1} + \lambda_2(k)\boldsymbol{A}_{\mathrm{M},2} + \lambda_3(k)\boldsymbol{A}_{\mathrm{M},3}, \tag{55}$$

$$\boldsymbol{K}(k) = \lambda_1(k)\boldsymbol{K}_1 + \lambda_2(k)\boldsymbol{K}_2 + \lambda_3(k)\boldsymbol{K}_3. \tag{56}$$

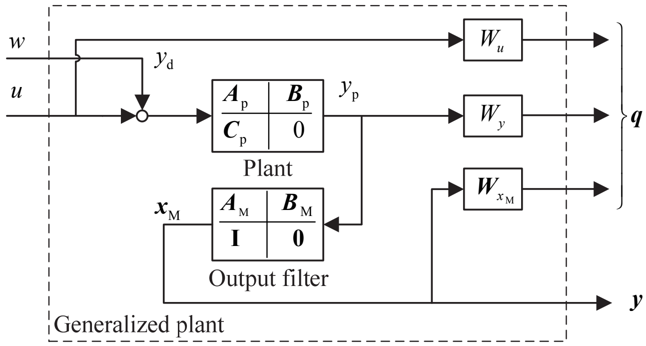

Figure 2. Control design setup.

Since the convex polytope $\boldsymbol{\Theta}$ is, in this case, a triangle and therefore a simplex in $\mathbb{R}^2$, for every $\boldsymbol{\theta}(k) \in \boldsymbol{\Theta}$ there exists only one coordinate vector $\boldsymbol{\lambda}(k)$ fulfilling (24), which is given by

$$\begin{bmatrix} \lambda_1(k) \\ \lambda_2(k) \\ \lambda_3(k) \end{bmatrix} = \begin{bmatrix} \theta_{1,\min} & \left(\theta_{1,\max} + \theta_{1,\min}\right)/2 & \theta_{1,\max} \\ \theta_{1,\min}^2 & \theta_{1,\max}\theta_{1,\min} & \theta_{1,\max}^2 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \\ 1 \end{bmatrix}. \tag{57}$$

For implementation, this can be written as

$$\boldsymbol{\lambda}(k) = \boldsymbol{M}\begin{bmatrix} \theta_1(k) \\ \theta_2(k) \end{bmatrix} + \boldsymbol{b}, \tag{58}$$

where $\boldsymbol{M}$ consists of the first and second column of the inverse matrix in (57), while $\boldsymbol{b}$ is the third column.

The matrix inverse in (57) is constant and can therefore be computed in advance. Hence, the online computation of the coordinate vector is done efficiently by a very simple matrix-vector multiplication. The resulting gain-scheduled controller structure is shown in Fig. 3.

If this approach works, it gives a time-varying output-feedback matrix $\boldsymbol{K}(k)$ and a constant Lyapunov matrix $\boldsymbol{X}$. In [19], this approach has been used successfully for the design of a reduced-order controller for the rejection of a disturbance consisting of three sinusoidal components for the same experimental setup as in this paper. There, the disturbance frequencies were not harmonically related and therefore, the approach for parameter reduction described in Sec. IV was not used. Instead, $\cos(2\pi f(k)T_{\mathrm{s}})$ has been used as time-varying parameter of the disturbance model and a hyper-rectangle with $2^3 = 8$ vertices was used for the controller design. In the application described in the present paper, however, the static output-feedback approach of [23] was found to be very conservative even for a fixed fundamental frequency in the sense that the resulting feedback matrix was stabilizing, but did not achieve good performance (in fact, it was commonly very small). For the pLPV design, no solution was found at all. To overcome the conservativeness of the approach, a simple, heuristic procedure is applied.

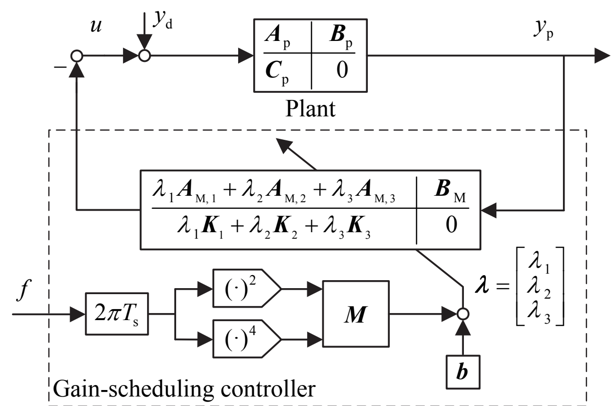

Figure 3. Closed-loop system with gain-scheduling controller structure.

First, one controller $\boldsymbol{K}_{j,0}$ is designed for each of the corners of the triangle $\boldsymbol{\Theta}$, but independently from one another, that is, with individual Lyapunov matrices $\boldsymbol{X}_{j,0}$, $j = 1,2,3$, using the static output-feedback approach of [23]. Then, the obtained gain matrices are used as initial values for a subsequent iteration, where the following calculation is performed independently for every $j = 1,2,3$:

- FOR $k = 1 : n_{\mathrm{iter}}$
  - Solve $\mathfrak{N}\left(\boldsymbol{X}_{j,k}, \boldsymbol{K}_{j,k-1}, \gamma\right) > \boldsymbol{0}$ for $\boldsymbol{X}_{j,k} = \boldsymbol{X}_{j,k}^{\mathrm{T}} > \boldsymbol{0}$, where

$$\mathfrak{N}\left(\boldsymbol{X}_{j,k}, \boldsymbol{K}_{j,k-1}, \gamma\right) = \begin{bmatrix} \boldsymbol{X}_{j,k} & * & * & * \\ \boldsymbol{0} & \gamma\mathbf{I} & * & * \\ \mathbf{A}(\boldsymbol{\theta}_j)\boldsymbol{X}_{j,k} - \boldsymbol{B}_u\boldsymbol{K}_{j,k-1}\boldsymbol{C}_y\boldsymbol{X}_{j,k} & \boldsymbol{B}_w & \boldsymbol{X}_{j,k} & * \\ \boldsymbol{C}_q\boldsymbol{X}_{j,k} - \boldsymbol{D}_{qu}\boldsymbol{K}_{j,k-1}\boldsymbol{C}_y\boldsymbol{X}_{j,k} & \boldsymbol{0} & \boldsymbol{0} & \gamma\mathbf{I} \end{bmatrix}.$$

  - Solve $\mathfrak{N}\left(\boldsymbol{X}_{j,k}, \boldsymbol{K}_{j,k}, \gamma\right) > \boldsymbol{0}$ for $\boldsymbol{K}_{j,k}$.
- END FOR
- Set $\boldsymbol{K}_j = \boldsymbol{K}_{j,n_{\mathrm{iter}}}$, $\boldsymbol{X}_j = \boldsymbol{X}_{j,n_{\mathrm{iter}}}$.

This approach converged to a final solution that resulted in a very good performance for the fixed fundamental frequencies at the corners of the parameter polytope. Next, bounds for the admissible parameter variation such that quadratic stability of the closed-loop system with the dynamic controller given by (53)-(56) with $\boldsymbol{K}_j$, $j = 1,2,3$, are found by a systematic procedure.

The parameter space is divided into overlapping sub-triangles for which common quadratic Lyapunov functions can be found. If no solution is found, the parameter space is divided into smaller sub-triangles until one individual Lyapunov matrix can be found for each region. This determines a maximum admissible parameter variation for one sampling step, since in this way stability can only be guaranteed as long as the parameter stays inside one triangle during one sampling step. In detail, the procedure is carried out as follows.

First, the frequency range $[f_{\min}, f_{\max}]$ is divided into N_Δ intervals $[f_{\min,i}, f_{\max,i}], i = 1,\ldots,N_\Delta$, with

$$f_{\min,i} = f_{\min} + (i-1)\Delta f,\ f_{\max,i} = f_{\min} + (i+1)\Delta f, \tag{59}$$

$$\Delta f = \left(f_{\max} - f_{\min}\right)/\left(N_\Delta + 1\right). \tag{60}$$

Since $\theta_1 = (2\pi f T_{\mathrm{s}})^2$, this results in N_Δ intervals for the range of θ_1 with

$$[\theta_{1,\min,i}, \theta_{1,\max,i}] = [(2\pi f_{\min,i} T_s)^2, (2\pi f_{\max,i} T_s)^2]. \quad (61)$$

For each of these intervals, a triangle is defined with the corners as in (44), with $\theta_{1,\min}$ and $\theta_{1,\max}$ replaced by $\theta_{1,\min,i}$ and $\theta_{1,\max,i}$, respectively. Controllers $\bar{\boldsymbol{K}}_{j,i}$ are calculated for each corner by interpolating the three originally obtained controllers. With these controllers, one common Lyapunov matrix $\bar{\boldsymbol{X}}_i$ is obtained for the vertices of the new, smaller triangles by solving $\mathfrak{P}\left(\bar{\boldsymbol{K}}_{j,i}\right) > \mathbf{0}$, $j = 1,2,3$, where

$$\mathfrak{P}\left(\bar{\boldsymbol{K}}_{j,i}\right) = \begin{bmatrix} \bar{\boldsymbol{X}}_i & * \\ (\boldsymbol{A}(\boldsymbol{\theta}_j) - \boldsymbol{B}_u \bar{\boldsymbol{K}}_{j,i} \boldsymbol{C}_y)^{\mathrm{T}} \bar{\boldsymbol{X}}_i & \bar{\boldsymbol{X}}_i \end{bmatrix}. \quad (62)$$

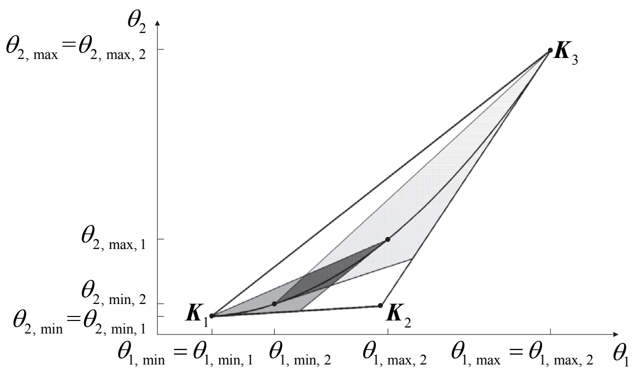

Figure 4. Division of the original triangle into overlapping sub-triangles.

If one Lyapunov matrix is found for every sub-triangle, quadratic stability is guaranteed for every parameter variation inside the sub-triangle. As a result, the closed-loop stability is guaranteed as long as the time-varying frequency does not change with a rate larger than $\Delta f / T_s$. If not, more intervals can be used, which means that the tolerable frequency variation Δf between two sampling instances must be reduced. This is a somewhat heuristic (but still systematic) approach characterized by first designing the controllers and then showing stability.

VI. Experimental Results

The controller obtained in the previous section is validated using an AVC test bed. The experimental setup is shown schematically in Fig. 5. Two shakers (inertia mass actuators) are attached to a steel cantilever beam. One shaker generates the multisine disturbance signal and the other shaker is driven by the control signal to counteract this disturbance. An accelerometer is used to measure the output signal. An anti-aliasing filter is applied to the output signal and a reconstruction filter to the control input.

Standard black-box system identification techniques were used to obtain the transfer function between output and input of the control unit. The identified transfer function is of 10th order (see the appendix). The disturbance signal is a sum of five harmonically related sine signals with a fundamental frequency between 100 Hz and 110 Hz. The five frequency components of the multisine disturbance signal are 1, 1.5, 2, 2.5 and 3 times the fundamental frequency.

The controller designed for the rejection of this disturbance is implemented on a rapid control prototyping unit (dSpace MicroAutoBox) using a sampling frequency of 1 kHz. The controller is of 10th order and a maximum frequency variation $\Delta f = 0.2$ Hz is obtained. Complete plant and controller data are given in the appendix.

Amplitude frequency responses of the controller and the open and closed loop disturbance transfer functions are shown in Figs. 6 and 7, respectively. As intended by the design, the controller has high gains at the disturbance frequencies.

Real-time results for the frequency variations in Fig. 8 are shown in Fig. 9. A very good disturbance rejection is achieved with the minimal order controller even for very fast changes. The last variation is a step change in the frequency, which is unrealistic in real applications.

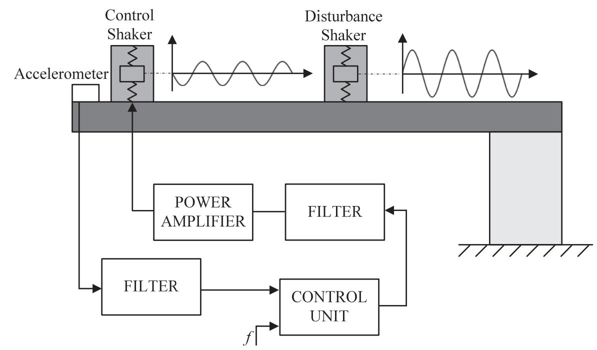

Figure 5. Schematic representation of the AVC system.

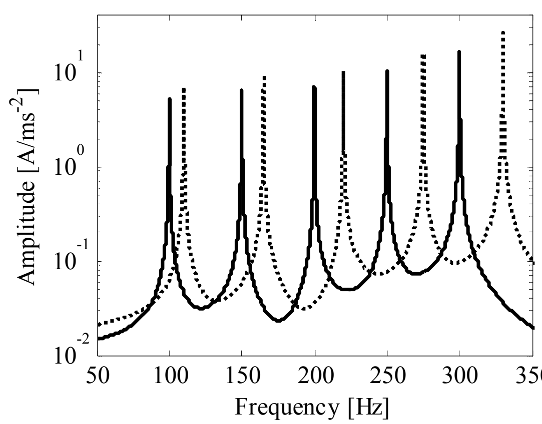

Figure 6. Amplitude frequency response of the controller for fixed disturbance frequencies of 100, 150, 200, 250 and 300 Hz (solid line) and 110, 165, 220, 275 and 330 Hz (dotted line).

VII. Extensions and Future Work

The approach presented in this paper can be extended in various directions. First, it is possible to use the exact disturbance model (34) instead of the simplified one by approximating also the sine function as

$$\sin h_i \Omega \approx b_{i,0} + b_{i,1}\Omega^2 + b_{i,2}\Omega^4. \quad (63)$$

In this way, it can be used without increasing the dimension of the controller or the number of pLPV parameters.

Second, the approach described here is a heuristic (although still systematic) one in the sense that controllers are first designed and then checked for stability. However, it can as well be uses as a design approach that guarantees stability. In that case, the triangle $\boldsymbol{\Theta}$ is first divided into overlapping sub-triangles. Then, the objective is to find gain matrices $\boldsymbol{K}_1, \boldsymbol{K}_2, \boldsymbol{K}_3$, and Lyapunov matrices $\boldsymbol{X}_i$, $i = 1,\ldots,N_\Delta$, through simultaneous solution of the appropriate LMIs for each sub-triangle, where the gain matrices for vertices of the sub-triangle are expressed as convex combinations of $\boldsymbol{K}_1, \boldsymbol{K}_2, \boldsymbol{K}_3$. However, this leads to a large number of LMIs in many variables, that have to be solved simultaneously.

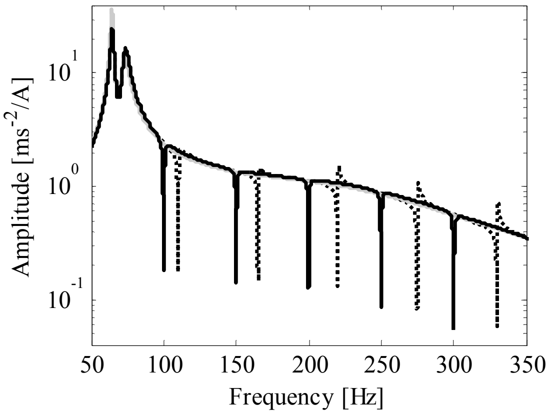

Figure 7. Amplitude frequency responses in open loop (gray) and closed loop (black) for fixed disturbance frequencies of 100, 150, 200, 250 and 300 Hz (solid line) and 110, 165, 220, 275 and 330 Hz (dotted line).

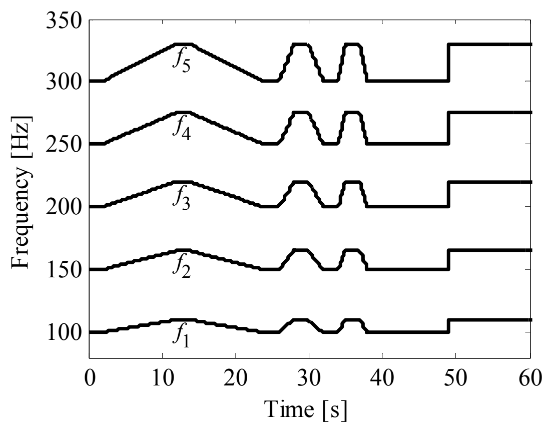

Figure 8. Variations of the disturbance frequencies.

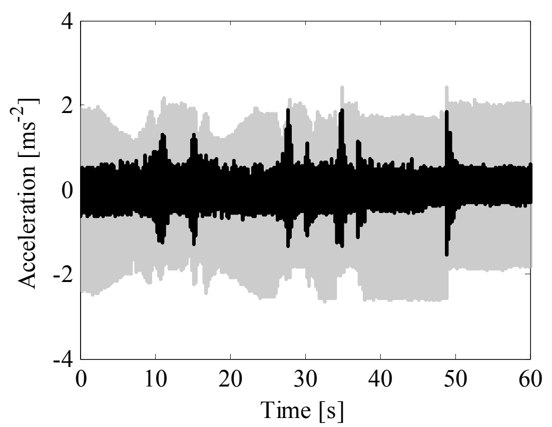

Figure 9. Measured acceleration (bottom) in open loop (gray) and closed loop (black).

VIII. Summary and Conclusions

A design method for a reduced-order two-parameter pLPV controller for the rejection of harmonically related multisine disturbances is presented. The control design results in a discrete-time controller, which has to be interpolated at each sampling time between three controllers. Experimental results show very good disturbance rejection even for very fast changes of five disturbance frequencies.

The proposed controller has several advantages compared to other methods. First of all, the resulting controller structure is very simple. Then, the order of the controller is only twice the number of frequency components of the multisine disturbance. In [16], for example, a controller of order 21 is used for the rejection of four disturbance frequencies. Furthermore, the time-varying controller matrices are obtained by interpolation between only three vertices (via simple matrix-vector multiplication), while other approaches require an interpolation between 2^{N_h} vertex controllers in every sampling instant [6, 7, 9, 16].

Appendix

Complete data for the closed-loop system shown in Fig. 3 are given. The transfer function of the plant is

$$G_\mathrm{p}(z) = k_\mathrm{p}\,\frac{(z - z_1)\ldots(z - z_{n_z})}{(z - p_1)\ldots(z - p_{n_\mathrm{p}})}$$

with gain factor $k_\mathrm{p} = -0.0038$, zeros

$$z_1 = -0.9567,\; z_{2,3} = 4.2889\, e^{\pm j0.9926},\; z_{4,5} = 2.6975\, e^{\pm j2.5827},$$
$$z_{6,7} = 1.0949\, e^{\pm j0.0663},\; z_{8,9} = 0.9847\, e^{\pm j0.4304},$$

and poles

$$p_1 = -0.4551,\; p_2 = 0.7592,$$
$$p_{3,4} = 0.2972\, e^{\pm j2.2389},\; p_{5,6} = 0.5586\, e^{\pm j1.3923},$$
$$p_{7,8} = 0.9954\, e^{\pm j0.4042},\; p_{9,10} = 0.9855\, e^{\pm j0.4587}.$$

The controller matrices are

$$\boldsymbol{A}_{\mathrm{M},j} = \mathrm{blockdiag}(\boldsymbol{A}_{\mathrm{M}_1,j}, \boldsymbol{A}_{\mathrm{M}_2,j}, \boldsymbol{A}_{\mathrm{M}_3,j}, \boldsymbol{A}_{\mathrm{M}_4,j}, \boldsymbol{A}_{\mathrm{M}_5,j}),$$
$$\boldsymbol{A}_{\mathrm{M}_i,j} = \begin{bmatrix} 0 & 1 \\ -0.999 & a_{\mathrm{M}_i,j} \end{bmatrix}$$

for $j = 1, 2, 3,\; i = 1,\ldots,5$, with

$$a_{\mathrm{M}_1,1} = 1.6172,\; a_{\mathrm{M}_2,1} = 1.1750,\; a_{\mathrm{M}_3,1} = 0.6177,$$
$$a_{\mathrm{M}_4,1} = 0,\;\; a_{\mathrm{M}_5,1} = -0.6178,$$
$$a_{\mathrm{M}_1,2} = 1.5785,\;\; a_{\mathrm{M}_2,2} = 1.095,\;\; a_{\mathrm{M}_3,2} = 0.4923,$$
$$a_{\mathrm{M}_1,3} = 1.5403,\;\; a_{\mathrm{M}_2,3} = 1.0176,\;\; a_{\mathrm{M}_3,3} = 0.3746,$$
$$a_{\mathrm{M}_4,3} = -0.3127,\;\; a_{\mathrm{M}_5,3} = -0.963,$$
$$a_{\mathrm{M}_4,2} = -0.1648,\;\; a_{\mathrm{M}_5,2} = -0.8056,$$

$$\boldsymbol{B}_\mathrm{M} = \begin{bmatrix} 0 & 1 & 0 & 1 & \cdots & 0 & 1 \end{bmatrix}^\mathrm{T},\text{ and}$$

$$\boldsymbol{K}_1 = \begin{bmatrix} -0.0048 \\ 0.0037 \\ 0.0060 \\ -0.0052 \\ 0.0064 \\ -0.0012 \\ -0.0039 \\ 0.0087 \\ -0.0119 \\ -0.0128 \end{bmatrix}^{\mathrm{T}}, \ \boldsymbol{K}_2 = \begin{bmatrix} -0.0058 \\ 0.0037 \\ 0.0025 \\ -0.0074 \\ 0.0072 \\ 0.0021 \\ -0.0099 \\ 0.0069 \\ -0.0085 \\ -0.0210 \end{bmatrix}^{\mathrm{T}}, \ \boldsymbol{K}_3 = \begin{bmatrix} -0.0061 \\ 0.0030 \\ 0.0042 \\ -0.0087 \\ 0.0062 \\ 0.0059 \\ -0.0146 \\ 0.0008 \\ -0.0004 \\ -0.0218 \end{bmatrix}^{\mathrm{T}}.$$

The coordinate vector is calculated via

$$\boldsymbol{\lambda}(k) = \boldsymbol{M} \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \end{bmatrix} + \boldsymbol{b}$$

with

$$\boldsymbol{M} = \begin{bmatrix} -139.0007 & 145.4929 \\ 253.8773 & -290.9859 \\ -114.8766 & 145.4929 \end{bmatrix}, \ \boldsymbol{b} = \begin{bmatrix} 33.1995 \\ -54.8753 \\ 22.6757 \end{bmatrix}.$$

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