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Analysis of Performance of a Liquid Level Process Controlled by the Super-Twisting Algorithm

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Abstract— **Second-order sliding mode control algorithms are a popular research area. In the present research, a study of performance of the Super-Twisting (STW) controller is done, with application to a liquid level process. Comparison of its performance with the performance of an optimally tuned PID controller is done. The study is carried out through analytical approach, simulations and experiments. From the comparison of the performance, conclusions are made about suitability and perspectives of the use of the STW controller for the considered process.**

I. INTRODUCTION

Higher-order and second-order sliding mode (SOSM) control algorithms in particular have been actively studied within the control research community since their introduction in 1990s [1]. One of popular control algorithms in this class is the super-twisting (STW) algorithm [1]. An attractive feature of the STW algorithm is the continuous control that it provides. It is worth noting that the continuity of control signal does now eliminate chattering in the system controlled by the STW algorithm [2] because of the existence of a relay and a Lipschitz-discontinuous nonlinearity in the algorithm. However, if some degree of chattering is allowed in a system then this algorithm can be considered as a control algorithm candidate. A number of attempts of using this algorithm were reported in the literature [3], [4], [5].

The STW algorithm was designed for controlling plants having relative degree one. In this case, ideal sliding mode occurs and finite-time convergence of error to zero is ensured. As was proved in [2], if relative degree is higher than one or delay is present in the loop then the system with STW features chattering, which is revealed as oscillation of a constant frequency and amplitude. Finding the frequency and amplitude can be done with the use of the describing function (DF) method, as in [2], or other approaches. In real life applications one can never encounter a plant of relative degree one because of the necessity of such devices as sensors and actuators that add some dynamics into the loop, so that relative degree becomes higher than one. In that aspect, it makes sense to refer to relative degree *one* as only applying to *principal dynamics* of the plant [6] keeping in mind the existence of *parasitic dynamics*. Despite the fact that chattering is impossible to avoid, in practical terms severity of chattering depends on the contributions of principal and parasitic dynamics.

Unfortunately, beside [2] and a few other publications by the same authors, chattering effect in the STW algorithm was, probably, analyzed only in [7] and [8]. The vast majority of publications provide analysis of systems with the STW on the basis of the consideration of only principal dynamics, without account of parasitic dynamics. Consequently, analysis of performance of these systems provides results that probably are more optimistic than those obtained for models with parasitic dynamics included.

In the present paper, a liquid level process is considered as a plant (process) in a system controlled by the STW algorithm. The principal dynamics of this process has relative degree one, which satisfied the prerequisites of the STW algorithm and makes it a good candidate for achieving high performance. Control of level process is considered to be one of the most common control problems in the process industry. In a wider scope, the process can be categorized into three different types according to applications and objectives:

- The process control in which maintaining the level to a certain set point by either controlling the inflow or the outflow is the primary objective.
- The process control in steam drums of utility boilers and steam generators.
- The process control in which maintaining a stable out flow is the primary objective and large level fluctuations are allowed (surge vessels) [9].

In this paper, only the first category is considered (Fig. 1). In this setup, the tank has a controlled inflow and uncontrolled outflow which is considered a disturbance. The disturbance is considered a result of the connection or disconnection of consumers.

The paper is organized as follows. First, the process model is provided. Then the STW algorithm along with approach to system analysis is given. Third, the problem of optimal design of the STW algorithm is considered. Forth, a benchmark controller, being optimally tuned PI controller, is presented. Fifth, experimental setup along with its model is presented. Sixth, results of optimal design of the STW controller are presented, and performance of the system is evaluated through simulations and experiments. And finally, some conclusions are given.

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II. LEVEL PROCESS

Principal dynamics of the level process is given by the following equation:

$$\dot{x} = \frac{1}{A}[F_{in} - F_{out}], \tag{1}$$

where x is level, A is the cross-sectional area of a vessel, F_{in} inflow rate, F_{out} outflow rate. Or in Laplace domain it is

$$X(s) = \frac{1}{As}[F_{in}(s) - F_{out}(s)] \tag{2}$$

From equation (2) and Figure 1, if the outflow is considered as a disturbance, the transfer function shows that the level process is an integrator process. Transfer function for the manipulated variable and disturbance are as follows:

$$G_p = \frac{X(s)}{F_{in}(s)} = \frac{1}{As} \tag{3}$$

$$G_D = \frac{X(s)}{F_{out}(s)} = -\frac{1}{As} \tag{4}$$

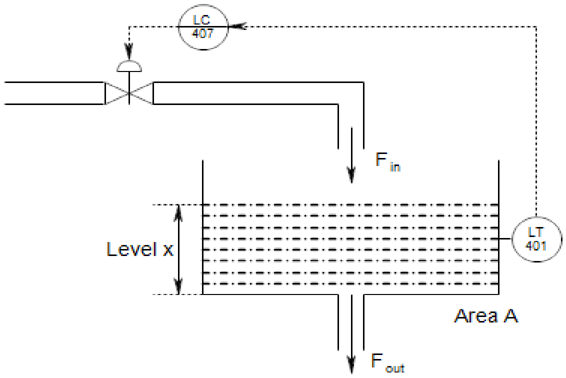

Figure 1. Considered Level Process

The set point is usually constant; therefore, a level controller is a regulator and the servo properties (set point tracking) are not specified. The main objective of the controller is to attenuate (reject) possible disturbances [2]. A disturbance is a change of outflow from a steady-state value, which may be caused by an abrupt change due to connection or disconnection of consumers. Therefore, the main control objective is usually to minimize the effect of this disturbance, which is manifested as level increase or decrease from the set point [10].

Normally, with the exception of the boilers and nuclear reactors (due to the shrink and swell effect), a level process is controlled by a PI or PID controller. Due to the necessary addition of the integral term in the PI/PID controller to remove any steady-state error it results in a double integrator in the loop leading to poor closed-loop response such as excessive overshoot and large settling time [10].

III. SUPER-TWISTING (STW) ALGORITHM

The so-called Super-Twisting Algorithm is one of the most popular second order sliding mode controllers that is designed for plants of relative degree one. Since the principal dynamics of the level process is an integrator with a relative degree one, this controller has the potential of showing a sufficient control performance in this application.

The control u for the super-twisting algorithm is given as a sum of two components [1]:

$$\begin{gathered} u(t) = u_1(t) + u_2(t) \\ \dot{u}_1(t) = -\gamma \mathrm{sign}(\sigma) \\ u_2 = \begin{cases} -\lambda |s_0|^{\rho} \mathrm{sign}(\sigma), if\ \ |\sigma| > s_0 \\ -\lambda |\sigma|^{\rho} \mathrm{sign}(\sigma), if\ \ |\sigma| \leq s_0 \end{cases} \end{gathered} \tag{5}$$

where s_0, ρ $(0.5 \leq \rho < 1)$, γ and λ are design parameters.

Block diagram of the STW controller is presented in Fig. 2. Whenever applied to linear plants with a relative degree greater than one, STW controlled systems always exhibit chattering in the form of periodic oscillations of the output variable [2]. Since the principal dynamics of the level process is an integrator, i.e. having relative degree one, and additional or parasitic dynamics always exists too, performance of the system and the chattering level depends on the contribution of the parasitic dynamics, which is often attributed to actuator and valve dynamics.

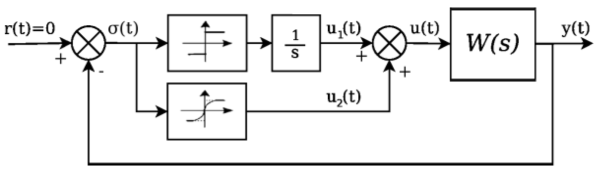

Figure 2. STW Controller

A. *Describing Function of STW*

DF of the STW algorithm was derived in [2]. The DF can be found through the sum of the two components. The first component is the result of the cascade connection of an ideal relay with an integrator:

$$N_1 = \frac{4\gamma}{\pi a_y} \frac{1}{j\omega}, \tag{6}$$

where a_y is the amplitude of the harmonic signal at the input to the STW algorithm (of chattering in the process output), ω is the frequency of the harmonic signal (chattering), γ is the relay amplitude. The DF of the second component is given as:

$$N_2 = \frac{2\lambda a_y^{\rho-1}}{\pi} \int_0^{\pi} (\sin\varphi)^{\rho+1}\, d\varphi$$

$$N_2 = \frac{2\lambda a_y^{p-1}}{\sqrt{\pi}} \frac{\Gamma\left(\frac{\rho}{2}+1\right)}{\Gamma\left(\frac{\rho}{2}+1.2\right)}, \quad 0 < \rho < 1 \tag{7}$$

For the value of ρ=0.5:

$$N_2 \approx \frac{1.1128\lambda}{\sqrt{a_y}} \tag{8}$$

Therefore, the DF of the STW for a value of ρ=0.5 is [2]:

$$N_{STW} = \frac{4\gamma}{\pi a_y} \frac{1}{j\omega} + \frac{1.1128\lambda}{\sqrt{a_y}} \tag{9}$$

DF (9) allows one to find the frequency and the amplitude of chattering in a system with the STW algorithm through solution of the harmonic balance equation:

$$N_{STW}(a_y,\Omega)W_p(j\Omega)=-1,$$

where Ω is the frequency of chattering $W_p(s)$ is the process transfer function.

B. STW Optimal Design and Tuning

Generally speaking, there are no any tuning rules for the STW controller. This is because the system under control by the STW algorithm is not linear, i.e. performance would depend not only on the plant model but also on initial conditions and specific performance requirements for a particular system. Therefore, controller design must be done for every particular plant/process depending on its dynamics and specific performance requirements. Yet, several researchers came up with different tuning algorithms to design the controller. For example, in [11] the STW is tuned manually by varying parameter values and observing overshoot, steady-state error and speed of convergence. However, this method suggests manual tuning aimed at reaching an acceptable performance (e.g. 5% overshoot), and other characteristics may not be tuned optimally. In addition, this method only provides general guidelines and might not lead to the same result for every process.

Other publications propose a tuning algorithm that allows shaping the characteristics of the chattering motion [7], [8]. This algorithm provides a constructive procedure to predetermine the desired frequency and the amplitude of the limit cycle and then find the controller parameters that would result in this chattering motion. This approach considers only the amplitude and the frequency of the limit cycle as characteristics of the system, which is not sufficient to ensure high performance of a system. Moreover, oscillations are present not only in the output but in other variables too, including the control signal. As it is shown in the present research, amplitude of chattering of the control signal is important and must be included in the design specifications too.

Consider main performance characteristics of the system that includes the liquid level process and the STW controller. As main performance measure we will consider reaction of level from the set point due to a step-wise change of the outflow. We shall evaluate performance in terms of two different cost functions for the level error signal:

- The Integral of the Absolute Error (IAE)

$$Q_{IAE}=\int_0^{\infty}|error(t)|\,dt \tag{10}$$

- The Integral of Time and the Absolute Error (ITAE)

$$Q_{ITAE}=\int_0^{\infty}t|error(t)|\,dt \tag{11}$$

The use of two different criteria is motivated by the considerations of reliability of our comparison with the benchmark controller. Only if both values (IAE and ITAE) of one controller are better (smaller) than those of the other one, we shall consider performance of the first controller higher. Because the system under STW control is nonlinear, its reaction to a change in outflow would depend on the magnitude of this change. Therefore, this magnitude must be selected as a practically reasonable value. In the considered example, the change is realized by opening (from fully closed position) or closing (from fully open position) of one of the two outlet valves on the water tank. The second valve has a constant opening and maintains a constant outflow from the tank, which is necessary to ensure proper functionality of the system (at fully closed outlet valves and positive error the required inflow to the tank would be a negative value). It is also worth noting that due to the existence of level fluctuations as a result of chattering we shall "subtract" the oscillatory component from the level trend when evaluating IAE or ITAE costs because otherwise both criteria become infinite values (see (10) and (11)).

We shall consider the constraint on the amplitude of level fluctuations due to chattering. Frequency of the oscillations is not an important parameter in this process while the amplitude is, because the primary objective of the controller is to maintain level constant, which is to minimize all fluctuations. We can consider that some level fluctuation, though undesirable, but can be tolerated. Therefore, some amplitudes of chattering (level fluctuations) below a_{ymax} are allowed.

We shall also consider the amplitude of chattering in the control signal as another constraint. The necessity of this constraint follows from the fact that there are physical limits on the control signal coming from the limits of the inflow rate. Inflow can be varied only between 0 and a certain maximum value. If control contains an oscillatory component then the effective regulation range becomes narrower by the double amplitude of the oscillations. Despite the fact that, although undesirable, but some amplitudes of chattering in the control signal can be allowed, we need to limit the possible amplitudes by a certain maximum value be a_{umax}.

We can now specify the requirements to system performance and formulate the design problem. The STW controller design for the liquid level process is formulated as the following nonlinear programming problem:

$$\text{minimize}\quad Q_{IAE(ITAE)}(\lambda,\gamma) \tag{12}$$

$$\text{subject to}\quad a_y(\lambda,\gamma)\le a_{y\max}, \tag{13}$$

$$\text{and}\quad a_u(\lambda,\gamma)\le a_{u\max}, \tag{14}$$

where parameters γ and λ are decision variables in the nonlinear programming problem, amplitude a_u is evaluated as:

$$a_u=a_y\left|N_{STW}(a_y,\Omega)\right|.$$

The problem (12) – (14) is separately solved for the IAE and ITAE cost functions.

IV. EXPERIMENTAL SETUP AND ITS MODEL

The setup that was used for this research is a multitank system produced by InTeco Ltd. The multitank system consists of upper, middle, and lower tanks equipped with level sensors, manual and control drain valves to stabilize the desired levels. In addition, there is a fourth tank in the lower part of the setup that acts like a reservoir and is connected to a variable speed pump that fills the upper tank depending on the control signal.

The multitank system is connected to a PC using the RT-DAC/PCI programmable Input/Output board and is controlled with the Matlab/Simulink Real Time Workshop software. In this research, only the upper tank is used since it has a constant cross-sectional area ($87.5 \text{x} 10^{-4}$ m^2).

A. System Model

The tank water level process model is comprised of the flow dynamic model relating controller command and water inflow rate and the level dynamic model relating inflow and outflow rates with the level (2). The flow model is given by the following transfer function [10]:

$$W_a(s) = K_v \frac{e^{-\tau s}}{T_a s + 1} \tag{18}$$

where K_v is a constant that relates steady states of the control signal and a flow signal, T_a is the time constant, τ is the delay.

The parameter values of the model (2), (18) were found through direct measurements (A and K_v) and identification (T_a and τ). Identification is done through the Modified Relay Feedback Test (MRFT) [12] and the use of the harmonic balance equation

$$N_{MRFT}(a_0) W_p(j\omega_0, T_a, \tau) = -1 \,. \tag{19}$$

Parameters of the oscillations in the MRFT a_0 and ω_0 were measured, and parameters of the model T_a and τ were found by solving the complex equation (19).

As a result, the model was found to have the following parameter values: cross-sectional area of the tank A=87.5 $\times 10^{-4}$ m^2, time constant of the inflow dynamics T_a=0.1556 s, delay of the inflow dynamics τ=0.0749 s, and the inflow dynamics gain K_v=0.0002 m^3/s. Static (steady state) flow dependence of the inflow rate on the controller command also includes a bias term, so the overall static dependence of the inflow on the controller command is given by the equation

$$F_{in} = 0.0002u - 0.000026$$

Due to the large amount of noise in the level signal caused by the turbulence of the inflow low-pass filtering was applied to the level signal. In particular, a fourth-order Butterworth low pass filter with a cut-off frequency of 10 rad/s was designed and implemented in the control loop. It was also included in the system model, so that the model of the process is given by Fig. 3.

B. Benchmark PI controller tuning

Tuning of the PI controller was done through use of the MRFT [12] along with level process-optimized tuning rules for the specification to a gain margin of 2 [10]. The parameter values for the PI controller were found to be K_c=69.1187, τ_I=2.2261 (or K_I =31.0498).

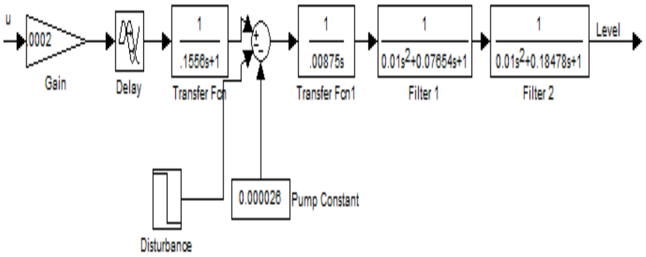

Figure 3. Process model

C. Optimal design of STW

A Matlab program was created to realize constrained parametric optimization (12) – (14). The optimization algorithm was based on the use of the Matlab function *fminsearch* and the Simulink model a part of which is shown in Fig. 3. A point of decision vector (λ, γ) is generated on every step, and through the solution of the differential equations of the system the reaction to a step change in the outflow is found, the cost (12) is computed, and satisfaction to the constraints (13), (14) is verified. At computing the cost (12), the oscillatory component was excluded from the cost. Iterations were carried out until the minimum cost was found, which produced the following optimal parameters of the STW algorithm: λ=6.8052 and γ=0.0602. It is worth mentioning that the optimization involved a high number of trials, which all lead to the same solution. Also, experiments were conducted with the meshing of parameter space (γ ,λ) and checking cost function values for every point (γ,λ) with over 100 different combinations in experiments. Both computer optimization and experimental search produced the same optimal solution.

V. RESULTS OF SIMULATION AND EXPERIMENTS

Experiments on the system and simulations of the corresponding model using Simulink were carried out. In addition to that, simulations of the system without low-pass filters were done, as well as simulations of the level model with parameters different from the parameters of the actual experimental setup. In simulations, instantaneous change in the outflow was considered the disturbance. In the actual experiments, this disturbance was applied through closing (opening) of one of the two drain valves installed on the tank. Level and controller output trends were recorded, and IAE and ITAE cost functions were automatically computed. Trends representing performance of the PI controller are given in Fig. 4 and 5, and trends representing performance of the STW controller are given in Fig. 6 and 7. Analysis and comparison of the performance is presented in the following sub-sections.

A. Actual Process Parameters

To ensure a fair comparison in terms of disturbance rejection between the controllers, the same disturbance was applied to all the controllers at the same time while calculating the ITAE and IAE values. First, the comparison was done for the actual process parameters in simulation and the low-pass filter in both experimental setup and the model. Both simulation and experimental results are shown in table 1.

TABLE 1
ACTUAL PARAMETERS SIMULATION & EXP. RESULTS

	PI		***STW***	
	ITAE	IAE	ITAE	IAE
Simulation	0.2940	0.0135	1.4821	0.0578
Experiment	0.5344	0.0231	0.9850	0.0412

One can see some mismatch between the experimental results and simulations, which can be explained partly by the existence of unfiltered fluctuations (low-pass filter cannot fully suppress the fluctuations caused by the flow turbulence and exciting the water surface in the tank), and partly by some residual mismatch between the model and the actual dynamics. However, in terms of comparison between the PI and STW controllers simulation results are very consistent with experiments. From both the simulation and experimental results one can see that that the STW algorithm shows a substantially worse performance than the optimally tuned PI controller (see Table 1 and Figs. 4-7). A possible explanation of this, which was accepted as a hypothesis, was the significant fluctuations of level in the tank and the necessity of the low-pass filter for their filtering. Another possible explanation might be the presence of relatively high delay in the flow dynamics, which was considered as the second hypothesis. Both hypotheses were verified in the following analysis.

B. Other Model Parameters

The same simulations were done using the model without low-pass filter. Both the PI and the STW controllers were optimally tuned again. The results are presented in Table 2 for τ/T_a=0.5 (row 1). In addition to that, the simulations were done for faster valve-actuator dynamics which was emulated by maintaining the same time constant and reducing the delay of the model. The results of this test are shown in Table. 2.

From the results, it can be seen that once the noise is not present and low-pass filter is not needed the STW still shows a worse performance than the PI controller. Yet, once the system gets a faster actuator-valve dynamics, the STW may show a better performance than the optimally tuned benchmark PI controller. This, however, is achieved at the value of the delay by 10 times smaller than the actual one. Feasibility of such dynamics in process control applications is questionable. Also, the existence of oscillations in the control signal of the STW algorithm (Figs. 6 – 7) narrows the regulation range. In the analysed system this reduction is approximately by 2 times. And what makes it especially inconvenient is the impossibility of providing small values of inflow: the control oscillations would be limited from below, which would disrupt the whole work of the control system.

TABLE 2
OTHER MODEL PARAMETERS IAE & ITAE SIMULATION RESULTS

τ/T_a ratio	***PI***		***STW***	
	ITAE	IAE	ITAE	IAE
0.5 ($\tau=0.08$)	0.0701	0.0032	0.1192	0.0049
0.2 ($\tau=0.032$)	0.0466	0.0011	0.0522	0.0021
0.1 ($\tau=0.016$)	0.0914	0.0021	0.0409	0.0016
0.05 ($\tau=0.008$)	0.0699	0.0016	0.0407	0.0016

VI. CONCLUSIONS

The objectives of this research were to optimally tune the STW controller for a level process, analyze its performance by comparison with that of an optimally tuned PI controller; to do the same analysis via simulations for other process model parameters aimed at finding a "break-even" point at which both controllers provide approximately the same performance, and verify results by experiments.

In the solution of the stated problem, a new approach was used to optimally design the STW controller. This design approach involves formulation and solution of the design problem as that of a nonlinear programming problem with parameters of the algorithm being the decision variables, criterion being an integral performance criterion of IAE or ITAE and a step change of the outflow being the disturbance. Inequality constraints on the amplitudes of chattering in level and controller output are also considered.

From the results obtained, the following conclusions can be made:

- In terms of design methodology: to encompass all possible aspects of controller design, the design of the STW controller should be formulated and solved as a nonlinear programming problem for a particular plant or process, with account of available control modes and disturbances.
- Experimental results show that the STW controller cannot outperform an optimally tuned PI controller when a substantial delay in the additional (parasitic) dynamics is present.
- If the delay were reduced, the STW controller could potentially outperform the optimally tuned PI controller (we disregard the problem of narrower regulation range of the STW controller in making this conclusion).
- However, process applications usually have a substantial delay in the loop, which makes the PI/PID controller more suitable.

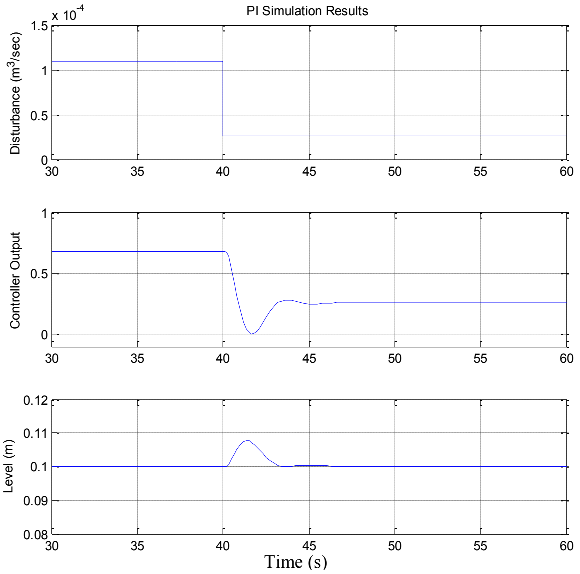

Figure 4. PI controller trends (simulations)

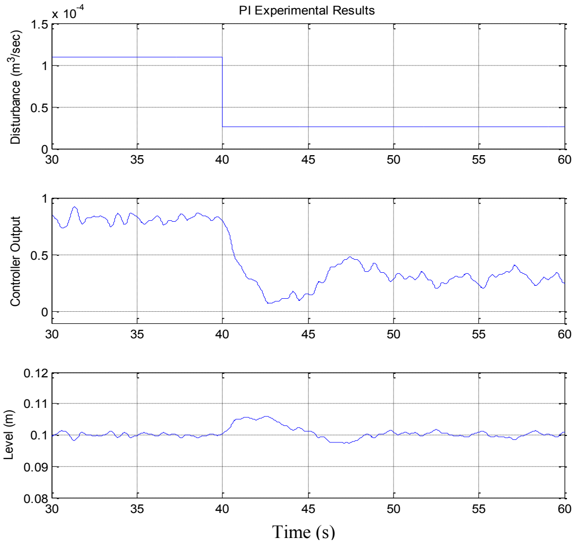

Figure 5. PI controller trends (experiment)

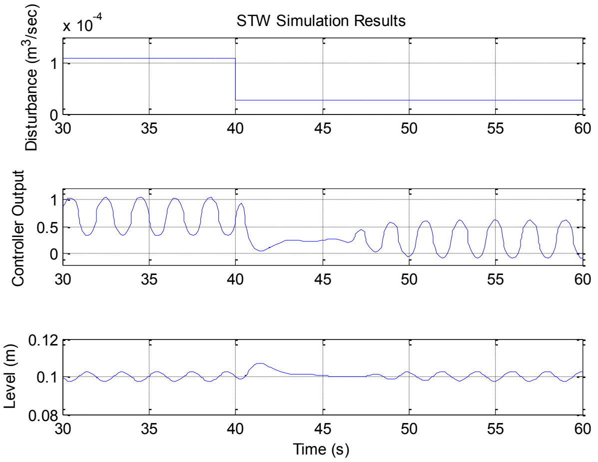

Figure 6. STW controller trends (simulations)

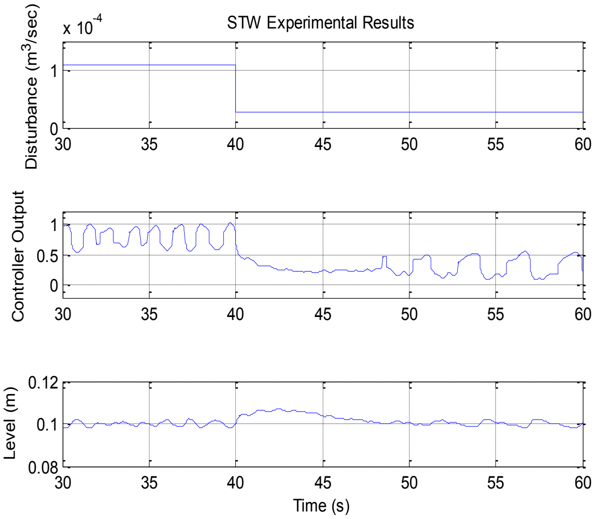

Figure 7. STW controller trends (experiments)

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