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Concurrent Nonlinear Predictive Control and Economic Management of Energy-Integrated Systems

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***Abstract*— We consider a class of process systems with significant energy recovery. Revisiting our previous results concerning the two time scale dynamics of such systems, we demonstrate that the fast dynamics is asymptotically stable in most physically-relevant cases. On this basis, we propose a hierarchical controller design, comprised of a simple, linear control system for the fast dynamics and a MISO NMPC for the slow dynamics. We prove that this composite controller ensures exponential stability for the two time scale system. We establish a parallel between this approach and the concept of economic model predictive control, and show its implications in optimal energy management. Finally, we provide an illustrative simulation case study.**

I. Introduction

Energy efficiency is a key concern in industry owing to rising energy prices and growing environmental concerns. Recovering energy from existing sources in the process and transferring to process energy sinks (assuming that such transfer is thermodynamically possible and economically feasible) is the basic tenet of process energy integration. Traditionally, energy recovery and integration in the chemical industry have targeted –with significant economic success (see *e.g.*, [1] for an overview)– thermal energy (heat or refrigeration). However, these principles are generally applicable to other forms of energy.

Implementing integration a chemical process (which comprises multiple unit operations that exchange material and energy) requires new physical connections between the process units. From a systems engineering perspective, such connections can give rise to feedback interactions in the process, which, in turn, increase the complexity of the process dynamics. Literature reports of such effects, which include the emergence of an inverse-response behavior and open-loop instability, abound in the literature (e.g., [1], [2], [3], [4]). With increasingly dynamic market conditions, which dictate that the control of integrated chemical processes go beyond simple regulation around a steady state, and towards efficient transient operation within a broader operating envelope, the control of such integrated processes is particularly challenging.

Motivated by the above, in this note we concentrate on the optimal control of energy-integrated process systems. Our target is a category of energy-integrated systems in which the rate of energy recovery is much larger than the rate of energy input to the process through any available external energy sources. Our previous efforts [1], [5] have revealed that the simultaneous presence of energy flows of different magnitudes causes model stiffness and is at the origin of a time scale separation in the system dynamics. Here, we use physical and control arguments to demonstrate that the fast component of the dynamics is stable in general and asymptotically stable in practical systems. On the basis of this property, we develop a composite controller design, consisting of i) a linear controller that guarantees the exponential stability of the fast dynamics and, ii) a NMPC for the slow dynamics. We show that this composite approach guarantees exponential stability for the overall system.

Subsequently, we utilize the proposed hierarchical control structure to formulate a strategy for optimal *energy management* at the system level, and establish a parallel between the NMPC introduced in this paper and the concepts economic model predictive control [6], [7]. We demonstrate these results with a simulation case study considering a reactor-Feed Effluent Heat Exchanger (FEHE) process.

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II. Systems with Energy Recovery

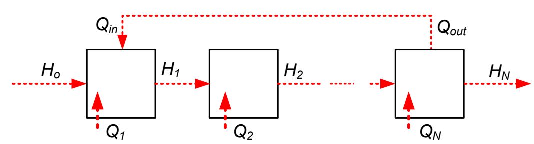

Fig. 1. Energy-integrated system [1]

We consider a system consisting of a series of N (lumped) units with feedback (Figure 1). The feedback connection represents energy integration via internal energy transfer (assumed to occur between the last and the first units). In the general case, we assume that energy transfer is thermodynamically feasible (i.e., our analysis is agnostic to the transfer *mechanism*) and that it occurs a rate $Q_{in} \equiv Q_{out}$.

Based on the structure of the prototype system, a unit i, $i = 2, \dots, N-1$ receives energy from the upstream unit $(i-1)$ at a rate H_{i-1}, and transfers energy downstream to unit $i+1$ at a rate H_i via a material stream. In the same manner, units 1 and N exchange energy with outside sources or sinks at rates H_o and H_N. Unit-level sources and sinks (e.g., local heating or cooling, chemical reactions) are accounted for by the energy flows Q_i, $i = 1, \dots, N$.

We are interested in investigating the effect of energy recycling on the system dynamics, as described by the (additive, extensive [8], [9]) unit enthalpies $\boldsymbol{\theta} = [\theta_1 \dots \theta_i \dots \theta_N]^T$, $\boldsymbol{\theta} \in \mathcal{Q} \subset \mathbb{R}^N$. Under standard assumptions (including those listed above), we can write a mathematical model of the process in the form [1], [5]:

$$\dot{\boldsymbol{\theta}} = \sum_{i=0,N} \boldsymbol{\gamma}_i(\boldsymbol{\theta})H_i + \sum_{i=1}^{N} \boldsymbol{\phi}_i(\boldsymbol{\theta})Q_i + \sum_{i=1}^{N-1} \boldsymbol{\gamma}_i(\boldsymbol{\theta})H_i + \boldsymbol{\gamma}_q^{in}(\boldsymbol{\theta})Q_{in} + \boldsymbol{\gamma}_q^{out}(\boldsymbol{\theta})Q_{out} \quad (1)$$

where $\gamma_i(\boldsymbol{\theta}), \gamma_q^j(\boldsymbol{\theta})$, $\phi_i(\boldsymbol{\theta})$ are vector functions.

We denote by $\omega_i = H_i/H_{i,s}$ the dimensionless variables corresponding to the energy flow rates H_i, $i = 1, \ldots, N$ (the subscript s denotes steady state values) and, correspondingly, $\omega_{in} = Q_{in}/Q_{in,s}$ and $\omega_{out} = Q_{out}/Q_{out,s}$. Let us also define the quantities $l_i = H_{i,s}/H_{1,s}$, $i = 1, \ldots, N-1$, $m_{in} = Q_{in,s}/H_{1,s}$ and $m_{out} = Q_{out,s}/H_{1,s}$. The model of the process in Figure 1 thus becomes:

$$\dot{\boldsymbol{\theta}} = H_{1,s}\Big[\boldsymbol{\gamma}_q^{in}(\boldsymbol{\theta})m_{in}\omega_{in} + \boldsymbol{\gamma}_q^{out}(\boldsymbol{\theta})m_{out}\omega_{out} + \sum_{i=1}^{N-1} \boldsymbol{\gamma}_i(\boldsymbol{\theta})l_i\omega_i\Big] + \sum_{i=0,N} \boldsymbol{\gamma}_i(\boldsymbol{\theta})H_i + \sum_{i=1}^{N} \boldsymbol{\phi}_i(\boldsymbol{\theta})Q_i \quad (2)$$

Let us now note some physical considerations. Clearly, to achieve economically meaningful results (i.e., effective energy integration) the amount of energy recovered from unit N and transferred to unit 1 should significantly exceed the amount of energy input to the system from external sources or local heat generation through chemical reactions. Moreover, the energy recovered from the process output should be effectively used in the process. Consequently, the rate at which energy is transferred *internally* between units 1 and n should be comparable in magnitude to the rate at which energy recovered. These observations can be formalized in the following assumptions [1], [5]:

Assumption 1: The internal energy flow rates $H_{i,s}$, $i = 1 \ldots N-1$ are of similar magnitude ($l_i = \frac{H_{i,s}}{H_{1,s}} = \mathcal{O}(1)$)

Assumption 2: The magnitude of $Q_{in} \equiv Q_{out}$ is similar to that of H_i, $i = 1 \ldots N-1$, and we have:

$$\frac{Q_{in,s}}{H_{1,s}} \equiv \frac{Q_{out,s}}{H_{1,s}} = m_{in} = m_{out} = \mathcal{O}(1) \quad (3)$$

Assumption 3: The internal energy flows are much larger than the inlet and outlet energy flows, that is:

$$\frac{H_{o,s}}{H_{1,s}} \ll 1 \text{ and } \frac{H_{N,s}}{H_{1,s}} \ll 1 \quad (4)$$

Assumption 4: The energy inputs Q_i, $i = 1 \ldots N$ to the individual units are of similar magnitude to the inlet energy flow. Equivalently, $\frac{Q_{i,s}}{H_{o,s}} = \mathcal{O}(1)$ and $\frac{Q_{i,s}}{H_{1,s}} \ll 1$.

We also make the following (non-restrictive, but convenient) assumption:

Assumption 5: Energy holdups θ_i are defined (*e.g.*, in terms of enthalpy or internal energy) such that $\theta_i \geq 0$.

Assumptions 3 and 4 formally capture the requirement of tight energy integration, stipulating that the amount of energy recovered from the process *exceed* the amount of energy input to the process from external sources. Based on Assumption 3, we can define a dimensionless energy recovery number [1] as a figure of merit for energy integration:

$$Rc = \frac{1}{\varepsilon} = \frac{H_{1,s}}{H_{o,s}} \quad (5)$$

Following these developments, the model (2) becomes

$$\dot{\boldsymbol{\theta}} = \frac{1}{\varepsilon}H_{o,s}\Big[\boldsymbol{\gamma}_q^{out}(\boldsymbol{\theta})m_{out}\omega_{out} + \boldsymbol{\gamma}_q^{in}(\boldsymbol{\theta})m_{in}\omega_{in} + \sum_{i=1}^{N-1} \boldsymbol{\gamma}_i(\boldsymbol{\theta})l_i\omega_i\Big] + \sum_{i=0,N} \boldsymbol{\gamma}_i(\boldsymbol{\theta})H_i + \sum_{i=1}^{N} \boldsymbol{\phi}_i(\boldsymbol{\theta})Q_i \quad (6)$$

Which we will write in a more compact form as:

$$\dot{\boldsymbol{\theta}} = \boldsymbol{\Gamma}^s(\boldsymbol{\theta})\boldsymbol{v}^s + \frac{1}{\varepsilon}\boldsymbol{\Gamma}^l(\boldsymbol{\theta})\boldsymbol{v}^l \quad (7)$$

where $\boldsymbol{v}^s \in \mathcal{U}^s \subset \mathbb{R}^{m^s}$ is a vector of scaled variables that correspond to the inlet and outlet energy flows, with $\boldsymbol{v}^s = [H_0/H_{0,s}\ H_N/H_{N,s}\ Q_1/Q_{1,s}\ Q_N/Q_{N,s}]^T$, and $\boldsymbol{v}^l \in \mathcal{U}^l \subset \mathbb{R}^{m^l}$ is a vector of scaled variables corresponding to the (much larger) internal and recycle energy flows, with $\boldsymbol{v}^l = [\omega_1 \ldots \omega_{N-1}\ \omega_{in}\ \omega_{out}]^T$. $\boldsymbol{\Gamma}^s$, $\boldsymbol{\Gamma}^l$ are matrices of appropriate dimensions.

Remark 1: Thus far, we have not considered material balance equations in the modeling framework developed above. These can be easily accommodated when necessary (see., e.g., [1]), and we have selected the case study presented later in the paper to illustrate this possibility.

The developments above (7) allow us to more clearly define tight energy integration at the process level; such processes are of very high interest from an economic point of view, since increased reliance on energy recuperated from the process reduces the use of external energy sources and lowers operating costs.. Using the energy recovery number and its reciprocal ε, we can isolate processes with tight energy integration as the class of systems for which $\varepsilon \ll 1$.

Mathematically, the property $\varepsilon \ll 1$ places the model (7) within the class of nonstandard singularly perturbed systems. The dynamics of such systems evolve over multiple time scales, a feature that must be accounted for in controller design (which, in principle, should be carried out separately in each time scale, an approach referred to as *composite control* [10], [11]).

III. Dynamic Analysis and Model Reduction

In order to elucidate the potential two time scale behavior of such systems, we extend the methods proposed in [1], [5], and proceed by defining a fast ("stretched") time scale $\tau = t/\varepsilon$ and rewriting the model (7) in this fast time scale to obtain a description of the fast dynamics [1], [5]:

$$\frac{d\boldsymbol{\theta}}{d\tau} = \varepsilon\boldsymbol{\Gamma}^s(\boldsymbol{\theta})\boldsymbol{v}^s + \boldsymbol{\Gamma}^l(\mathbf{x}, \boldsymbol{\theta})\boldsymbol{v}^l \quad (8)$$

We continue with using singular perturbation arguments and consider the limit $\varepsilon \to 0$, which, from a physical perspective, corresponds to the ideal case of total heat integration via an infinitely high energy recycle. In this limit, we obtain a description of the fast dynamics of the process of the form:

$$\frac{d\boldsymbol{\theta}}{d\tau} = \boldsymbol{\Gamma}^l(\boldsymbol{\theta})\boldsymbol{v}^l \quad (9)$$

Based on the additivity property of the inventories $\boldsymbol{\theta}$, we can also define the total energy stored in the system as:

$$\theta_{tot} = \sum_{i=1}^{N} \theta_i \quad (10)$$

θ_{tot} is governed exclusively by small external energy flows and energy generation terms (i.e., the vector $\boldsymbol{v}^s$, which captures energy exchange between the system and the environment). Equivalently, the (large) internal $\boldsymbol{v}^l$ flows do not affect the total energy stored in the system. It is also easy to verify that the dynamics of θ_{tot} are independent of $\boldsymbol{v}^l$; moreover, the fast dynamics in (9) are independent of $\boldsymbol{v}^s$. Based on this observation, we can state our first result.

Proposition 1: The fast component of the dynamics of systems with significant energy recycle is stable.

Proof: Let $V^l = \theta_{tot}$. Based on Assumption 5, we have $V(\boldsymbol{\theta}) > 0$ and $V^l(\mathbf{0}) = 0$, and therefore the total energy holdup of the system is (evidently) a Lyapunov function. Our reasoning above indicates that $\frac{dV^l}{d\tau} = 0$ and therefore the fast dynamics in (9) are stable. ■

In what follows, we will make a physical argument and a control argument to derive stronger stability results.

Corollary 1: Considering the structure of the system and the developments above, we can write a differential equation describing the evolution of the total enthalpy of the process in the fast time scale as:

$$\frac{d\theta_{tot}}{d\tau} = \frac{1}{H_{1,s}}(Q_{in} - Q_{out}) \tag{11}$$

Our initial assertion that $Q_{in} \equiv Q_{out}$ is based on the ideal case where all energy recovered from the outlet of the system can be recycled to the inlet. This, however, may not be true for real systems, where thermodynamic limitations (e.g., heat transfer) would preclude the *entire* quantity of energy recovered from the last unit to be transferred to the first. We would thus have $Q_{in} < Q_{out}$ and, assuming that $H_{1,s} > 0$, $\frac{dV^l}{d\tau} \leq 0$. Consequently, the fast dynamics of real physical systems with significant energy recycling is asymptotically stable.

Corollary 2: The inputs $\boldsymbol{v}^l$ can be set, e.g., via a linear state feedback law

$$\boldsymbol{v}^l = \mathbf{K}\boldsymbol{\theta} \tag{12}$$

Assuming that the gains $\mathbf{K}$ can be chosen so that the matrix $\boldsymbol{\Gamma}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta}$ is Hurwitz for any $\boldsymbol{\theta} \in \mathcal{Q}$, the fast dynamics of systems with significant energy recycling becomes exponentially stable.

In the sequel, we will use the singular perturbation arguments in [5], [1] to derive an expression for the slow dynamics of the process. Observe that, since there exists a variable (the total energy holdup) that does not depend on the internal energy flows, the differential equations in the fast subsystem (9) are not linearly independent. This suggests the existence of a slow component of the system dynamics, and also indicates that the solution of the system of algebraic equations that describes the fast dynamics quasi-steady state,

$$0 \quad = \quad \boldsymbol{\Gamma}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta} \tag{13}$$

is not an set of isolated equilibrium points, but rather an equilibrium subspace (or manifold) where the slow dynamics evolves. Based on the physical arguments above (also, see [5]), this subspace is at most one-dimensional. To proceed with our analysis, we must characterize this manifold and ensure that a set of linearly independent constraints can be extracted from (13). To this end, we assume: [5], [1]

Assumption 6: There exist a full column rank matrix $\mathbf{B}(\boldsymbol{\theta}) \in \mathcal{B} \subset \mathbb{R}^{\mathcal{N}\times\mathcal{N}-1}$ and a matrix $\tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta}) \subset \mathbb{R}^{(\mathcal{N}-1)\times m^l}$ with linearly independent rows, such that $\boldsymbol{\Gamma}^l(\boldsymbol{\theta})$ can be rewritten as

$$\boldsymbol{\Gamma}^l(\boldsymbol{\theta}) = \mathbf{B}(\boldsymbol{\theta})\tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta}) \tag{14}$$

We can now substitute (14) in the original model (7), and consider the model in the same limit case, $\varepsilon \to 0$, in the original time scale t and under the linearly independent constraints isolated from (13), obtaining [5], [1]:

$$\begin{aligned} \dot{\boldsymbol{\theta}} &= \boldsymbol{\Gamma}^s(\boldsymbol{\theta})\boldsymbol{v}^s + \mathbf{B}(\boldsymbol{\theta}) \lim_{\varepsilon\to 0} \frac{1}{\varepsilon}\tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta} \\ 0 &= \tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta} \end{aligned} \tag{15}$$

The terms $\lim_{\varepsilon\to 0} \frac{1}{\varepsilon}\tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta}$ (which, based on Equation (6), represent differences between large internal energy flows), become indeterminate in the slow time scale. These terms do, however, remain finite, and constitute an additional set of algebraic (rather than differential) variables in the model of the slow dynamics. Defining $\mathbf{z} = \lim_{\varepsilon\to 0} \frac{1}{\varepsilon}\tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta}$, the reduced-order representation of the slow dynamics becomes:

$$\begin{aligned} \dot{\boldsymbol{\theta}} &= \boldsymbol{\Gamma}^s(\boldsymbol{\theta})\boldsymbol{v}^s + \mathbf{B}(\boldsymbol{\theta})\mathbf{z} \\ 0 &= \tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta} \end{aligned} \tag{16}$$

Equation (16) is a (high-index) differential-algebraic representation of the process slow dynamics. In order to compute $\mathbf{z}$ and, implicitly, to obtain an ODE representation of the slow dynamics, the algebraic constraints of Equation (16) must be differentiated with respect to the state variables. After one differentiation (the assumption in Corollary 2 ensures $L_{\mathbf{B}(\boldsymbol{\theta})}\left(\tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta}\right)$ is invertible), we obtain:

$$\mathbf{z} = -L_{\mathbf{B}(\boldsymbol{\theta})}\left(\tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta}\right)^{-1} L_{\boldsymbol{\Gamma}^s(\boldsymbol{\theta})}\left(\tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta}\right)\boldsymbol{v}^s \tag{17}$$

Substituting (17) in (16), we obtain a (non-minimal order) ODE representation of the slow dynamics. A minimal order state-space realization of the slow dynamics can be obtained via a coordinate transformation [10] involving the energy balance equations and constraints:

$$\begin{bmatrix} \boldsymbol{\zeta} \\ \boldsymbol{\eta} \end{bmatrix} = \boldsymbol{T}(\boldsymbol{\theta}) \begin{bmatrix} \boldsymbol{\delta}(\boldsymbol{\theta}) \\ \tilde{\boldsymbol{\Gamma}}^l(\boldsymbol{\theta})\mathbf{K}\boldsymbol{\theta} \end{bmatrix} \tag{18}$$

under which the reduced-order model of the slow dynamics becomes:

$$\begin{aligned} \dot{\boldsymbol{\zeta}} &= \frac{\partial \boldsymbol{\delta}}{\partial \boldsymbol{\theta}}\boldsymbol{\Gamma}^s(\boldsymbol{\theta}) \mid_{\boldsymbol{\theta}=\boldsymbol{T}^{-1}(\boldsymbol{\zeta})} \boldsymbol{v}^s + \frac{\partial \boldsymbol{\delta}}{\partial \boldsymbol{\theta}}\mathbf{B}(\boldsymbol{\theta})\mathbf{z} \mid_{\boldsymbol{\theta}=\boldsymbol{T}^{-1}(\boldsymbol{\zeta})} \\ \boldsymbol{\eta} &= 0 \end{aligned} \tag{19}$$

From the preceding discussion regarding the size of the equilibrium subspace corresponding to the fast dynamics we can infer that, in effect, $\boldsymbol{\zeta} \in \mathcal{Q}^1 \subset \mathbb{R}^1$. We can rewrite (19) as:

$$\dot{\boldsymbol{\zeta}} \quad = \quad \hat{\mathbf{f}}(\boldsymbol{\zeta}) + \hat{\mathbf{G}}(\boldsymbol{\zeta})\boldsymbol{v}^s \tag{20}$$

which constitutes a (multiple-input, single output) representation of the slow dynamics of the system.

IV. SUPERVISORY NONLINEAR MPC

With the exponential stability of the fast dynamics ensured by the feedback controller (12), we can –as stipulated by standard results in nonlinear systems theory – design a controller for the entire system in the *slow time scale*, using the slow model (20). This approach has historically been referred to as *composite control* [11].

We formulate this system-wide control problem as a Nonlinear Model Predictive Controller (NMPC), aimed at computing the optimal inputs $\boldsymbol{v}^{s*}(t)$ that provide exponential stability to the slow dynamics (20). For the NMPC, we define the stage cost, terminal cost, and cost function as [12]:

$$\mathcal{L}(\boldsymbol{\zeta}(t), \boldsymbol{v}^s(t)) = \frac{1}{2}\left[S\boldsymbol{\zeta}(t)^2 + \boldsymbol{v}^s(t)^T\mathbf{R}\boldsymbol{v}^s(t)\right] \tag{21}$$

$$F(\boldsymbol{\zeta}(t)) = \frac{1}{2}P\boldsymbol{\zeta}(t)^2 \tag{22}$$

$$J(t, \boldsymbol{\zeta}, \boldsymbol{v}^s) = F(\boldsymbol{\zeta}(t+T)) + \int_t^{t+T} \mathcal{L}(\boldsymbol{\zeta}(s), \boldsymbol{v}^s(s))ds \tag{23}$$

where T is the prediction time horizon, $P > 0$ and $S > 0$ are penalty coefficients and $\mathbf{R} \succ 0$ is a positive-definite matrix that captures the *cost* associated with each energy input from external sources. We thus compute the control signal $\boldsymbol{v}^{s*}$ via the optimization problem

$$\begin{aligned} \boldsymbol{v}^{s*} &= \arg\min_{\boldsymbol{v}^s} J \qquad (24) \\ s.t.\ \ \dot{\boldsymbol{\zeta}} &= \hat{\mathbf{f}}(\boldsymbol{\zeta}) + \hat{\mathbf{G}}(\boldsymbol{\zeta})\boldsymbol{v}^s, \\ & \boldsymbol{v}^s \in \mathcal{U}^s \subset \mathbb{R}^{m^s}, \\ & \boldsymbol{\zeta} \in Z \subset \mathbb{R}, \\ & \boldsymbol{\zeta}(t+T) \in Z_f \subset \mathbb{R} \end{aligned}$$

where the last three constraints require that the inputs and the state belong to their respective admissible sets, and the state at the end of the prediction time horizon belong to the terminal set Z_f [12].

Proposition 2: Assuming that:

1) the set Z_f is positively invariant with respect to $\boldsymbol{v}^s$
2) there exists a local feedback controller $\boldsymbol{v}^s_k = k(\boldsymbol{\zeta})$ defined in Z_f such that F is a local Lyapunov function, i.e., $\dot{F}(\boldsymbol{\zeta}) + \mathcal{L}(\boldsymbol{\zeta}) \leq 0$
3) the prediction horizon T is sufficiently long for $\boldsymbol{\zeta}^*(t + T) \subset Z_f$, where the $*$ symbol denotes the closed-loop state trajectory under the controller (24)
4) the function $\psi(\boldsymbol{\zeta}, \boldsymbol{v}^s) = \hat{\mathbf{f}}(\boldsymbol{\zeta}) + \hat{\mathbf{G}}(\boldsymbol{\zeta})\boldsymbol{v}^s$ is Lipschitz in $\boldsymbol{\zeta}$, with Lipschitz constant L, $\|\psi(\boldsymbol{\zeta}, \boldsymbol{v}^s)\| \leq L\|\boldsymbol{\zeta}\|$

the slow component of the system dynamics (20) is exponentially stable under the control law (24).

Proof: See Appendix. ■

Following these developments, we can state:

Theorem 1: Consider a system with high energy recovery of the type (1) under the composite control consisting of the linear feedback (12) and the supervisory control (24). Then, there exists $\varepsilon^* > 0$ such that the origin ($\boldsymbol{\theta} = 0$) of the full order system is exponentially stable for all $0 < \varepsilon < \varepsilon^*$.

Proof: The arguments presented in Theorem 11.4 in [13] can be used to show that there exists an $\varepsilon^* > 0$, such that the origin of (7) under the composite control (12 -24) inherits the exponential stability property of the closed-loop slow system, $\forall \varepsilon < \varepsilon^*$. ■

Remark 2: The control structure in (24) can fulfil a dual role depending on the relative values of the penalty coefficients P, R and S. Thus, (24) can be construed as either a NMPC controller designed for disturbance rejection or setpoint tracking (this is the case, for example, when $P/S = \mathcal{O}(1)$ and $R = \mathtt{diag}[\lambda_1 \ldots \lambda_{m^s}]$, with $\lambda_i/\lambda_j = \mathcal{O}(1)$ and $P/\lambda_i = \mathcal{O}(1)$), or a real-time optimization of energy use (if $P/S = \mathcal{O}(1)$ and $P/\lambda_i \ll 1$).

Remark 3: A purely economic control objective that only accounts for the cost of the external energy inputs (i.e., having $P = S = 0$) can be employed to develop a version of the supervisory controller. However, this approach will only guarantee the asymptotic stability [6] in the slow time scale and the overall stability result in Theorem 1 is no longer valid. Note, however, that stability is still likely to be achieved in practical situations.

V. EXAMPLE: CONTROL OF A REACTOR FEED-EFFLUENT HEAT EXCHANGER PROCESS

Consider the process in Figure 2, consisting of an adiabatic continuously stirred tank reactor (CSTR), a feed-effluent heat exchanger (FEHE) and a fired heater, with the latter used to regulate the inlet temperature of the reactor feed at a desired value that ensures, e.g., maximum conversion. A bypass stream is available as a means to adjust the operation of the FEHE. The component A is fed to the process at a volumetric flow rate F_{in}, with the inlet stream having concentration c_{Ao}. The reaction $A \rightarrow B$, which is first-order with rate constant $k = k_o e^{(-E/RT)}$, occurs in the reactor, and B is the product of interest. Under standard assumptions

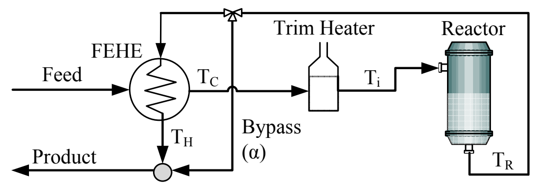

Fig. 2. Reactor-FEHE process.

(minimal dependence of physical properties on temperature and composition, no phase change in the FEHE or elsewhere in the process), we can write the mathematical model of the process as:

$$\begin{aligned} \frac{dV}{dt} &= F_{in} - F \\ \frac{dc_A}{dt} &= \frac{F_{in}}{V}(c_{Ao} - c_A) - kc_A \\ \frac{dT_R}{dt} &= \frac{F_{in}}{V}(T_i - T_R) - \frac{1}{\rho\, C_p} kc_A \Delta H \\ \frac{\partial T_H}{\partial t} &= -v_H \frac{\partial T_H}{\partial z} - \frac{UA}{\rho\, C_p}\frac{T_H - T_C}{V_H} \\ \frac{\partial T_C}{\partial t} &= -v_C \frac{\partial T_C}{\partial z} + \frac{UA}{\rho\, C_p}\frac{T_H - T_C}{V_C} \\ \frac{dT_i}{dt} &= \frac{F_{in}}{V_f}(T_{C_{z=L}} - T_i) + \frac{Q_H}{\rho\, C_p V_f} \qquad (25) \end{aligned}$$

The temperature in the FEHE is distributed in the axial (z) direction, and we define the boundary conditions for T_H, the temperature in the hot leg of the exchanger, and, respectively, for T_C, the temperature in the cold leg, as $T_{H_{z=L}} = T_R$, $T_{C_{z=0}} = T_{in}$, with T_{in} being the temperature of the feed stream, and T_R the reactor temperature. We denote by T_{exit} the temperature of the product stream, and by T_i the temperature of the stream leaving the trim heater. Let $v_H = F_{in}(1-\alpha)/A_H$ and $v_C = F_{in}/A_C$, where α is the bypass ratio. Finally, we denote the holdups of the reactor, the FEHE cold and hot sides, and the trim furnace by V, V_H, V_C and V_f, respectively, and the duty of the furnace by Q_H. The nominal values of the process variables and parameters are presented in Table I. Note that the difference between the warm leg outlet and the feed stream temperatures is very small, which is indicative of significant energy recovery.

We define the energy recovery number as $Rc = H_{rec,s}/[F_{in}\rho\, C_p(T_{in} - T_{ref})]_s$, with T_{ref} being a reference temperature. Following the developments in Section III, we also define the following $\mathcal{O}(1)$ quantities, where $j \in \{R, i, C\}$:

$$
\begin{aligned}
k_j &= \frac{[F_{in}\rho\, C_p(T_j - T_{ref})]_s}{H_{rec,s}} \\
u_j &= \frac{F_{in}\rho\, C_p(T_j - T_{ref})}{[F_{in}\rho\, C_p(T_j - T_{ref})]_s}
\end{aligned} \tag{26}
$$

and $u_{rec} = H_{rec}/H_{rec,s}$.

Defining the stretched, fast time variable $\tau = t/\varepsilon$ and considering the limit $\varepsilon \to 0$ yields a description of the fast dynamics of the process in the form of Eq. (9):

$$
\begin{aligned}
\frac{dV}{d\tau} &= 0 \\
\frac{dc_A}{d\tau} &= 0 \\
\frac{dT_R}{d\tau} &= \frac{[F_{in}(T_{in} - T_{ref})]_s}{V}(k_i u_i - k_R u_R) \\
\frac{dT_H}{d\tau} &= \frac{[F_{in}(T_{in} - T_{ref})]_s}{V_H}(k_R u_R - u_{rec}) \\
\frac{dT_C}{d\tau} &= \frac{[F_{in}(T_{in} - T_{ref})]_s}{V_C}(-k_C u_C + u_{rec}) \\
\frac{dT_i}{d\tau} &= \frac{[F_{in}(T_{in} - T_{ref})]_s}{V_f}(k_C u_C - k_i u_i)
\end{aligned} \tag{27}
$$

Subsequently, using the analysis tools presented earlier in the paper, a description of the slow dynamics of the process can be obtained [1] in the form :

$$
\begin{aligned}
\dot{V} &= F_{in} - F \\
\dot{c_A} &= \frac{F_{in}}{V}(c_{Ao} - c_A) - k_o e^{(-E/RT_R)} c_A \\
\dot{T_R} &= [-k c_A T_R H V\, UA + \rho C_p T_{in}\, UA(F_{in} - F) \\
&+ 8\,(\rho C_p)^2 F F_{in}\, T_R(\alpha - 1) \,+\, \rho C_p\, F\, T_{in}\, \alpha\, UA \\
&- \alpha\, \rho C_p\, T_R\, F\, UA + QH\, UA]\, /\text{DEN} \\
\text{DEN} &= \rho C_p\, [UA(V_C + V_f + V) + 8\, V_h\, F_{in}\, \rho C_p (1 - \alpha)]
\end{aligned} \tag{28}
$$

which constitutes a state-space realization of the form (20), with $\zeta = T_R$ and $\omega^s = QH$.

We simulated a production increase scenario, with a 7% feed flow rate increase at $t = 1h$, along with an increase in the feed temperature to $310K$. We considered both a tracking controller (the tuning parameters in (24) were set to $P = S = 10$ and $R = 10^{-6}$ as discussed in Remark 2), and an economics-oriented optimization, with $R = 10$ and $P = S = 10^{-6}$. The prediction horizon was $T = 10$ minutes. We solved the problem using a standard "delta-input" formulation, with $|dQ/dt| < 0.3$kW/min as an additional constraint.

To carry out numerical simulations, the partial derivatives in (25) were discretized using a backwards finite difference scheme on a grid of 301 nodes, while the supervisory controller was based on the reduced-order model of the slow dynamics in Eq. (28). In simulating the temperature tracking scenario, we also considered a $2K$ rise in the reactor temperature setpoint, $T_{R,sp}$, to compensate the conversion decrease associated with the rise in throughput. In both cases, lower and upper bounds of, respectively, $910K$ and $940K$ are imposed on the reactor temperature T_R. To achieve offset-free tracking, we utilized a model extension [14], [15], which effectively endows the NMPC controller with integral action. We note that this approach does not change the dynamic analysis and control results presented above.

Figure 3 shows the response of the reactor-FEHE process

k_o	1.2667×10^7	s^{-1}	α	0.1	
E	142870.0	J/mol	T_R	922.0	K
ΔH	-54.828	kJ/mol	T_i	909.62	K
$\rho\, C_p$	4.184×10^6	$J/m^3/K$	$T_{H,z=0}$	301.83	K
UA	83680	W/K	$T_{C,z=L}$	864.59	K
c_A	55.19	mol/m^3	T_{in}	300.0	K
Q_H	108.7	kW	V	0.1	m^3
F_{in}	5.77×10^{-4}	m^3/s	V_H	0.1	m^3
F	5.77×10^{-4}	m^3/s	V_C	0.09	m^3
c_{Ao}	1000.0	mol/m^3	V_f	0.01	m^3
L	5.0	m			

TABLE I
PROCESS PARAMETERS (ADAPTED FROM [1])

to these disturbances under the supervisory NMPC using the tracking objective defined in the previous section. Note that the controller exhibits excellent performance. The simulation results obtained using the economic objective function are shown in Figure 4. As expected, the reactor temperature is at the lower acceptable bound, which, intuitively, translates into minimizing the heating rate QH in the furnace.

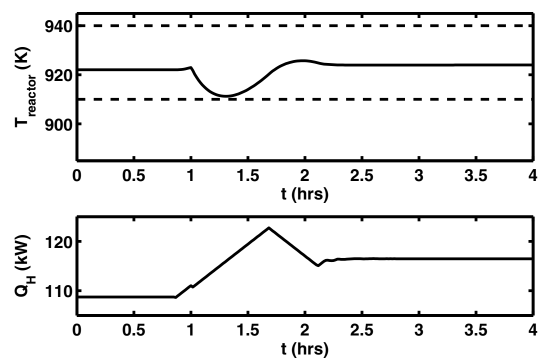

Fig. 3. Evolution of the reactor temperature and furnace heat duty using a tracking objective

VI. CONCLUSIONS

In this paper, we considered a class of systems with significant energy recovery. Extending our previous results, we demonstrated that the fast component of the dynamics is asymptotically stable in practical systems. We then developed a hierarchical controller design, consisting of i) a linear controller to exponentially stabilize the fast dynamics and, ii) a NMPC for the slow dynamics, proving that it guarantees exponential stability for the overall system. Our NMPC approach is versatile and we showed that, through a transparent definition of the penalty terms in the objective function, the proposed controller can be translated into a strategy for optimally managing process energy use. Further, we established a parallel between our approach and economic MPC. Finally, we illustrated our theoretical developments with a simulation case study.

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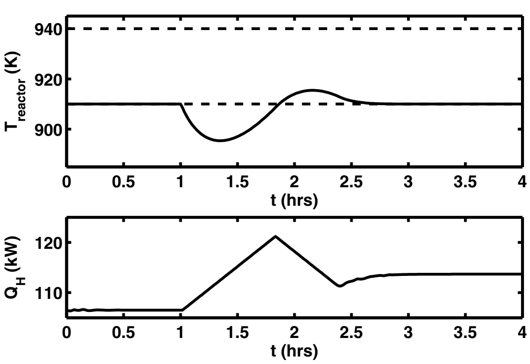

Fig. 4. Evolution of the reactor temperature and furnace heat duty using an economic objective

APPENDIX

Proof of Proposition 2: The proof follows the developments made in [16] for SISO systems, and expands the reasoning to the MISO case of interest. Let us consider the Lyapunov function candidate:

$$V(t,\boldsymbol{\zeta}) \quad = \quad \underset{\boldsymbol{v}^s}{min} J(t,\boldsymbol{\zeta},\boldsymbol{v}^s) \tag{29}$$

We compute $\dot{V}$, as:

$$\dot{V} \quad = \quad \lim_{\Delta t\to 0}\frac{V(t+\Delta t,\boldsymbol{\zeta}(t+\Delta t)) - V(t,\boldsymbol{\zeta}(t))}{\Delta t} \tag{30}$$

Let $\hat{\boldsymbol{v}}(s),\ s\in[t,\infty)$ be the input sequence consisting of the optimal input $\boldsymbol{v}^{s*}$, for the period of time $s\in[t,t+T)$, and the input $\boldsymbol{v}_k^s(s)$ computed by the local controller at times $s\geq t+T$. Then, since $V(t+\Delta t,\boldsymbol{\zeta})\leq J(t+\Delta t,\boldsymbol{\zeta},\hat{\boldsymbol{v}}^s)$, we have

$$\dot{V}\leq\lim_{\Delta t\to 0}\frac{J(t+\Delta t,\boldsymbol{\zeta}(t+\Delta t),\hat{\boldsymbol{v}}^s) - V(t,\boldsymbol{\zeta}(t))}{\Delta t} \tag{31}$$

Now,

$$\begin{aligned} J(t+\Delta t,\boldsymbol{\zeta}(t+\Delta t),\hat{\boldsymbol{v}}^s) &= F(\boldsymbol{\zeta}(t+\Delta t+T)) \\ &+\textstyle\int_{t+\Delta t}^{t+T}\mathcal{L}(\boldsymbol{\zeta},\boldsymbol{v}^{s*})ds+\int_{t+T}^{t+\Delta t+T}\mathcal{L}(\boldsymbol{\zeta},\boldsymbol{v}_k^s)ds \end{aligned}$$

and therefore

$$\begin{aligned} J(t+\Delta t,\boldsymbol{\zeta},\hat{\boldsymbol{v}}^s)-V(t,\boldsymbol{\zeta}) &= F(\boldsymbol{\zeta}(t+\Delta t+T)) \\ +\textstyle\int_{t+T}^{t+\Delta t+T}\mathcal{L}(\boldsymbol{\zeta},\boldsymbol{v}_k^s)ds - F(\boldsymbol{\zeta}(t+T)) &- \textstyle\int_{t}^{t+\Delta t}\mathcal{L}(\boldsymbol{\zeta},\boldsymbol{v}^{s*})ds \end{aligned}$$

Based on Assumption 2 in the proposition, we have $\dot{F}(\boldsymbol{\zeta})+\mathcal{L}(\boldsymbol{\zeta},\boldsymbol{v}_k^s)\leq 0,\ \forall_{s\in[T,\infty)}$. Using this expression, we can write:

$$\textstyle\int_{t+T}^{t+\Delta t+T}\dot{F}(\boldsymbol{\zeta}(s))ds+\int_{t+T}^{t+\Delta t+T}\mathcal{L}(\boldsymbol{\zeta},\boldsymbol{v}_k^s)ds\leq 0$$

and thus

$$F(\boldsymbol{\zeta}(t+\Delta t+T))-F(\boldsymbol{\zeta}(t+T))+\textstyle\int_{t+T}^{t+\Delta t+T}\mathcal{L}(\boldsymbol{\zeta},\boldsymbol{v}_k^s)\leq 0$$

Substituting all of the above in (31), we get

$$\begin{aligned} \dot{V} &\leq \lim_{\Delta t\to 0}\frac{\int_t^{t+\Delta t}-\mathcal{L}(\boldsymbol{\zeta},\boldsymbol{v}^{s*})ds}{\Delta t} \\ &\leq -\frac{1}{2}S\boldsymbol{\zeta}(t)^2-\frac{1}{2}\boldsymbol{v}^{s*T}\mathbf{R}\boldsymbol{v}^{s*} \\ &\leq -\frac{1}{2}S\boldsymbol{\zeta}(t)^2 \end{aligned}$$

Consequently (see Theorem 4.9 in [13]) the slow component of the system dynamics is *asymptotically* stable.

Then, we have $\boldsymbol{v}^s(s)^T\mathbf{R}\boldsymbol{v}^s(s) > 0$ ($\mathbf{R}$ is positive definite) and $F(\boldsymbol{\zeta}(t+T)) > 0$ and thus $V(t,\boldsymbol{\zeta})\geq\frac{1}{2}\int_t^{t+T}S\boldsymbol{\zeta}(s)^2ds$. Since ψ is Lipschitz, we have that $\|\boldsymbol{\zeta}(s)\|\geq\|\boldsymbol{\zeta}(t)\|\exp[-L(s-t)],\ s\in[t,\infty)$, and:

$$\begin{aligned} V(t,\boldsymbol{\zeta}) &\geq \frac{1}{2}\textstyle\int_t^{t+T}S\boldsymbol{\zeta}(s)^2ds \\ &\geq \frac{1}{2}S\boldsymbol{\zeta}(t)^2\textstyle\int_t^{t+T}e^{-2L(s-t)}ds \\ &\geq \frac{1-e^{-2LT}}{4L}S\boldsymbol{\zeta}(t)^2 \end{aligned} \tag{32}$$

Let $\bar{\boldsymbol{\zeta}}(s)$ be the state at time s starting from an initial condition $\boldsymbol{\zeta}_o$ and with zero energy input, $\boldsymbol{v}^s(t)=\mathbf{0}$. As $V(t,\boldsymbol{\zeta})$ is the minimum of J, $V(t,\boldsymbol{\zeta})\leq J(t,\bar{\boldsymbol{\zeta}},0)$. From the Lipschitz property of ψ, it follows that $\|\bar{\boldsymbol{\zeta}}(s)\|\leq\|\boldsymbol{\zeta}(t)\|\exp[L(s-t)],\ s\in[t,\infty)$. Consequently, we have:

$$\begin{aligned} V(t,\boldsymbol{\zeta}) &\leq\frac{1}{2}P\bar{\boldsymbol{\zeta}}(t+T)^2+\frac{1}{2}\textstyle\int_t^{t+T}S\bar{\boldsymbol{\zeta}}(s)^2ds \\ &\leq\frac{1}{2}Pe^{2LT}\boldsymbol{\zeta}(t)^2+\frac{1}{2}S\boldsymbol{\zeta}(t)^2\textstyle\int_t^{t+T}e^{2L(s-t)}ds \\ &\leq\left[\frac{1}{2}Pe^{2LT}+S\frac{e^{2LT}-1}{4L}\right]\boldsymbol{\zeta}(t)^2 \end{aligned} \tag{33}$$

With the bounds (32)-(33), the function V satisfies the conditions of Theorem 4.10 in [13], and therefore the controller (24) guarantees the exponential stability of the slow dynamics (20).