2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Application of an Infinite-Dimensional Observer for Drilling Systems Incorporating Kick and Loss Detection

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Abstract— **We apply a PDE observer to a hydraulic system of a managed pressure drilling rig. The observer can be used to detect and quantify an in- or outflux. We show that the PDE model of the hydraulic system can be expressed as a 2×2 linear hyperbolic system of PDEs with spatially varying coefficients coupled with an ODE at the inlet boundary that models the in- or outflux. Using the method of backstepping, we design an observer which is exponentially stable at the origin in the $\mathscr{L}^2$-norm while relying on measurements taken at the outlet boundary, only. Simulation results verify the validity of the observer.**

I. INTRODUCTION

Detection and quantification of leaks in pipelines have been a subject of research for many years. We look at an analog problem concerning detection and quantification of in-/outflux during drilling of oil and gas wells. In the drilling community, an outflux to the formation from the wellbore is called a loss or ”lost returns”, while an influx is called a kick or ”gaining fluid”. These two unwanted events are a result of the pressure in the wellbore being either too high or too low compared to the surrounding formation. Even with proper pressure control and seismic surveys, loss and gain are bound to happen during the development of new fields. Drilling of exploration wells is a dangerous task where the crew runs the risk of drilling into a high-pressure gas-pocket or possibly losing/gaining fluid to/from the formation. This can lead to loss of control of the well, which again can result in damage to the well, or in worst case, a catastrophic blowout. The consequences are not only monetary and environmental, but also concern the safety of the rig personnel.

Managed Pressure Drilling (MPD) [1] is a new technology gaining more and more interest. MPD is an enabling technology to drill challenging wells into depleted reservoirs with narrow pressure margins. Also, MPD has the economical benefit of reduced rig time and health, safety and environmental benefits of improved well control. The concept includes a rig pump pumping drilling mud into the drillstring, through the bit at the bottom of the well, and up through the annulus. The annulus is sealed and the pressure in the closed volume is controlled by a choke. This is illustrated in Figure 1. By controlling the flow rate from the pump and the choke

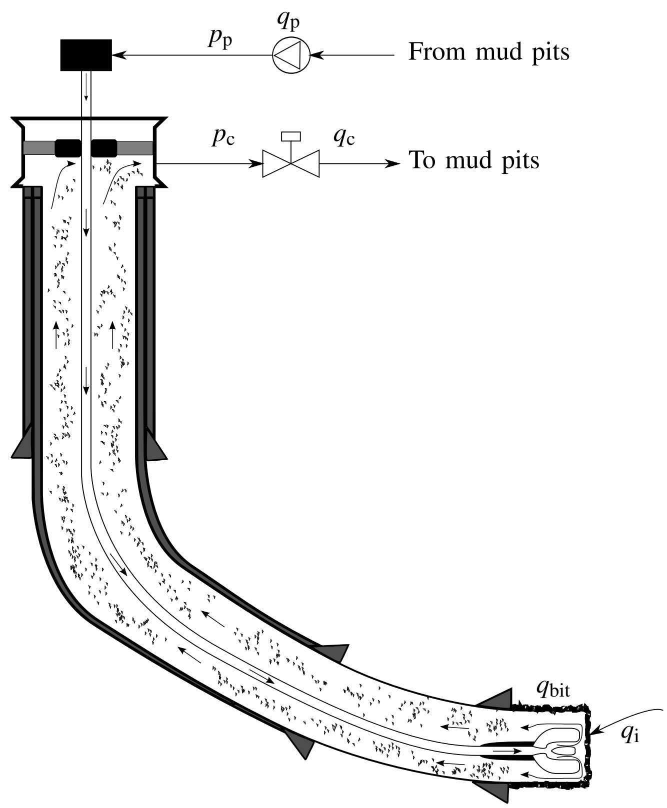

Fig. 1. Schematic of a managed pressured drilling system.

opening, it is possible to regulate the pressure at a point in the well to a predetermined value within a few seconds.

Due to the harsh conditions in the wellbore, the most reliable measurements of pressure and flow rate are the ones located at the rig. And subsequently, a monitoring system for in-/outflux has to be model based. The ODE model in [1] is applied to gas kick detection and mitigation in [2], where the concept of switched control of the bottomhole pressure is presented. An MPD system is considered where a controller manipulating the choke and backpressure pump switches between a combination of pressure and flow control when there is no in-flux and pure flow control when a kick is detected. In [3] another model based in-/outflux detection and mitigation scheme is presented. The adaptive observer is shown to be globally exponentially stable and is capable of quantifying and locating an in-/outflux. The choke controller does not rely upon switching for stopping the in-/outflux. An early example of model-based leak detection can be found in [4] where a correlation technique is used to

This work was supported by the Research Council of Norway and Statoil ASA

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detect, quantify and locate the leak. Using the water hammer equations based on a collocation method, a model based leak detection scheme using an extended Kalman filter observer is implemented in [5]. In [6], it is shown that multiple leaks in a pipeline can be detected if the system is sufficiently excited through the boundary conditions. The work of [7] covers a PDE-observer which is able to adapt to the friction of a horizontal pipeline before quantifying and locating a leak. The observer uses non-reflective boundaries for robustness to initial conditions. The boundary conditions are found by linearization of the nonlinear PDEs. The observer for the linearized system is proven to be exponentially stable at the origin in the $\mathscr{L}^2$-norm.

In this paper we consider a PDE observer for the well hydraulics. The model of the hydraulics is a system of linear 2×2 first-order hyperbolic PDEs which can be transformed into the system

$$u_t = -\varepsilon_1(x)u_x + c_1(x)v, \tag{1}$$

$$v_t = \varepsilon_2(x)v_x + c_2(x)u, \tag{2}$$

$$u(0,t) = qv(0,t) + v_{\mathrm{i}}(t) + CX(t), \tag{3}$$

$$v(1,t) = U(t), \tag{4}$$

$$\dot{X} = AX, \tag{5}$$

where $(x,t) \in [0,1] \times [0,\infty)$, $\varepsilon_1(x) > 0$, $\varepsilon_2(x) > 0$, $q \neq 0$, $v_{\mathrm{i}}(t)$ is a known injection at the boundary, $X(t)$ is a n-dimensional vector, A is a $n \times n$ matrix and C is a $1 \times n$ matrix. $U(t)$ is the control input, while $u(1,t)$ is the measurement. The subscripts t and x denote the time- and spatial-derivative, respectively. The objective is to demonstrate an application of an exponentially stable observer for (1)–(5) with measurements only at the boundary $x = 1$. Similar efforts have previously been made for heat exchangers, [8]; and irrigation canals, [9], [10]. The observer is designed by combining results from [11] and [12] which both use the method of backstepping. In [11], backstepping boundary control is applied to first-order hyperbolic PDEs by using invertible Volterra integral transformations together with boundary feedback. The joint papers [12] and [13] consider boundary backstepping stabilization for a linear hyperbolic system, and a quasilinear hyperbolic system, respectively. Both of the systems considered are non-uniform. We use a similar observer structure and transformation as in [12] for (1)–(4) and an observer similar to the observer for ODE systems with sensor delays in [11] for (5).

This article is structured as follows. Section II shows how the model of the well hydraulics can be transformed into (1)–(5). In Section III we briefly present the observer design. An application of the observer with the well hydraulics is illustrated with simulation results in Section IV. Conclusions are given in Section V which is followed by Section VI concerning further work.

II. Well hydraulics

Consider a drilling rig, illustrated in Figure 1. We model the fluid dynamics of the return side (from the bit, up the annulus to the choke) in the standard way for vertical hydraulic pipe flow [14] as

$$p_t(z,t) = -\frac{\beta}{A_{\mathrm{a}}} q_z(z,t), \tag{6}$$

$$q_t(z,t) = -\frac{A_{\mathrm{a}}}{\rho} p_z(z,t) - \frac{F}{\rho} q(z,t) - A_{\mathrm{a}} g, \tag{7}$$

$$q(0,t) = q_{\mathrm{bit}}(t), \tag{8}$$

$$p(l,t) = p_{\mathrm{c}}(t), \tag{9}$$

where $(z,t) \in [0,l] \times [0,\infty)$, p is the pressure along the annulus, q is the volumetric flow rate through the annulus, l denotes the total length of the annulus, β is the isothermal bulk modulus, A_{a} the area of the annulus, F a friction coefficient, g the gravitational constant, and ρ the density of the drilling mud. $p_{\mathrm{c}}(t)$ is the pressure upstream the choke and the controlled input, and $q_{\mathrm{c}}(t) = q(l,t)$ is the volumetric flow rate through the choke, which is measured. We can remove the contribution of the hydrostatic head from the momentum equation by defining

$$\bar{p}(z,t) = p(z,t) - \rho g (l - z), \tag{10}$$

which gives

$$\bar{p}_t(z,t) = p_t(z,t) = -\frac{\beta}{A_{\mathrm{a}}} q_z(z,t), \tag{11}$$

$$\begin{aligned} q_t(z,t) &= -\frac{A_{\mathrm{a}}}{\rho}\left(\bar{p}(z,t) + \rho g (l-z)\right)_z - \frac{F}{\rho} q(z,t) - A_{\mathrm{a}} g \\ &= -\frac{A_{\mathrm{a}}}{\rho}\bar{p}_z(z,t) + A_{\mathrm{a}} g - \frac{F}{\rho} q(z,t) - A_{\mathrm{a}} g \\ &= -\frac{A_{\mathrm{a}}}{\rho}\bar{p}_z(z,t) - \frac{F}{\rho} q(z,t), \end{aligned} \tag{12}$$

and

$$\bar{p}(l,t) = p(l,t) = p_{\mathrm{c}}(t). \tag{13}$$

We denote the volumetric flow rate and pressure at the bit with, q_{bit} and p_{bit}, respectively, and get

$$q_{\mathrm{bit}}(t) = q_{\mathrm{p}}(t) + q_{\mathrm{i}}(t), \tag{14}$$

$$p_{\mathrm{bit}}(t) = p(0,t), \tag{15}$$

where $q_{\mathrm{p}}(t)$ is the volumetric flow rate from the rig pump, which is measured, and q_{i} is the volumetric in-/outflux. This gives

$$\bar{p}_t = -\frac{\beta}{A_{\mathrm{a}}} q_z, \tag{16}$$

$$q_t = -\frac{A_{\mathrm{a}}}{\rho}\bar{p}_z - \frac{F}{\rho} q, \tag{17}$$

$$q(0,t) = q_{\mathrm{p}}(t) + q_{\mathrm{i}}(t), \tag{18}$$

$$\bar{p}(l,t) = p_{\mathrm{c}}(t). \tag{19}$$

We now want to transform (16)–(19) into the form (1)–(5), which is needed for the observer design. A similar method of transformation was carried out in [15], but we highlight the steps for sake of the reader. Consider the change of variables

$$\bar{u}(z,t) = \frac{1}{2}\left(q(z,t) + \frac{A_\mathrm{a}}{\sqrt{\beta\rho}}\bar{p}(z,t)\right), \tag{20}$$

$$\bar{v}(z,t) = \frac{1}{2}\left(q(z,t) - \frac{A_\mathrm{a}}{\sqrt{\beta\rho}}\bar{p}(z,t)\right). \tag{21}$$

Taking the time-derivative of (20) and (21) gives

$$\begin{aligned}\bar{u}_t &= \frac{1}{2}\left(q_t(z,t) + \frac{A_\mathrm{a}}{\sqrt{\beta\rho}}\bar{p}_t\right)\\ &= -\sqrt{\frac{\beta}{\rho}}\bar{u}_z - \frac{F}{2\rho}(\bar{u}+\bar{v}).\end{aligned} \tag{22}$$

and

$$\begin{aligned}\bar{v}_t &= \frac{1}{2}\left(q_t(z,t) - \frac{A_\mathrm{a}}{\sqrt{\beta\rho}}\bar{p}_t(z,t)\right)\\ &= \sqrt{\frac{\beta}{\rho}}\bar{v}_z - \frac{F}{2\rho}(\bar{u}+\bar{v}).\end{aligned} \tag{23}$$

The boundary conditions are

$$\bar{u}(0,t) = \frac{1}{2}\left(q_\mathrm{p}(t) + q_\mathrm{i}(t) + \bar{u}(0,t) - \bar{v}(0,t)\right), \tag{24}$$

$$\bar{v}(l,t) = \frac{1}{2}\left((\bar{u}(l,t) + \bar{v}(l,t)) - \frac{A_\mathrm{a}}{\sqrt{\beta\rho}}p_\mathrm{c}(t)\right), \tag{25}$$

so we get

$$\bar{u}(0,t) = -\bar{v}(0,t) + q_\mathrm{p}(t) + q_\mathrm{i}(t), \tag{26}$$

$$\bar{v}(l,t) = \bar{u}(l,t) - \frac{A_\mathrm{a}}{\sqrt{\beta\rho}}p_\mathrm{c}(t). \tag{27}$$

Next, consider the transformation $x = z/l$, and define

$$u(x,t) = \bar{u}(xl,t)e^{ax}, \tag{28}$$

$$v(x,t) = \bar{v}(xl,t)e^{-ax}, \tag{29}$$

where a is to be defined. Taking the time-derivative of (28) gives

$$u_t = -\sqrt{\frac{\beta}{\rho}}\bar{u}_z e^{ax} - \frac{F}{2\rho}\left(u(x,t) + v(x,t)e^{2ax}\right). \tag{30}$$

The spatial-derivative of (28) is

$$u_x(x,t) = l\bar{u}_z(xl,t)e^{ax} + a\bar{u}(xl,t)e^{ax}, \tag{31}$$

so

$$\bar{u}_z(xl,t) = \frac{1}{l}u_x(x,t)e^{-ax} - \frac{a}{l}u(x,t)e^{-ax}. \tag{32}$$

Inserting (32) into (30) yields

$$u_t = -\frac{1}{l}\sqrt{\frac{\beta}{\rho}}u_x(x,t) + \left(\frac{a}{l}\sqrt{\frac{\beta}{\rho}} - \frac{F}{2\rho}\right)u - \frac{F}{2\rho}e^{2ax}v(x,t). \tag{33}$$

Choosing

$$a = \frac{Fl}{2\sqrt{\beta\rho}} \tag{34}$$

gives

$$u_t = -\frac{1}{l}\sqrt{\frac{\beta}{\rho}}u_x(x,t) - \frac{F}{2\rho}e^{2ax}v(x,t). \tag{35}$$

Similar steps for (29) gives

$$v_t(x,t) = \frac{1}{l}\sqrt{\frac{\beta}{\rho}}v_x(x,t) - \frac{F}{2\rho}e^{-2ax}u(x,t). \tag{36}$$

We can now write the boundary conditions as

$$u(0,t) = -v(0,t) + q_\mathrm{p}(t) + q_\mathrm{i}(t), \tag{37}$$

and

$$v(1,t) = u(1,t)e^{-2a} - \frac{A_\mathrm{a}e^{-a}}{\sqrt{\beta\rho}}p_\mathrm{c}(t). \tag{38}$$

In summary, the model is

$$u_t = -\frac{1}{l}\sqrt{\frac{\beta}{\rho}}u_x - \frac{F}{2\rho}e^{2ax}v, \tag{39}$$

$$v_t = \frac{1}{l}\sqrt{\frac{\beta}{\rho}}v_x - \frac{F}{2\rho}e^{-2ax}u, \tag{40}$$

$$u(0,t) = -v(0,t) + q_\mathrm{p}(t) + q_\mathrm{i}(t), \tag{41}$$

$$v(1,t) = u(1,t)e^{-2a} - \frac{A_\mathrm{a}e^{-a}}{\sqrt{\beta\rho}}p_\mathrm{c}(t), \tag{42}$$

which corresponds to (1)–(5) with

$$\varepsilon_1(x) = \varepsilon_2(x) = \frac{1}{l}\sqrt{\frac{\beta}{\rho}}, \tag{43}$$

$$c_1(x) = -\frac{F}{2\rho}e^{2ax}, \tag{44}$$

$$c_2(x) = -\frac{F}{2\rho}e^{-2ax}, \tag{45}$$

$$q = -1, \tag{46}$$

$$v_\mathrm{i}(t) = q_\mathrm{p}(t) \tag{47}$$

$$U(t) = u(1,t)e^{-2a} - \frac{A_\mathrm{a}e^{-a}}{\sqrt{\beta\rho}}p_\mathrm{c}(t), \tag{48}$$

where a is given by (34), and (A,C) such that

$$\dot{X} = AX, \tag{49}$$

$$q_\mathrm{i}(t) = CX(t). \tag{50}$$

The original variables are obtained as

$$q(z,t) = u\left(\frac{z}{l},t\right)e^{-\frac{a}{l}z} + v\left(\frac{z}{l},t\right)e^{\frac{a}{l}z}, \tag{51}$$

$$\begin{aligned}p(z,t) = &\frac{\sqrt{\beta\rho}}{A_\mathrm{a}}\left(u\left(\frac{z}{l},t\right)e^{-\frac{a}{l}z} - v\left(\frac{z}{l},t\right)e^{\frac{a}{l}z}\right)\\ &+ \rho g(l-z).\end{aligned} \tag{52}$$

In the presence of a constant in-/outflux, we can simply set $A = 0$ and $C = 1$.

III. Observer design

In this section we present an exponentially stable observer for (1)–(5). The observer is based on [12], [13], where an observer without the disturbance at x=1 was designed using backstepping. While a detailed derivation of the observer augmented with the disturbance was carried out in [16], here we repeat some of the steps for clarity. Consider the observer

$$\hat{u}_t = -\varepsilon_1(x)\hat{u}_x + c_1(x)\hat{v} + p_1(x)\tilde{u}(1,t), \tag{53}$$
$$\hat{v}_t = \varepsilon_2(x)\hat{v}_x + c_2(x)\hat{u} + p_2(x)\tilde{u}(1,t), \tag{54}$$
$$\hat{u}(0,t) = q\hat{v}(0,t) + v_{\mathrm{i}}(t) + C\hat{X}(t), \tag{55}$$
$$\hat{v}(1,t) = U(t), \tag{56}$$
$$\dot{\hat{X}} = A\hat{X} + e^{A\phi(0)}L\tilde{u}(1,t), \tag{57}$$

where

$$\phi(x) = \int_x^1 \frac{1}{\varepsilon_1(\gamma)}d\gamma, \tag{58}$$

and

$$\tilde{u}(1,t) = u(1,t) - \hat{u}(1,t). \tag{59}$$

The functions $p_1(x)$ and $p_2(x)$, and the matrix L are output injection gains to be designed. Forming error equations by subtracting (53)–(57) from (1)–(5), denoted by a tilde, we have

$$\tilde{u}_t = -\varepsilon_1(x)\tilde{u}_x + c_1(x)\tilde{v} - p_1(x)\tilde{u}(1,t), \tag{60}$$
$$\tilde{v}_t = \varepsilon_2(x)\tilde{v}_x + c_2(x)\tilde{u} - p_2(x)\tilde{u}(1,t), \tag{61}$$
$$\tilde{u}(0,t) = q\tilde{v}(0,t) + C\tilde{X}(t), \tag{62}$$
$$\tilde{v}(1,t) = 0, \tag{63}$$
$$\dot{\tilde{X}} = A\tilde{X} - e^{A\phi(0)}L\tilde{u}(1,t). \tag{64}$$

In [12], the following backstepping transformation

$$\tilde{u}(x,t) = \tilde{\alpha}(x,t) - \int_x^1 P^{uu}(x,\xi)\tilde{\alpha}(\xi,t)d\xi - \int_x^1 P^{uv}(x,\xi)\tilde{\beta}(\xi,t)d\xi, \tag{65}$$
$$\tilde{v}(x,t) = \tilde{\beta}(x,t) - \int_x^1 P^{vu}(x,\xi)\tilde{\alpha}(\xi,t)d\xi - \int_x^1 P^{vv}(x,\xi)\tilde{\beta}(\xi,t)d\xi. \tag{66}$$

was used for observer design in the case without disturbance, where the kernels are given by the system of equations[1]

$$\varepsilon_1(x)P_x^{uu}(x,\xi) + \varepsilon_1(\xi)P_\xi^{uu}(x,\xi) = -\varepsilon_1'(\xi)P^{uu}(x,\xi) + c_1(x)P^{vu}(x,\xi), \tag{67}$$
$$\varepsilon_1(x)P_x^{uv}(x,\xi) - \varepsilon_2(\xi)P_\xi^{uv}(x,\xi) = \varepsilon_2'(\xi)P^{uv}(x,\xi) + c_1(x)P^{vv}(x,\xi), \tag{68}$$
$$\varepsilon_2(x)P_x^{vu}(x,\xi) - \varepsilon_1(\xi)P_\xi^{vu}(x,\xi) = \varepsilon_1'(\xi)P^{vu}(x,\xi) - c_2(x)P^{uu}(x,\xi), \tag{69}$$
$$\varepsilon_2(x)P_x^{vv}(x,\xi) + \varepsilon_2(\xi)P_\xi^{vv}(x,\xi) = -\varepsilon_2'(\xi)P^{vv}(x,\xi) - c_2(x)P^{uv}(x,\xi), \tag{70}$$

on $\mathscr{T} = \{(x,\xi) : 0 \le \xi \le x \le 1\}$ with boundary conditions

$$P^{uu}(0,\xi) = qP^{vu}(0,\xi), \tag{71}$$
$$P^{uv}(x,x) = \frac{c_1(x)}{\varepsilon_1(x) + \varepsilon_2(x)}, \tag{72}$$
$$P^{vu}(x,x) = -\frac{c_2(x)}{\varepsilon_1(x) + \varepsilon_2(x)}, \tag{73}$$
$$P^{vv}(0,\xi) = \frac{1}{q}P^{uv}(0,\xi). \tag{74}$$

It was shown in [12] that there exists a unique solution to (67)–(74) which is $C(\mathscr{T})$. We have the following result for our error system (60)–(64) (the proof is similar to that in [17] but omitted due to page limitation).

Lemma 1: Let $(P^{uu}, P^{uv}, P^{vu}, P^{vv})$ be the solution to (67)–(74). If

$$p_1(x) = Ce^{A\phi(x)}L - P^{uu}(x,1)\varepsilon_1(1) - \int_x^1 P^{uu}(x,\xi)Ce^{A\phi(\xi)}Ld\xi, \tag{75}$$
$$p_2(x) = -P^{vu}(x,1)\varepsilon_1(1) - \int_x^1 P^{vu}(x,\xi)Ce^{A\phi(\xi)}Ld\xi, \tag{76}$$

then the transformation (65)–(66) maps (60)–(64) into the system

$$\tilde{\alpha}_t(x,t) = -\varepsilon_1(x)\tilde{\alpha}_x(x,t) - Ce^{A\phi(x)}L\tilde{\alpha}(1,t), \tag{77}$$
$$\tilde{\beta}_t = \varepsilon_2(x)\tilde{\beta}_x(x,t), \tag{78}$$

with boundary conditions

$$\tilde{\alpha}(0,t) = q\tilde{\beta}(0,t) + C\tilde{X}, \tag{79}$$
$$\tilde{\beta}(1,t) = 0, \tag{80}$$

and

$$\dot{\tilde{X}} = A\tilde{X} - e^{A\phi(0)}L\tilde{\alpha}(1,t). \tag{81}$$

It was shown in [12] that the transformation (65)–(66) is invertible, so that stability properties of (60)–(64) and (77)–(81) are equivalent. We therefore analyze stability of the origin of (77)–(81). The system (77)–(81) can be viewed as a cascade consisting of the parts

$$\tilde{\beta}_t = \varepsilon_2(x)\tilde{\beta}_x(x,t), \tag{82}$$
$$\tilde{\beta}(1,t) = 0, \tag{83}$$

and

$$\tilde{\alpha}_t(x,t) = -\varepsilon_1(x)\tilde{\alpha}_x(x,t) - Ce^{A\phi(x)}L\tilde{\alpha}(1,t), \tag{84}$$
$$\tilde{\alpha}(0,t) = q\tilde{\beta}(0,t) + C\tilde{X}, \tag{85}$$
$$\dot{\tilde{X}} = A\tilde{X} - e^{A\phi(0)}L\tilde{\alpha}(1,t), \tag{86}$$

where the former affects the latter only through the boundary condition (85). It is clear from (82)–(83) that $\tilde{\beta}(x,t)$ converges to 0 in finite time, so it suffices to consider stability of the origin of

$$\tilde{\alpha}_t(x,t) = -\varepsilon_1(x)\tilde{\alpha}_x(x,t) - Ce^{A\phi(x)}L\tilde{\alpha}(1,t), \tag{87}$$
$$\tilde{\alpha}(0,t) = C\tilde{X}, \tag{88}$$
$$\dot{\tilde{X}} = A\tilde{X} - e^{A\phi(0)}L\tilde{\alpha}(1,t). \tag{89}$$

[1] Note that there are typos in equations (24)–(25) in [12].

TABLE I

VALUES OF PARAMETERS USED IN THE SIMULATION.

Parameter	Value	Unit
ρ	$1.250 \cdot 10^3$	kg/m^3
l	$2.000 \cdot 10^3$	m
F	$1.500 \cdot 10^2$	kg/sm^3
A_a	$2.530 \cdot 10^{-2}$	m^2
β	$2.000 \cdot 10^{-9}$	Pa

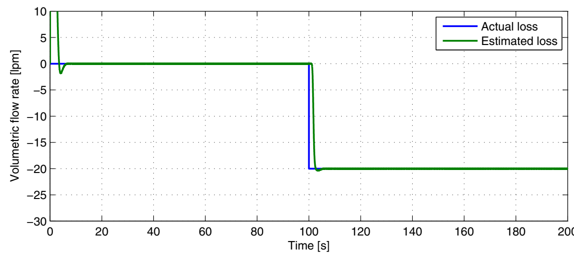

Fig. 2. The actual and estimated loss rate.

Based on [11], we obtain the following result (see [16] for proof).

Lemma 2: Let L be chosen such that $A - LC$ is Hurwitz. The origin of (87)–(89) is exponentially stable in the norm

$$\left(\left| \tilde{X}(t) \right|^2 + \int_0^1 \tilde{\alpha}^2(x,t)\, dx \right)^{1/2}. \tag{90}$$

The following theorem summarizes the observer design.

Theorem 3: Consider the observer (53)–(57) with initial conditions $\hat{u}_0$ and $\hat{v}_0$, with output injection kernels given by (75)–(76), where P^{uu} and P^{vu} are found from (67)–(74), and where L is chosen such that $A - LC$ is Hurwitz. Under the assumptions

$$\varepsilon_1, \varepsilon_2 \in \mathscr{C}^1([0,1]),\, c_1, c_2 \in \mathscr{C}([0,1]),\, u_0, v_0 \in \mathscr{L}^2([0,1]), \tag{91}$$

and $\varepsilon_1(x), \varepsilon_2(x) > 0$, it is guaranteed that $\hat{X}$, $\hat{u}$, $\hat{v}$ exponentially converge to X, u, v, i.e., more specifically, that the observer error system is exponentially stable in the sense of the norm (90).

Proof: This follows from Lemma 1 and Lemma 2 and the fact that the transformation (65)–(66) is invertible. ■

IV. SIMULATIONS

We demonstrate the use of the observer with simulations of the model of the well hydraulics. The well is vertical and 2000 m deep, and the flow rate from the rig pump is 1000 lpm. At 100 seconds a loss of 20 lpm occurs, i.e. X is changed from 0 to -20 lpm. We consider the particularly simple case when there is no dynamics associated with X, i.e., $A = 0$ and $C = 1$. We choose $L = 5$. The well parameters are listed in Table I. In Figure 2 we see that the loss rate is accurately estimated 10 seconds after the loss has occurred. The initial discrepancy is due to the observer being initialized

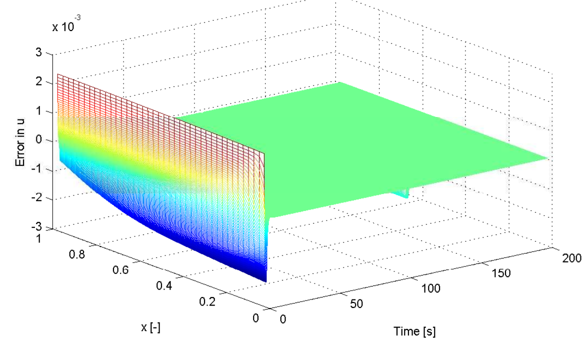

Fig. 3. The error $\tilde{u}(x,t)$.

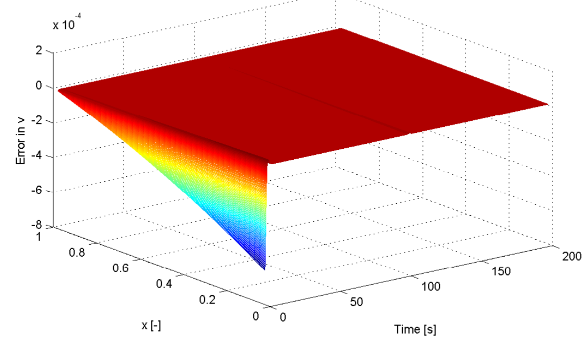

Fig. 4. The error $\tilde{v}(x,t)$.

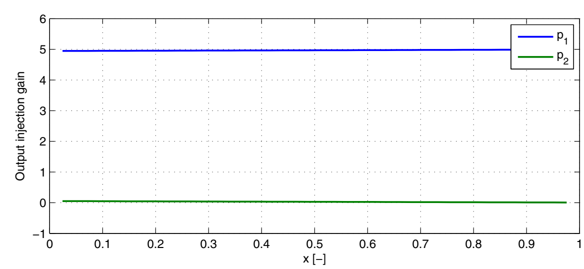

Fig. 5. The output injection terms $p_1(x)$ and $p_2(x)$.

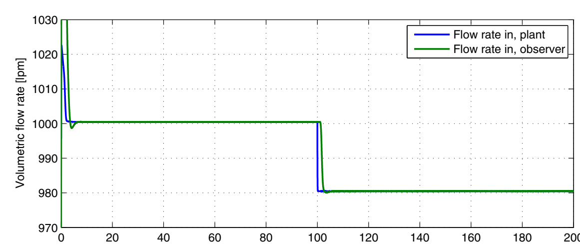

Fig. 6. The flow rate at the inlet.

with values different from the plant. Figure 3 and Figure 4 illustrate that the observer converges to the value of the plant throughout the well. Note that the plotted errors are in the transformed coordinates. However, the invertibility of the transformation ensures that the system also converges in its original coordinates. The output injection gains $p_1(x)$ and

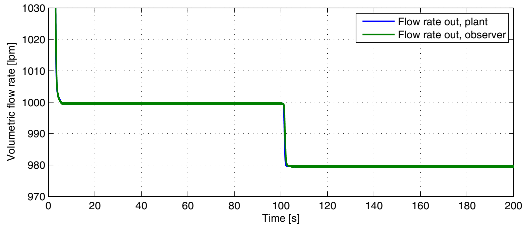

Fig. 7. The flow rate at the outlet.

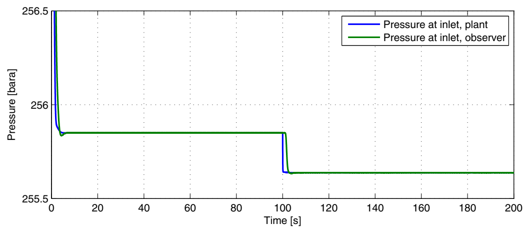

Fig. 8. The pressure at the inlet.

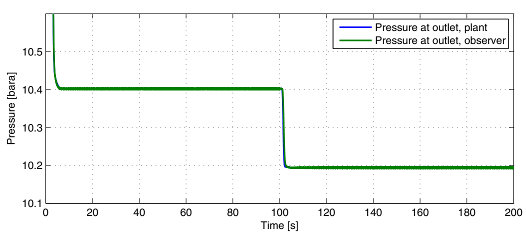

Fig. 9. The pressure at the outlet.

$p_2(x)$ are shown in Figure 5. The flow rates and pressures at the inlet and outlet of the plant and observer are plotted in Figure 6 through Figure 9. The slight oscillations seen at the observer boundaries in these figures are due to the numerical scheme. The results can be improved by refining the spatial discretization grid.

V. Conclusions

The main result in this paper is the application of a PDE observer for a hydraulic system of an MPD rig. The hydraulics can be expressed as a 2×2 linear hyperbolic system of PDEs coupled with an ODE at the inlet boundary. By using the method of backstepping, we are able to design an observer which is exponentially stable at the origin in the $\mathscr{L}^2$-norm. A simulation of the mud circulation system at a drilling rig is used to demonstrate an application of the observer.

VI. Further work

When an in-/outflux occurs, it is of interest to attenuate it by automatic control. Towards that end, it would be beneficial to model the in-/outflux as a pressure dependent disturbance by defining

$$\dot{X} = AX + Bw, \tag{92}$$

$$q_{\mathrm{i}}(t) = CX + Dw, \tag{93}$$

$$w(t) = u(0,t) + sv(0,t). \tag{94}$$

Extending the results in [16] to incorporate disturbances of the form (92)–(94), and developing a controller for the attenuation of undesired in-/outfluxes, would be valuable contributions.

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