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Hybrid MPC approach to reconfiguration of building heating system

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Abstract— **This paper describes the application of Hybrid Model Predictive Control (HMPC) to a building heating system. The hybrid model contains continuous variables corresponding to physical quantities as well as discrete variables serving as indices of a Linear Time Invariant (LTI) model in action. Two LTI models are considered, each describing different configurations of the building heating system. The application of HMPC allows efficient handling of disturbances by reconfiguration of the heating system. The proposed HMPC strategy improves comfort and reduces energy demands. An exact solution of HMPC is computationally demanding; therefore, three suboptimal solutions are suggested. These take into account specifics of the building heating system and significantly reduce the computational complexity. All presented strategies are compared by means of a numerical simulations using real weather data and a model of a real building.**

I. INTRODUCTION

In recent years, there has been an intensive effort to achieve significant energy savings. This has been demonstrated by the governments of many developed countries. For instance, the European Union (EU) presented targets concerning energy cuts defining goals by 2020 [1]: *i)* Reduction in the EU greenhouse gas emissions at least 20 % below the 1990 levels, *ii)* 20 % of the EU energy consumption to come from renewable resources, *iii)* 20 % reduction in primary energy use compared to projected levels to be achieved by improving energy efficiency.

As buildings account for about 40 % of total final energy consumption [2] and more than half is lost in Heating, Ventilation and Air Conditioning (HVAC) systems, an efficient building climate control can significantly contribute to reduction of the power demands and lowering of the greenhouse gas emissions. Energy savings with minimal additional cost can be achieved by the improvement of algorithms of Building Automation Systems (BAS), which has been shown by leading academic and industrial teams [3], [4], [5], [6].

Most researchers in this area adopt a Model Predictive Control (MPC) framework. The main principle of such a controller is a trade-off between energy savings and user welfare making use of predictions of disturbances acting on the system (ambient temperature, solar radiation, occupancy, etc.). From a wide variety of results, a few instances can be listed. A MPC controller *i)* takes disturbance predictions (occupancy, weather etc.) into account, thus adjusts control actions appropriately [7], [8], *ii)* can utilize the thermal mass of a building in a better way compared to the conventional control strategies (e.g heating/cooling curve or rule based control) [9], [10], *iii)* is able to deal with variable energy price that can easily be included in a formulation of an optimization problem [11], [12], *iv)* can be formulated with the aid of thermal comfort indices instead of indoor operative temperature [13], [14], *v)* can handle the minimization of energy peaks within a certain time frame [15], [16] (beneficial because of both the possibility of tariff selection and lowering operational costs). There have also been some experimental setups of MPC, which have proven the energy savings potential [4], [7], [17](15–30 % compared to the conventional control strategies).


Fig. 1. Building of the Czech Technical University in Prague

This work follows up a project that focused on the application of MPC on a real building in Prague. Current heating season is already the fifth successive season when the pilot building of the Czech Technical University (CTU) is controlled by MPC. The results are encouraging, MPC saved 15-28 % of energy when compared with the previous control strategy that was based on heating curve [7], [17], [18]. The MPC strategy that is currently used for CTU building control is denoted as *normal MPC* in this paper.

In spite of the fact that normal MPC was superior to heating curve strategy in terms of energy usage as well as comfort violations [7], there are situations when normal MPC could use less energy and improve occupants' comfort. This paper introduces a technique that allows MPC to take advantage of an unexploited potential by utilization of excess heat that can accumulate in a part of the building. The effective use of the excess heat is based on the reconfiguration of the heating system in such a way that the heating water circulates through all heating circuits and redistributes the excess heat. This results in equalization of temperatures of return water from all circuits. This heating system regime is denoted as *temperature equalization* (EQ) regime. During a sunny day,

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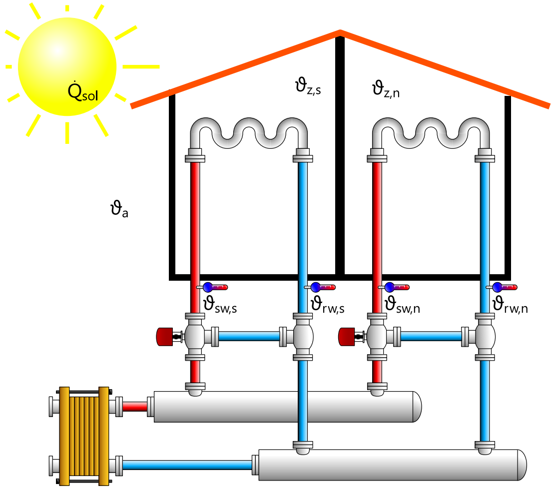

Fig. 2. Simplified schematic of the ceiling radiant heating system

the south oriented façade of the building is exposed to solar radiation most of a day, while the north oriented façade is in shade. The EQ regime enables redistributing the excess heat from the south to the north part of the building. The EQ regime was originally introduced by CTU building operators. It was switched on manually according to zone temperatures and a weather forecast. This paper introduces an optimization approach to switching of the heating system regime based on HMPC. It is shown that HMPC introduced in this paper makes use of the excess heat more efficiently than the normal MPC.

This paper is further organized as follows: Technical details of the building under investigation are outlined in Section II, then Section III introduces simplified mathematical models suitable for MPC which is described in Section IV. Section V is devoted to the description of the experiments that have been carried out and to discussion of the results. Finally, Section VI draws conclusions and states possibilities for future work.

II. DESCRIPTION OF THE BUILDING

The CTU building uses Crittall [19] type ceiling radiant heating and cooling system. In this system, the heating (or cooling) beams are embedded into the concrete ceiling. Such heating and cooling systems are usually denoted as Thermally Activated Building Systems (TABS).

A simplified schematic of the ceiling radiant heating system is depicted in Fig. 2.

The source of the heat is a vapor-liquid heat exchanger, which supplies hot water to a distributor. There are five heating circuits in the CTU building. For the sake of simplicity, only two heating circuits are considered in this paper. Supply water temperature is controlled individually for each heating circuit by a three-way valve with a servo drive. The valve mixes hot water from the distributor and cooled return water. The return water that is not used for a mixture flows to a collector and is heated by the vapor-liquid heat exchanger.

The heating system can also be switched to the EQ regime when no heat is supplied to the vapor-liquid heat exchanger and all the three-way valves are fully opened. It means that the return water from all heating circuits is mixed in the collector, flows to the distributor and finally to all the heating circuits.

TABLE I

NOTATION OF THE QUANTITIES IN THE SYSTEM

Notation	Unit	Description
$\vartheta_{z,s}$	°C	Zone temperature – south
$\vartheta_{z,n}$	°C	Zone temperature – north
$\vartheta_{rw,s}$	°C	Return water temperature – south
$\vartheta_{rw,n}$	°C	Return water temperature – north
$\vartheta_{rw,\varnothing}$	°C	Mean return water temperature
$\vartheta_{sw,s}$	°C	Supply water temperature – south
$\vartheta_{sw,n}$	°C	Supply water temperature – north
ϑ_a	°C	Ambient temperature
$\dot{Q}_{sol}$	W/m^2	Solar radiation from the south

The EQ regime cannot be achieved by the normal MPC. The normal MPC manipulates only the supply water temperature setpoints, it cannot directly influence valve positions nor operation of the heat exchanger. Advantage of HMPC with EQ regime can be illustrated by the following scenario. The required supply water temperature is 24 °C for both façades and return water temperatures are 22 °C and 26 °C for the north façade and south façade respectively. In case of normal MPC, some heat has to be supplied into the north heating circuit in order to achieve the required supply water temperature. In case of HMPC, the heat exchanger can be switched into the EQ regime and no heat is needed because mean of the return water temperatures is equal to the required supply water temperature.

Note also that during both regimes, the water circulation pump has to be in operation and therefore the EQ regime does not increase energy consumption of the pump.

III. MATHEMATICAL MODELS

In the following, two mathematical models of the studied system are presented. The first one represents the normal operation of the heating system, while the other represents the EQ regime.

The models are derived from a simplified physical description based on resistance-capacitance (RC) schemes in the analogue of electrical circuits [20]. Both of the models have the same state vector $x \in \mathbb{R}^n$ that is composed as

$$x = [\vartheta_{z,s},\, \vartheta_{rw,s},\, \vartheta_{z,n},\, \vartheta_{rw,n},\, \vartheta_{rw,\varnothing}]^T. \tag{1}$$

The system input $u \in \mathbb{R}^m$ involves supply water temperatures, i.e.

$$u = [\vartheta_{sw,s},\, \vartheta_{sw,n}]^T, \tag{2}$$

and finally, the disturbances acting on the system are included in the vector of disturbances $v \in \mathbb{R}^p$

$$v = [\vartheta_a,\, \dot{Q}_{sol}]^T. \tag{3}$$

For meaning of the quantities, refer to Table I.

A. Normal operation model

As already stated, the model of the normal operation is based on a simplified RC scheme and the first principles of the building physics. The model has the following structure

$$
\begin{aligned}
x_{k+1} &= A_1 x_k + B_1 u_k + V_1 v_k \\
&= \begin{bmatrix} a_{11,s} & a_{12,s} & 0 & 0 & 0 \\ a_{21,s} & a_{22,s} & 0 & 0 & 0 \\ 0 & 0 & a_{11,n} & a_{12,n} & 0 \\ 0 & 0 & a_{21,n} & a_{22,n} & 0 \\ \frac{1}{2}a_{21,s} & \frac{1}{2}a_{22,s} & \frac{1}{2}a_{21,n} & \frac{1}{2}a_{22,n} & \frac{1}{2}(b_{2,s}+b_{2,n}) \end{bmatrix} x_k \\
&+ \begin{bmatrix} b_{1,s} & 0 \\ b_{2,s} & 0 \\ 0 & b_{1,n} \\ 0 & b_{2,n} \\ 0 & 0 \end{bmatrix} u_k + \begin{bmatrix} v_{1,s} & q_{1,s} \\ v_{2,s} & 0 \\ v_{1,n} & 0 \\ v_{2,n} & 0 \\ 0 & 0 \end{bmatrix} v_k. \qquad (4)
\end{aligned}
$$

The parameters $a_{ij,\bullet}$ describe the dynamics of the respective building part (i.e. heat transfers between heating water and the particular zone), the last row in A_1 represents evolution of the weighted average of both of the return water temperatures, in our case with the factor $\frac{1}{2}$. Water flow rates in the heating circuits are assumed to be constant and mutually equivalent and thus the terms are pre-multiplied by a weighting factor $\frac{1}{2}$, otherwise, an appropriate scaling would have to be used. $b_{i,\bullet}$ are parameters standing for the impact of the supply water temperatures on both zone and return water temperatures and finally $v_{i,\bullet}$ and $q_{1,s}$ are parameters defined in a similar way. Some model parameters can be set in advance (e.g. $\vartheta_{sw,s}$ does not affect $\vartheta_{z,n}$ at all, weak thermal coupling between south and north part of the building, etc.).

Non-zero parameters of the presented model were then estimated using least squares technique that preserves prespecified model structure. For details of the identification algorithm and validation of the resulting model cf. [7, Section 3.1.2]. Note that the mean return water temperature $\vartheta_{rw,\varnothing}$ was not considered in the identification setup and the elements in the last row of A_1 were derived based on the rest of A_1.

B. Temperature equalization model

Now, the EQ regime model is proposed. It describes the heating system when the supply water is cut off and the heating water circulates in the pipes. This action can be realized, in terms of mathematical operations over the model of a normal operation, by the reorganization of elements in B_1 towards the last column of A_1 (the last column stands for the effect of mean return water temperature on other states). The EQ model then reads

$$
\begin{aligned}
x_{k+1} &= A_2 x_k + B_2 u_k + V_2 v_k \\
&= \begin{bmatrix} a_{11,s} & a_{12,s} & 0 & 0 & b_{1,s} \\ a_{21,s} & a_{22,s} & 0 & 0 & b_{2,s} \\ 0 & 0 & a_{11,n} & a_{12,n} & b_{1,n} \\ 0 & 0 & a_{21,n} & a_{22,n} & b_{2,n} \\ \frac{1}{2}a_{21,s} & \frac{1}{2}a_{22,s} & \frac{1}{2}a_{21,n} & \frac{1}{2}a_{22,n} & \frac{1}{2}(b_{2,s}+b_{2,n}) \end{bmatrix} x_k \\
&+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u_k + \begin{bmatrix} v_{1,s} & q_{1,s} \\ v_{2,s} & 0 \\ v_{1,n} & 0 \\ v_{2,n} & 0 \\ 0 & 0 \end{bmatrix} v_k. \qquad (5)
\end{aligned}
$$

Note that in this mode, the input matrix vanishes and the system becomes autonomous driven only by disturbances.

IV. OPTIMAL CONTROL PROBLEM FORMULATION

In this section, the HMPC problem formulation is outlined. The aim of HMPC is to design a control input that minimizes the energy consumption (or operational costs) while guaranteeing that comfort requirements are met. To do so, two artificial system outputs are introduced

$$
\begin{aligned}
y_k &= C x_k \qquad (6) \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} x_k, \\
e_k &= F x_k + G u_k \qquad (7) \\
&= \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u_k.
\end{aligned}
$$

The former one $y_k \in \mathbb{R}^l$ stands for zone temperatures, thus $C \in \mathbb{R}^{l \times n}$ is a selection matrix that picks up an appropriate system state, l refers to the number of outputs, i.e. zone temperatures. On the other hand, the latter output $e_k \in \mathbb{R}^m$ is proportional to the energy delivered to the respective heating circuit. In this study, the energy is computed for each heating circuit as the difference between supply and return water temperatures (due to the constant mass flow rate in the heating system). This is achieved by an appropriate selection of matrices $F \in \mathbb{R}^{m \times n}$ and $G \in \mathbb{R}^{m \times m}$.

A. General hybrid MPC formulation

These artificial outputs help us to simplify the notation of the optimization problem setup that comprises a trade-off between these outputs. The HMPC problem setup then reads

$$
\begin{aligned}
\min_{u_0^{N-1}, i_0^{N-1}, z_0^{N-1}} \quad & \sum_{k=0}^{N-1} |Q(y_k - z_k)| + |Re_k| \\
\text{s.t.} \quad & \underline{r}_k \le z_k \le \overline{r}_k, \\
& x_0 = x_{init}, \\
& H_k u_k \le h_k, \qquad (8) \\
& x_{k+1} = A_{i_k} x_k + B_{i_k} u_k + V_{i_k} v_k, \quad i_k \in \{1,2\}, \\
& y_k = C x_k, \\
& e_k = F x_k + G u_k, \\
& e_k \ge 0.
\end{aligned}
$$

The trade-off between the precision of reference tracking and the energy consumption is expressed by a proportion of positive semidefinite matrices $Q \in \mathbb{R}^{l \times l}$ and $R \in \mathbb{R}^{m \times m}$. The goal here is to penalize only violations of a comfort range defined by the lower and upper reference trajectories i.e. $\underline{r}_k \in \mathbb{R}^l$ and $\overline{r}_k \in \mathbb{R}^l$, respectively. This is achieved by the aid of slack variables $z_k \in \mathbb{R}^l$ constrained by the comfort range boundaries. As far as energy minimization is concerned, the accumulation of the delivered energy is to be minimized, hence the absolute value of e_k arise in the cost function.

The system inputs have to come from a polytopic region defined by matrices H_k and vectors h_k. This type of constraint is common, and, for instance, can be used to place upper/lower bounds on system variables.

In this setup, it is assumed that the profile of the disturbances is perfectly known over the prediction horizon of length N and the full state measurement x_{init} is available.

Finally, the decision variables arising in the optimization problem are of two types:

1) Continuous variables: these are the system inputs u_k and the slack variables z_k

2) Integer variables: represented by the variables i_k. This kind of variable serves as a boolean indicator of what system model is active at a certain time instant.

In total, the decision variables are grouped, for example in case of the system inputs, as $u_0^{N-1} = \{u_0, u_1, \ldots, u_{N-1}\}$. The remainder of the variables is denoted in a similar way.

When the EQ regime is in action, no energy is delivered i.e. $e_k = 0$, u_k has no effect on the zone or return water temperatures because all elements of matrix B_2 are zero. Hence, we always obtain an analytical solution of (7) as $u_k = -G^{-1}Fx_k$, which basically means that supply water temperature equals to the weighted average of return water temperatures.

YALMIP optimization toolbox [21] with Mixed-Integer Linear Programming (MILP) routine from the CPLEX package were used to define and solve the optimization problem.

B. Hybrid MPC approximations

Buildings usually possess very slow dynamics, compared to the frequency of changes in the reference trajectories or disturbance variables. Thus, the prediction horizon needs to be sufficiently long. On the other hand, the number of combinations of binary variables grows exponentially with the prediction horizon length (the number of possible combinations is 2^N) and the problem setup becomes computationally demanding for longer prediction horizons.

That is why several approximation techniques that enable solution of even larger problems are presented in the following. These techniques are based on reducing of the integer search space, which helps the MILP solver to prune it more effectively.

1) Fixed switching times: Some time steps are more preferable for regime switching due to varying ambient conditions during a day. A set of time steps, when the switching is allowed, can be set a priory based on expert knowledge about the controlled building. The problem setup (8) is then extended by the following constraint

$$|i_k - i_{k+1}| \begin{cases} \geq 0, & k \in ST, \\ = 0, & \text{otherwise}, \end{cases} \tag{9}$$

where ST is a set of time indices when switching is allowed. The number of combinations of binary variables is reduced to $2^{card(ST)}$, where $card(ST)$ denotes the cardinality of set ST.

2) Limited number of switchings: Another possibility to reduce computational time is to limit the number of allowed regime switchings. The problem setup (8) is extended by the following constraint

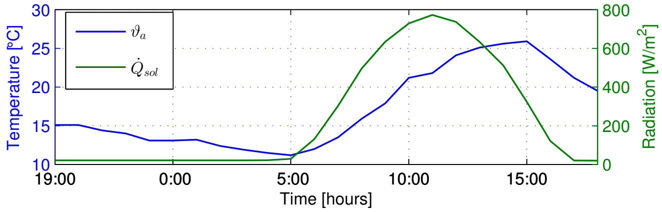

Fig. 3. Disturbance profiles on the prediction horizon

$$\sum_{k=0}^{N-1} |i_k - i_{k+1}| \leq SN, \tag{10}$$

where SN stands for a maximal allowed number of switchings. Note that SN is determined by an expert knowledge.

3) Regime comparison: Besides the suboptimal strategies listed above, one straightforward approach can be used as well. A simulation of the EQ regime is performed over the prediction horizon and the result is then compared with the normal MPC operation. The EQ regime always has zero control cost. Therefore, the EQ regime is selected when the comfort range violation penalty is smaller than the total control cost of the normal MPC operation. The main advantage of this suboptimal strategy is that no mixed integer solver is needed and the normal MPC operation is compared to the simulation of the EQ regime only. Performance improvement when compared to normal MPC operation can be expected especially during fall and spring when only a little heat has to be supplied to a building.

V. NUMERICAL EXPERIMENT

The numerical experiment is based on the building model that is used for the normal MPC in [7]. The model was identified using measured data and HMPC models were derived using the procedure described in Section III. Sampling time of the model is one hour, prediction horizon is one day ($N = 24$), weighting matrices are $R_k = I_{m\times m}$, $Q_k = 100 \cdot I_{l\times l}$ and input constraint matrices are $H_k = [-1, 0; 0, 1], h_k = [-15, 50]^T$, $k = 0, \ldots, N-1$. The upper reference trajectory is not considered (i.e. $\overline{r}_k = \infty$) and the lower reference trajectory is defined as follows

$$\underline{r}_k = \begin{cases} 22, & 8 < k < 17, \\ 20, & \text{otherwise}. \end{cases} \tag{11}$$

Note that the lower reference trajectory depends on the starting time. Initial conditions as well as disturbances are set according to on site measurements taken from March 29, 2011, 7 am till March 30, 2011, 6 pm. Disturbance profiles are depicted in Fig. 3.

The numerical example shows a situation where regime switching helped to improve energy efficiency by making use of the predicted solar gains that affect south façade only. The optimal solution of HMPC is depicted in Fig. 4. The EQ regime is in action as long as possible except a short period when heat is supplied to the north heating circuit and circulates there for five hours, i.e. normal configuration is

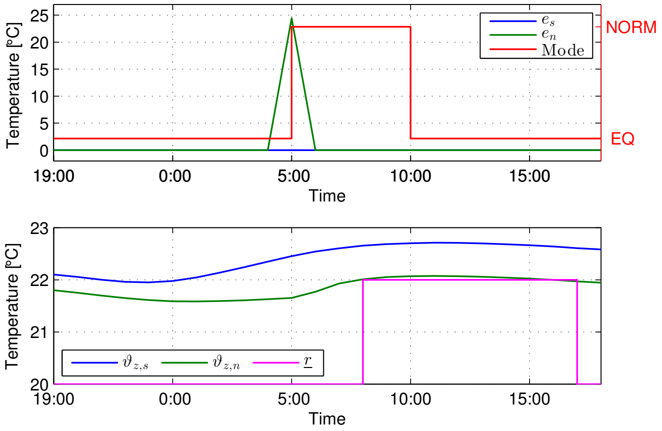

Fig. 4. General hybrid MPC formulation. In the top chart, the left y-axis represents the difference between supply water temperature and return water temperature. This quantity is proportional to delivered energy because flow rate is constant. The same description applies to Fig. 5 - Fig. 8.

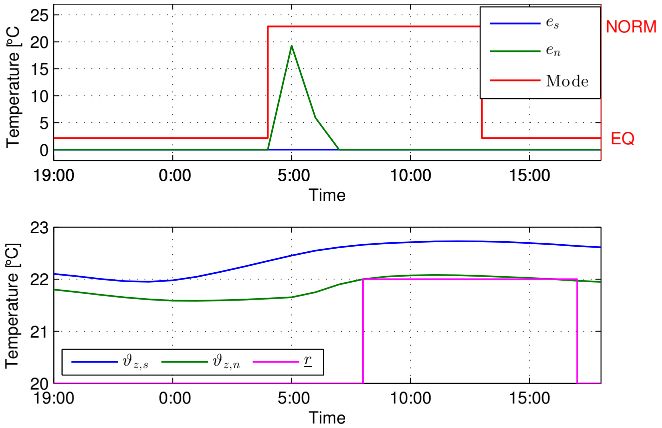

Fig. 5. Hybrid MPC with fixed switching times

active (in the figures referred to as NORM). Energy savings provided by HMPC are 14 %, see summary in Table II. Similar results are obtained in case of the suboptimal strategy based on fixed switching times with $ST = \{4\,\mathrm{am}, 1\,\mathrm{pm}, 6\,\mathrm{pm}, 8\,\mathrm{pm}\}$, see Fig. 5. The only difference is that the normal operation regime is in action for a longer time period. As a consequence all solar gains from the south façade cannot be utilized and more heat has to be supplied into the north heating circuit. A different profile of regime switching was designed by the suboptimal solution based on a fixed number of switchings (see Fig. 6), with $SN = 1$. Energy is delivered into the north heating circuit in the first step. It increases the north façade zone temperature and the solar gains from the south façade are utilized without any comfort violations. Note that the selection of $SN = 1$ is not practical and it was motivated by the demonstration of different suboptimal strategies, however, this strategy is still more energy efficient than the normal MPC.

In case of $SN \geq 2$, the result is the same as in the case of HMPC. The last suboptimal strategy does not provide any improvement compared to the normal operation since the EQ regime causes significant comfort range violations, see Fig. 8, and the normal MPC operation provides better results, see Fig. 7.

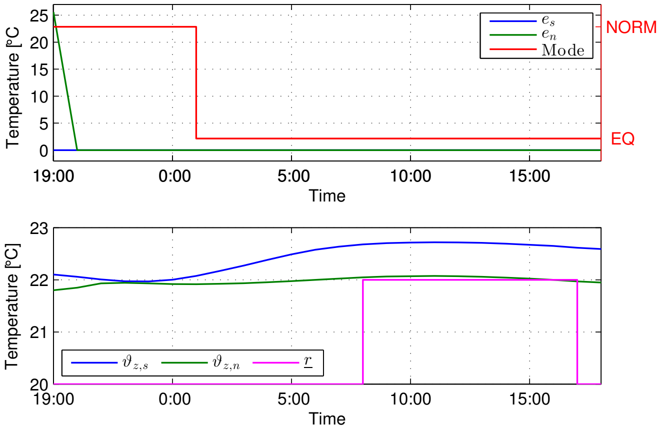

Fig. 6. Hybrid MPC with limited number of switchings

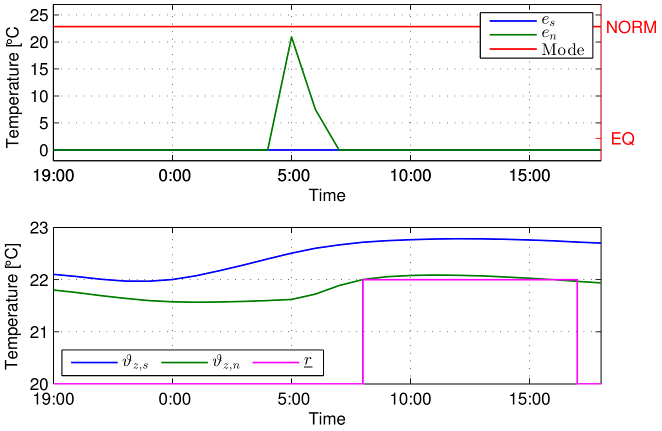

Fig. 7. Normal MPC formulation

The differences between the proposed control strategies may seem to be insignificant. However, the CTU building is a large building equipped with TABS heating system that possesses very slow dynamics [17]. Therefore, in spite of small changes in the zone temperature and energy demand profiles, meaningful heating cost reduction can be achieved.

Finally, all proposed control strategies were compared with respect to the computational time. Table II summarizes the computational time needed to compute solution to the respective optimization problems. The statistics were obtained from several runs of each respective algorithm. Moreover, for each control strategy, two costs are included. The first one is imposed by the heating demand, while the second one is the cost caused by comfort violations. Note that energy cost gradually increases with the level of approximation.

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper describes an application of Hybrid MPC to a building heating system. The hybrid model constitutes two linear time invariant models of a building heating system: a model of the normal operation regime and a model of the temperature equalization regime. The temperature equalization regime makes use of the excess heat accumulated in one part of the building. The goal of Hybrid MPC is to minimize energy usage while respecting occupants' comfort

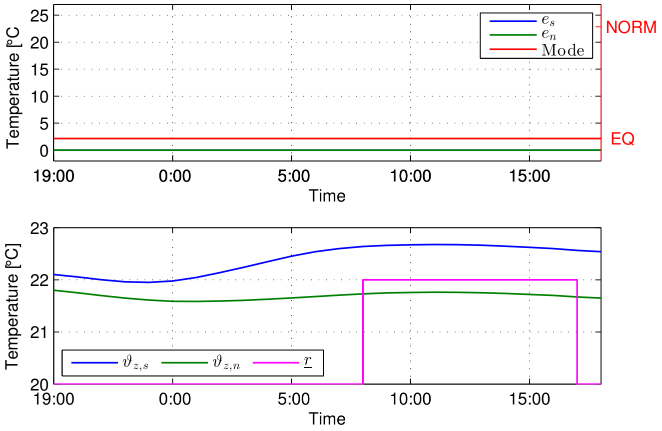

Fig. 8. One day run of the EQ model

TABLE II

COST FUNCTION VALUES AND COMPUTATIONAL TIMES FOR VARIOUS PROBLEM SETUPS

Problem	J_e	J_z	CPU time	Relative energy savings (compared to Normal MPC)
HMPC	24.44	0	206.67 s	14%
HMPC + SN	25.58	0	11.17 s	10%
HMPC + ST	25.18	0	3.63 s	11%
EQ regime	0	233.04	0.01 s	100%
Normal MPC	28.43	0	0.68 s	0%

$J_e = \sum_{k=0}^{N-1} |R_k e_k|$ (energy cost), $J_z = \sum_{k=0}^{N-1} |Q_k(y_k - z_k)|$ (comfort violation cost), HMPC+SN denotes strategy with limited number of switchings and HMPC+ST stands for fixed switching times suboptimal strategy

by designing an optimal trajectory of the continuous and discrete inputs. It was shown that additional energy savings could be achieved when two regimes of heating system were considered.

The technique described in this paper was applied to the model of the CTU building in Prague, however, this technique can be applied to almost any building that has several heating circuits connected through one heat exchanger. There are no special modeling requirements, the temperature equalization regime model can be easily derived from a model describing normal operation of a heating system. Therefore, the proposed technique is advantageous mainly for buildings that are already operated by MPC where the Hybrid MPC formulation can readily be applied.

B. Future work

Ongoing research will focus on application of the proposed technique on a real building and evaluation of energy saving potential based on a long-term operation of Hybrid MPC. In this case, imperfect predictions of disturbances and model error will deteriorate simulated performance of HMPC.

The temperature equalization regime was considered for heating only, however, it can be applied to cooling as well. In case of cooling, a reasonable amount of unwanted heat can be redistributed using temperature equalization regime.

The aim of this paper was to prove the concept of reusing the excess heat on a very simple mathematical model of a building. The model, however, did not account for internal gains in the building, whose effect may also be studied.

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