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Disturbance Feedback for Handling Uncertainty in Air Traffic Flow Management

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***Abstract*— This paper presents the novel application of disturbance feedback optimization techniques for uncertainty management in Air Traffic Flow Management (ATFM) problems. The efficiency of ATFM optimizations in preventing local demand-capacity imbalances is reliant on predictions of future capacity states. However these predictions are inherently uncertain due to factors such as weather effects and unscheduled demand.**

A pre-existing ATFM flow based model is augmented to include feedback on the disturbances which perturb the weather scenario away from the nominal. Two formulations for modelling the feedback disturbance signal are explored.

Results are presented demonstrating the benefits, in terms of reduced delays, of incorporating feedback on the problem solutions over single-solution approaches. Some initial studies of the relative computational scaling properties are also presented, demonstrating that taking advantage, within the formulation, of linearly related scenarios can yield computational advantages. Directions for further computational improvement are also discussed.

I. INTRODUCTION

Increasing levels of demand in global air traffic over the last few decades have begun to stretch the air traffic management (ATM) system [1]. This trend is set to continue with the Federal Aviation Administration (FAA) predicting in 2009 that commercial passenger numbers within the US will reach one billion by 2021 [2]. In order to meet the predicted traffic levels, improvements are needed in all areas of ATM.

In order to facilitate the growth in air traffic, optimization techniques have been applied to many aspects of ATM. The focus in this paper is on the subset of ATM dealing with allocating airspace resources such that the balance between capacity and demand is maintained subject to both enroute and airport capacity constraints. This is known as Air Traffic Flow Management (ATFM) and many studies, including References [3]–[8], have applied optimization to solve the problem.

The ATFM optimization problem involves finding the 'best' sector usage policy (subject to some objective), whilst preventing capacity violations within the problem airspace. The solution consists of a sector-time plan for each flight. Initial ideal flight plans are altered via control actions. Control actions available are delays to the arrival, departure, and sector crossing times within the flight plans.

In this paper a Eulerian-Lagrangian or flow-based model of the ATFM problem is used. These models aggregate the traffic into flows and do not track individual aircraft plans [5], [9], [10] within the optimization. However separating the flows by destination allows some knowledge of individual flights [6], [8], ensuring all flights reach the correct destination. As flights are aggregated a separate optimization stage is required to assign individual flights to the flow solution delays. This is not discussed explicitly in this paper but has been extensively covered previously, for example in [11].

One of the difficulties in improving the performance of ATFM through optimization is the presence of uncertainty. Future capacity state predictions are inherently uncertain due to factors such as weather effects and unscheduled demand. Our goal is to see if improvements can be made in ATFM performance by explicitly incorporating information about uncertainty within the optimization.

Feedback Model Predictive Control (MPC) methods offer a well studied approach to dealing with uncertainty. These formulations introduce, in the control optimization, the notion that one can have a fundamental plan based on the idea that feedback is present in the control implementation, meaning that the resulting action will take into account the effects of possible future disturbances [12].

In this paper we are particularly interested in linear feedback on disturbances on the airspace capacity. Disturbance feedback has been explored in the MPC community [13]. The idea here is to design, within the optimization, reactions to disturbances which may perturb the capacity state from the predicted values. This is less conservative than single-plan approaches allowing solutions which respond to the differing capacity realizations.

In this paper a pre-existing Eulerian-Lagrangian optimization model of ATFM is augmented to include disturbance feedback. Two formulations of the disturbance signal to feedback are explored. Firstly a scenario tree option is considered, this approach is well studied in the scenario based MPC literature [12], [14], [15]. The disadvantage of this type of formulation is that the computational demands of the feedback algorithms can be very high. As a result a second uncertainty formulation is considered which introduces virtual aircraft traversing the airspace to represent lost capacity. The advantage of this formulation is that, computationally, it scales more favourably with increasing numbers of scenarios, given that some of the scenarios are linear combinations of others.

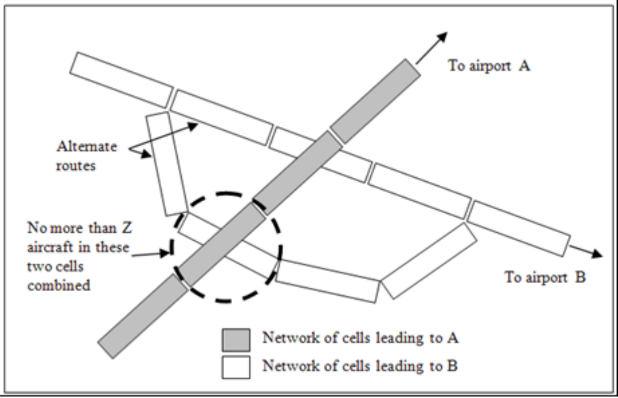

Fig. 1. Eulerian-Lagrangian Network Model

The paper begins with a detailed outline of the ATFM baseline flow model, to which the disturbance feedback is later appended. Section III explains the uncertainty model and describes in more detail the two formulations of feedback explored. Results demonstrating the effects of the feedback on the solutions are then presented, along with the computational scaling under increasing numbers of flights, time periods and scenarios. Finally suggestions for further computational improvement are presented followed by some concluding remarks.

II. BASE ATFM MODEL

The baseline Eulerian-Lagrangian, or flow based, model implemented is a slight reformulation of the model presented in Sun and Bayen [6] which was inspired by techniques of modelling road traffic [16]. Instead of modelling individual motions, it models flows of aircraft between cells in a network, using integer optimization to ensure only integer numbers of aircraft are present.

A. Problem Definition Parameters

The parameters needed to define the problem fall into three categories: Airspace Information, Flight Plan Information and Capacity Information.

- **Airspace Information** A set of enroute sectors, $\mathcal{S}$, and airport sectors, $\mathcal{A}$, partitioning all of the operating airspace into a set of non-overlapping volumes on which capacity restrictions are applied.
  The flow is modelled on a series of parallel, separate networks of 'cells', each with a different destination ensuring that the right number of flights arrive at each destination. To construct these networks each flight path is broken down into a series of 'cells', one for each sector it passes through. The flight paths are then grouped by destination so that cells representing the same route portion are not repeated within the network for any given destination. This results in the final set of cells, $\mathcal{C}$ as illustrated in Figure 1. This parallel network construction means that where a sector in physical space is shared by aircraft heading to two different destinations, the model includes two separate cells, without any routing connection between them. The capacity constraints for any given physical volume can be easily applied by summing over the set of cells, $\mathcal{B}(v)$, associated with the airspace volume, v.
- **Flight Plan Information** For each flight, f in the set of flights, $\mathcal{F}$, to be considered in the problem, a flight plan is provided. This is converted into the network parameters:
  - $\mathcal{P}^i$ set of possible preceding cells to cell i
  - $\mathcal{L}^i$ set of possible subsequent cells to cell i
  - $\mathcal{B}(s)$ Set of cells in sector/airport, s

  and the driving inputs and costs:
  - $f^i(k)$ No. of aircraft scheduled to enter via cell i at time k
  - $x^i(0)$ No. of aircraft in cell i at time 0
  - c_a cost of 1 time period of airborne delay
  - c_g cost of 1 time period of ground delay
- **Nominal Capacity Information** For each sector, j in the set of sectors, $\mathcal{S}$, in each time period, k, in the set of time periods, $\mathcal{T}$, in the planning horizon of problem, a nominal capacity $C_s(k)$ is defined. Similarly airport arrival and departure capacities, $C_a^{\text{arr}}(k)$ $C_a^{\text{dep}}(k)$, are also defined. The fluctuations in capacity caused by uncertainty are modelled separately, as will be discussed in Section III.

B. Decision Variables

The decision variables introduced by the Sun and Bayen [6] model are integer variables which encode the control actions introduced.

$$\forall k \in \mathcal{T}$$
$$u^{i,j}(k) = \text{No. flights moving, cell } i \rightarrow j \text{ in time } k \quad (1)$$

Note the $u^{i,j}$ are only defined for indices where they are variable, i.e. for cell indices which represent a pair of connected cells, including (i, i). Sun and Bayen also use a cell state variable, $x^i(k)$, which represents the aircraft count in each cell, i, at each time period, k. We have reformulated to eliminate this variable to aid the inclusion of feedback.

C. Objective

The objective to be minimized is a weighted combination of airborne delay and ground based delay, allowing the imbalance in the costs to be represented.

$$J = \min \sum_{k \in \mathcal{T}} \left(\sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{B}(s)} c_a u^{i,i}(k) + \sum_{a \in \mathcal{A}} \sum_{i \in \mathcal{B}(a)} c_g u^{i,i}(k) \right) \quad (2)$$

Where c_a and c_g are the weightings on airborne delay and ground delay respectively.

D. System Validity Constraints

The following constraints ensure that the solution created is valid. The conservation of aircraft in cell i from time $k-1$

to time k is maintained via

$$\forall i \in \mathcal{C}, k \in \mathcal{T} : k > 1$$
$$\sum_{j \in \mathcal{P}^i} u^{j,i}(k) = f^i(k) + \sum_{j \in \mathcal{P}^i} u^{j,i}(k-1)$$

with the initial number given by

$$\sum_{j \in \mathcal{P}^i} u^{j,i}(1) = f^i(1) + x^i(0) \qquad \forall i \in \mathcal{C}. \tag{3}$$

Moreover, at any step, the number of aircraft that can be controlled in each cell must be in the set of non-negative integers. This in enforced by Eqn 4.

$$u^{j,i}(k) \geq 0 \qquad \forall i \in \mathcal{C}, j \in \mathcal{P}^i \tag{4}$$

In a departure from the Sun and Bayen model [6], we consider that the flow of aircraft has a minimum number of time periods required to cross each cell, T^i, allowing us to use fewer cells. The minimum dwell time within each cell is then enforced by the following constraint, inspired by a constraint presented in Sun et al. [11]:

$$\forall i \in \mathcal{C}, k \in \mathcal{T} :$$
$$f^i(1) + \sum_{t \in \mathcal{T}: t < k} \left(f^i(t+1) + \sum_{j \in \mathcal{P}^i} u^{j,i}(t) \right) \geq \sum_{j \in \mathcal{L}^i} \sum_{\substack{t \in \mathcal{T}: \\ t \geq T^i \\ t < k + T^i}} u^{i,j}(t) \tag{5}$$

A final constraint ensures that the flights make a plan which finishes within the problem horizon.

$$\sum_{i \in \mathcal{B}(a)} u^{i,i}(\bar{t}) \geq N_d(a) \qquad \forall a \in \mathcal{A} \tag{6}$$

Where $\bar{t}$ is the final time step in the problem horizon and $N_d(a)$ is the number of aircraft whose destination is airport a.

E. Capacity Constraints

The Eulerian-Lagrangian model deals with airborne sector capacity restrictions across the whole time period by considering all flights in the sector at the beginning of time period k, as well as those which might enter during time period k:

$$\forall s \in \mathcal{S}, k \in \mathcal{T} : k > 1$$
$$\sum_{i \in \mathcal{B}(s)} \left(\sum_{j \in \mathcal{P}^i} u^{j,i}(k-1) + \sum_{\substack{j \in \mathcal{P}^i \\ i \neq j}} u^{j,i}(k) \right) \leq C_s(k) \tag{7}$$

Similarly airport arrival capacities are handled thus:

$$\forall a \in \mathcal{A}, k \in \mathcal{T} : k > 1$$
$$\sum_{i \in \mathcal{B}(a)} \sum_{\substack{j \in \mathcal{P}^i \\ i \neq j}} u^{j,i}(k) \leq C_a^{\text{arr}}(k) \tag{8}$$

and airport departure capacities are handled thus:

$$\forall a \in \mathcal{A}, k \in \mathcal{T} : k > 1$$
$$\sum_{i \in \mathcal{B}(a)} \sum_{\substack{j \in \mathcal{L}^i \\ i \neq j}} u^{i,j}(k) \leq C_a^{\text{dep}}(k) \tag{9}$$

III. Uncertainty Modelling

Two feedback models are considered in this paper, each providing a different feedback mechanism. Both models begin with the assumption that uncertainty in the airspace capacity is handled by considering a series of possible outcomes or scenarios. A scenario is a realization of the uncertain parameters in the given time horizon. In order to take into account the capacity uncertainty a set of capacity reduction scenarios, $\mathcal{E}$, are defined each having a given probability of occurrence and an associated set of capacity reductions.

Defining uncertainty in terms of scenarios is a popular approach and much previous work has been done on the definition of capacity reduction scenarios based on weather forecast data and airspace configuration data [17]–[19]. Recently this work has been brought together by Taylor et al. [20] to allow a representative sample of weather impact scenarios to be developed with associated probabilities. It is therefore assumed that such scenarios are available.

Some ATFM models have previously been explored which create tailored solutions for each scenario, such as that of Agustin et al. [21], however a feedback formulation was not used. The first feedback model presented here gathers the scenarios into a tree structure and feedback then acts on the binary branching nodes of the tree. In the second model, the storms are represented as virtual aircraft taking up capacity within the sectors and their uncontrollable movement actions are fed back to generate the solutions. In both models the objective is augmented to take into account the delays introduced in the feedback solutions as well as the nominal case.

$$\min \sum_{e \in \mathcal{E}} \epsilon_e J^e$$

Note that coefficients ϵ_e are weighted such that they relate to the probability of their respective scenarios occurring. Each model is described in more detail below.

A. Scenario Tree

In the scenario tree model, predefined scenarios are grouped into a tree structure, as shown in Figure 2. In this structure each root-to-leaf path represents an individual scenario. Each white branching node represents a point in time at which the scenarios diverge. Beyond the branching points there is a difference between the separate scenario branches, such as uncertainty in the speed, strength and path of disruptive storms.

The branching points are represented mathematically for each scenario, e, by the binary disturbance signals $W_n(e)$. Future decisions, to be made after $t_w(n)$, can then vary using feedback from signal W_n. The four scenarios in the example tree shown in Figure 2 are represented by 3 binary decisions.

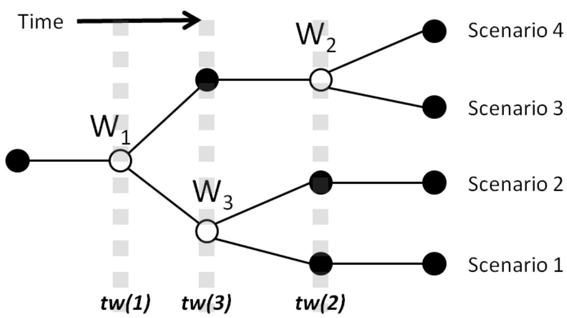

Fig. 2. Scenario Tree

1) Incorporating Feedback: The following equation describes how feedback can be incorporated such that the decision variables in the models are allowed to vary with the disturbance signal $W_n(e)$.

$$\forall i \in \mathcal{C}, j \in \mathcal{L}^i, e \in \mathcal{E}, k \in \mathcal{T}:$$
$$u^{i,j}(k,e) = v^{i,j}(k) + \sum_{n:t_w(n)<k}^{Nw} N_k^{i,j}(n) W_n(e) \quad (10)$$

where the new decision variables are $v^{i,j}(k)$, an affine or nominal decision, and $N_k^{i,j}(n)$, the feedback term relating uncertainty signal W_n to decision i at time k. Substituting (10) into the Sun and Bayen model [6] yields the final form of the flow model constraints with disturbance feedback. The capacity constraint of Eqn 7 is augmented to also include a term representing the capacity reduction, $q(e,s,k)$, for each time period, scenario, and sector, as follows.

$$\forall s \in \mathcal{S}, e \in \mathcal{E}, k \in \mathcal{T}: k > 1$$
$$\sum_{i \in \mathcal{B}(s)} \left(\sum_{j \in \mathcal{P}^i} u^{j,i}(k-1,e) + \sum_{\substack{j \in \mathcal{P}^i \\ i \neq j}} u^{j,i}(k,a) \right) \leq C_s(k) - q(e,s,k) \quad (11)$$

B. Virtual Aircraft

In this model the capacity uncertainty is represented as virtual aircraft taking up capacity within the sectors. Each capacity reduction within each scenario is represented as an uncontrollable virtual aircraft which traverses a series of arcs between the sectors and airports. Instead of control actions defining the movements, similar yet predefined actions are defined based on interpretation of the known capacity scenarios.

$$u_d^{i,j}(k,e) = \begin{cases} 1 & \text{if a virtual aircraft moves, cell } i \to j \\ & \text{in time } k, \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Note that $u_d^{i,j}$ is binary, unlike $u^{i,j}$ in order to avoid non-integer numbers of aircraft being fed back. As a result, to represent multiple capacity reductions, multiple parallel virtual arcs are defined.

1) Incorporating Feedback: In this formulation the uncontrollable movement actions, $u_d^{i,j}(k,e)$, provide the feedback mechanism within the optimizer for the solution.

$$\forall i \in \mathcal{C}, j \in \mathcal{L}^i, e \in \mathcal{E}, k \in \mathcal{T}: \qquad u^{i,j}(k,e) =$$
$$v^{i,j}(k) + \sum_{i' \in \mathcal{C}} \sum_{j' \in \mathcal{P}_d^{i'}} \sum_{k'}^{k'<k} N_{i',j'}^{i,j}(k,k') u_d^{i',j'}(k',e) \quad (13)$$

This formulation results in the same solutions as Scenario Tree. However, we can reduce the number of feedback variables, N, by only having feedback from independent uncertainty variables, i.e. by ignoring those values of $u_d^{i,j}$ that are perfectly correlated with earlier values. Using linear algebra, we can identify a matrix F that maps all $u_d^{i,j}$ values to a unique independent set of values $\mathcal{I}$, such that:

$$\forall i \in \mathcal{C}, j \in \mathcal{L}_d^i, e \in \mathcal{E}, k \in \mathcal{T}:$$
$$u_d^{i,j}(k,e) = \sum_{(i',j',k') \in \mathcal{I}} F(i,j,k,i',j',k') u_d^{i',j'}(k',e) \quad (14)$$

where $|\mathcal{I}| \leq |\mathcal{E}|$. This implies that any linear feedback of the form (13) is equivalent to a different linear feedback from an earlier variable $u_d^{i,j}(k)$ with $(i,j,k) \in \mathcal{I}$. Hence we can remove feedback from all variables other than those in $\mathcal{I}$, thus reducing the number of variables in the problem. This reduction may result in some difference in the solutions when compared to the Scenario Tree method.

The capacity constraint previously described in Eqn 7 now includes the virtual aircraft so becoming:

$$\forall s \in \mathcal{S}, e \in \mathcal{E}, k \in \mathcal{T}: k > 1$$
$$\sum_{i \in \mathcal{B}(s)} \left(\sum_{j \in \mathcal{P}^i} u^{j,i}(k-1,e) + \sum_{\substack{j \in \mathcal{P}^i \\ i \neq j}} u^{j,i}(k,e) \right) + \sum_{i \in \mathcal{B}_d(s)} \left(\sum_{j \in \mathcal{P}_d^i} u_d^{j,i}(k-1,e) + \sum_{\substack{j \in \mathcal{P}_d^i \\ i \neq j}} u_d^{j,i}(k,e) \right) \leq C_s(k) \quad (15)$$

IV. Results

A. Implementation

The optimization was translated into the AMPL modelling language [22]. An AMPL model file contains the constraint forms for all instances, while the data is written to an AMPL data file by a Matlab script. CPLEX 10.1 optimization software is used on a 3.4GHz PC with 2.98GB of RAM to solve all problems.

B. Basic Problem Set-Up

The example problems presented in this work are all drawn from a set of 30 flights between 5 airports, and across the 17 sectors shown in Figure 3. Each flight has an associated

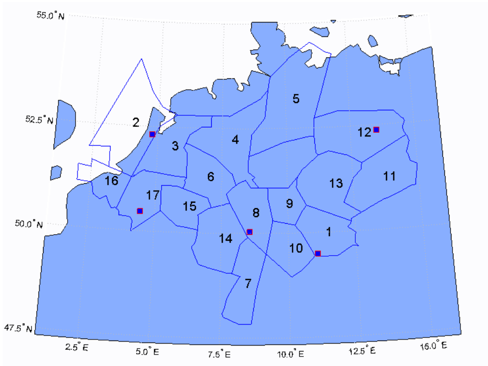

Fig. 3. Problem Scope

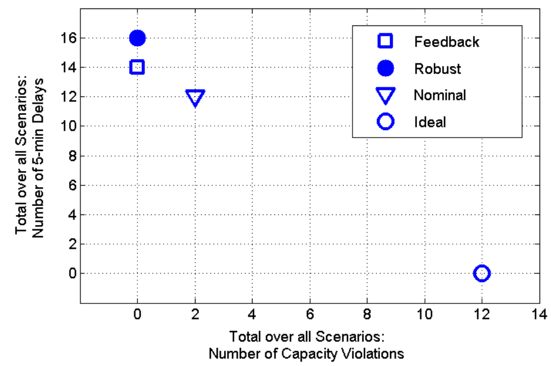

Fig. 4. Trade Plot: Delays Introduced vs Capacity Violations Observed

ideal trajectory and plan. All airspace sectors involved in the problem are considered to have a 5 aircraft capacity in any given 5-minute time window.

C. Benefits of Feedback

In this section the benefits of feedback in the solution of a 10 flight case are considered. The 10 flights were selected from the 30-flight set chronologically from the start of the set. A set of 4 capacity reduction scenarios was defined involving two storms, one with uncertain speed passing through the problem airspace. Optimizations were conducted with objective weightings $c_a = 2$ and $c_g = 1$. Four solutions will be compared. Firstly the ideal plan in which no delays are introduced will be analysed, then the solution considering only the most likely or "nominal" weather impact scenario, e_1. A single-plan robust solution and finally the disturbance feedback solution will also be compared. It is important to note that for this problem the feedback formulations both produced the same solution. The trade plot in Figure 4 summarises the statistics being discussed.

1) Ideal: If the ideal flight plans are accepted capacity violations were found to occur in a total of 12 sector-times across all 4 scenarios. As would be expected in this case there is no ground or airborne delay.

2) Nominal (most likely): If only the most likely scenario, e_1, is considered in the optimization, as would be expected delays are introduced in order that no capacity violations occur when this scenario is enacted. However, across the other 3 scenarios, $e_2 - e_4$, capacity violations were found to occur in a total of 2 sector-times.

3) Robust: In the robust case one plan is made to satisfy all scenarios. As a result no capacity violations occur. However the price paid for this is that in all scenarios are subject to the most conservative level of delay required in any one scenario. In more dense problem cases it is also highly likely that a robust solution is infeasible.

4) Disturbance Feedback: The disturbance feedback case also incurs no capacity violations. However, as tailored feedback solutions are developed, the minimum amount of delay needed for each scenario can be applied.

D. Computational Scaling

In optimizations of this type computation times can grow rapidly with problem size, therefore it is important to consider how the feedback formulations perform when scaling up the problem scope. Here we consider scaling in two main dimensions to compare computational performance. Firstly the number of flights and length of the problem horizon are jointly considered before focus shifts to scaling with the number of scenarios to be considered. Note that for all problems considered here the feedback formulations both produced the same solution.

1) Scaling with Flights and Problem Length: Three flight sets of increasing size were selected from the 30-flight set chronologically from the start of the set. The 10 flight problem discussed above which contains 64 time periods, a 20 flight problem containing 99 time periods and the full 30 flight set containing 139 time periods. Each flight set was solved for a set of 4 capacity scenarios, and computation comparison results are presented in Table I. Solve times for the Virtual Flights formulation are significantly higher than for the Scenario Tree formulation. This is an interesting observation, which will be investigated further in future work.

	10 Flights 64 Time Steps	20 Flights 99 Time Steps	30 Flights 139 Time Steps
Scenario Tree	4.2(s)	18.7(s)	56.7(s)
Virtual Flights	19.2(s)	280.5(s)	862.2(s)

TABLE I
COMPUTATIONAL SCALING WITH FLIGHTS AND TIME PERIODS

It can also be seen that the solve times increase relatively rapidly with problem size and may become impractical. A possibility for reducing these computation times is to defined the variable $u^{i,j}(k)$ only for a subset of time periods, $\mathcal{T}^*(i) \subset \mathcal{T}$, defined as those times in which flights can physically be present to be controlled in cell i.

2) Scaling with Scenarios: Using the 20 flight problem defined above a separate study was conducted into increasing scenario numbers. In order to assess the potential computational benefits of the Virtual Aircraft formulation when some scenarios are linear combinations of others, two additional scenarios were created which were linear combinations of the four defined for the previous test. Table II summarizes the computational results along with the number of feedback mechanisms required.

From this it can be seen that whilst in the Scenario Tree formulation the number of W's required is always $|\mathcal{E}| - 1$, due to the linear relation between scenarios the number of independent arc-times, $|\mathcal{I}|$ remains unchanged by the introduction of the additional scenarios. As a result the percentage increase in solve time between 4 and 6 scenarios for the Virtual Aircraft formulation is lower than that for the Scenario Tree.

V. Conclusions and Future Work

Two feedback formulations for solution of the ATFM problem under uncertain capacities have been outlined in this paper. Results have demonstrated the benefits, in terms of reduced delays, of the disturbance feedback approach over a single, robust or nominal, plan. The scalability of computation of the two formulations has been explored, with results demonstrating that the Scenario Tree method is faster but the Virtual Aircraft method has the potential for better scaling with number of scenarios in scenario sets where some scenarios are linear combinations of others.

It has also been noted that further computational saving should be available within both the formulations presented by eliminating control variables for time periods in which aircraft are not physically able to be present.

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	4 Scenarios		6 Scenarios	
Scenario Tree	No. W's:	3	No. W's	5
	Solve Time(s):	18.72	Solve Time(s):	29.45
			Time Increase	57%
Virtual Flights	$\|\mathcal{I}\|$:	3	$\|\mathcal{I}\|$:	3
	Solve Time(s):	280.47	Solve Time(s):	357.16
			Time Increase	28%

TABLE II
Scaling with Number of Scenarios

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