2013 European Control Conference (ECC)
July 17-19, 2013, Zürich, Switzerland.

Passive and Active FTC Comparison for Polytopic LPV Systems

Damiano Rotondo, Fatiha Nejjari and Vicenç Puig

Abstract— **Fault-tolerant control (FTC) allows to preserve performance and stability despite the presence of faults. The literature considers two main groups of techniques: the passive and the active FTC techniques. In case of the passive techniques, the fault is taken into account as a system perturbation, so that the control law has fault capabilities that allow the system to cope with the fault presence. On the other hand, in the case of the active FTC techniques, the control law uses some information given by a Fault Detection and Isolation (FDI) module, so that through some automatic adjustment in the control loop, the fault is tolerated with minimum performance degradation. In this paper, a linear parameter-varying (LPV)/linear matrix inequalities (LMIs)-based technique is used to achieve fault tolerance and to compare benefits and drawbacks of passive and active FTC. The proposed approach is applied to a two-wheel differential robot.**

Keywords: Fault tolerant systems, Linear parameter-varying systems, Autonomous Robots

I. INTRODUCTION

Fault-tolerant control (FTC) allows to maintain current performance close to desirable ones and preserve stability conditions in the presence of component and/or instrument faults [1]. Accommodation capability of a control system depends on many factors such as severity of the failure, the robustness of the nominal control system and mechanisms that introduce redundancy in sensors and/or actuators.

From the point of view of the control strategies, the literature considers two main groups of techniques, namely the *active* and the *passive*. The *passive FTC techniques* are control laws that take into account the fault appearance as a system perturbation. Thus, within certain margins, the control law has inherent fault capabilities, allowing the system to cope with the fault presence. On the other hand, the *active FTC techniques* adapt the control law using the information given by a Fault Detection and Isolation (FDI) module. With this information, some automatic adjustments in the control loop are done after the fault appearance trying to satisfy the control objectives with minimum performance degradation. The development and characteristics of both active and passive approaches have been reviewed by the survey papers appeared in the last decades [2]–[4]. On the other hand, a comparative study between the two approaches is done in [5] and [6]. In these works, similarities and differences are studied from both philosophical and practical point of view, and advantages and limitations of each approach are examined using a wind turbine and an aircraft control system, respectively.

In this paper, a linear parameter-varying (LPV)/linear matrix inequalities (LMIs)-based technique is used to achieve fault tolerance and to further compare benefits and drawbacks of passive and active FTC. The use of LPV systems theory for FTC dates back to the end of the 80s, when the concept of a self-repairing control system was introduced in the form of a reconfigurable multivariable feedback, where a gain scheduling design procedure was used to stabilize a collection of plant models representing the aircraft in various control failure modes [7]. In [8], an LPV controller is synthesized using a quasi-LPV model of the Boeing 747-100/200 longitudinal axis. In faulty situation, the controller is scheduled using a fault signal generated by an FDI algorithm. In [9], a polytopic LPV representation is used to handle multiple actuator failures. A Static Output Feedback (SOF) technique is used to design multiple controllers such that closed-loop stability can be maintained for any combination of multiple actuator failures. In recent years, a lot of effort has been put into combining LPV and FTC with other approaches, e.g. admissible model matching [10], virtual sensors [11] and virtual actuators [12].

This paper has the following structure: in Section II, the LPV and fault representations are presented. In Section III, the fault tolerant control strategies using the passive LPV approach and the active LPV approach are described. In Section IV, the design by means of LMI pole placement and the polytopic approximation are presented. In Section V, the application example, namely a two-wheel differential robot, is described and a quasi-LPV model is obtained. In Section VI, the comparison between the passive and active LPV approaches is performed and discussed, and the main conclusions are finally summarized in Section VII.

Following the notation used by [13], σ stands for the Laplace variable s in the continuous-time (C-T) case and for the Z-transform variable z in the discrete-time (D-T) case. Similarly, τ will stand for the time $t \in \mathbb{R}^+$ in the continuous-time case and for the time samples $k \in \mathbb{Z}^+$ in the discrete-time case. The notation $\sigma.x(\tau)$ stands for $\dot{x}(t)$ for continuous-time signals and for $x(k+1)$ for discrete-time signals. For real symmetric matrices M, the notation $M > 0$ stands for *positive definite* and indicates that all the eigenvalues of M are positive. Similarly, $M < 0$ means *negative definite*, that is, all the eigenvalues of M are negative. Given a vector

This work has been funded by the Spanish MINECO through the project CYCYT SHERECS (ref. DPI2011-26243), by the European Commission through contract i-Sense (ref. FP7-ICT-2009-6-270428) and by UPC through the grant FPI-UPC E-01104.

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$v = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix}$, diag($v$) is defined as follows:

$$\text{diag}(v) = \begin{pmatrix} v_1 & 0 & \dots & 0 \\ 0 & v_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & v_n \end{pmatrix}$$

II. LPV System and Fault Definition

An LPV system is defined by state-space equations whose matrices depend on some vector of varying parameters $\vartheta(\tau) \in \Theta$:

$$\sigma.x(\tau) = A(\vartheta(\tau))x(\tau) + B(\vartheta(\tau))u(\tau) \quad (1)$$

$$y(\tau) = C(\vartheta(\tau))x(\tau) + D(\vartheta(\tau))u(\tau) \quad (2)$$

where $x(\tau) \in \mathbb{R}^{n_x}$ represents the state vector, $u(\tau) \in \mathbb{R}^{n_u}$ denotes the control input and $y(\tau) \in \mathbb{R}^{n_y}$ are the sensor outputs. In this paper, we consider that the state is directly measured and available for the controller design, that is, $C(\vartheta(\tau)) = I$ and $D(\vartheta(\tau)) = 0$.

Two types of faults are considered: parametric faults $f_A(\tau) \in \mathbb{R}^{n_{fA}}$ affecting the matrix A and actuator faults $f_B(\tau) \in \mathbb{R}^{n_u}$ affecting the matrix B. Then, under fault occurrence, (1) becomes:

$$\sigma.x(\tau) = A_f(\vartheta(\tau), f_A(\tau))x(\tau) + B_f(\vartheta(\tau), f_B(\tau))u(\tau) \quad (3)$$

where the actuator faults are embedded in the matrix $B_f(\vartheta(\tau), f_B(\tau))$ as follows:

$$B_f(\vartheta(\tau), f_B(\tau)) = B(\vartheta(\tau))\,\text{diag}(f_B(\tau)) \quad (4)$$

Each element of the vector $f_B(\tau)$ can take values between 0 and 1, representing the effectiveness of the actuators. For example, $f_{B,i}$ represents the effectiveness of the i^{th} actuator, such that $f_{B,i} = 1$ represents the healthy condition and $f_{B,i} = 0$ the total loss of the i^{th} actuator.

On the other hand, it is assumed that the parametric faults $f_A(\tau)$ affect the state matrix in such a way that they can be considered as additional variables that schedule the system. It is also assumed that they are bounded, and that a lower and upper bound for each of its elements are known a priori.

III. Fault Tolerant Control Strategy

A. Passive LPV Approach

In the passive LPV approach, it is assumed that no information about the faults is available on-line, thus the controller cannot be scheduled by an estimation of the faults affecting the system. However, there is some knowledge available *a priori* about the boundedness of the faults, that can be exploited in the controller design phase. Hence, if a state-feedback control with tracking reference input, as proposed in [14], is used, the feedback control law results in the following structure:

$$u(\tau) = u_r(\tau) + K_{pas}(\vartheta(\tau))(x(\tau) - x_r(\tau)) \quad (5)$$

where the state reference $x_r(\tau)$ and the feedforward control action $u_r(\tau)$ correspond to an equilibrium point for the reference $r(\tau)$.

The controller gain $K_{pas}(\vartheta(\tau)) \in \mathbb{R}^{n_u \times n_x}$ should be designed in such a way that some desired specifications, among which stability, should be satisfied for any value of $\vartheta \in \Theta$, and for any possible value of the faults $f_A(\tau)$ and $f_B(\tau)$.

B. Active LPV Approach

In the active LPV approach, an estimation of the faults is provided by some fault estimation algorithm, and this information can be used to schedule accordingly the controller. Hence, if a state-feedback control with tracking reference input [14] is used, the control law is expressed as:

$$u(\tau) = u_r(\tau) + K_{act}(\vartheta(\tau), f_A(\tau), f_B(\tau))(x(\tau) - x_r(\tau)) \quad (6)$$

with the state reference $x_r(\tau)$ and the feedforward control law $u_r(\tau)$ corresponding to an equilibrium for the reference $r(\tau)$. The controller gain $K_{act}(\vartheta(\tau), f_A(\tau), f_B(\tau))$ should be designed so as to satisfy the desired specifications for any possible value of $\vartheta(\tau)$, $f_A(\tau)$ and $f_B(\tau)$.

C. LPV Integral Action

Integral control is needed in order to eliminate the steady-state errors due to inherent uncertainty in the modeling of the system to be controlled. The fault occurrence gives rise to a model mismatch that influences the steady-state behavior of the system. For example, in the passive LPV approach, even though some properties as stability and desired dynamical performance can be assured despite the faults thanks to the robustness of the control law, the tracking performances could exhibit a degradation because the state reference $x_r(\tau)$ and the feedforward control action $u_r(\tau)$ are calculated on the basis of the nominal fault-free model of the system. Thus, under fault occurrence, an adjustment of $x_r(\tau)$ and $u_r(\tau)$ could be needed, but this is not achievable as it is assumed that no information about the fault is available. On the other hand, if an active LPV approach is used, such an adjustment is possible and the addition of an integral action would not be needed. However, steady-state errors in the fault estimation could lead to a similar problem than the one arisen in the passive LPV case, although less critical. This motivates the use of LPV integral action.

In this paper, a method based on state augmentation is used to add an integral action to the control law [15]. More specifically, the state of the system (1)-(2) is augmented with $x_I(\tau)$, the integral of the error $e(\tau) = y(\tau) - r(\tau)$, therefore arriving to the augmented plant model:

$$\begin{bmatrix} \sigma.x(\tau) \\ \sigma.x_I(\tau) \end{bmatrix} = \begin{bmatrix} A(\vartheta(\tau)) & 0 \\ C(\vartheta(\tau)) & A_I \end{bmatrix} \begin{bmatrix} x(\tau) \\ x_I(\tau) \end{bmatrix} + \begin{bmatrix} B(\vartheta(\tau)) \\ 0 \end{bmatrix} u(\tau) - \begin{bmatrix} 0 \\ I \end{bmatrix} r(\tau) \quad (7)$$

where $A_I = 0$ in the C-T case and $A_I = I$ in the D-T case.

Notice that the addition of extra poles will typically lead to a deteriorated response compared to the one obtained without integral control.

IV. Design using LMI Pole Placement

A. LMI Pole Placement for LPV Systems

An LMI approach for the design by pole placement constraints is described in [16]. The main motivation for seeking pole clustering in specific regions of the complex plane is that, by constraining the eigenvalues to lie in a predefined region, a satisfactory transient response can be ensured. A subset $\mathcal{D}$ of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha = [\alpha_{kl}] \in \mathbb{R}^{m\times m}$ and a matrix $\beta = [\beta_{kl}] \in \mathbb{R}^{m\times m}$ such that:

$$\mathcal{D} = \{z \in \mathbb{C} : f_{\mathcal{D}}(z) < 0\} \tag{8}$$

with

$$f_{\mathcal{D}}(z) := \alpha + z\beta + \bar{z}\beta^T = [\alpha_{kl} + \beta_{kl}z + \beta_{lk}\bar{z}]_{1\leqslant k,l\leqslant m} \tag{9}$$

Using Gutman's theorem for LMI regions [17], pole location in a given LMI region can be characterized in terms of the $m \times m$ block matrix

$$\begin{aligned} M_{\mathcal{D}}(A,X) &= \alpha \otimes X + \beta \otimes (AX) + \beta^T \otimes (AX)^T \\ &= \left[\alpha_{kl}X + \beta_{kl}AX + \beta_{lk}XA^T\right]_{1\leq k,l\leq m} \end{aligned} \tag{10}$$

Then, a matrix A is $\mathcal{D}$-stable (that is, all its eigenvalues lie in $\mathcal{D}$) if and only if there exists a symmetric matrix X such that:

$$\begin{cases} M_{\mathcal{D}}(A,X) < 0 \\ X > 0 \end{cases} \tag{11}$$

where $M_{\mathcal{D}}(A,X)$ in (10) and $f_{\mathcal{D}}(z)$ in (9) are related by the substitution $(X, AX, XA^T) \leftrightarrow (1, z, \bar{z})$.

Consider the LPV system (1)-(2) under state-feedback control law $u(\tau) = K(\vartheta(\tau))x(\tau)$. The problem to be solved consists in finding a state-feedback gain K scheduled by ϑ that places the closed-loop poles of (1)-(2) in some LMI region $\mathcal{D}$ with characteristic function (9).

Notice that, following [18] and with a little abuse of language, the poles of an LPV system are defined as the set of all the poles of the LTI systems obtained by freezing $\vartheta(\tau)$ to some value ϑ^*.

Hence, the pole-placement constraint is satisfied if and only if there exists $X = X^T > 0$ such that:

$$\left\{\alpha_{kl}X + \beta_{kl}\left[A(\vartheta) + B(\vartheta)K(\vartheta)\right]X + \beta_{lk}X\left[A(\vartheta) + B(\vartheta)K(\vartheta)\right]^T\right\}_{1\leq k,l\leq m} < 0 \tag{12}$$

is satisfied for all possible values of ϑ.

By means of the auxiliary variable $\Gamma(\vartheta) = K(\vartheta)X$, the matrix inequality (12) becomes an LMI, that can be solved through convex optimization techniques:

$$\left\{\alpha_{kl}X + \beta_{kl}\left[A(\vartheta)X + B(\vartheta)\Gamma(\vartheta)\right] + \beta_{lk}\left[A(\vartheta)X + B(\vartheta)\Gamma(\vartheta)\right]^T\right\}_{1\leq k,l\leq m} < 0 \tag{13}$$

B. Polytopic Approximation

From a practical point of view, (13) is useless because it imposes an infinite number of constraints to be solved. One effective way to solve this problem is reducing the LMI constraints to a finite number through polytopic LPV modeling.

An LPV system is called *polytopic* when it can be represented by state-space matrices whose dependence on the parameter vector ϑ, that ranges over a polytope, is affine [13]:

$$\begin{aligned} &\begin{pmatrix} A(\vartheta(\tau)) \\ B(\vartheta(\tau)) \end{pmatrix} \in Co\left\{\begin{pmatrix} A_i \\ B_i \end{pmatrix}\right\} \\ &= \left\{\sum_{i=1}^{N} \alpha_i(\vartheta)\begin{pmatrix} A_i \\ B_i \end{pmatrix}, \alpha_i(\vartheta) \geq 0, \sum_{i=1}^{N} \alpha_i(\vartheta) = 1\right\} \end{aligned} \tag{14}$$

In this case, under the assumptions that the input matrix is constant, that is, $B(\vartheta) = B$, and that the controller structure is polytopic as well, that is, $K(\vartheta(\tau)) = \sum_{i=1}^{N} \alpha_i(\vartheta(\tau))K_i$, the problem reduces to computing a single matrix $X = X^T > 0$ and N matrices $\Gamma_i = K_iX$ such that:

$$\left[\alpha_{kl}X + \beta_{kl}(A_iX + B\Gamma_i) + \beta_{lk}(A_iX + B\Gamma_i)^T\right]_{1\leq k,l\leq m} < 0 \tag{15}$$

Once the LMIs system (15) is solved, the controller gains K_i can be determined as $K_i = \Gamma_i X^{-1}$.

Given an LPV system, starting with the assumption that measurements of ϑ are available in real-time and the range of ϑ is known a priori, a polytopic representation of the LPV system can be obtained using the technique described in [19]. This modeling approach is referred to as *bounding box method* because the convex hull generated by such approach has always the shape of a rectangular bounding box, that is, an hyperrectangle.

Remark: If the assumption that the input matrix $B(\vartheta) = B$ is constant were not satisfied, the difficulty could be alleviated by pre-filtering the inputs u, as proposed in [13]. Specifically, define a new input vector $\tilde{u}$ by:

$$\sigma.x_{\tilde{u}}(\tau) = A_{\tilde{u}}x_{\tilde{u}}(\tau) + B_{\tilde{u}}\tilde{u}(\tau) \tag{16}$$

$$u(\tau) = C_{\tilde{u}}x_{\tilde{u}}(\tau) \tag{17}$$

where $A_{\tilde{u}}$ is stable. The resulting LPV plant is described by:

$$\begin{pmatrix} \sigma.x(\tau) \\ \sigma.x_{\tilde{u}}(\tau) \end{pmatrix} = \begin{pmatrix} A(\vartheta(\tau)) & B(\vartheta(\tau))C_u \\ 0 & A_{\tilde{u}} \end{pmatrix}\begin{pmatrix} x(\tau) \\ x_{\tilde{u}}(\tau) \end{pmatrix} + \begin{pmatrix} 0 \\ B_{\tilde{u}} \end{pmatrix}\tilde{u}(\tau) \tag{18}$$

that satisfies the assumption.

V. Application Example

The application example used in this paper is a two-wheel differential robot simulator. The simulated robot has a circular shape with a diameter $d = 2r = 0.34\,m$ and a mass $m = 2.92\,kg$. The vehicle is driven by two differential drive wheels that can reach the maximum speed of $\dot{z}_{\max} = 0.5\,m/s$. By altering the speed of the individual wheels, the direction of the movements can be changed.

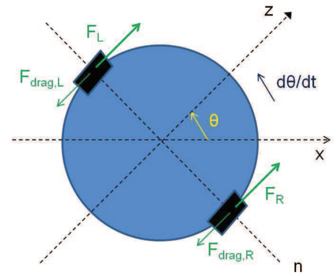

Fig. 1. The robotic platform used in the example.

A mathematical model of the robot (Fig. 1) can be obtained through a balance of the forces and the moments acting on the system:

$$\left\{ \begin{array}{l} m\ddot{z} = \left(F_L + F_R - F_{drag,L} - F_{drag,R}\right) \\ \frac{1}{2}mr^2\ddot{\theta} = \left(F_R - F_L + F_{drag,L} - F_{drag,R}\right) r \end{array} \right. \tag{19}$$

where:

$$\begin{array}{cc} F_{drag,L} = k_{drag} v_L \left|v_L\right| & F_{drag,R} = k_{drag} v_R \left|v_R\right| \\ v_L = \dot{z} - r\dot{\theta} & v_R = \dot{z} + r\dot{\theta} \end{array}$$

with $k_{drag} = 11\,kg/m$ being the drag coefficient in the normal operating conditions, z the total covered distance, $\dot{z}$ the linear velocity, θ the yaw angle and $\dot{\theta}$ the angular velocity. The system can be controlled using the available control input F_L and F_R, that are the forces acting on the left and the right wheel, respectively.

By considering the state vector $x = \left[\begin{array}{cccc} z & \dot{z} & \theta & \dot{\theta} \end{array}\right]^T$, the input vector $u = \left[\begin{array}{cc} F_L & F_R \end{array}\right]^T$, and embedding the non-linearities in the parameters, (19) can be put in a quasi-LPV form as follows:

$$\left(\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{array}\right) = \left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & \vartheta_1(x) & 0 & \vartheta_2(x) \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2}{r^2}\vartheta_2(x) & 0 & \vartheta_3(x) \end{array}\right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array}\right) + B \left(\begin{array}{c} u_1 \\ u_2 \end{array}\right) \tag{20}$$

with:

$$B = \left(\begin{array}{cccc} 0 & 1/m & 0 & -2/(mr) \\ 0 & 1/m & 0 & 2/(mr) \end{array}\right)^T$$

and vector of varying parameters $\vartheta(x) = \left[\begin{array}{ccc} \vartheta_1(x) & \vartheta_2(x) & \vartheta_3(x) \end{array}\right]^T$, where $\vartheta_1(x)$, $\vartheta_2(x)$ and $\vartheta_3(x)$ are defined as follows:

$$\vartheta_1(x) = a_{22}(x_2,x_4) = \left\{ \begin{array}{ll} -2k_{drag}x_2/m & x_2 \geq |rx_4| \\ -2rk_{drag}x_4/m & -rx_4 < x_2 < rx_4 \\ 2rk_{drag}x_4/m & rx_4 < x_2 < -rx_4 \\ 2k_{drag}x_2/m & x_2 \leq |rx_4| \end{array} \right.$$

$$\vartheta_2(x) = a_{24}(x_2,x_4) = \left\{ \begin{array}{ll} -2k_{drag}r^2x_4/m & x_2 \geq |rx_4| \\ -2rk_{drag}x_2/m & -rx_4 < x_2 < rx_4 \\ 2rk_{drag}x_2/m & rx_4 < x_2 < -rx_4 \\ 2k_{drag}r^2x_4/m & x_2 \leq |rx_4| \end{array} \right.$$

$$\vartheta_3(x) = a_{44}(x_2,x_4) = \left\{ \begin{array}{ll} -4k_{drag}x_2/m & x_2 \geq |rx_4| \\ -4rk_{drag}x_4/m & -rx_4 < x_2 < rx_4 \\ 4rk_{drag}x_4/m & rx_4 < x_2 < -rx_4 \\ 4k_{drag}x_2/m & x_2 \leq |rx_4| \end{array} \right.$$

VI. RESULTS

The robot can be affected by parametric faults (unexpected change in the drag coefficient k_{drag}), and sensor/actuator faults, of either additive or multiplicative faults. In this paper, we focus on the parametric fault, that can be quite critical if no measures are taken. In Fig. 2, the time response of the robot with different values of the drag coefficient is compared under the following conditions:

- the controller is designed to put the closed-loop poles in a circle of center $(-5.5, 0)$ and radius 4.5 without taking into account the possibility of faults occurrence; this choice guarantees a decay rate $\lambda \in [1s, 10s]$, a minimum damping ratio $\zeta = 0.5750$ and a maximum undamped natural frequency $\omega_d = 8.1816\,rad/s$;
- a bounding box characterization of the quasi-LPV model is used for designing the controller, using the following extreme values for the state variables affecting the quasi-LPV model parameters[1]:

$$x_2^{\min} = -0.5 \quad x_2^{\max} = 0.5 \quad x_4^{\min} = -3 \quad x_4^{\max} = 3$$

- the reference is chosen as follows:

$$\left[x_2^{ref}(t), x_4^{ref}(t)\right] = \left\{ \begin{array}{ll} [0.1, 0.2]^T & t \leq 15s \\ [0.4, 0.3]^T & 15s < t \leq 30s \\ [-0.3, -0.7]^T & 30s < t \leq 60s \end{array} \right.$$

$$x_1^{ref}(t) = \int_0^t x_2^{ref}(t)dt \quad x_3^{ref}(t) = \int_0^t x_4^{ref}(t)dt$$

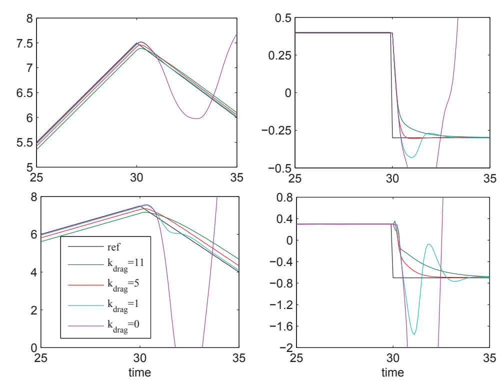

Fig. 2. Time response of the robot under fault occurrence.

It can be seen that the controller has an intrinsic robustness against faults and can tolerate them until a certain magnitude without losing the system stability (e.g. $k_{drag}^f = 5\,kg/m$, corresponding to the red line). However, taking a look at the position of the poles under fault occurrence (Fig. 3), it can be seen that even though the fault $k_{drag}^f = 5\,kg/m$ does not affect the system stability, it compromises its performance, as the desired specification in terms of poles location is no longer respected.

[1]These extreme values have been found by simulating the non-linear model applying the maximum and minimum possible inputs $F_{min} = -2.7N$ and $F_{max} = 2.7N$.

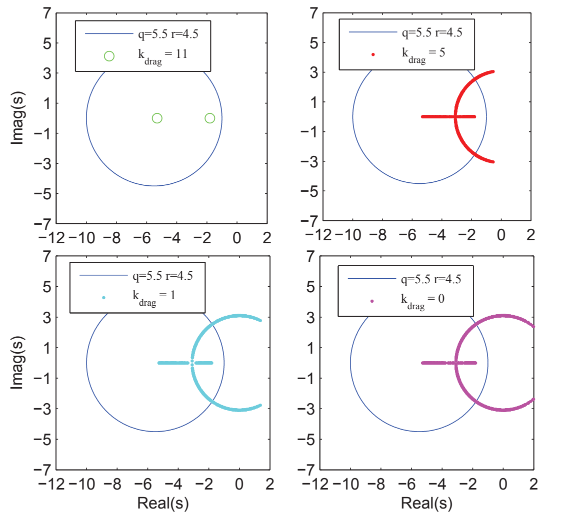

Fig. 3. Pole placement of the robot under fault occurrence without FTC for different possible values of the faulty drag coefficient.

Using passive FTC, it is possible to enforce the robustness of the controller under fault occurrence. Once a desired region of the complex plane has been chosen, it is possible to find a lower bound for the faulty drag coefficient that makes the design LMIs feasible. In this example, for the circle of center $(-5.5, 0)$ and radius 4.5, a limit value of $k^f_{drag} = 7\,kg/m$ has been found. Fig. 4 shows a comparison between the closed-loop poles without FTC and the ones with passive FTC with such a value of the faulty drag coefficient, for 10000 different realizations of the robot state matrix. It can be seen that if no fault tolerance is enforced during the design phase, the poles can escape from the desired region. The proposed approach avoids such undesired behavior.

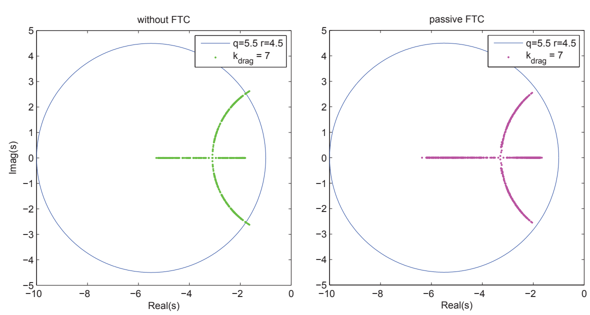

Fig. 4. Robot under fault occurrence: comparison between pole placement without FTC and pole placement with passive FTC ($K_{drag^f} = 7\,kg/m$).

On the other hand, active FTC is less conservative than passive FTC because the fault is dealt with as it were a scheduling variable and not an additional uncertainty against which robustness must be enforced. A lesser conservativeness can be seen analyzing the lower bound for the faulty drag coefficient that makes the design LMIs feasible for the desired region of the complex plane. In Fig. 5, it is shown that active FTC is able to satisfy the desired specifications for a circle of center $(-5.5, 0)$ and radius 4.5 for any value of the faulty drag coefficient until $0\,kg/m$. The position of the closed-loop poles does not depend on the specific realization of the robot state matrix.

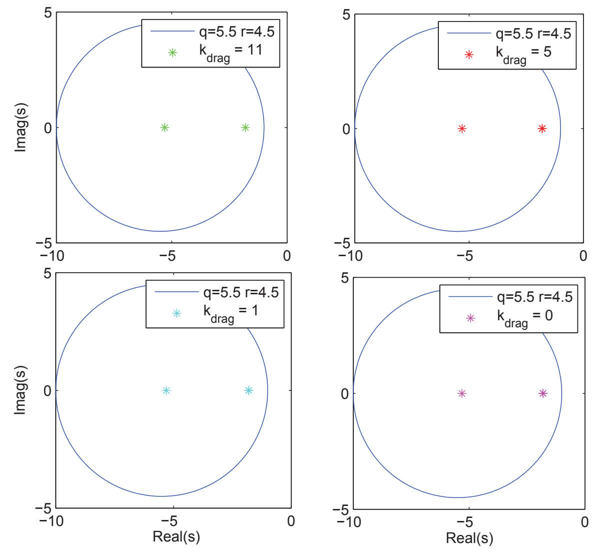

Fig. 5. Pole placement of the robot under fault occurrence with active FTC for different possible values of the faulty drag coefficient.

However, a drawback of the active FTC methods is that the precision of the fault estimation can affect the performances in terms of fault tolerance. This is shown in Fig. 6, where it can be seen that as the uncertainty in the fault estimation grows, so does the variation of the closed-loop poles position[2]. This effect may even cause the closed-loop poles to leave the desired region of the complex plane, as in the case of an estimation error bigger than 80% of the real value of the faulty drag coefficient.

The comparison between passive FTC and active FTC has been carried out for different regions of the complex plane, all expressed as circles with a certain center $(-q, 0)$ and a certain radius r. For a lack of space, the obtained results are not shown in a graphical form, but summarized in Table I, where the lower bound on the value of k^f_{drag} for which fault tolerance is guaranteed is given for each circle in both the passive and the active FTC cases. Moreover, for the active FTC case, the tolerated uncertainty is shown too. It can be seen that there is a trade-off between performances and tolerable fault in the passive FTC case, and between performance and tolerable uncertainty in the active FTC case. It appears clearly that the designer of the FTC system should choose the strategy according to the availability of an estimation of the fault magnitude and the goodness of this estimation.

[2]In this work, the problem of fault estimation is not addressed. Hence, the uncertainty in the fault estimation is modeled as random noise uniformly distributed around the real value of the drag coefficient.

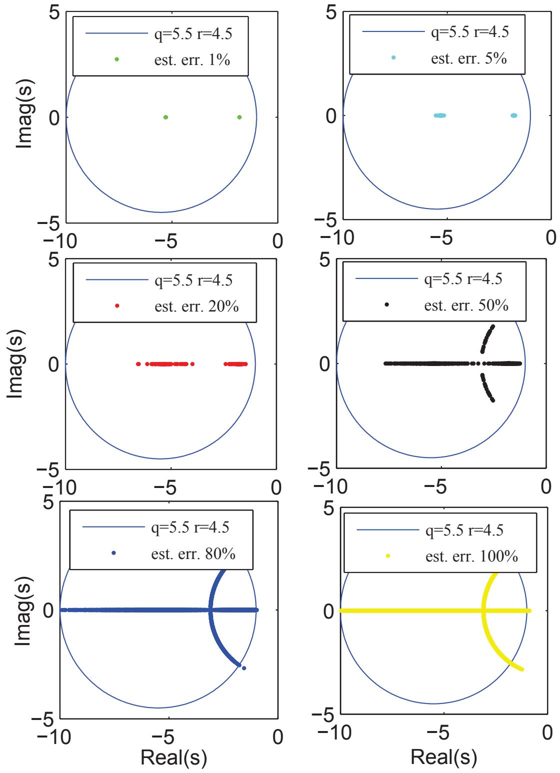

Fig. 6. Pole placement of the robot under fault occurrence with active FTC for different possible uncertainties of the fault estimation ($k^f_{drag} = 5\,kg/m$).

TABLE I

COMPARISON BETWEEN PASSIVE FTC AND ACTIVE FTC

Center	Radius	k^f_{drag} pas.	k^f_{drag} act.	Tol. unc. ($k^f_{drag} = 5\,kg/m$)
$(-10.5,0)$	10.5	$0\,kg/m$	$0\,kg/m$	210%
$(-9.5,0)$	9.5	$0.5\,kg/m$	$0\,kg/m$	190%
$(-8.5,0)$	8.5	$1.6\,kg/m$	$0\,kg/m$	160%
$(-7.5,0)$	7.5	$2.6\,kg/m$	$0\,kg/m$	150%
$(-6.5,0)$	6.5	$3.8\,kg/m$	$0\,kg/m$	130%
$(-5.5,0)$	5.5	$5.9\,kg/m$	$0\,kg/m$	100%
$(-5.5,0)$	4.5	$7\,kg/m$	$0\,kg/m$	80%
$(-5.5,0)$	3.5	$8.6\,kg/m$	$0\,kg/m$	20%
$(-5.5,0)$	2.5	$9.9\,kg/m$	$0\,kg/m$	10%
$(-5.5,0)$	1.5	$10.6\,kg/m$	$0\,kg/m$	6%
$(-5.5,0)$	0.5	$11\,kg/m$	$0\,kg/m$	0.6%

VII. CONCLUSIONS

In this paper, a linear parameter-varying (LPV)/linear matrix inequalities (LMIs)-based technique has been used to achieve fault tolerance and to compare benefits and drawbacks of passive and active FTC.

Results have been obtained designing passive and active FTC for a two-wheel differential robot simulator subject to a parametric fault, namely a change in the drag coefficient with respect to normal operating conditions.

The obtained controllers have been compared in terms of pole placement specifications. Such comparison has shown that the passive FTC conservativeness results in a bigger lower bound of the faulty drag coefficient for which fault tolerance can be achieved, with respect to the active FTC. On the other hand, the latter is sensitive to uncertainties in the fault estimation that can reduce or even eliminate the benefits of such approach.

It seems clear that an important direction for further research is to investigate the design of *hybrid* FTC controllers that can merge the benefits of active and passive FTC controllers, while at the same time reduce or even nullify their respective drawbacks.

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