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Building semi-physical modeling: On selection of the model complexity

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Abstract— **Buildings account for significant amount of final energy consumption and therefore there is an intensive research aimed at its optimization. Predictive control has become a very popular approach in many industries buildings included. The main bottleneck of this method is a need for a good model.**

There are many different identification frameworks, plenty of methods and approaches, some of them more or less suitable for the building modeling. A common situation is that there are number of models at hand (often of different complexities), and there is a need for selection of the "best" model for predictive control. A logical choice is to start testing the statistical significance of an additional complexity (in a sense of the structure and number of disturbance inputs) of the model. This paper proposes a systematic way of building-up the model, starting from a simple structure. Then, more complex models are considered in an iterative manner. In each iteration, the statistical significance of the additional information due to the more complex model is checked. The procedure stops when selecting more complex models brings no quality improvements. In this paper, a semi-physical modeling using CTSM[1] and model selection based on statistical tests are presented. Finally, the properties of the proposed algorithm are investigated.

I. Introduction

The Model Predictive Control (MPC) used in Building Automation Systems or control of Heating Ventilation Air Conditioning (HVAC) has been widely studied (see e.g. [2], [3]) and also applied to real systems as well [4], [5], [6], [7]. The control part is almost a standard solution, however, the way how to get a building model suitable for control is still under investigation [8], [9], [10].

This work follows the previous papers by [1], [10], [11], [12], where an idea of a model selection according to statistical tests based on the information content was introduced.

A semi-physical modeling (in the paper solved by CTSM) making use of Maximum Likelihood (ML) estimation is utilized in this paper. For models obtained by maximization of likelihood, there are several statistical tests [13], [14] at hand, which evaluate the statistical relevancy of adding more parameters to the model, i.e. the model selection is performed in an iterative manner when in each iteration a more complex model is selected and the increase in the model quality statistically evaluated. This approach is extended, when broadening the model complexity means both adding the disturbance inputs and system states. In the presented paper, an approach of systematic building-up the model through growing model complexity is introduced. In the first stage, a minimum set of disturbance inputs maximizing the model quality is selected (full-complex inner structure is considered) and then, for a fixed set of inputs, a minimum set of states using the same logic as in the previous stage is selected. The model resulting from this procedure is less complex and of comparable quality with the "full" (model with full set of disturbance inputs and all the system states considered) model. As a result, the selected model is (when used for control) computationally less demanding, which is of crucial significance. Moreover, lower number of states and disturbance inputs means less sensors, and, in cases when the disturbances are provided as a service (weather forecasts for building climate control) this leads to significant financial savings.

Building simulation software Trnsys is used for creating a detailed building model. This model is afterwards considered as a simulator of a real building as it contains a full physical description of the building. In contrary to the real building, the measurements of almost arbitrary quantities are available, imposing thus no additional technical and economical costs.

The paper is organized as follows. The building under investigation, a brief survey of the heat transfers as well as formulation of a semi-physical modeling of a building are described in Section II. Short note on parameter estimation methods is outlined in Section III. Section IV is devoted to the description of the model selection, validation and statistical tests. Performance of the aforementioned methods and approaches is discussed and summarized in Section V. Finally, the last section concludes the paper.

II. Test building

Medium weight building with a concrete wall separating two zones (both having same area of $5 \times 5 \times 3\,\mathrm{m}$ and south oriented wall with a window of $3.75\,\mathrm{m}^2$) is an objective of the investigation of this paper and is schematically sketched in Fig. 1. Transient properties between zones are considered in the following investigation. The HVAC system is of an active layer type and consists of a set of metal pipes placed in the ceiling[2] distributing supply water which then performs thermal exchange with concrete core. Each zone has a unique heating circuit with constant mass flow rate of the supply water, thus its temperature is the only manipulated variable in particular heating circuit.

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[1]Continuous time stochastic modeling software, implemented by [1].

[2]Although it might seem awkward for someone from USA to see ceiling in the heating, it was very popular way of heating in 60s in Europe, see e.g. [15]

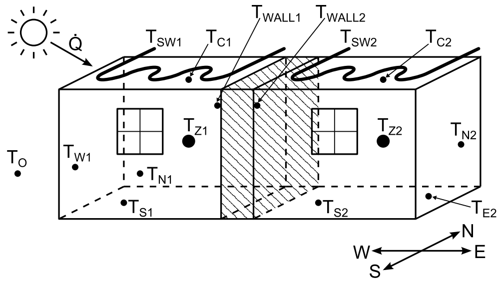


Fig. 1. A scheme of modeled building

A. Trnsys as a simulation tool

Trnsys is an energy simulation software primarily used in the fields of renewable energy and building simulation. For control engineers, a Trnsys model can be used as a data generator for construction of a building model or for validation of control algorithms. The Trnsys model as such cannot be used directly in the optimization, i.e. in predictive control, because the model is in implicit form and thus general nonlinear solvers would have to be utilized, which is computationally intractable. Various components were employed in the modeling of the building 1) Type56 for the building construction, 2) Type15 for simulation of the weather profile corresponding to Prague, Czech Republic, 3) For purposes of model identification, the link between Trnsys and Matlab was established based on Trnsys Type155. The communication link was used to generate identification data in order to excite the system properly. Pseudo random binary sequence was used as the excitation input signal. Time-step of the simulation was set to $T_s = 1/4\,\mathrm{h}$. This time-step guarantees proper convergence of Trnsys internal algorithms.

B. Heat transfer in a building

Several ways of heat transfer are recalled here.

- Conduction, which can be expressed as $\dot{T}_2 \approx \frac{T_1 - T_2}{C_2} US \approx \dot{Q}$, represents a heat transfer through solid body. T_1 and T_2 are the temperatures (both in K) of a source and a measured entity, respectively, $C_2[J/K]$ is heat capacity, $U[W/m^2K]$ is a heat transfer coefficient and $S[m^2]$ stands for area; $\dot{Q}$ is a heat flux in W. One can then write conduction time constant as $k_{cd} = \frac{C_2}{US}$ $[s]$.
- Convection, characterized as $\dot{T}_2 \approx \frac{T_1 - T_2}{k_{cv}} \cdot \sqrt[4]{\frac{T_1 - T_2}{T_1 + T_2}}$, corresponds to a heat transfer through the air, k_{cv} is a convection time constant and is derived analogically. It can be approximated by $\dot{T}_2 \approx \frac{T_1 - T_2}{K_{cv}}$ as $\sqrt[4]{\frac{T_1 - T_2}{T_1 + T_2}}$ is considered constant for building heating process [16].
- Radiation, specified by $\dot{T}_2 \approx \frac{T_1^4 - T_2^4}{k_{ra}}$, is a heat transfer through the air, k_{ra} is a constant.

C. Formulation of a model

Heat transfers from the heating pipes to ceiling surface as well as the transfer through the walls can be represented by conduction. Heat transfers between walls surfaces and zone air corresponds to both convection and radiation. For the sake of simplicity, functions $\mathcal{D}$ for conduction and $\mathcal{R}$ for convection and radiation are defined as

$$\mathcal{D}(T, D) = \sum_{T_d \in D} \frac{T_d - T}{k_i}, \tag{1}$$

$$\mathcal{R}(T, R) = \sum_{T_r \in R} \frac{T_r - T}{k_i} + \frac{T_r^4 - T^4}{k_j}, \tag{2}$$

where D, R are sets of all appropriate sources of heat, T is the temperature and k_i, k_j are unknown time constants, indices i, j means that all used constants are different (even for every use of $\mathcal{D}$ and $\mathcal{R}$). The building's physics can be described by a set of non-linear equations utilizing the heat transfer relations as described in Section II-B and by Eqns. (1)–(2). The following non-linear equations are considered as fully describing the Trnsys model.

$$
\begin{aligned}
\dot{T}_{c_1} &= \mathcal{D}(T_{c_1}, \{T_{sw_1}, T_o\}) + \mathcal{R}(T_{c_1}, \{T_{z_1}\}), \\
\dot{T}_{wall_1} &= \mathcal{D}(T_{wall_1}, \{T_{wall_2}\}) + \mathcal{R}(T_{wall_1}, \{T_{z_1}\}), \\
\dot{T}_{s_1} &= \mathcal{D}(T_{s_1}, \{T_{os_1}\}) + \mathcal{R}(T_{s_1}, \{T_{z_1}\}), \\
\dot{T}_{w_1} &= \mathcal{D}(T_{w_1}, \{T_{ow_1}\}) + \mathcal{R}(T_{w_1}, \{T_{z_1}\}), \\
\dot{T}_{n_1} &= \mathcal{D}(T_{n_1}, \{T_{on_1}\}) + \mathcal{R}(T_{n_1}, \{T_{z_1}\}), \\
\dot{T}_{z_1} &= \mathcal{R}(T_{z_1}, \{T_{c_1}, T_{wall_1}, T_{s_1}, T_{w_1}, T_{n_1}, T_o, T_{sky}\}) + \\
&\quad + \frac{\dot{Q}_s}{k_i} + \frac{\dot{Q}_{bs}}{k_i}, \\
\dot{T}_{c_2} &= \mathcal{D}(T_{c_2}, \{T_{sw_2}, T_o\}) + \mathcal{R}(T_{c_2}, \{T_{z_2}\}), \\
\dot{T}_{wall_2} &= \mathcal{D}(T_{wall_2}, \{T_{wall_1}\}) + \mathcal{R}(T_{wall_2}, \{T_{z_2}\}), \\
\dot{T}_{s_2} &= \mathcal{D}(T_{s_2}, \{T_{os_2}\}) + \mathcal{R}(T_{s_2}, \{T_{z_2}\}), \\
\dot{T}_{e_2} &= \mathcal{D}(T_{e_2}, \{T_{oe_2}\}) + \mathcal{R}(T_{e_2}, \{T_{z_2}\}), \\
\dot{T}_{n_2} &= \mathcal{D}(T_{n_2}, \{T_{on_2}\}) + \mathcal{R}(T_{n_2}, \{T_{z_2}\}), \\
\dot{T}_{z_2} &= \mathcal{R}(T_{z_2}, \{T_{c_2}, T_{wall_2}, T_{s_2}, T_{e_2}, T_{n_2}, T_o, T_{sky}\}) + \\
&\quad + \frac{\dot{Q}_s}{k_i} + \frac{\dot{Q}_{bs}}{k_i}, \\
\dot{T}_{os_1} &= \mathcal{D}(T_{os_1}, \{T_{s_1}\}) + \mathcal{R}(T_{s_1}, \{T_o, T_{sky}\}) + \\
&\quad + \frac{\dot{Q}_s}{k_i} + \frac{\dot{Q}_{bs}}{k_i}, \\
\dot{T}_{ow_1} &= \mathcal{D}(T_{ow_1}, \{T_{w_1}\}) + \mathcal{R}(T_{w_1}, \{T_o, T_{sky}\}) + \\
&\quad + \frac{\dot{Q}_w}{k_i} + \frac{\dot{Q}_{bw}}{k_i}, \\
\dot{T}_{on_1} &= \mathcal{D}(T_{on_1}, \{T_{n_1}\}) + \mathcal{R}(T_{n_1}, \{T_o, T_{sky}\}) + \\
&\quad + \frac{\dot{Q}_n}{k_i} + \frac{\dot{Q}_{bn}}{k_i}, \\
\dot{T}_{os_2} &= \mathcal{D}(T_{os_2}, \{T_{s_2}\}) + \mathcal{R}(T_{s_2}, \{T_o, T_{sky}\}) + \\
&\quad + \frac{\dot{Q}_s}{k_i} + \frac{\dot{Q}_{bs}}{k_i}, \\
\dot{T}_{oe_2} &= \mathcal{D}(T_{oe_2}, \{T_{e_2}\}) + \mathcal{R}(T_{e_2}, \{T_o, T_{sky}\}) + \\
&\quad + \frac{\dot{Q}_e}{k_i} + \frac{\dot{Q}_{be}}{k_i}, \\
\dot{T}_{on_2} &= \mathcal{D}(T_{on_2}, \{T_{n_2}\}) + \mathcal{R}(T_{n_2}, \{T_o, T_{sky}\}) +
\end{aligned}
$$

$$+ \frac{\dot{Q}_n}{k_i} + \frac{\dot{Q}_{bn}}{k_i}. \quad (3)$$

The respective symbols are defined in Table I and Table II. All radiation terms of Eq. (2) can be linearized around operating point P_0, which is naturally chosen as stable state with maximum entropy $P_0 = [T, T_r]$, where $T_r = T$ for all $T_r \in R$. Then $\sum_{T_r \in R} \frac{\partial \mathcal{R}(T,R)}{\partial T_r}\Big|_{P_0} \approx \sum_{T_r \in R} \frac{T_r - T}{\overline{k}_i}$, where $\overline{k}_i$ is a constant of proportionality influencing k_i in linear term of Eq. (2). Then the linear approximation of Eq. (3) can be written in the same form, but $\mathcal{R}(T, R)$, which is reformulated as

$$\mathcal{R}(T, R) = \sum_{T_r \in R} \frac{T_r - T}{K_i}. \quad (4)$$

Note that constants K_i in Eq. (4) and k_i in Eq. (2) have different meanings and $\frac{1}{K_i} = \frac{1}{k_i} + \frac{1}{\overline{k}_i}$.

TABLE I
NOTATION OF THE SYSTEM INPUTS AND MEASURED DISTURBANCES

Notation	ID	Description
T_{sw_1}	1	Supply water temperature, zone 1
T_{sw_2}	2	Supply water temperature, zone 2
T_o	3	Ambient temperature
$\dot{Q}_s$	4	Total solar radiation on south side
$\dot{Q}_w$	5	Total solar radiation on west side
$\dot{Q}_n$	6	Total solar radiation on north side
$\dot{Q}_e$	7	Total solar radiation on east side
$\dot{Q}_{bs}$	8	Direct solar radiation on south side
$\dot{Q}_{bw}$	9	Direct solar radiation on west side
$\dot{Q}_{bn}$	10	Direct solar radiation on north side
$\dot{Q}_{be}$	11	Direct solar radiation on east side
T_{sky}	12	Sky temperature

III. Estimation of the model parameters

A number of modeling approaches suitable for buildings has been developed over the years, from a wide variety of results, some instances can be listed as follows *i)* probabilistic semi-physical modeling [12], [10], *ii)* grey box modeling using the resistance capacitance (RC) in an analogue to electric circuit [11], [17], [18], [19], *iii)* subspace identification methods (4SID) [5], *iv)* MPC relevant identification (MRI) [8]. In the following, semi-physical modeling used for parameter estimation in this paper, is discussed in detail.

TABLE II
NOTATION OF THE SYSTEM STATES USED IN DESCRIBED MODELS

Notation	ID	Description
T_{c_1}	1	Ceiling core temperature, zone 1
T_{wall_1}	2	Core temperature of common wall, zone 1
T_{s_1}	3	Core temperature on south side, inside, zone 1
T_{w_1}	4	Core temperature on west side, inside, zone 1
T_{n_1}	5	Core temperature on north side, inside, zone 1
T_{z_1}	6	Zone temperature, zone 1
T_{c_2}	7	Ceiling core temperature, zone 2
T_{wall_2}	8	Core temperature of common wall, zone 2
T_{s_2}	9	Core temperature on south side, inside, zone 2
T_{e_2}	10	Core temperature on east side, inside, zone 2
T_{n_2}	11	Core temperature on north side, inside, zone 2
T_{z_2}	12	Zone temperature, zone 2
T_{os_1}	13	Core temperature on south side, outside, zone 1
T_{ow_1}	14	Core temperature on west side, outside, zone 1
T_{on_1}	15	Core temperature on north side, outside, zone 1
T_{os_2}	16	Core temperature on south side, outside, zone 2
T_{oe_2}	17	Core temperature on east side, outside, zone 2
T_{on_2}	18	Core temperature on north side, outside, zone 2

A. Semi-physical modeling

Having a physical description of the system Eq. (3), it is possible to estimate the model parameters directly from Eq. (3) making use of e.g Maximum Likelihood (ML) estimate formulated as follows

$$\begin{aligned} \theta^*_{ML} &= \arg\max_\theta \{\ln(L(\theta, Y_1^N))\}, \quad (5) \\ L(\theta, Y_1^N) &= \prod_{k=1}^{N} \frac{\exp(-\frac{1}{2}\varepsilon_k^T R_{k|k-1}^{-1} \varepsilon_k)}{(\sqrt{2\pi})^m \sqrt{\det(R_{k|k-1})}} p(y_o|\theta). \end{aligned}$$

Following the standard notation, L is likelihood function, Y_1^N stands for N measurements, y_o is a vector of the initial conditions, m is a dimension of the problem (number of outputs), θ is vector of unknown parameters, $p(y_o|\theta)$ is conditional probability of initial conditions on parameters, ε_k are residuals and $R_{k|k-1}$ is residual covariance matrix. It must be noted here that the problem can be solved only in iterative manner, when ε_k and $R_{k|k-1}$ are computed given estimate $\hat{\theta}$ of θ. However, to compute $\hat{\theta}$, the knowledge of the noise properties must be assumed. The estimation of both parameters and covariance matrix is performed using EM algorithm [20], [21]. The alternative procedure estimating covariance by Kalman filter in a recursive manner is implemented in CTSM [1].

IV. Model selection and validation

A. Criteria for model selection and evaluation of its quality

In case that models with different number of disturbance inputs or states are at hand, the natural question arises, how to select the better model. There are three statistical tests available for solving this issue, namely Wald test (WT), Lagrange multipliers test (LMT) and likelihood ratio test (LRT). When using LMT, the question is what parameters (if any) should be added to improve the performance of less complex model, whilst in case of WT, the task is an exact contrary, i.e. having the more complex model, the objective is to test if there are any parameters of those currently used which could be fixed to zero without worsening the model performance significantly. The basic idea of the LRT, which is used in this paper, is to compare the amount of information contained in a given model and its submodel (restricted model). If the performance of the model is not significantly (in a statistical sense) better, then it is possible to use less complex submodel. The statistical relevancy of model quality increase is based on a computation of λ defined as

$$\lambda(Y_1^N) = \frac{\max_{\theta_0 \in \Theta_0} L(\theta_0, Y_1^N)}{\max_{\theta \in \Theta} L(\theta, Y_1^N)}, \quad (6)$$

where $\theta_0 \in \Theta_0$ are the parameters of the submodel, r is a number of parameters of the model, c is a number of parameters of submodel, i.e. $dim(\Theta_0) = c$, $dim(\Theta) = r$. Then, under hypothesis $H_0 : \theta_0 \in \Theta_0$, the test statistics can be expressed as $-2\ln(\lambda(Y_1^N))$ which asymptotically follows $\chi^2(r-c)$. If the corresponding p-value is lower than the selected significance level α, then H_0 is rejected, i.e. the increase in the model quality is still significant.

B. Model performance and validation

To evaluate model in a quantitative sense, the following measures are defined as follows.

$$FIT_j = 100 \times \left(1 - \frac{||y_j - \hat{y}_j||}{||y_j - \varepsilon(y_j)||}\right) [\%], \tag{7}$$

$$R_j = 1 - \frac{var(y_j - \hat{y}_j)}{var(y_j)}, \tag{8}$$

where y_j denotes the j^{th} output of the model and $\varepsilon(x)$ denote the mean of x. The former measure is called fit factor (100% represents zero error, i.e. even noise is fully described by the model), while the latter measure is a coefficient of determination. Note that Eq. (7) and Eq. (8) are unreliable in case of purely statistically based methods such as 4SID applied to estimation of the dynamic models intended for control as they usually have the far higher fit, but for the cost of spoiled system structure.

The qualitative validation is based on testing the model residuals as they need to satisfy the assumption of whiteness [22]. Several possibilities are at hand. *i)* Autocorrelation function for visual inspection. *ii)* Statistical tests, when the null hypothesis H_0 assumes that the residuals are generated by the zero order AR process (residuals comprise white sequences) against the hypothesis that the AR process is of higher order. For details of the test, refer to [23], [24]. *iii)* Cumulative periodogram, for quantity $x_k, k = 1, \dots, n$ defined as [25]

$$\hat{F}_x(\omega_j) = \frac{\sum_{i=1}^{j} \hat{I}_x(\omega_i)}{\sum_{i=1}^{m} \hat{I}_x(\omega_j)}, \quad j = 1, \dots, m, \; m = \frac{n-1}{2} \tag{9}$$

with $\hat{I}_x(\omega) = \frac{1}{2\pi n}\left|\sum_{k=1}^{n} x_k e^{-i\omega k}\right|^2$, evaluated at ω of the form $\omega_j = 2\pi j/n$, that is $\omega \in [0, 0.5]$ of Nyquist frequency. The cumulative periodogram compares ideal white noise against the actual residuals, which is statistically tested using two-sample KS test [26], [27], where an L_∞ vector norm is used. *iv)* The last criterion for quality measure was developed for the purposes of this paper as

$$T = \sum_{j=1}^{N} \min\left\{\left\|\hat{F}_x(\omega_j) - \left(\hat{F}_{wn}(\omega_j) \pm \frac{1.36}{\sqrt{\frac{N}{2}+1}}\right)\right\|\right\} \tag{10}$$

where $\hat{F}_{wn}(\omega_j)$ and $\hat{F}_x(\omega_j)$ are values of periodogram of white noise and tested residuals at ω_j, respectively; $1.36/\sqrt{\frac{N}{2}+1}$ is KS statistics corresponding to 5% significance level, N is a number of considered samples. In fact, Eq. (10) corresponds to sum of two-norms of distances between actual $\hat{F}_x(\omega_j)$ periodogram and permitted range $\hat{F}_{wn} \pm 1.36/\sqrt{\frac{N}{2}+1}$ for white noise. Note that Eq. (10) is proportional to the area between two tested functions. Yet another possibility is to use the cross-validation techniques, see e.g. [28].

V. RESULTS

True non-linear model Eq. (3) is linearized first and the model of the following form is obtained

$$dx_t = (A(\theta)x_t + B(\theta)u_t)dt + \sigma(\theta)d\omega_t, \tag{11}$$

$$y_t = C(\theta)x_t + D(\theta)u_t + e_t, \tag{12}$$

where ω_t is an n-dimensional Wiener process and e_t is a white noise process, $e_t \sim \mathcal{N}(0, S(\theta))$. $t \in \mathbb{R}$ is time, $x_t \in X \subset \mathbb{R}^n$ is a state vector, $u_t \in U \subset \mathbb{R}^m$ is an input vector, $y_k \in Y \subset \mathbb{R}^l$ is an output vector, $\theta \in \Theta \subset \mathbb{R}^p$ is a vector of parameters, $A(\bullet)$, $B(\bullet)$, $\sigma(\bullet)$, $C(\bullet)$ $D(\bullet)$ and $S(\bullet)$ are nonlinear functions of parameters. Then, the systematic way of building-up the low-complex model is initiated. The model selection procedure is performed in two stages in an iterative manner. In the first stage, the minimum set of inputs maximizing the model quality is selected as follows. All the control inputs are taken as a fixed set and then as many models as disturbance inputs are created, each of them containing the fixed set of control inputs and one of the disturbances. The model with highest increase of quality is selected and its λ is evaluated and compared to the model from the previous iteration using LRT (see Section IV-A). If the increase in model quality is statistically significant, the input of the model under consideration is added, thus the fixed set increases by one. Thereafter, the whole procedure is repeated and continues until the increases in the model quality are statistically significant. Then, the second stage of selecting the system states follows. It proceeds in the very same manner as for the case of system inputs. The quantitative results recorded in Table III and Table IV show that both the first and the second stages had two iterations and thereafter the procedure stopped as the increase in the model quality in compare to the model was statistically insignificant. The inputs and states of the corresponding submodel are denoted U and X, respectively and are defined by indices from Table I and Table II. Models depicted by red color had the biggest value of the likelihood. The model selected as the best contains only 4 inputs (out of 12, namely both supply water temperatures, ambient temperature and total solar radiation on south surface) and only 6 states (out of 18). The results of the statistical testing as well as the values of the corresponding likelihood functions are provided in Table V. The performance of the model described by fit factor and coefficient of determination is evaluated in Table VI. These two values were computed for 192 step-ahead predictions (2 days). T from the same table was defined by Eq. (10) and KS represents L_∞ norm of the difference between the ideal white noise and periodograms of the actual residuals. The quantitative as well as qualitative

TABLE III

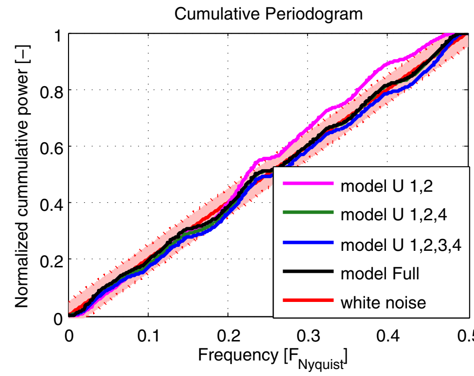
TWO-STAGE PROCEDURE OF MODEL COMPLEXITY SELECTION

Inputs analysis					
Iteration 1			Iteration 2		
model	L	r	model	L	r
U 1,2,3	11 157.8	66	U 1,2,3,4	11 964.1	70
U 1,2,4	11 600.6	60	U 1,2,4,5	11 668.5	61
U 1,2,5	11 182.0	57	U 1,2,4,6	11 669.7	62
U 1,2,6	11 159.2	58	U 1,2,4,7	11 607.3	61
U 1,2,7	11 170.7	57	U 1,2,4,8	11 605.2	64
U 1,2,8	11 536.1	60	U 1,2,4,9	11 746.9	61
U 1,2,9	11 166.3	57	U 1,2,4,10	11 746.9	62
U 1,2,10	11 166.3	58	U 1,2,4,11	11 765.3	61
U 1,2,11	11 168.0	57	U 1,2,4,12	11 631.3	68
U 1,2,12	11 175.6	64	-	-	-
States analysis					
Iteration 1			Iteration 2		
model	L	r	model	L	r
X 1,6,7,12	11 842.2	20	X 1,2,6,7,8,12	11 948.3	26
X 2,6,8,12	11 094.8	18	X 1,3,6,7,9,12	11 832.0	30
X 3,6,9,12	11 081.4	22	X 1,4,6,7,10,12	11 831.9	28
X 4,6,10,12	6 692.6	20	X 1,5,6,7,11,12	11 831.9	28
X 5,6,11,12	6 692.0	20	X 1,6,7,12,13,16	11 867.2	30
X 6,12,13,16	6 698.5	22	X 1,6,7,12,14,17	11 867.3	29
X 6,12,14,17	6 698.5	20	X 1,6,7,12,15,18	11 867.3	29
X 6,12,15,18	6 698.5	20	-	-	-

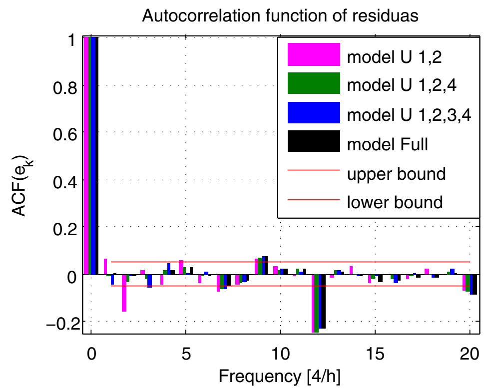
results of the model selected as a satisfactory representative of the full model has recorded indeed very good results and is almost indistinguishable from the full model as far as value of likelihood function, properties of the residuals and prediction properties. The results of the analysis of residuals are depicted in Fig. 2. The red line in Figs. 2(a) and 2(c) represents the cumulative periodograms for white noise and dotted red lines correspond to $\pm 5\%$ significance level from the white noise line. As was said, in each iteration an input (state) is added, which corresponds to one periodogram of corresponding model residuals. It can be seen, that addition of the inputs (states) causes the approach of the periodogram curves towards the tolerance range for white noise. The selected model (depicted in blue) is well within the tolerance range, i.e. its residuals are a white noise sequences, which are further tested as described in Section V. Figs. 2(b) and 2(d) depicts the autocorrelation functions of residuals coming from the successive adding disturbance inputs and system states. The red horizontal lines correspond to 5% tolerance range. It can be seen, that the values for different frequency bins decrease with the more complex models and for the model selected as a final candidate (depicted in blue) replacing the full model are within the tolerance range for the white noise sequence.

VI. CONCLUSION

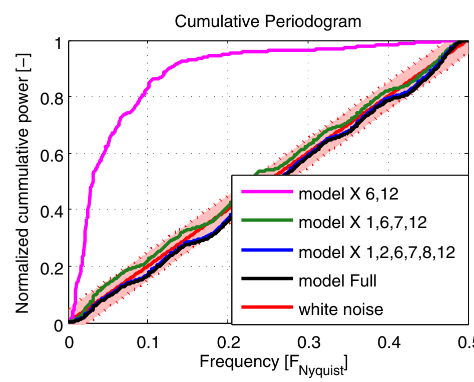
The paper presented a two stage procedure for the model complexity selection. The method advances in an iterative manner adding disturbance inputs and systems states. The presented model was a multi-zone building with temperatures



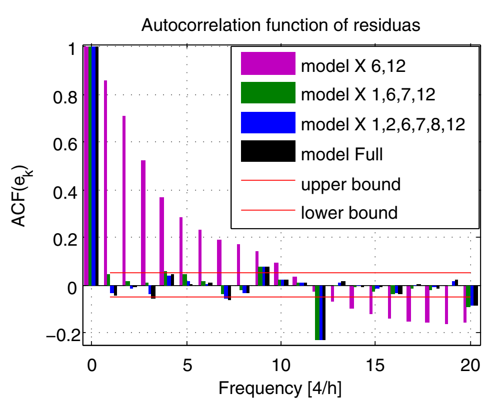
(a) Cumulative periodograms, increasing input set.



(b) Autocorrelations, increasing input set.



(c) Cumulative periodograms, increasing state set.



(d) Autocorrelations, increasing state set.

Fig. 2. Periodograms and partial autocorrelations

TABLE VI

QUANTITATIVE RESULTS OF TWO-STAGE MODEL COMPLEXITY SELECTION PROCEDURE

Model	Criterion			
	T [-]	KS [-]	Fit [%]	R [-]
U 1,2	80.80	1.00E-09	64.44	0.87
U 1,2,4	5.69	6.86E-02	72.66	0.93
U 1,2,3,4	1.25	2.07E-01	78.97	0.96
X 6,12	281.89	2.00E-139	79.25	0.96
X 1,6,7,12	1.11	1.85E-01	87.05	0.98
X 1,2,6,7,8,12	0.00	4.17E-01	89.33	0.99
Full	0.00	9.00E-01	89.37	0.99

TABLE IV

MODEL COMPLEXITY SELECTION: INITIAL AND FULL MODELS

Other models			
	model	L	r
The simplest input structure	U 1,2	11 166.7	56
The simplest state structure	X 6,12	6 692.0	12
The most complex model	Full	11 964.7	90

TABLE V

LIKELIHOOD RATIO TEST FOR IMPROVEMENTS OF MODEL QUALITY DUE TO ADDITION OF INPUTS/STATES

model	L	r	p-value
U 1,2	11 166.7	56	0.0000
U 1,2,4	11 600.6	60	0.0000
U 1,2,3,4	11 964.1	70	1.0000
X 6,12	6 692.0	12	0.0000
X 1,6,7,12	11 842.2	20	0.0000
X 1,2,6,7,8,12	11 948.3	26	0.9993

in respective zones being affected by those from the other zones. The procedure stops when there is no statistically significant quality improvement. The ultimately selected model containing only 4 inputs and 6 states has similar properties as the model with the full set of inputs and states. The resulting model is much simpler, thus computationally tractable for calculation of the optimal control problem for large systems.

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REFERENCES

[1] N. Kristensen, H. Madsen, and S. Jørgensen, "Parameter estimation in stochastic grey-box models," *Automatica*, vol. 40, no. 2, pp. 225–237, 2004.

[2] Y. Ma, F. Borrelli, B. Hencey, B. Coffey, S. Bengea, and P. Haves, "Model predictive control for the operation of building cooling systems," in *American Control Conference (ACC), 2010*, jun. 2010, pp. 5106–5111.

[3] J. Cigler, S. Prívara, Z. Váňa, E. Žáčeková, and L. Ferkl, "Optimization of predicted mean vote index within model predictive control framework: Computationally tractable solution," *Energy and Buildings*, vol. 52, no. 0, pp. 39 – 49, 2012.

[4] J. Široký, F. Oldewurtel, J. Cigler, and S. Prívara, "Experimental analysis of model predictive control for an energy efficient building heating system," *Applied Energy*, vol. 88, no. 9, pp. 3079–3087, 2011.

[5] S. Prívara, J. Široký, L. Ferkl, and J. Cigler, "Model predictive control of a building heating system: The first experience," *Energy and Buildings*, vol. 43, no. 2, pp. 564–572, 2011.

[6] S. Prívara, J. Cigler, Z. Váňa, F. Oldewurtel, and E. Žáčeková, "Use of partial least squares within the control relevant identification for buildings," *Control Engineering Practice*, vol. 21, pp. 113–121, January 2013.

[7] S. Prívara, J. Cigler, Z. Váňa, F. Oldewurtel, C. Sagerschnig, and E. Žáčeková, "Building modeling as a crucial part for building predictive control," *Energy and Buildings*, vol. 56, pp. 8–22, January 2013.

[8] E. Žáčeková, S. Prívara, and Z. Váňa, "Model predictive control relevant identification using partial least squares for building modeling," in *Australian Control Conference (AUCC), 2011*. IEEE, 2011, pp. 422–427.

[9] C. Verhelst and L. Helsen, "Low-order state space models for borehole heat exchangers," *HVAC&R Research*, vol. 17, no. 6, pp. 928–947, 2011.

[10] P. Bacher and H. Madsen, "Identifying suitable models for the heat dynamics of buildings," *Energy and Buildings*, vol. 43, no. 7, pp. 1511–1522, 2011.

[11] M. Jimenez, H. Madsen, and K. Andersen, "Identification of the main thermal characteristics of building components using MATLAB," *Building and Environment*, vol. 43, no. 2, pp. 170–180, 2008.

[12] K. Andersen, H. Madsen, and L. Hansen, "Modelling the heat dynamics of a building using stochastic differential equations," *Energy and Buildings*, vol. 31, no. 1, pp. 13–24, 2000.

[13] C. Gourieroux, A. Holly, and A. Monfort, "Likelihood ratio test, wald test, and kuhn-tucker test in linear models with inequality constraints on the regression parameters," *Econometrica: Journal of the Econometric Society*, pp. 63–80, 1982.

[14] D. Dickey and W. Fuller, "Likelihood ratio statistics for autoregressive time series with a unit root," *Econometrica: Journal of the Econometric Society*, pp. 1057–1072, 1981.

[15] R. G. Crittall and J. L. Musgrave, "Heating and cooling of buildings," GB Patent No. 210880, April 1927.

[16] M. Barták, *Úvod do přenosových jevu pro inteligentní budovy*. Prague, Czech Republic: CTU Prague, 2010.

[17] G. Hudson and C. Underwood, "A simple building modelling procedure for MATLAB/SIMULINK," vol. 99, no. 2, pp. 777–83, 1999.

[18] G. Levermore, *Building energy management systems: an application to heating and control*. E & FN Spon, 1992.

[19] S. Wang and X. Xu, "Simplified building model for transient thermal performance estimation using GA-based parameter identification," *International Journal of Thermal Sciences*, vol. 45, no. 4, pp. 419–432, 2006.

[20] T. Louis, "Finding the observed information matrix when using the em algorithm," *Journal of the Royal Statistical Society. Series B (Methodological)*, pp. 226–233, 1982.

[21] G. McLachlan and T. Krishnan, *The EM algorithm and extensions*. John Wiley and Sons, 2008, vol. 382.

[22] L. Ljung, *System identification*. Wiley Online Library, 1999.

[23] T. Pukkila and P. Krishnaiah, "On the use of autoregressive order determination criteria in multivariate white noise tests," *Acoustics, Speech and Signal Processing, IEEE Transactions on*, vol. 36, no. 9, pp. 1396–1403, sep 1988.

[24] P. Matisko and V. Havlena, "Optimality tests and adaptive kalman filter," in *Preprint submitted to 16th IFAC Symposium on System Identification*. IFAC, 2012.

[25] G. Box, G. Jenkins, and G. Reinsel, *Time series analysis*. Holden-day San Francisco, 1970.

[26] I. Young, "Proof without prejudice: use of the kolmogorov-smirnov test for the analysis of histograms from flow systems and other sources." *Journal of Histochemistry & Cytochemistry*, vol. 25, no. 7, p. 935, 1977.

[27] G. Fasano and A. Franceschini, "A multidimensional version of the kolmogorov-smirnov test," *Monthly Notices of the Royal Astronomical Society*, vol. 225, pp. 155–170, 1987.

[28] M. A. Lukas, "Strong robust generalized cross-validation for choosing the regularization parameter," *Inverse Problems*, vol. 24, no. 3, p. 034006, 2008.