

\mathcal{H}_∞ LPV filtering for discrete-time linear systems subject to additive and multiplicative uncertainties in the measurement

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Abstract—This paper is concerned with the problem of \mathcal{H}_∞ linear parameter-varying (LPV) filter design for discrete-time linear systems where the measurement of the scheduling parameters may be affected by additive and multiplicative uncertainties. By conveniently modeling the uncertainties and the time-varying parameters, new robust linear matrix inequality (LMI) conditions for the existence of a full order LPV filter assuring a prescribed \mathcal{H}_∞ performance, irrespective of the uncertainties affecting the measures, are given. The design procedure can simultaneously handle time-invariant uncertainties and arbitrary time-varying parameters as well. The problem is solved through LMI relaxations based on homogeneous polynomial matrices of arbitrary degree. A numerical experiment illustrates the performance of the proposed LPV filter when compared to other filters obtained with methods from the literature.

I. INTRODUCTION

In the last years, the problems of gain-scheduling control [1–7] and filter design [8–11] have received a lot of attention. In most of the cases, however, the methods are not able to deal with the presence of uncertainties in the measurements. Some recent works have addressed this issue, as in the gain-scheduled control design for continuous-time systems with parameters subject to multiplicative [12] and additive [13] uncertainties, or in the filter design method [14] where additive uncertainties affect the scheduling parameters. In the discrete-time case, the design of output feedback controllers taking into account additive uncertainties in the scheduling parameters has been investigated in [15].

This paper is concerned with the problem of LPV filter design for discrete-time linear systems subject to inexactly measured scheduling parameters. The scheduling parameters, i.e., the ones used for the filter implementation, are supposed to be affected by both additive (representing a bias) and multiplicative (representing a percentage) bounded errors in the measures. The parameters and the uncertainties, supposed to be bounded, can be time-invariant or time-varying, and the time-varying ones can have known bounds on their rate of variation or vary arbitrarily fast. Each class of parameters is adequately modeled through a multi-simplex representation, i.e., the Cartesian product of simplexes [16, 17].

As main contribution, new robust linear matrix inequality (LMI) conditions for the existence of a full order LPV filter assuring a prescribed \mathcal{H}_∞ performance, irrespective of the

uncertainties affecting the measures, are given. By imposing a particular structure to the extra parameter-dependent matrices, the problem can be solved through LMI relaxations based on homogeneous polynomial matrices of arbitrary degree that are constructed in terms of the multi-simplex representation, accordingly to the different classes of parameters and uncertainties affecting the system. The advantages of the proposed method when compared to other techniques from the literature are illustrated through a numerical example.

II. PROBLEM DEFINITION

Consider the LPV discrete-time system

$$\begin{aligned} x(k+1) &= A(\theta(k))x(k) + B_1(\theta(k))w(k) \\ z(k) &= C_1(\theta(k))x(k) + D_{11}(\theta(k))w(k) \\ y(k) &= C_2(\theta(k))x(k) + D_{21}(\theta(k))w(k) \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state, $w(k) \in \mathbb{R}^r$ is the noise input, $z(k) \in \mathbb{R}^p$ is the signal to be estimated, $y(k) \in \mathbb{R}^q$ is the measured output, and $\theta(k) = (\theta_1(k), \dots, \theta_M(k))'$ is the vector of time-varying parameters that satisfy

$$|\theta_i(k)| \leq a_i, \quad a_i \in \mathbb{R}^+, \quad i = 1, \dots, M$$

For simplicity of notation, the dependence of $\theta(k)$ on k is omitted hereafter. All the matrices of system (1) are considered as affine parameter-dependent, i.e., any matrix in (1) can be represented as

$$Z(\theta) = Z_0 + \sum_{i=1}^M \theta_i Z_i \quad (2)$$

This paper deals with the problem of finding a full order parameter-dependent filter subject to inexactly measured scheduling parameters. Actually, the measured parameter is given by

$$\tilde{\theta}_i = (1 + \rho_i)(\theta_i + \delta_i), \quad i = 1, \dots, M \quad (3)$$

where θ_i is the real parameter, δ_i is the additive uncertainty and ρ_i is the multiplicative uncertainty satisfying

$$|\delta_i| \leq b_i, \quad |\rho_i| \leq c_i, \quad b_i, c_i \in \mathbb{R}^+, \quad i = 1, \dots, M$$

Note that (3) represents a more general modeling for the uncertainties, that can consider, as particular cases, only additive, as in [14], by imposing $\rho_i = 0$, or only multiplicative uncertainties, as in [12], by setting $\delta_i = 0$.

The full order filter to be designed is given by

$$\begin{aligned} x_f(k+1) &= A_f(\tilde{\theta})x_f(k) + B_f(\tilde{\theta})y(k) \\ z_f(k) &= C_f(\tilde{\theta})x_f(k) + D_f(\tilde{\theta})y(k) \end{aligned} \quad (4)$$

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where $x_f(k) \in \mathbb{R}^{n_f}$, $n_f = n$, is the filter state and $z_f(k) \in \mathbb{R}^p$ is the estimated output, such that the error dynamics is asymptotically stable and the energy gain from the disturbance input $w(k)$ to the error $e(k) = z(k) - z_f(k)$ (i.e., the \mathcal{H}_∞ norm) is bounded. Matrices $A_f(\tilde{\theta})$, $B_f(\tilde{\theta})$, $C_f(\tilde{\theta})$ and $D_f(\tilde{\theta})$ in (4) present an affine dependence on $\tilde{\theta}_i$ and can be generically represented as follows

$$Z_f(\tilde{\theta}) = Z_{f_0} + \sum_{i=1}^M ((1 + \rho_i)(\theta_i + \delta_i)Z_{f_i}) \quad (5)$$

III. UNCERTAINTIES MODELING

Let the unit simplex (of dimension r) be given by

$$\Lambda_r = \left\{ \zeta \in \mathbb{R}^r : \sum_{i=1}^r \zeta_i = 1, \zeta_i \geq 0, i = 1, \dots, r \right\} \quad (6)$$

Definition 1 (Multi-Simplex): [16] A multi-simplex Λ_N is the Cartesian product $\Lambda_{N_1} \times \dots \times \Lambda_{N_m}$ of a finite number of simplex. The dimension of Λ_N is defined as the index $N = (N_1, \dots, N_m)$. A given element α of Λ_N is decomposed as $(\alpha_1, \alpha_2, \dots, \alpha_m)$, according to the structure of Λ_N and subsequently, each α_i is decomposed in the form $(\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{iN_i})$.

In order to explore the benefits and advantages of the multi-simplex representation, the following change of variables is applied

$$\begin{aligned} \alpha_{i1} &= \frac{\theta_i + a_i}{2a_i}, \quad \alpha_{i2} = 1 - \alpha_{i1}, \quad \tilde{\alpha}_{i1} = \frac{\delta_i + b_i}{2b_i}, \\ \tilde{\alpha}_{i2} &= 1 - \tilde{\alpha}_{i1}, \quad \hat{\alpha}_{i1} = \frac{\rho_i + c_i}{2c_i}, \quad \hat{\alpha}_{i2} = 1 - \hat{\alpha}_{i1}, \\ \alpha_i, \tilde{\alpha}_i, \hat{\alpha}_i &\in \Lambda_2, \quad i = 1, \dots, M \end{aligned} \quad (7)$$

Using the proposed change of variables it is possible to transform the original affine dependence into a multi-simplex representation. In this case the variables of the multi-simplex are $\tilde{\alpha} = (\alpha, \tilde{\alpha}, \hat{\alpha})$, where $\alpha = (\alpha_1, \dots, \alpha_M)$, $\tilde{\alpha} = (\tilde{\alpha}_1, \dots, \tilde{\alpha}_M)$, $\hat{\alpha} = (\hat{\alpha}_1, \dots, \hat{\alpha}_M)$ and $\alpha_i = (\alpha_{i1}, \alpha_{i2}) \in \Lambda_2$ is related to θ_i , $\tilde{\alpha}_i = (\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}) \in \Lambda_2$ is related to δ_i and $\hat{\alpha}_i = (\hat{\alpha}_{i1}, \hat{\alpha}_{i2}) \in \Lambda_2$ is related to ρ_i , for $i = 1, \dots, M$. For instance, if $M = 1$, (5) yields

$$Z_f(\tilde{\theta}) = Z_{f_0} + (\theta_1 + \delta_1)Z_{f_1} + \rho_1(\theta_1 + \delta_1)Z_{f_1} \quad (8)$$

With the proposed changes of variables, (8) can be rewritten in terms of $\tilde{\alpha}$ as

$$\begin{aligned} Z_f(\tilde{\alpha}) &= Z_{f_0} + (c_1 - 1)(a_1 + b_1)Z_{f_1} + (2a_1 - 2a_1c_1)\alpha_{11}Z_{f_1} \\ &\quad + (2b_1 - 2b_1c_1)\tilde{\alpha}_{11}Z_{f_1} - 2c_1(a_1 + b_1)\hat{\alpha}_{11}Z_{f_1} \\ &\quad + 4a_1c_1\alpha_{11}\hat{\alpha}_{11}Z_{f_1} + 4c_1b_1\tilde{\alpha}_{11}\hat{\alpha}_{11}Z_{f_1} \end{aligned} \quad (9)$$

and applying a homogenization procedure for polynomials in the multi-simplex (see [16] for details) one has a polynomial matrix with parameters in a multi-simplex of dimension $N = (2, 2, 2)$

$$\begin{aligned} Z_f(\tilde{\alpha}) &= \alpha_{11}\tilde{\alpha}_{11}\hat{\alpha}_{11}T_{111} + \alpha_{11}\tilde{\alpha}_{11}\hat{\alpha}_{12}T_{112} + \alpha_{11}\tilde{\alpha}_{12}\hat{\alpha}_{11}T_{121} \\ &\quad + \alpha_{11}\tilde{\alpha}_{12}\hat{\alpha}_{12}T_{122} + \alpha_{12}\tilde{\alpha}_{11}\hat{\alpha}_{11}T_{211} + \alpha_{12}\tilde{\alpha}_{11}\hat{\alpha}_{12}T_{212} \\ &\quad + \alpha_{12}\tilde{\alpha}_{12}\hat{\alpha}_{11}T_{221} + \alpha_{12}\tilde{\alpha}_{12}\hat{\alpha}_{12}T_{222} \end{aligned} \quad (10)$$

whose matrix valued coefficients are

$$\begin{aligned} T_{111} &= Z_{f_0} + (a_1 + b_1 + c_1a_1 + c_1b_1)Z_{f_1} \\ T_{112} &= Z_{f_0} + (a_1 + b_1 - c_1a_1 - c_1b_1)Z_{f_1} \\ T_{121} &= Z_{f_0} + (a_1 - b_1 + c_1a_1 - c_1b_1)Z_{f_1} \\ T_{122} &= Z_{f_0} + (a_1 - b_1 - c_1a_1 + c_1b_1)Z_{f_1} \\ T_{211} &= Z_{f_0} + (-a_1 + b_1 - c_1a_1 + c_1b_1)Z_{f_1} \\ T_{212} &= Z_{f_0} + (-a_1 + b_1 + c_1a_1 - c_1b_1)Z_{f_1} \\ T_{221} &= Z_{f_0} + (-a_1 - b_1 - c_1a_1 - c_1b_1)Z_{f_1} \\ T_{222} &= Z_{f_0} + (-a_1 - b_1 + c_1a_1 + c_1b_1)Z_{f_1} \end{aligned}$$

This procedure can be extended to cope with any polynomial matrix with $M \geq 1$, yielding (11) (top of next page). In this way, the filter matrices $A_f(\tilde{\alpha})$, $B_f(\tilde{\alpha})$, $C_f(\tilde{\alpha})$ and $D_f(\tilde{\alpha})$ have appropriate dimensions and depend on the time-varying parameter $\tilde{\alpha}$ as $Z_f(\tilde{\alpha})$ in (11).

Note that the additive and the multiplicative uncertainties do not affect the system matrices in (1). Nevertheless, equation (11) can be used to compute the system matrices in the multi-simplex form. It suffices to impose $b = 0$ and $c = 0$ in (11), to produce

$$A(\tilde{\alpha}), B_1(\tilde{\alpha}), C_1(\tilde{\alpha}), D_{11}(\tilde{\alpha}), C_2(\tilde{\alpha}), D_{21}(\tilde{\alpha}) \quad (12)$$

IV. PARAMETER VARIATION MODELING

The parameter variation is modeled as in [18]. The time-varying parameters α , $\tilde{\alpha}$, $\hat{\alpha}$ have bounded rate of variations given by b , \tilde{b} and \hat{b} respectively, with $b, \tilde{b}, \hat{b} \in [0, 1]$. To avoid repetitions, only the modeling for the parameter α is presented, since the parameters $\tilde{\alpha}$ and $\hat{\alpha}$ can be modeled following the same procedure.

The case $b = 0$ corresponds to the classical time-invariant uncertain systems [19], and $b = 1$ corresponds to the systems with arbitrary time-varying parameters [20]. In time-varying discrete-time systems with limited rate of variation, i.e., the case $0 < b < 1$, the maximum variation $\Delta\alpha(k)$ depends on the actual value of $\alpha(k)$ as shown in Figure 1. Any feasible pair $(\alpha_{im}, \Delta\alpha_{im})$, $m = 1, 2$, belongs to the polytope Γ_{im} , $i = 1, \dots, M$, given by

$$\Gamma_{im} \triangleq \left\{ \delta \in \mathbb{R}^2 : \delta = \sum_{j=1}^6 \gamma_j h_i^{(j)}, \quad \gamma \in \Lambda_6 \right\}$$

with

$$\left[h_i^{(1)} \quad \dots \quad h_i^{(6)} \right] = \begin{bmatrix} 0 & 0 & 1-b & 1 & 1 & b \\ 0 & b & b & 0 & -b & -b \end{bmatrix}$$

that is, Γ_{im} is the convex hull of vertices of the feasible area related to the parameter α_{im} .

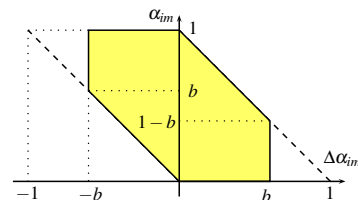


Fig. 1. Region on the plane $\Delta\alpha_{im} \times \alpha_{im}$ where $\Delta\alpha_{im}$ can assume values as a function of α_{im} .

$$Z_f(\tilde{\alpha}) = \sum_{i_1=1}^2 \cdots \sum_{i_M=1}^2 \sum_{k_1=1}^2 \cdots \sum_{k_M=1}^2 \sum_{\ell_1=1}^2 \cdots \sum_{\ell_M=1}^2 \alpha_{i_1} \cdots \alpha_{M i_M} \bar{\alpha}_{1 k_1} \cdots \bar{\alpha}_{M k_M} \hat{\alpha}_{1 \ell_1} \cdots \hat{\alpha}_{M \ell_M} T_{i_1 \dots i_M k_1 \dots k_M \ell_1 \dots \ell_M} \quad (11)$$

$$T_{i_1 \dots i_M k_1 \dots k_M \ell_1 \dots \ell_M} = Z_{f_0} + \sum_{j=1}^M \left[(-1)^{i_j+1} a_j + (-1)^{k_j+1} b_j + (-1)^{\ell_j+i_j} c_j a_j + (-1)^{k_j+\ell_j} c_j b_j \right] Z_{f_j}$$

To model the parametric space $(\alpha_{im}, \Delta\alpha_{im})$, $m = 1, 2$, the Cartesian product of $\Gamma_{i1} \times \Gamma_{i2} = \Gamma_i$ must be taken into account, discarding all the vertices that do not respect the constraints

$$\sum_{m=1}^2 \alpha_{im} = 1, \quad \alpha_{im} \geq 0, \quad \sum_{m=1}^2 \Delta\alpha_{im} = 0, \quad i = 1, \dots, M, \quad m = 1, 2 \quad (13)$$

The resulting polytope Γ_i is given by

$$\Gamma_i \triangleq \{ \delta \in \mathbb{R}^4 : \delta = \sum_{j=1}^6 \gamma_j h_i^{(j)}, \quad \gamma \in \Lambda_6 \} \quad (14)$$

where the $h_i^{(j)} \in \mathbb{R}^4$ (conveniently reordered) are given by

$$\begin{bmatrix} h_i^{(1)} & \dots & h_i^{(6)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & b & (1-b) \\ 0 & 0 & 1 & 1 & (1-b) & b \\ 0 & -b & 0 & b & -b & b \\ 0 & b & 0 & -b & b & -b \end{bmatrix} = \begin{bmatrix} f_{i1} \\ f_{i2} \\ g_{i1} \\ g_{i2} \end{bmatrix}$$

From (14), any fixed $(\alpha_{i1}, \alpha_{i2}, \Delta\alpha_{i1}, \Delta\alpha_{i2})' \in \Gamma_i$ is related to $\gamma \in \Lambda_6$ through the linear transformation

$$\begin{bmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \Delta\alpha_{i1} \\ \Delta\alpha_{i2} \end{bmatrix} = \begin{bmatrix} f_{i1} \\ f_{i2} \\ g_{i1} \\ g_{i2} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix} \quad (15)$$

The main benefit of this approach is that $(\alpha_{im}, \Delta\alpha_{im})$, $m = 1, 2$, are jointly considered in an augmented space of dimension four. The parameters $(\alpha, \hat{\alpha}, \bar{\alpha})$ are written as in (15), being represented in the domain $\Gamma = \Gamma_1 \times \dots \times \Gamma_M$ by $(\gamma, \hat{\gamma}, \bar{\gamma})$. Hereafter, the symbols ‘hat’ and ‘bar’ are used to identify variables related to $\hat{\alpha}$ or $\bar{\alpha}$, while variables with no superscript are related to α . The system matrices, filter variables and Lyapunov matrix are represented through the multi-simplex in an augmented space Γ . The variables of the multi-simplex are $\tilde{\gamma} = (\gamma, \bar{\gamma}, \hat{\gamma})$, where $\gamma = (\gamma_1, \dots, \gamma_M)$, $\bar{\gamma} = (\bar{\gamma}_1, \dots, \bar{\gamma}_M)$, $\hat{\gamma} = (\hat{\gamma}_1, \dots, \hat{\gamma}_M)$ and $\gamma_i = (\gamma_{i1}, \dots, \gamma_{i6}) \in \Lambda_6$ is related to α , $\bar{\gamma}_i = (\bar{\gamma}_{i1}, \dots, \bar{\gamma}_{i6}) \in \Lambda_6$ is related to $\bar{\alpha}$ and $\hat{\gamma}_i = (\hat{\gamma}_{i1}, \dots, \hat{\gamma}_{i6}) \in \Lambda_6$ is related to $\hat{\alpha}$, for $i = 1, \dots, M$. For instance, considering $M = 1$, writing α , $\bar{\alpha}$, $\hat{\alpha}$ as in (15) as a function of γ , $\hat{\gamma}$, $\bar{\gamma}$ and replacing in (10) one has

$$Z_f(\tilde{\gamma}) = \begin{bmatrix} T'_{111} \\ T'_{112} \\ T'_{121} \\ T'_{122} \\ T'_{211} \\ T'_{212} \\ T'_{221} \\ T'_{222} \end{bmatrix}' \begin{bmatrix} f_{11} \otimes \bar{f}_{11} \otimes \hat{f}_{11} \\ f_{11} \otimes \bar{f}_{11} \otimes \hat{f}_{12} \\ f_{11} \otimes \bar{f}_{12} \otimes \hat{f}_{11} \\ f_{11} \otimes \bar{f}_{12} \otimes \hat{f}_{12} \\ f_{12} \otimes \bar{f}_{11} \otimes \hat{f}_{11} \\ f_{12} \otimes \bar{f}_{11} \otimes \hat{f}_{12} \\ f_{12} \otimes \bar{f}_{12} \otimes \hat{f}_{11} \\ f_{12} \otimes \bar{f}_{12} \otimes \hat{f}_{12} \end{bmatrix} \begin{bmatrix} \gamma_{11} \bar{\gamma}_{11} \hat{\gamma}_{11} \\ \gamma_{11} \bar{\gamma}_{11} \hat{\gamma}_{12} \\ \vdots \\ \gamma_{11} \bar{\gamma}_{11} \hat{\gamma}_{16} \\ \vdots \\ \gamma_{16} \bar{\gamma}_{16} \hat{\gamma}_{15} \\ \gamma_{16} \bar{\gamma}_{16} \hat{\gamma}_{16} \end{bmatrix} = \tilde{T}' \tilde{F} \tilde{\Gamma} \quad (16)$$

where \bar{f}_{im} and \hat{f}_{im} , $i = 1, \dots, M$, $m = 1, 2$, can be obtained as f_{im} . The construction of matrices \tilde{T} , \tilde{F} and $\tilde{\Gamma}$ can be systematically extended to the case $M > 1$. From (16), a generic procedure to obtain $Z_f(\tilde{\gamma})$ is given by (17) (top of next page), where $T_{i_1 \dots i_M k_1 \dots k_M \ell_1 \dots \ell_M}$ is as in (11), $\tilde{F}(r, c)$ is the entry of matrix \tilde{F} at row r and column c , with r and c given by

$$r = \sum_{i=i_1}^{i_M} (i-1)2^2 + \sum_{k=k_1}^{k_M} (k-1)2 + \sum_{\ell=\ell_1}^{\ell_M} (\ell-1) + 1$$

$$c = \sum_{j=j_1}^{j_M} (j-1)6^2 + \sum_{w=w_1}^{w_M} (w-1)6 + \sum_{z=z_1}^{z_M} (z-1) + 1$$

The filter matrices $A_f(\tilde{\gamma})$, $B_f(\tilde{\gamma})$, $C_f(\tilde{\gamma})$ and $D_f(\tilde{\gamma})$ have appropriate dimensions and depend on the parameter $\tilde{\gamma}$ as $Z_f(\tilde{\gamma})$ in (17). Applying the transformation in (15) to the system matrices in (12) to produce $A(\tilde{\gamma})$, $B_1(\tilde{\gamma})$, $C_1(\tilde{\gamma})$, $D_{11}(\tilde{\gamma})$, $C_2(\tilde{\gamma})$ and $D_{21}(\tilde{\gamma})$, an augmented system can be defined

$$\begin{bmatrix} x(k+1) \\ x_f(k+1) \end{bmatrix} = \begin{bmatrix} A(\tilde{\gamma}) & 0 \\ B_f(\tilde{\gamma})C_2(\tilde{\gamma}) & A_f(\tilde{\gamma}) \end{bmatrix} \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix} + \begin{bmatrix} B_1(\tilde{\gamma}) \\ B_f(\tilde{\gamma})D_{21}(\tilde{\gamma}) \end{bmatrix} w(k)$$

$$e(k) = \begin{bmatrix} C_1(\tilde{\gamma}) - D_f(\tilde{\gamma})C_2(\tilde{\gamma}) & -C_f(\tilde{\gamma}) \end{bmatrix} \begin{bmatrix} x(k) \\ x_f(k) \end{bmatrix} + [D_{11}(\tilde{\gamma}) - D_f(\tilde{\gamma})D_{21}(\tilde{\gamma})] w(k)$$

with $\tilde{x}(k)' = [x(k)' \quad x_f(k)']$, yielding

$$\begin{aligned} \tilde{x}(k+1) &= \tilde{A}(\tilde{\gamma})\tilde{x}(k) + \tilde{B}(\tilde{\gamma})w(k) \\ e(k) &= \tilde{C}(\tilde{\gamma})\tilde{x}(k) + \tilde{D}(\tilde{\gamma})w(k) \end{aligned} \quad (18)$$

where

$$\tilde{A}(\tilde{\gamma}) = \begin{bmatrix} A(\tilde{\gamma}) & 0 \\ B_f(\tilde{\gamma})C_2(\tilde{\gamma}) & A_f(\tilde{\gamma}) \end{bmatrix} \in \mathbb{R}^{2n \times 2n}, \quad (19)$$

$$\tilde{B}(\tilde{\gamma}) = \begin{bmatrix} B_1(\tilde{\gamma}) \\ B_f(\tilde{\gamma})D_{21}(\tilde{\gamma}) \end{bmatrix} \in \mathbb{R}^{2n \times r}, \quad (20)$$

$$\tilde{C}(\tilde{\gamma}) = [C_1(\tilde{\gamma}) - D_f(\tilde{\gamma})C_2(\tilde{\gamma}) \quad -C_f(\tilde{\gamma})] \in \mathbb{R}^{p \times 2n}, \quad (21)$$

$$\tilde{D}(\tilde{\gamma}) = [D_{11}(\tilde{\gamma}) - D_f(\tilde{\gamma})D_{21}(\tilde{\gamma})] \in \mathbb{R}^{p \times r} \quad (22)$$

and vector $\tilde{\gamma} = (\gamma, \bar{\gamma}, \hat{\gamma})$, belongs to the multi-simplex Λ_N , for all $k \geq 0$.

To determine any matrix at the instant $k+1$, i.e., $(\alpha + \Delta\alpha)$ one has to apply the transformations in (15), replacing \tilde{F} by

$$Z_f(\tilde{\gamma}) = \sum_{j_1=1}^6 \cdots \sum_{j_M=1}^6 \sum_{w_1=1}^6 \cdots \sum_{w_M=1}^6 \sum_{z_1=1}^6 \cdots \sum_{z_M=1}^6 \gamma_{j_1} \cdots \gamma_{M j_M} \tilde{\gamma}_{1 w_1} \cdots \tilde{\gamma}_{M w_M} \hat{\gamma}_{1 z_1} \cdots \hat{\gamma}_{M z_M} X_{j_1 \dots j_M w_1 \dots w_M z_1 \dots z_M} \quad (17)$$

$$X_{j_1 \dots j_M w_1 \dots w_M z_1 \dots z_M} = \sum_{i_1=1}^2 \cdots \sum_{i_M=1}^2 \sum_{k_1=1}^2 \cdots \sum_{k_M=1}^2 \sum_{\ell_1=1}^2 \cdots \sum_{\ell_M=1}^2 T_{i_1 \dots i_M k_1 \dots k_M \ell_1 \dots \ell_M} \tilde{F}(r, c)$$

\tilde{G} in (16), with

$$\tilde{G} = \begin{bmatrix} (f_{11} + g_{11}) \otimes (\tilde{f}_{11} + \tilde{g}_{11}) \otimes (\hat{f}_{11} + \hat{g}_{11}) \\ (f_{11} + g_{11}) \otimes (\tilde{f}_{11} + \tilde{g}_{11}) \otimes (\hat{f}_{12} + \hat{g}_{12}) \\ (f_{11} + g_{11}) \otimes (\tilde{f}_{12} + \tilde{g}_{12}) \otimes (\hat{f}_{11} + \hat{g}_{11}) \\ (f_{11} + g_{11}) \otimes (\tilde{f}_{12} + \tilde{g}_{12}) \otimes (\hat{f}_{12} + \hat{g}_{12}) \\ (f_{12} + g_{12}) \otimes (\tilde{f}_{11} + \tilde{g}_{11}) \otimes (\hat{f}_{11} + \hat{g}_{11}) \\ (f_{12} + g_{12}) \otimes (\tilde{f}_{11} + \tilde{g}_{11}) \otimes (\hat{f}_{12} + \hat{g}_{12}) \\ (f_{12} + g_{12}) \otimes (\tilde{f}_{12} + \tilde{g}_{12}) \otimes (\hat{f}_{11} + \hat{g}_{11}) \\ (f_{12} + g_{12}) \otimes (\tilde{f}_{12} + \tilde{g}_{12}) \otimes (\hat{f}_{12} + \hat{g}_{12}) \end{bmatrix} \quad (23)$$

Furthermore, a generic expression can be formulated as in (17) by changing \tilde{F} by \tilde{G} and using appropriately the coefficients T obtained from the homogenization procedure.

V. PARAMETER-DEPENDENT LMI CONDITIONS

First, a parameter-dependent matrix inequality condition with parameter-dependent slack variables is presented. By imposing a particular structure to the variables, sufficient parameter-dependent conditions that assure an \mathcal{H}_∞ performance bound for the augmented system (18) are obtained.

Lemma 1: The maximum energy gain from $w(k)$ to $e(k)$ in system (18) is limited by μ , and $\tilde{A}(\alpha)$ is asymptotically stable for all $\tilde{\gamma} \in \Lambda_N$ if there exist parameter-dependent symmetric positive definite matrices $W(\tilde{\gamma}) \in \mathbb{R}^{2n \times 2n}$ and $\tilde{W}(\tilde{\gamma}) \in \mathbb{R}^{2n \times 2n}$, parameter-dependent matrices $K(\tilde{\gamma}) \in \mathbb{R}^{2n \times 2n}$, $E(\tilde{\gamma}) \in \mathbb{R}^{2n \times 2n}$, $Q(\tilde{\gamma}) \in \mathbb{R}^{r \times 2n}$ and $F(\tilde{\gamma}) \in \mathbb{R}^{p \times 2n}$ such that

$$\Psi(\tilde{\gamma}) + \Theta_d(\tilde{\gamma}) > 0, \quad \forall \tilde{\gamma} \in \Lambda_N \quad (24)$$

with¹

$$\Theta_d(\alpha) = \text{diag}(W(\tilde{\gamma}), -\tilde{W}(\tilde{\gamma}), \mathbf{I}_r, \mu^2 \mathbf{I}_p) \quad (25)$$

and

$$\Psi(\tilde{\gamma}) = \begin{bmatrix} K(\tilde{\gamma})\tilde{A}(\tilde{\gamma}) + \tilde{A}(\tilde{\gamma})'K(\tilde{\gamma})' & \star & \star & \star \\ -K(\tilde{\gamma})' + E(\tilde{\gamma})\tilde{A}(\tilde{\gamma}) & -E(\tilde{\gamma}) - E(\tilde{\gamma})' & \star & \star \\ \tilde{B}(\tilde{\gamma})'K(\tilde{\gamma})' + Q(\tilde{\gamma})\tilde{A}(\tilde{\gamma}) & \tilde{B}(\tilde{\gamma})'E(\tilde{\gamma})' - Q(\tilde{\gamma}) & \star & \star \\ F(\tilde{\gamma})\tilde{A}(\tilde{\gamma}) + \tilde{C}(\tilde{\gamma}) & -F(\tilde{\gamma}) & \star & \star \\ \star & \star & \star & \star \\ \tilde{B}(\tilde{\gamma})'Q(\tilde{\gamma})' + Q(\tilde{\gamma})\tilde{B}(\tilde{\gamma}) & \star & \star & \star \\ F(\tilde{\gamma})\tilde{B}(\tilde{\gamma}) + \tilde{D}(\tilde{\gamma}) & 0 & \star & \star \end{bmatrix} \quad (26)$$

Proof: Suppose that (24) holds. Then, multiplying it by Π on the left and by Π' on the right, with

$$\Pi = \begin{bmatrix} \mathbf{I}_{2n} & \tilde{A}(\tilde{\gamma})' & 0 & 0 \\ 0 & \tilde{B}(\tilde{\gamma})' & \mathbf{I}_r & 0 \\ 0 & 0 & 0 & \mathbf{I}_p \end{bmatrix} \quad (27)$$

¹The symbol \star means a symmetric block.

and by applying Schur complement one gets

$$\begin{bmatrix} W(\tilde{\gamma}) & \tilde{A}(\tilde{\gamma})'\tilde{W}(\tilde{\gamma}) & 0 & \tilde{C}(\tilde{\gamma})' \\ \star & \tilde{W}(\tilde{\gamma}) & \tilde{W}(\tilde{\gamma})B(\tilde{\gamma}) & 0 \\ \star & \star & \mathbf{I}_r & \tilde{D}(\tilde{\gamma})' \\ \star & \star & \star & \mu^2 \mathbf{I}_p \end{bmatrix} > 0 \quad (28)$$

This can be recognized as the bounded real lemma for discrete time-varying systems [21]. Therefore, $\tilde{A}(\tilde{\gamma})$ is asymptotically stable and the energy gain from w to e is bounded by μ , $\forall \tilde{\gamma} \in \Lambda_N$. $\tilde{W}(\tilde{\gamma})$ is obtained from $W(\tilde{\alpha} + \Delta\tilde{\alpha})$. ■

Lemma 1 presents a sufficient condition assuring a bound μ to the energy gain from $w(k)$ to $e(k)$ of system (18), where the slack variables are used to separate the Lyapunov matrix from the system matrices.

However, the parameter-dependent condition exhibits nonlinear terms, involving products between the filter matrices and slack variables. In order to linearize the conditions presented in Lemma 1, structural constraints are imposed to the parameter-dependent matrices $K(\tilde{\gamma})$, $E(\tilde{\gamma})$, $Q(\tilde{\gamma})$ and $F(\tilde{\gamma})$ (similarly to the approaches in [19, 22, 23]) as follows

$$K(\tilde{\gamma}) = \begin{bmatrix} K_{11}(\tilde{\gamma}) & \lambda_1 \hat{K} \\ K_{21}(\tilde{\gamma}) & \lambda_2 \hat{K} \end{bmatrix}, \quad E(\tilde{\gamma}) = \begin{bmatrix} E_{11}(\tilde{\gamma}) & \hat{K} \\ E_{21}(\tilde{\gamma}) & \hat{K} \end{bmatrix}, \\ Q(\tilde{\gamma}) = [Q_1(\tilde{\gamma}) \quad 0], \quad F(\tilde{\gamma}) = [F_1(\tilde{\gamma}) \quad 0] \quad (29)$$

where $\hat{K} \in \mathbb{R}^{n \times n}$ is a constant matrix and λ_1, λ_2 are scalar variables to be determined. Matrix $W(\tilde{\gamma})$ and $\tilde{W}(\tilde{\gamma})$, are partitioned in $n \times n$ blocks as

$$W(\tilde{\gamma}) = \begin{bmatrix} W_{11}(\tilde{\gamma}) & W_{12}(\tilde{\gamma}) \\ \star & W_{22}(\tilde{\gamma}) \end{bmatrix} \quad (30)$$

and the following change of variables

$$K_1(\tilde{\gamma}) = \hat{K}A_f(\tilde{\gamma}), \quad K_2(\tilde{\gamma}) = \hat{K}B_f(\tilde{\gamma})$$

is applied, implying that $K_1(\tilde{\gamma})$ and $K_2(\tilde{\gamma})$ have the same form of $A_f(\tilde{\gamma})$ and $B_f(\tilde{\gamma})$, that is, they are represented as in (17). With the above particular choices for the decision variables, parameter-dependent LMI conditions that are sufficient to assure a solution to Lemma 1 can be formulated in such a way that the filter matrices $A_f(\tilde{\gamma})$, $B_f(\tilde{\gamma})$, $C_f(\tilde{\gamma})$ and $D_f(\tilde{\gamma})$ are readily computed.

Theorem 1: If there exist symmetric parameter-dependent positive definite matrices $W(\tilde{\gamma})$, $\tilde{W}(\tilde{\gamma})$, parameter-dependent matrices $K(\tilde{\gamma})$, $E(\tilde{\gamma})$, $Q(\tilde{\gamma})$, $F(\tilde{\gamma})$ as in (29), $K_1(\tilde{\gamma}) \in \mathbb{R}^{n \times n}$, $K_2(\tilde{\gamma}) \in \mathbb{R}^{n \times q}$, $C_f(\tilde{\gamma}) \in \mathbb{R}^{p \times n}$, $D_f(\tilde{\gamma}) \in \mathbb{R}^{p \times q}$, $\mu > 0$ and scalars λ_1, λ_2 , such that

$$\bar{\Psi}(\tilde{\gamma}) + \Theta_d(\tilde{\gamma}) > 0, \quad \forall \tilde{\gamma} \in \Lambda_N \quad (31)$$

with $\Theta_d(\tilde{\gamma})$ given by

$$\Theta_d(\tilde{\gamma}) = \text{diag} \left(\begin{bmatrix} W_{11}(\tilde{\gamma}) & W_{12}(\tilde{\gamma}) \\ \star & W_{22}(\tilde{\gamma}) \end{bmatrix}, \begin{bmatrix} -\tilde{W}_{11}(\tilde{\gamma}) & -\tilde{W}_{12}(\tilde{\gamma}) \\ \star & -\tilde{W}_{22}(\tilde{\gamma}) \end{bmatrix}, \begin{bmatrix} \mathbf{I}_r & 0 \\ 0 & \mu^2 \mathbf{I}_p \end{bmatrix} \right) \quad (32)$$

and $\bar{\Psi}$ given by (33) (top of next page), then

$$A_f(\tilde{\gamma}) = \hat{K}^{-1} K_1(\tilde{\gamma}), B_f(\tilde{\gamma}) = \hat{K}^{-1} K_2(\tilde{\gamma}), C_f(\tilde{\gamma}), D_f(\tilde{\gamma}) \quad (34)$$

are the matrices of the parameter-dependent filter that assures an \mathcal{H}_∞ performance for system (18) bounded by μ .

Proof: Follows the steps of the proof of Lemma 1. ■

VI. IMPLEMENTATION ISSUES

Theorem 1 presents a sufficient condition in terms of a robust LMI, i.e., a parameter-dependent LMI condition that need to be verified for all $\tilde{\gamma} \in \Lambda_N$. Parameter-dependent LMIs can be solved, for instance, by a sequence of LMI relaxations as proposed in [16, 24]. The Lyapunov matrix $W(\tilde{\gamma})$, matrices $K(\tilde{\gamma})$, $E(\tilde{\gamma})$, $Q(\tilde{\gamma})$ and $F(\tilde{\gamma})$ in Theorem 1 are assumed as homogeneous polynomials in Λ_N . The systematic way to derive the LMI relaxations in this case can be found in [16]. It is worth to emphasize that the filter matrices A_{f_i} and B_{f_i} , $i = 0, \dots, M$, are obtained from (34) by imposing to the decision variables of Theorem 1, $K_1(\tilde{\gamma})$ and $K_2(\tilde{\gamma})$ (as well as directly to $C_f(\tilde{\gamma})$ and $D_f(\tilde{\gamma})$), the structure in (17).

The degrees of the decision polynomial variables used in the numerical experiments are defined as follows.

1) The partial degrees associated to the Lyapunov matrix $W(\tilde{\gamma})$ are given by vector $g = (g_1, \dots, g_M)$, with $g_i = (g_{\theta_i}, g_{\delta_i}, g_{\rho_i})$, $i = 1, \dots, M$.

- a) for arbitrarily fast time-varying parameters $g_i = (0, 0, 0)$ (i.e., quadratic stability);
- b) time-invariant parameters and time-varying parameters that have known bounds can use g_{θ_i} , g_{δ_i} and g_{ρ_i} of arbitrary degree.

2) The slack variables $K_{11}(\tilde{\gamma})$, $K_{21}(\tilde{\gamma})$, $E_{11}(\tilde{\gamma})$, $E_{21}(\tilde{\gamma})$, $Q_1(\tilde{\gamma})$ and $F_1(\tilde{\gamma})$ are chosen as polynomials of degree f (for simplicity, all the slack variables are considered to have the same degree).

3) The partial degrees related to polynomial variables $K_1(\tilde{\gamma})$, $K_2(\tilde{\gamma})$, $C_f(\tilde{\gamma})$ and $D_f(\tilde{\gamma})$ are denoted by $s = (s_1, \dots, s_M)$, with $s_i = (s_{\theta_i}, s_{\delta_i}, s_{\rho_i})$, $i = 1, \dots, M$. In this paper s_{θ_i} , s_{δ_i} and s_{ρ_i} can assume values 1 (affine filter) or 0 (robust filter).

- a) affine filters can be obtained when the parameters are supposed to be available for measurements under additive $s_i = (1, 1, 0)$, multiplicative $s_i = (1, 0, 1)$ or both uncertainties $s_i = (1, 1, 1)$ in real time;
- b) when the parameter is not available for measurement, a robust filter can be designed, $s_i = (0, 0, 0)$.

VII. NUMERICAL EXPERIMENT

The objective of the experiments is to compare the results provided by the conditions proposed in this paper with other methods from the literature. The routines were implemented in MATLAB, version 7.1.0.246 (R14) SP 3 using the packages

Yalmip [25] and SeDuMi [26]. Line searches in λ_i , $i = 1, 2$, could further improve the \mathcal{H}_∞ bounds, but $\lambda_1 = \lambda_2 = 0$ have been used in this paper with good results.

Consider the discrete time-varying system from [27]

$$A(\theta) = \begin{bmatrix} 0.265 - 0.165\theta_1(k) & 0.45(1 + \theta_1(k)) \\ 0.5(1 - \theta_1(k)) & 0.265 - 0.215\theta_1(k) \end{bmatrix},$$

$$B_1(\theta) = \begin{bmatrix} 1.5 - 0.5\theta_1(k) \\ 0.1 \end{bmatrix}, \quad C_1(\theta) = \mathbf{I}_2, \quad D_{11}(\theta) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$C_2(\theta) = [1 \ 0], \quad D_{21}(\theta) = [1]$$

with $|\theta_1(k)| \leq 1$, with additive uncertainty $|\delta| \leq \xi$, multiplicative uncertainty $|\rho| \leq \varphi$ and bounds for the rate of variation given by b , \bar{b} and \hat{b} .

First, only the additive uncertainty has been considered. Table I presents the \mathcal{H}_∞ bounds obtained through LMI relaxations based on Theorem 1 for $g = (1, 1, 0)$, $f = 1$, $s = (1, 1, 0)$ and $b = \bar{b}$. It is important to note that Theorem 1 can provide smaller \mathcal{H}_∞ bounds for $b < 1$, emphasizing the importance of taking into account the bounded time-varying parameters, mainly for larger values of ξ . For this example, the method in [27] (that cannot deal with uncertainties in the measurement) provides an LPV filter with guaranteed cost \mathcal{H}_∞ of 1.22 for arbitrarily fast parameter variation ($b = 1$). As can be seen in Table I, the proposed method is no more conservative than the one in [27].

TABLE I
 \mathcal{H}_∞ BOUNDS OBTAINED THROUGH LMI RELAXATIONS BASED ON THEOREM 1 FOR DIFFERENT VALUES OF b AND ξ .

ξ	b				
	0	0.2	0.5	0.8	1
0	1.104	1.138	1.168	1.213	1.214
0.01	1.106	1.139	1.171	1.222	1.244
0.1	1.121	1.163	1.252	1.519	3.267
0.2	1.160	1.282	1.582	2.281	4.681
0.4	1.964	2.038	2.404	3.350	6.225
0.5	2.233	2.328	2.681	3.701	6.750
1	2.423	2.875	3.420	4.835	8.166

Table II presents the results obtained considering additive and multiplicative uncertainties for $g = (1, 1, 1)$, $s = (1, 1, 1)$, $f = 1$, $b = \bar{b} = \hat{b} = 1$ and different values of ξ and φ . In this case the parameters and the uncertainties can vary arbitrarily fast. Note that the presence of multiplicative uncertainty results in larger values for the \mathcal{H}_∞ performance bounds.

VIII. CONCLUSIONS

LMI conditions for the design of \mathcal{H}_∞ LPV filters for discrete-time systems subject to inexactly measured scheduling parameters were proposed. The multi-simplex representation combined with the use of homogeneous polynomially parameter-dependent variables of independent partial degrees and the parameter variation modeling provided a flexible tool for filter design, taking into account the different types of parameters and uncertainties in the model (time-invariant, arbitrarily time-varying or time-varying with bounded rates of variation) in a systematic way.

$$\Psi(\tilde{\gamma}) = \begin{bmatrix} K_{11}(\tilde{\gamma})A(\tilde{\gamma}) + A(\tilde{\gamma})'K_{11}(\tilde{\gamma})' & A(\tilde{\gamma})'K_{21}(\tilde{\gamma})' + \lambda_2 C_2(\tilde{\gamma})'K_2(\tilde{\gamma})' + \lambda_1 K_1(\tilde{\gamma}) & -K_{11}(\tilde{\gamma}) + A(\tilde{\gamma})'E_{11}(\tilde{\gamma})' \\ + \lambda_1 (C_2(\tilde{\gamma})'K_2(\tilde{\gamma})' + K_2(\tilde{\gamma})C_2(\tilde{\gamma})) & \lambda_2 (K_1(\tilde{\gamma}) + K_1(\tilde{\gamma})') & + C_2(\tilde{\gamma})'K_2(\tilde{\gamma})' \\ * & * & -K_{21}(\tilde{\gamma}) + K_1(\tilde{\gamma})' \\ * & * & -E_{11}(\tilde{\gamma}) - E_{11}(\tilde{\gamma})' \\ * & * & * \\ * & * & * \\ -\lambda_1 \hat{K} + A(\tilde{\gamma})'E_{21}(\tilde{\gamma})' + C_2(\tilde{\gamma})'K_2(\tilde{\gamma})' & K_{11}(\tilde{\gamma})B_1(\tilde{\gamma}) + \lambda_1 K_2(\tilde{\gamma})D_{21}(\tilde{\gamma}) + A(\tilde{\gamma})'Q_1(\tilde{\gamma})' & A(\tilde{\gamma})'F_1(\tilde{\gamma})' + C_1(\tilde{\gamma})' - C_2(\tilde{\gamma})'D_f(\tilde{\gamma})' \\ -\lambda_2 \hat{K} + K_1(\tilde{\gamma})' & K_{21}(\tilde{\gamma})B_1(\tilde{\gamma}) + \lambda_2 K_2(\tilde{\gamma})D_{21}(\tilde{\gamma}) & -C_f(\tilde{\gamma})' \\ -\hat{K} - E_{21}(\tilde{\gamma})' & E_{11}(\tilde{\gamma})B_1(\tilde{\gamma}) + K_2(\tilde{\gamma})D_{21}(\tilde{\gamma}) - Q_1(\tilde{\gamma})' & -F_1(\tilde{\gamma})' \\ -\hat{K} - \hat{K}' & E_{21}(\tilde{\gamma})B_1(\tilde{\gamma}) + K_2(\tilde{\gamma})D_{21}(\tilde{\gamma}) & 0 \\ * & Q_1(\tilde{\gamma})B_1(\tilde{\gamma}) + B_1(\tilde{\gamma})'Q_1(\tilde{\gamma})' & B_1(\tilde{\gamma})'F_1(\tilde{\gamma})' + D_{11}(\tilde{\gamma})' - D_{21}(\tilde{\gamma})'D_f(\tilde{\gamma})' \\ * & * & 0 \end{bmatrix} \quad (33)$$

TABLE II

\mathcal{H}_∞ BOUNDS OBTAINED THROUGH LMI RELAXATIONS BASED ON THEOREM 1, FOR $b = 1$ AND DIFFERENT VALUES OF φ AND ξ .

ξ	φ					
	0.001	0.01	0.02	0.04	0.05	0.1
0.01	1.249	1.314	1.463	1.996	2.286	3.460
0.1	3.285	3.459	3.636	3.947	4.085	4.660
0.2	4.691	4.777	4.870	5.046	5.130	5.511
0.4	6.229	6.272	6.321	6.412	6.456	6.657
0.5	6.753	6.784	6.817	6.882	6.912	7.057
1	8.166	8.166	8.166	8.166	8.166	8.166

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