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Distributed Model Predictive Control for Energy Distribution

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Abstract— **This work presents a distributed model predictive control scheme for energy distribution. The energy is supposed to be supplied by a renewable power system whose energy production is limited and demanded by several consumers. Therefore, in some cases the produced energy cannot fulfill the energy requirements of the consumers. The proposed controller allows to distribute equitably the produced energy to these consumers without harming any of them. Moreover, simulated results based on a real case are presented in order to assess the proposed control strategy.**

I. INTRODUCTION

World energy consumption has become a major concern to scientific and political communities. According to recent studies, energy consumption in buildings represents a 40% rate of the worldwide energy, of which more than a half is used by HVAC (Heating, Ventilation and Air Conditioning) systems [1], [2], [3]. For this reason, one of the most promoted and supported measures from the administration side is the design of energy-efficient buildings [4], [5]. In line with this measure, energy efficient management of building systems will be in the near future a keystone in the aim to minimize the overall energy consumption and costs.

When the subject of HVAC control systems is treated, one must take into account the complexity of dynamics involved, sometimes making the operation in real-time a challenge. The MPC is a viable alternative in such cases because of its well-known advantages, such as the provision of optimal control, and a systematic way to add constraints to the system, among others [6], [7], [8].

However, the number of control variables and signals from sensors and actuators can grow rapidly depending on the complexity of HVAC systems controlled simultaneously. Thus, the use of this type of controller may also be compromised, mainly due to the lack of computational capacity.

An alternative in such cases may be Distributed Model Predictive Control (DMPC), which advocates the distribution of sensing and control while preserving the same features of standard MPC, using local controllers that operate jointly by exchanging information, in order to decide their control actions [9], [10], [11], [12].

This work deals with the development of a DMPC control scheme to manage the energy produced in a renewable energy source and distribute this energy among a set of HVAC systems. Differently from other works which deal with the thermal comfort control problem through the use of predictive controllers [1], [13], the main objective in this paper is to present a control scheme for scenarios where the produced energy is not sufficient to fulfill the energy requirements of the whole set of HVAC systems.

In order to evaluate the proposed strategy, the operation of an air-conditioning system in an efficient building was explored, where a solar plant is responsible for generating energy to a set of HVAC systems (corresponding to the rooms of the building). Simulated results based on a real case study are presented, showing the effectiveness of the proposed strategy to provide an equitable energy distribution.

The rest of the paper is organized as follows: The problem to be solved is described in Section 2. Some details and theory related to the DMPC controller are provided in Section 3. Section 4 is devoted to show some promising results of the proposed strategy. The paper ends in Section 5 with some conclusions and directions for a future research.

II. CASE STUDY

This section describes the main modeling issues of the process used to test the proposed control technique. The scenario consisted of a solar thermal system with the structure depicted in Figure 1. The system has two main parts, composed by a solar plant and a set of subsystems. These subsystems refer to the set of HVAC systems (corresponding to the rooms of the building).

This system is a simplification of the solar plant of the Centro de Investigación de Energía Solar (CIESOL) located at the University of Almería (Spain) [14], which has been adapted in this work to implement the proposed distributed strategy. This plant can produce either hot or chilled water to heat or cool the rooms of the building. In this work, only the chilled water production was considered.

The solar plant has a flat solar collector field, a hot water storage system which is used either to accumulate hot water when there is no cooling demand or as a buffer to smooth variations on the outlet temperature of the solar collector caused by disturbances, a gas heater and an absorption machine.

In the solar plant scheme there are two circuits, where the primary one includes the collector field, tanks, gas heater,

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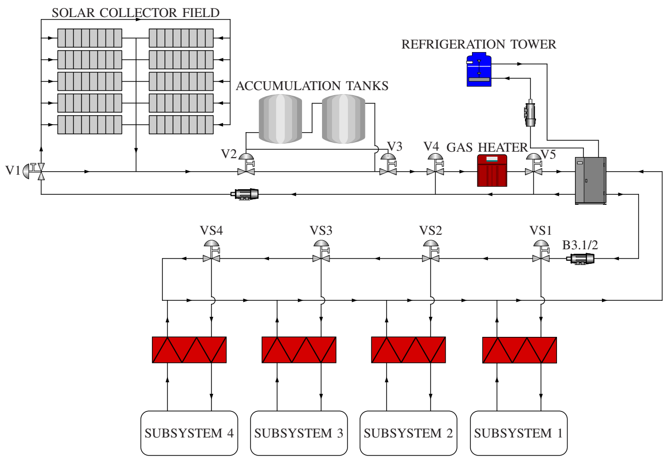

Fig. 1. Solar cooling/heating installation scheme

an absorption machine, a refrigeration tower, and an external component of the absorption machine; and the second one includes a valve for each subsystem of the network and a pump. Physical and operation details of this system can be found in [14], [15]. Some of the main features of this system are:

- B3.1/2 $\rightarrow$ is a pump that regulates the flow of fluid energy in the circuit.
- VS1-4 $\rightarrow$ are valves that divert part of the flow amount, by this way the energy input in each subsystem can be controlled.
- The cycle of the fluid energy transmitted is closed, that is, returns to the pump after passing through the subsystems.

In order to simulate the energy demand, each HVAC system will be modeled as a parallel-flow heat exchanger (see Figure 2), where the two fluids, in this case chilled water and air, enter the heat exchanger at the same end, and travel in parallel to one another to the other side. The heat-exchanger dynamic is given by the following model [16]:

$$\frac{dT_{wo}}{dt} = -\dot{q}_w \frac{T_{wo} - T_{wi}}{V_w} + \frac{1}{\tau_w}(T_{a\,imp} - T_{wo}) \quad (1a)$$

$$\frac{dT_{a\,imp}}{dt} = -v_{imp} \frac{T_{a\,imp} - T_{a\,ret}}{L} - \frac{1}{\tau_a}(T_{a\,imp} - T_{wo}) \quad (1b)$$

where in the previous equations, the subscripts w and a refer to the water and the air, respectively; whereas the subscripts i, o, imp and ret refer to input, output, impulse and return, respectively.

This model has two inputs used as manipulated variables, the water flow ($\dot{q}_w$) and the air speed (v_{imp}); two other inputs considered as disturbances, the input air temperature, *i.e.*, the return temperature (T_{ret}), and the inlet water temperature (T_{wi}); and two output variables, the output water temperature (T_{wo}) and the output air temperature, *i.e.* the impulse air temperature ($T_{a\,imp}$). The remaining variables are listed in Table I. In this work, the output air temperature is controlled by the water flow, keeping the air speed constant. This option became viable for the realization of distributed control.

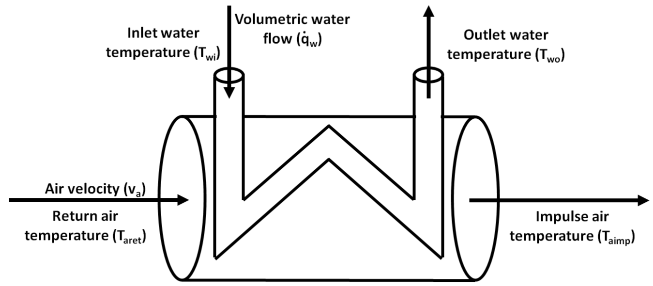

Fig. 2. Heat exchanger scheme adopted for each subsystem

TABLE I

MODEL VARIABLES AND PARAMETERS

Parameters	Description	SI units
A_a	cross-sectional area for air	m^2
A_w	cross-sectional area for water	m^2
Cp_a	air specific heat capacity	$J/kg°C$
Cp_w	water specific heat capacity	$J/kg°C$
d_a	diameter of the pipe for air	m
d_w	diameter of the pipe for water	m
h_a	air convective heat-transfer coefficient	$W/m^{2°}C$
h_w	water convective heat-transfer coefficient	$W/m^{2°}C$
L	heat exchanger length	m
$\dot{q}$	water volumetric flow	m^3/s
ρ_a	air density	kg/m^3
ρ_w	water density	kg/m^3
T_a	air temperature	$°C$
T_w	water wall temperature	$°C$
v_{imp}	impulse air velocity	m/s
V_w	water pipe volume	m^3
$\tau_a = \frac{A_a \rho_a Cp_a}{\pi d_a h_a}$	T_a-related fundamental time constant against T_w variations	s
$\tau_w = \frac{A_w \rho_w Cp_w}{\pi d_w h_w}$	T_w-related fundamental time constant against T_a variations	s

Given the model of the HVAC system for each room of the building, it is possible to build a model that represents the entire system. One possibility is through linear dynamic networks, which approximates the dynamic model of a distributed system by the interconnection of subsystems where the local control actions influence the dynamics of neighboring subsystems. Examples of systems that can be modeled as linear dynamic networks are urban traffic networks [17] and electric power grids [18].

Observing the part after the pump B3.1/2 in Figure 1, it can be noticed that the set of HVAC systems can be represented by a linear dynamic network. In this representation, the nodes are valves and the arcs are flows of energy. Figure 3 shows this system representation.

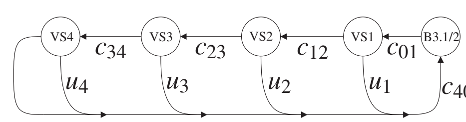

Fig. 3. Graph representing the set of HVAC systems

In this scheme, u_i is the control action of subsystem i and c_{ij} is the remaining flow from i to j. The system output is given by y_i, but it has been omitted in the diagram for simplicity.

In this case, a single energy source is responsible for a set of HVAC systems with a variable energy consumption. In each of these subsystems, a controller is responsible for regulating the local consume of chilled water.

These systems treated previously can operate independently, where the local dynamics only depends on the temperature and amount of chilled water being consumed. However, knowing that the solar plant depends on climatic conditions, there may be situations where it is impossible to provide the total flow demanded by the subsystems, and a strategy of local controllers may not be so effective, degrading the operation of the subsystems farthest from the energy source. Therefore, it is necessary to use a strategy of coordination between the controllers, like DMPC, to provide an optimized consumption that will ensure a reasonable operation to all subsystems.

III. DISTRIBUTED PREDICTIVE CONTROL

In this section, the control problem of the set of HVAC systems described previously is formalized. The aim is to formulate the control problem using the technique of DMPC, and subsequently solve this set of problems through a network of agents. These agents will operate jointly in the search for control actions that result in a smooth operation of all subsystems and a fair distribution of the available energy [19], [20].

Distributed predictive control is a technique that may be appropriate when there are difficulties in applying a centralized control and the process has a distributed structure. In this case study, the couplings of subsystems have only one direction, making viable the use of a distributed strategy. Following, the centralized control problem for the set of HVAC systems is presented, and then the distributed formulation is given, discussing the differences between the formulations.

A. Centralized Control Problem

The centralized control problem P is given in (2), where the objective function aims to reduce the tracking error and the control effort, with constraints that limit the output of the system and the consumption of each valve. Furthermore, there is a physical limitation of the system where the sum of the output signals of the valves cannot exceed the maximum water flow supplied by the pump B3.1/2:

$$
\begin{aligned}
P : \min\ f_0 = & \sum_{m\in\mathcal{M}} \sum_{j=N_1}^{N_2} \|w_m(k+j) - \hat{y}_m(k+j\ |k)\|^2 \\
& + \sum_{m\in\mathcal{M}} \sum_{j=1}^{N_u} \|\Delta u_m(k+j-1\ |k)\|^2_{\lambda_m} \qquad (2a) \\
\text{s. t.}: \quad & \sum_{m\in\mathcal{M}} (u_m(k-1)+\Delta u_m(k)) \le c^{\max} \qquad (2b) \\
& \forall\ m\in\mathcal{M}, \text{for } j=1,\ldots,N_u: \\
& u_m^{\min} \le u_m(k+j\ |k) \le u_m^{\max} \qquad (2c) \\
& \Delta u_m^{\min} \le \Delta u_m(k+j\ |k) \le \Delta u_m^{\max} \qquad (2d) \\
& \forall\ m\in\mathcal{M}, \text{for } j=N_1,\ldots,N_2: \\
& y_m^{\min} \le \hat{y}_m(k+j\ |k) \le y_m^{\max} \qquad (2e)
\end{aligned}
$$

where $\mathcal{M}$ represents the set of subsystems that form the linear dynamic network, k is the discrete time, $\hat{y}_m(k+j|k)$ is the output prediction of the air temperature ($T_{a\,imp}$) and $u_m(k+j|k)$ is the water flow control signal, both for the subsystem m at time $k+j$ based on the information acquired in k, $w_m(k+j)$ is the future reference of the temperature for the output of subsystem m at time $k+j$, $\Delta u(k)$ refers to variation in the control action at time k, $\|x\|^2_{\lambda_m}$ means $x'\lambda_m x$, $c^{\max}$ is the maximum flow that the pump can provide, N_1 and N_2 are the minimum and maximum prediction horizons, N_u is the control horizon and λ_m is the relative weight between the control actions and the reference tracking. From now on, the notation "$|k$" is dropped from all variables for the sake of simplicity.

As previously stated, the subsystems can operate independently, provided that the amount of energy produced by the solar plant is sufficient to fulfill all subsystem demands. In the control problem, the coupling between the subsystems only appears in constraint (2b), which limits the sum of the outputs of the valves. This constraint is important for a centralized controller, but it is an obstacle for distributed control applications because it requires that all subsystems communicate with each other to exchange information, creating an excess of communication in the network. A distributed protocol and some modifications in the original problem that enables the agents to reach an optimal compromise will be presented below.

First, the original problem will be manipulated to present a more compact form. Defining a vector with all the predictions of the output of a subsystem m for a given horizon, represented as [7]:

$$\hat{\mathbf{y}}_m = G_m \Delta\mathbf{u}_m + \mathbf{f}_m \qquad (3)$$

where $\mathbf{f}$ includes the terms of past outputs and past control actions, called the free response of the system, and G_m is the subsystem m dynamic matrix, also called the forced response matrix [7].

Using equation (3) to expand the terms given in (2a), it is possible to notice the cross relations between variables and constants for a given subsystem m. These terms are grouped as follows:

$$H_m = G_m' G_m + \lambda_m$$
$$\mathbf{h}_m = (\mathbf{w_m} - \mathbf{f}_m) G_m$$
$$\alpha_m = (\mathbf{w_m} - \mathbf{f}_m)'(\mathbf{w_m} - \mathbf{f}_m)$$

where $\mathbf{w}_m$ is a vector containing the local future reference for the prediction horizon.

Thus, problem (2) can be expressed as a convex quadratic problem:

$$P: \ \min \ f_0 = \sum_{m \in \mathcal{M}} \left(\frac{1}{2}\Delta\mathbf{u}_m{}' H_m \Delta\mathbf{u}_m + \mathbf{h}_m' \Delta\mathbf{u}_m + \frac{\alpha_m}{2}\right) \quad (4a)$$

s. t. :

$$\sum_{m \in \mathcal{M}} (u_m(k-1) + \Delta u_m(k)) \leq c^{\max} \quad (4b)$$

$$\forall \ m \in \mathcal{M}:$$

$$\mathbf{u}_m^{\min} \leq u_m(k-1)\mathbf{1}_{Nu} + \Gamma\Delta\mathbf{u}_m \leq \mathbf{u}_m^{\max} \quad (4c)$$

$$\Delta\mathbf{u}_m^{\min} \leq \Delta\mathbf{u}_m \leq \Delta\mathbf{u}_m^{\max} \quad (4d)$$

$$\mathbf{y}_m^{\min} \leq G_m \Delta\mathbf{u}_m + \mathbf{f}_m \leq \mathbf{y}_m^{\max} \quad (4e)$$

where $\mathbf{u}_m$ has all the control signals of a given subsystem m for the horizon N_u, $\mathbf{1}_{Nu}$ is a vector of N_u unitary elements, and Γ is a lower triangular matrix with unitary elements.

B. Distributed Control Problem

Following the steps of the methodology developed in [19], the problem P is decomposed into a set of subproblems $\{P_m\}$, where in each subproblem m appears the influence of neighboring subsystems on the local inputs and outputs. However, in this particular case, the methodology makes the purpose of the distributed controller compromised, because constraint (2b) is the only coupling element that appears between subsystems, causing each agent to coordinate its actions with all of other agents to ensure feasibility and convergence.

To avoid this excessive need of communication, a practical solution is proposed, where the shared constraint (2b) is dropped, and each subsystem introduces in its control problem a term modeling its influence on the dynamics of the next subsystem. This influence is approximated, considering the structure of the HVAC systems, and is given in a way that only geographically neighboring subsystems $(m, m+1)$ will be coupled through the objective of the control problem, eliminating the coupling of the whole network.

The problem to be solved in each agent is given by:

$$P_m: \min_{\Delta\mathbf{u}_m} \ f_m = \frac{1}{2}\Delta\mathbf{u}_m' H_m \Delta\mathbf{u}_m + \mathbf{h}_m' \Delta\mathbf{u}_m + \frac{\alpha_m}{2} + \gamma_m \phi_m \quad (5a)$$

s. t. :

$$\mathbf{u}_m^{\min} \leq u_m(k-1)\mathbf{1}_{Nu} + \Gamma\Delta\mathbf{u}_m \leq \mathbf{u}_m^{\max} \quad (5b)$$

$$\Delta\mathbf{u}_m^{\min} \leq \Delta\mathbf{u}_m \leq \Delta\mathbf{u}_m^{\max} \quad (5c)$$

$$\mathbf{y}_m^{\min} \leq G_m \Delta\mathbf{u}_m + \mathbf{f}_m \leq \mathbf{y}_m^{\max} \quad (5d)$$

$$u_m(k-1)\mathbf{1}_{Nu} + \Gamma\Delta\mathbf{u}_m \leq \hat{\mathbf{c}}_{m-1,m} \quad (5e)$$

where $\hat{\mathbf{c}}_{m-1,m}$ is the flow prediction passed by the previous subsystem $m-1$ to the current subsystem m, ϕ_m is the objective term which refers to the coupling between neighboring subsystems, and γ_m is a trade-off parameter.

In the distributed problem P_m, there is no longer the constraint on the sum of the outputs of the valves. However, one of the necessary replacements to circumvent this problem is in constraint (5e). It is an inheritance of the control action constraint, given by the previous neighbor, which informs the agent of how much flow is still available for consumption. In this case, the constraint was extended to the entire prediction horizon N_u.

In the term ϕ_m appears a forced coupling between subsystems, given as the minimization of the tracking error and control effort of the downstream neighboring agent, mathematically represented as follows:

$$\phi_m = \|\mathbf{w}_{m+1} - \hat{\mathbf{y}}_{m+1}\|^2 + \|\Delta\mathbf{u}_{m+1}\|^2_{\lambda_{m+1}} \quad (6)$$

For this particular case, when the energy flow reaches the upper bound, a subsystem agent is able to directly influence the tracking error of the downstream agents. This choice of ϕ_m seems appropriate for the way that the subsystems are interconnected; moreover, the information of the tracking error of a given agent m propagates through the upstream agents.

Now, expanding the remaining terms in such a way that the control signal of the agent m appears in ϕ_m, the relation of the flows of chilled water represented in a simplified way is obtained as $flow_{in} - flow_{out} = flow_{consumed}$.

Applying this relation for a given prediction horizon with the predicted flow passed between the subsystems, the equation defining the flow of a certain subsystem $m+1$ can be given by:

$$\hat{\mathbf{c}}_{m,m+1} - \hat{\mathbf{c}}_{m+1,m+2} = \mathbf{u}_{m+1} + \Gamma\Delta\mathbf{u}_{m+1} \quad (7)$$

The part of the equation which defines the incoming flow may be replaced by a similar equation applied to subsystem m. Performing the replacements of the terms to display the control signal of the agent m and isolating $\Delta\mathbf{u}_{m+1}$ results in:

$$\Delta\mathbf{u}_{m+1} = \Gamma^{-1}(\hat{\mathbf{c}}_{m-1,m} - \mathbf{u}_m - \Gamma\Delta\mathbf{u}_m - \hat{\mathbf{c}}_{m+1,m+2} - \mathbf{u}_{m+1}) \quad (8)$$

Now replacing (8) for $\Delta\mathbf{u}_{m+1}$ in (3) for a subsystem $m+1$, it is obtained a relation in terms of $\Delta\mathbf{u}_m$ which can be applied in (6) to yield:

$$\begin{aligned}\phi_m = \|&\mathbf{w}_{m+1} - \mathbf{f}_{m+1} \\ &- G_{m+1}\Gamma^{-1}(\hat{\mathbf{c}}_{m-1,m} - \mathbf{u}_m - \Gamma\Delta\mathbf{u}_m - \mathbf{u}_{m+1} - \hat{\mathbf{c}}_{m+1,m+2})\|^2 \\ &+ \|\Gamma^{-1}(\hat{\mathbf{c}}_{m-1,m} - \mathbf{u}_m - \Gamma\Delta\mathbf{u}_m - \mathbf{u}_{m+1} - \hat{\mathbf{c}}_{m+1,m+2})\|^2_{\lambda_{m+1}}\end{aligned}$$

This equation can be regrouped, in order to leave it in a quadratic form:

$$\phi_m = \frac{1}{2}\Delta\mathbf{u}_m' \tilde{H}_m \Delta\mathbf{u}_m + \tilde{\mathbf{h}}_m' \Delta\mathbf{u}_m + \frac{\tilde{\alpha}_m}{2} \quad (9)$$

where the notation ˜ is just to differentiate from similar terms in (5). It is possible to note that the quadratic form of ϕ_m can be easily combined with the terms in the objective of P_m.

It is important to highlight that the problem to be solved by each agent is much simpler than the one used in the centralized formulation, so the distributed agents must perform a sequence of steps before a solution of the set of subproblems $\{P_m\}$ is reached [21]. Notice that the optimal solution of the set $\{P_m\}$ may be different from the optimal solution of the problem P, given the modifications in the constraints and objective function presented above.

C. Distributed Algorithm

This section briefly describes the distributed algorithm for the agents to achieve a stationary point for the set of subproblems $\{P_m\}$. In this case, the control problems can be solved with the distributed barrier method, which is an iterative algorithm that generates a sequence of solutions that will arrive at a stationary solution. This algorithm is described in detail in [20], [22].

The algorithm starts with a control vector $\Delta\mathbf{u}(k)^0$ and produces a series $\Delta\mathbf{u}(k)^l$ of control signals for the entire system. The notation k is used for discrete time and the notation l is used to indicate iteration in the same period of time k. Let $P_m(\varepsilon)$ be the centering problem for P_m for a given tunable parameter ε:

$$P_m(\varepsilon) : \min f_m(\Delta\mathbf{u}_m) + \varepsilon\theta_m(\Delta\mathbf{u}_m) \quad (10)$$

where $\theta_m(\Delta\mathbf{u}_m)$ is the logarithmic barrier function of the constraints given in (5) [23].

In each iteration l, each agent m receives the decisions of its neighbors, coordinates its iterations, and calculates its own control actions $\Delta\mathbf{u}_m(k)^l$ by solving problem $P_m(\varepsilon)$. This process is repeated for a decreasing sequence of $\varepsilon \to 0^+$. The algorithm runs until a given stopping criterion is reached, and the obtained control values are applied in the subsystems.

Two stopping criteria were used together, being a limit on the number of messages exchanged between agents and a measure of the control change, stopping when the changes are lower than a given parameter σ. A measurement of the control change was given by the norm of the difference between the calculated control $\Delta\mathbf{u}_m(k)^l$ and the previous control $\Delta\mathbf{u}_m(k)^{l-1}$. For the first iteration, the control is given by the result of the previous sampling period, then, it is updated in every iteration l between agents.

An assumption for the operation of this algorithm is that neighboring agents cannot update their control decisions simultaneously to prevent the agents to reach infeasible solutions, as well as ensure a steady decrease on the objective value towards the optimum. In this case, the agents could operate in parallel two by two, but a sequential resolution of the control problems was used to illustrate the worst case for transferring the information in the agent network.

Algorithm 1 gives the pseudo-code of the distributed algorithm applied to the control of the set of HVAC systems.

Algorithm 1: Distributed algorithm

```
given: initial parameters, 0 < μ < 1, initial conditions
       and current capacity of the pump.
while loop stop condition not satisfied do
    Define flag for internal loop test δ :=false;
    while internal loop test δ =false do
        each agent m receives the available flow from
        agent m − 1 and data from agent m + 1;
        each agent m computes Δu_m(k)^l and the
        prediction of available flow;
        agents in the neighborhood of m maintain their
        decisions;
        if {Δu_i(k)^l} is a stationary point for {P_i(ε)},
        then δ := true ;
    loop stop test and parameters are updated;
    ε := με;
    l := l + 1;
apply: calculated control in the system
```

IV. SIMULATION RESULTS

This section presents the results of simulations with a numerical example demonstrating the effectiveness of the proposed control strategy. In this case, the system presented in Section II was considered.

To perform the simulations a simulator of a solar plant located at the University of Almería (Spain) was used [14], connected to a simulator of a set of HVAC systems, exchanging information of flow and temperature of the produced chilled water. Some of the parameters considered in these examples were:

$$\begin{array}{cc} \text{Sampling period} = 3\ (s) & N_u = 10 \\ N_1 = 1 & N_2 = 30 \\ \lambda_i = 2 & \gamma_i = 1 \\ \Delta\mathbf{u}_i^{\min} = -1\ (m^3/h) & \Delta\mathbf{u}_i^{\max} = 1\ (m^3/h) \end{array}$$

for $i = \{1,2,3,4\}$. The algorithm was implemented in Matlab®, in a PC with Intel Core 2 Duo CPU 2.13GHz, where all the control problems were solved below the adopted time for the sampling period. To avoid over-communication, the DMPC strategy only acts when some units cannot follow their temperature reference.

The proposed tests illustrate the possible cases where DMPC may be advantageous. Experiments have the following sequence:

- $t \leq 80$: the controllers are turned off, with all the valves of subsystems fixed at 5 m^3/h and the maximum output of the pump of B3.1/2 is at 24.5 m^3/h.
- $t > 80$: DMPC is connected with the output references of $T_{a\,imp}$ in 10 ^oC for the subsystems.
- $t > 150$: subsystem 3 changes its reference to 9 ^oC, in which case the pump flow is insufficient to meet the references of all subsystems.
- $t > 250$: subsystem 3 returns its reference to 10 ^oC.
- $t > 350$: the maximum output flow of pump B3.1/2 drops to 22.5 m^3/h.

Figures 4 and 5 show the responses obtained in these experiments, comparing a case of decentralized controllers with independent MPC in each subsystem versus the case using DMPC to distribute energy in the network, respectively.

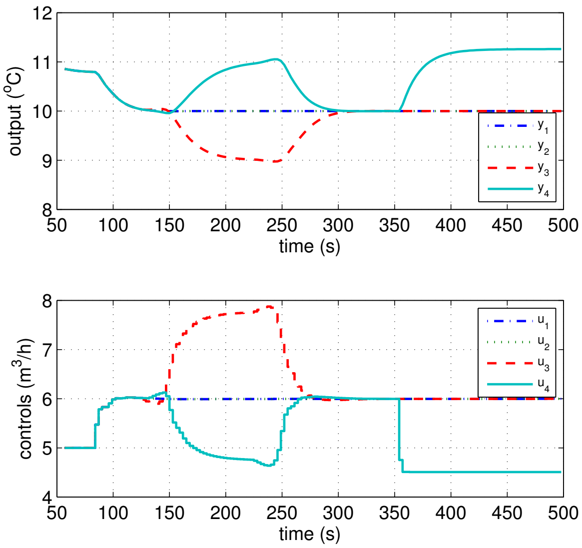

Fig. 4. Outputs and controls of the subsystems with independent MPC controllers

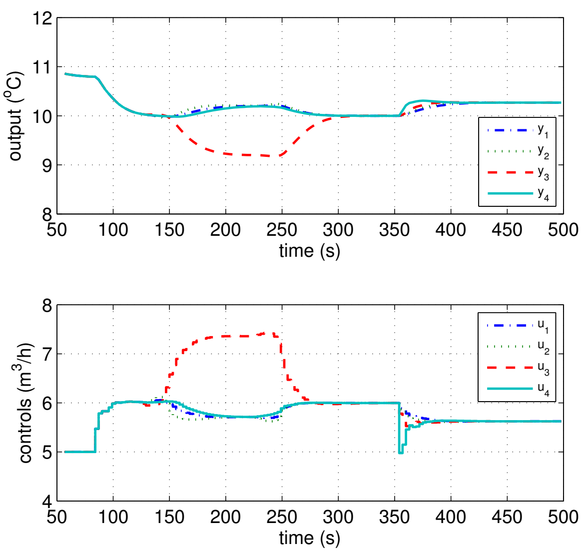

Fig. 5. Outputs and controls of the subsystems with DMPC controllers

The first observed result was that the behaviors of independent controllers and using the DMPC strategy were similar while there was enough chilled water for all subsystems reach their references. Figures 4 and 5 show this behavior in $t < 80$ and $250 < t \leq 350$. With this result it can be seen that the set of HVAC systems could be operated with independent controllers if the solar plant producing chilled water is oversized.

When a subsystem requires more flow of chilled water to achieve its reference, as it happens for $t > 150$ and $t \leq 250$, the last subsystem was always the first to be harmed, since the entire production deficit was reflected in this subsystem. However, in a network using DMPC, all the subsystems changed its consumptions, coming as close as possible to the desired operation points while releasing chilled water flow to assist the downstream subsystems.

A change in the maximum chilled water flow delivered by the pump B3.1/2 in a situation in which the system is working close to saturation affects directly the subsystems that are farther from the source of energy in the case of independent controllers, as in $t > 350$ in Figure 4. The same situation presents a much better result when using DMPC, where the last subsystem is the first to be affected by the shortage of energy, but as the other subsystems receive information about that the last subsystem is working away from its desired point, they begin to release energy flow until all subsystems come into balance.

In the studied cases it can be seen that a strategy of independent controllers harmed the last subsystem whenever the amount of pump flow was insufficient. In this comparison, the advantage of DMPC is evident, where all the subsystems cooperate, sharing fairly the available resources.

A critical issue when using a distributed control strategy is in relation to the number of messages exchanged between agents. In the case studied two stopping criteria were considered, as stated in the previous section, being the maximum number of message exchange established at 10, and the control error in relation to the previous iteration established at $\sigma = 0.001$.

Figure 6 represents the output and the number of iterations between agents in each sample time for the entire experiment for a stopping criterion 10 times greater than that used in previous simulations, namely $\sigma = 0.01$.

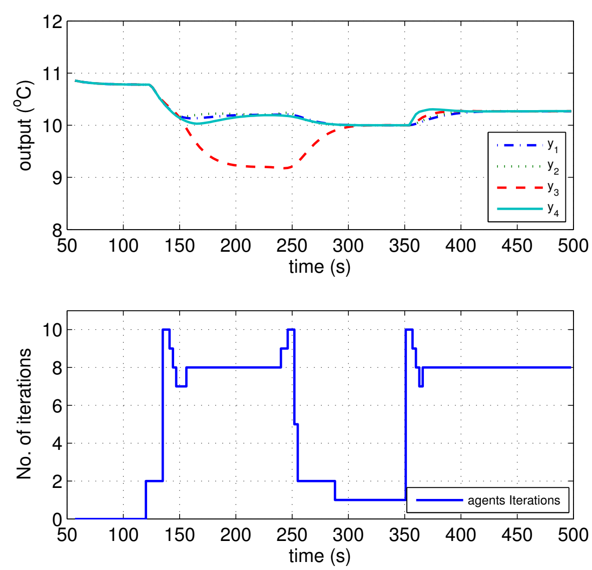

Fig. 6. Outputs and iterations required for convergence of the subsystems with DMPC controllers

In this case, it is possible to note that the number of exchanged messages until convergence was almost always below the limit of 10 messages, while the output barely changed in relation to the results observed in Figure 5. Thus, it is possible to say that a study of maximum number of exchanged messages and stopping criterion σ in the distributed strategy is relevant, reflecting a trade-off between response quality and computational effort of the agent network.

The comparisons made in this section were not in relation to a centralized controller that encompasses all subsystems because the original control problem P became different from the set of subproblems $\{P_m\}$ when the constraints were removed and objective terms were added to the problem.

However, since this is a stable and noncritical system, the development of controllers that provide approximate solutions is plausible. The results showed that the distributed strategy can achieve satisfactory performance, in addition to having the advantages of modularization and the possibility of modifying the structure of the network without modifying the entire control problem, only readjusting the neighborhood of the modified subsystems.

V. CONCLUSIONS

This paper deals with the control of energy distribution carried out through distributed model predictive controllers. In the case study, a solar plant generated a chilled water flow to be properly shared among a set of HVAC systems.

A distributed control strategy was proposed and implemented in such a way that local agents in the HVAC systems operated together, exchanging information about outputs and control signals, making joint decisions on the use of the available resources, especially when the production of chilled water is insufficient for all desired operation points. In this study, the centralized control problem was replaced by a modified control problem that allows the application of an efficient distributed strategy, circumventing physical characteristics that made the system fully coupled.

Simulation results have been presented to demonstrate that this strategy can be effective, especially in the case of limited resources. The results obtained were quite satisfactory, with the advantage that the distributed problem is simpler when compared to the centralized problem, and easily extendable to a larger set of HVAC systems.

Future works will provide improvements in models of subsystems by incorporating other situations that may occur in real cases. Furthermore, experiments will be performed in a real solar plant, providing flows and temperatures subject to disturbances from the external environment.

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