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A local approach framework for black-box and gray-box LPV system identification

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***Abstract*— This paper presents a comparison of two techniques dedicated to the identification of LPV systems by using local experiments only. Such an approach can be justified by the fact that, in many practical cases, exciting the scheduling variables persistently is not conceivable for safety/economic reasons. According to the prior information available on the system, a black-box and a gray-box model-based technique are described and compared through a simulation example. More precisely, a new version of the algorithm suggested in [1] is compared with a gray-box model-based technique consisting in interpolating local re-structured LTI state-space models, whose basis coherence is ensured thanks to prior knowledge about the system to identify. This contribution shows that prior information can be really helpful when the problem of coherent basis selection arises.**

I. INTRODUCTION

The basic idea of the main techniques dedicated to the identification of non-linear systems consists in following the well-known and efficient "divide and conquer" strategy [2], *i.e.*, breaking down the whole non-linear functioning domain of the system into many local domains where

- the local behavior of the system is more linear than the global one,
- the system can be approximated by a simple but reliable local model.

This idea of combining (in different ways) several local and (quasi-)linear models in order to capture the whole dynamics of a non-linear system is widely used in system identification [2]. Among all the multi-model structures available in the literature, a particular attention has been payed to the linear parameter-varying (LPV) models during the last two decades (see Chapter 1 of [3] for an historical presentation of LPV identification). This interest can be mainly explained by the following reasons. First, an LPV model can be seen as a combination of local models with parameters evolving as a function of measurable variables (called the scheduling variables) which can be related to the different operating points of the system. By this way, the model structure is close to the standard LTI one but with a structural flexibility able to picture time-varying, even non-linear behaviors. Second, the development of LPV models is linked to control engineering where a controller must be designed in order to guarantee a suitable closed-loop performance for a given plant in different operating conditions. A well-known example of controller design technique using this "divide and conquer" basic idea is the gain scheduling approach [4].

As far as the determination of LPV models is concerned, two broad classes of methods can be considered [5]: first, the analytic methods consisting in converting the available non-linear physical model of the system into an LPV representation; second, the experimental methods aiming at determining LPV models of the plant under study from the available input-output data. The first class, which probably gathers the initial solutions to the LPV modeling problem of physical systems [4], mainly resorts to extensions of the familiar notions of linearization (see [3, Chapter 7] for an overview of the main analytic developments). The second family of methods can be broken down into two sub-classes generally called the global approach and the local approach respectively. Historically, the first developments focused on the global procedure and assumed that one global experiment can be performed during which the control inputs as well as the scheduling variables can be both excited [6], [7]. By this way, all the non-linearities of the system are excited in one shot by passing through a large number of operating points. On the other hand, the recent methods are based on a multi-step procedure where

1) local experiments are carried out in which the operating points (corresponding to fixed values of the scheduling variables) are held constant and the control inputs are (persistently) excited,
2) local LTI models are estimated by using these sets of local input/output (I/O) measurements,
3) an interpolation phase is performed in order to derive a final global parameter-dependent model.

It is obvious that this multi-step approach is a lot closer to the standard procedure used for non-linear system identification or the one dedicated to gain scheduling. Such a viewpoint has been considered, *e.g.*, in [8], [9], [10]. Both classes of methods have advantages and drawbacks. From a practical point of view, the global approach can suffer from the difficulty to satisfy the "rich" excitation of the control inputs and the scheduling variables simultaneously. It is obvious that such an experimental procedure may not be

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reasonable for specific applications mainly for safety reasons. On the contrary, applying small variations around particular operating points, as considered by the local approach, is more conceivable in many practical cases. That is the reason why the following developments will focus on the local approach. Despite its practical simplicity, as pointed out first in [11], then in [12], the interpolation step involved in the local approach can lead to a global LPV model with an inaccurate dynamic behavior even if the local LTI models are consistent. Thus, a number of issues deserves attention as far as the relation between the local models is concerned, as shown, *e.g.*, [3]. Some of them are going to be introduced and solved in this contribution. A particular attention will be payed to affine LPV models and a comparison of techniques dedicated to black-box and gray-box LPV models will be handled. More precisely

- thanks to recent extensions that improves the numerical reliability of the local models, efficient black-box LPV models can be estimated when slow variations of the operating conditions are considered (see [13], [14], [15], [16] as well as Section III for recent contribution),
- the prior knowledge about the non-linear equations governing the behavior of the system can be used to improve the local approach by choosing the structure of the final LPV model and by adapting the identification strategy consequently (see Section IV for details).

In the following, after the problem statement in Section II, a description of both strategies is given in Section III and in Section IV respectively. Both techniques are then compared via a simulation example in Section V. Section VI concludes this paper.

II. PROBLEM STATEMENT

In this article, LPV systems described as a state-space representation are considered. More precisely, the following form are handled

$$\delta \mathbf{x}(t) = \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t) \tag{1a}$$

$$\mathbf{y}(t) = \mathbf{C}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{D}(\mathbf{p}(t))\mathbf{u}(t) \tag{1b}$$

where $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ are the input signals, $\mathbf{y}(t) \in \mathbb{R}^{n_y}$ are the output signals, $\mathbf{x}(t) \in \mathbb{R}^{n_x}$ is the state vector and $t \in \mathbb{R}$ or $\mathbb{Z}$. Contrary to the standard LTI state-space forms, the matrices $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ are functions of measurable time-varying signals, gathered into the vector $\mathbf{p}(t) \in \mathbb{P} \subseteq \mathbb{R}^{n_p}$ and called the scheduling variables, *i.e.*, $\mathbf{A} : \mathbb{P} \longmapsto \mathbb{R}^{n_x \times n_x}$, $\mathbf{B} : \mathbb{P} \longmapsto \mathbb{R}^{n_x \times n_u}$, $\mathbf{C} : \mathbb{P} \longmapsto \mathbb{R}^{n_y \times n_x}$ and $\mathbf{D} : \mathbb{P} \longmapsto \mathbb{R}^{n_y \times n_u}$. $\mathbb{P}$ is the so-called scheduling space [17]. Herein, δ stands for the forward shift operator when discrete-time systems are considered or for the differential operator when continuous-time systems are handled.

Whatever the time domain, the state-space matrices are assumed to be continuous functions of the scheduling variables $\mathbf{p}$. First, this assumption ensures that the dynamics of the model vary smoothly with respect to (w.r.t.) $\mathbf{p}$ [18]. Second, it implies boundedness of the coefficients composing these matrices for $\mathbf{p} \in \mathbb{P}$ [18]. Hereafter, in order to simplify the explanations, it is assumed that the afore-introduced matrices $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ have affine dependence on $\mathbf{p}$, *i.e.*,

$$\clubsuit(\mathbf{p}) = \clubsuit_0 + \sum_{i=1}^{n_p} \clubsuit_i p_i \tag{2}$$

where $\clubsuit_\bullet$ is a constant matrix standing for $\mathbf{A}_\bullet$, $\mathbf{B}_\bullet$, $\mathbf{C}_\bullet$ and $\mathbf{D}_\bullet$ and where $\mathbf{p} = \begin{bmatrix} p_1 & \cdots & p_{n_p} \end{bmatrix}^\top \in \mathbb{R}^{n_p}$. Notice that an extension of the developments suggested in Section IV to linear fractional representations is under study. Thus, the matrices $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D})$ are only dependent on the instantaneous value of the scheduling variables. In this case, the functional dependence is called static [3].

Herein, the local approach is applied because, from a practical point of view, well-exciting the scheduling variables is not conceivable for many systems. This is the case with the system used in Section V.

As shown first in [11], the main difficulty with the local approach for LPV state-space model identification is the determination of a suitable coherent basis for all the local estimated models so that the interpolated LPV state-space representation is able to capture the dynamics of the process to identify. Many attempts to solve this problem are suggested in the literature. In the black-box framework, *i.e.*, when no prior information about the system to identify is available, interesting solutions have been developed, *e.g.*, in [8], [9], [10], [19] during the last decade. Among them, the developments suggested in [1] (and shortly recalled in Section III) and based on local balanced realizations can be viewed as one of the most numerically reliable techniques. Unfortunately, as explained in Sub-Section III-B and recently in [19], whatever the black-box model-based technique used to estimate an LPV model from local experiments, getting an LPV model able to picture the dynamic dependence on the scheduling variables is not an easy task even if the LPV system to identify satisfies a static dependence on $\mathbf{p}$. In order to bypass this difficulty, an efficient solution consists in resorting to the prior information about the structure of the LPV system in order to ensure that (i) the static or dynamic dependence on $\mathbf{p}$ of the estimated LPV model is satisfied, and (ii) the local models are estimated w.r.t. a coherent basis. Both strategies (black-box and gray-box) are introduced in the following two sections. As far as the black-box model-based technique is concerned, the method suggested in [1] is considered. A more general version of the subspace-based procedure developed in the aforementioned article (with a solution to the manual sorting step required in [1]) is more precisely given. Notice however that the comments made in Sub-Section III-B are true for all the local black-box model-based techniques developed until now.

Whatever the technique used for the identification, it is assumed that (i) a set of constant scheduling variables for $N_{op} \in \mathbb{N}^*$ local working points is given beforehand, (ii) this set of local experiments is well-chosen, *i.e.*, all the working range of the system is covered (*i.e.*, N_{op} is large enough) and the "distance" between two constant working points is small enough.

III. LOCAL BLACK-BOX MODEL-BASED APPROACH

A. A balanced subspace-based technique for local LPV model identification

As most of the local techniques, the black-box model-based technique used in Section V is composed of the three main steps recalled in Section I. The technique introduced in [1] and shortly presented herein differs from the other local methods mainly when the second step is carried out. More precisely, for step (2) (see Section I), (i) the LTI state-space models are estimated for each user-defined operating point by using a dedicated[1] subspace-based algorithm, so enabling the treatment of MIMO as well as SISO systems by resorting to state-space forms and, possibly, by using both time-domain or frequency-domain data, (ii) the identified local models are balanced (in Moore's sense [21]) if the used subspace-based identification technique does not yield a balanced form directly, (iii) if necessary, the local DT balanced models are converted into continuous-time state-space representations with a bilinear transformation (see [21] for more details). Then, as shown in [1], [22, Chapter 8], the last step, *i.e.*, the interpolation of the state-space matrices of the local models, is improved by using the good properties of the balanced realizations. Besides the good numerical properties of the balanced realizations [21], the main reason why balanced realizations are used in this local approach is the uniqueness of the similarity transformation involved in the balancing procedure [21]. Indeed,

- if the eigenvalues of the product of the reachability and observability Gramians are distinct, the similarity transformation $\mathbf{T}$ is essentially unique, *i.e.*, is unique up to a post-multiplication by any matrix of the form $\mathrm{diag}(\pm 1, \pm 1, \cdots, \pm 1)$,
- if two or more eigenvalues are repeated, their corresponding eigenvectors can be rotated arbitrarily in the corresponding eigen-space.

As far as LPV system identification is concerned, this property implies that, as long as the eigenvalues are distinct, if the true system exhibits a smooth dependence from the scheduling variables, then the overall parameter dependent model can be reconstructed directly from the identified local models. As shown in Section V, because the determination of the similarity transformation matrix is only possible up to the sign, the behavior of the elements of the state-space matrices may have sign changes when passing from one operating point to another. Such a problem can be solved by introducing a similarity transformation, diagonal and almost equal to the identity matrix, with -1 at the position corresponding to the required changes of sign. In [23, Chapter 2], an algorithm, based on a smart analysis of the components of the matrices of two successive local models, is available to deal with this problem of non-uniqueness of the similarity transformation involved in the construction of a balanced form. This automatic sorting procedure, associated with improvements described in [22, Chapter 8], answers the criticisms raised in [19].

[1] According to the disturbances acting on the system, different subspace-based algorithms are available in the literature to yield consistent state-space estimates [20].

B. Discussion and comments

As shown initially in [11], more recently in [19], the local black-box model-based approach, which only uses constant values of the scheduling variables, cannot lead to an interpolated LPV model able to capture the dynamic behavior of an LPV system depending on $\mathbf{p}$ dynamically, *i.e.*, on $\mathbf{p}$, $\dot{\mathbf{p}}$, $\ddot{\mathbf{p}}$, ..., because, by construction, the local black-box models do not contain any information about the dynamic parameter dependence of the LPV system.

Such a problem may also come out when static dependent state-space LPV systems are handled. It is well-known that a state-space representation is not unique but known up to a similarity transformation. This idea is used locally to express the local LTI models w.r.t. a coherent basis. Unfortunately, such a procedure, in a black-box LPV framework, may also lead to a loss of information. Indeed, as soon as a similarity transformation is applied for the local LTI models, an equivalent coordinate change should be carried out on the LPV system to identify. Contrary to the LTI framework, the similarity transformations used for LPV systems may dynamically depend on time derivatives of $\mathbf{p}$ [3]. Thus, even if the initial model is static w.r.t. $\mathbf{p}$, the transformed one may depend on $\mathbf{p}$, $\dot{\mathbf{p}}$, $\ddot{\mathbf{p}}$, ... Hence, the problem highlighted previously may happen even if static dependent LPV state-space systems are considered.

In order to bypass this problem, it can be assumed that the variations of $\mathbf{p}$ are slow w.r.t. the dynamics of the system. This assumption, which is system dependent, is compulsory (but, unfortunately not sufficient) to perform an efficient interpolation [24] when the only source of information is restricted to measured data. On the contrary, when prior information about the LPV system structure is available, better results can be obtained, as shown in the following section.

IV. LOCAL GRAY-BOX MODEL-BASED APPROACH

It is not rare that prior information about the parameter-dependent structure of the system is available when a process is identified. Such information may appear from an initial study of the non-linear equations governing the behavior of the plant under study. As shown hereafter, this information can be used to determine a suitable coherent basis when static parameter-dependent LPV models are concerned and that, without raising the problem pointed out previously.

In order to apply this local gray-box model-based approach, let us assume that a prior study of the process to identify has been performed leading to a static parameter-dependent structure of the LPV system such as an affine or a polynomial $\mathbf{p}$-dependence. The estimation of the unknown parameters composing the physically-structured LPV model could be performed theoretically by optimizing a global cost function gathering all of the information available locally as performed in [25]. Unfortunately, such a "glocal" procedure requires a reliable initial parameter vector to ensure the convergence to the global optimum of the optimized criterion. In order to circumvent this difficulty, the procedure developed in this article consists in (i) selecting $N \in \mathbb{N}^*$ local working points and (ii) identifying, locally, N consistent LTI state-space models.

Remark 4.1: As shown in [24], when a polynomial dependence is considered, $N \in \mathbb{N}^*$ must be chosen w.r.t. the order of the involved polynomial function.

For each local working point, a fully-parameterized continuous-time LTI state-space form $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C})$ can be estimated consistently by resorting to a dedicated[2] subspace-based technique [20]. Then, by following the idea suggested in [26], these local models can be re-parameterized so that the re-structured LTI state-space models satisfy the structure of the corresponding "frozen" LPV system [3], *i.e.*, the structure of the LPV process to identify with $\mathbf{p}(t) = \bar{\mathbf{p}} = cte$. By doing so, thanks to the physical meaning of the underlying state vector, coherence of all the local state coordinates can be ensured.

The technique developed in [26] can be summarized as follows. By having access to a fully-parameterized continuous-time LTI state-space form $(\mathfrak{A}, \mathfrak{B}, \mathfrak{C})$ as well as a known state-space structure $(\mathbf{A}(\boldsymbol{\theta}), \mathbf{B}(\boldsymbol{\theta}), \mathbf{C}(\boldsymbol{\theta}))$ depending on an unknown parameter vector $\boldsymbol{\theta}$, the procedure consists in determining the similarity transformation $\mathbf{T}$ as well as the vector $\boldsymbol{\theta}$ satisfying $\mathfrak{A}\mathbf{T} = \mathbf{T}\mathbf{A}(\boldsymbol{\theta}),\ \mathfrak{B} = \mathbf{T}\mathbf{B}(\boldsymbol{\theta}),\ \mathfrak{C}\mathbf{T} = \mathbf{C}(\boldsymbol{\theta})$. As shown in [26], this set of bilinear equations can be easily rewritten as a null-space-based problem, *i.e.*,

$$\boldsymbol{\Delta}\boldsymbol{\tau} = \mathbf{0}_{n_x^2 + n_x(n_u+n_y)} \tag{3}$$

where $\boldsymbol{\Delta} \in \mathbb{R}^{(n_x^2+n_x(n_u+n_y))\times(2n_x^2+n_x(n_u+n_y)+1)}$ is composed of known coefficients, while $\boldsymbol{\tau} \in \mathbb{R}^{(2n_x^2+n_x(n_u+n_y)+1)}$ gathers the unknown parameters. Such a null-space problem can be solved by resorting to non-convex optimization techniques such as the limited-BFGS algorithm [27]. More interesting, by assuming that the initial fully-parameterized realization of the system is consistent and the final physically-structured state-space form is identifiable, uniqueness of the solution can be guaranteed [26].

Such a procedure is interesting because, contrary to the black-box model-based approach, the use of prior information helps the user ensuring the consistency of the coherent basis (thanks to the physical meaning of the involved state vector for each working point) at least for static dependent LPV systems.

V. SIMULATION EXAMPLE

In order to illustrate the identification procedures introduced previously, a three compartment e-nose sensor model is used . For such a system, $s_i(t)$ and k_{ij} are, respectively, the concentration of a chemical compound at time t (in mg/l) and the kinetic rate constant in units of time^{-1}, associated with the mass leaving the j^{th} compartment and arriving at the i^{th} compartment. The input $u(t)$ is a gas released at $t = 0$ and allowed to disperse through the sensor chamber (see [26] for details about this system). Under specific practical uses, this process can satisfy an LPV structure. More precisely, the kinetic rates involved in the behavior of an e-nose sensor may depend on an external measurable signal, used hereafter as a scheduling variable p. Associated with the conservation of mass principle which states that the rate of change of mass accumulation within the i^{th} compartment is equal to the mass flow rate-in minus the mass flow rate-out of the compartment at time t, we have three state equations of the form

$$\dot{s}_1(t) = u(t) - (k_{01} + k_{21})s_1(t) + k_{12}s_2(t) \tag{4a}$$
$$\dot{s}_2(t) = k_{21}s_1(t) - (k_{12} + k_{32})s_2(t) + k_{23}s_3(t) \tag{4b}$$
$$\dot{s}_3(t) = k_{32}s_2(t) - k_{23}s_3(t), \tag{4c}$$

where $k_{12} = 0.1 + p(t)$, $k_{21} = 0.2 + p(t)$, $k_{23} = 0.15 + p(t)$ and with the initial conditions are $s_i(0) = 0$, for $i = 1, 2, 3$, the system being initially at rest. The output of the system is the sum of the concentrations in each compartment, *i.e.*, $y(t) = f(s_1(t) + s_2(t) + s_3(t))$, where $f = 0.05 + p(t)$ is a proportion of the total concentration of the gas in the system. The state and output equations can be written as an affine parameter-dependent vector-matrix form given by Eq. (2) where

$$\mathbf{A}_0 = \begin{bmatrix} -0.3 & 0.1 & 0 \\ 0.2 & -0.5 & 0.15 \\ 0 & 0.4 & -0.15 \end{bmatrix} \qquad \mathbf{A}_1 = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{B}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{C}_0^\top = \begin{bmatrix} 0.05 \\ 0.05 \\ 0.05 \end{bmatrix} \quad \mathbf{C}_1^\top = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

The first step of the local procedures studied in this paper consists in selecting N local operating points, then estimating N local LTI state-space models. Herein, p is chosen to evolve in the range $[0.02 : 0.02 : 0.2]$, with a step equal to 0.02. Then, $N = 10$. By having access to the LPV equations satisfied by this e-nose sensor, 10 analytic local LTI models can be easily extracted and used for the validation of the identification techniques developed in this paper. More precisely, these local analytic models satisfy the frequency responses given above in Fig. 1 as well as local $(\mathbf{A}(\bar{p}), \mathbf{B}(\bar{p}), \mathbf{C}(\bar{p}))$ coefficients evolving w.r.t. p in the range $[0.02 : 0.02 : 0.2]$ as shown in Fig. 2a for $\mathbf{A}(\bar{p})$. The second step aims at estimating local LTI fully-parameterized state-space models $(\mathfrak{A}_i, \mathfrak{B}_i, \mathfrak{C}_i)$, $i \in [1, N]$. To get these initial models, for each working point, the e-nose system is excited with a Gaussian pulse input signal (in order to be realistic with respect to a standard use of such a system). Second, a MOESP algorithm [20] is applied to the acquired data set. Because this subspace-based technique leads to discrete-time models, their continuous-time counterparts are computed by applying the iterative conversion algorithm available in [21]. These initial models are easily validated by comparing the frequency responses of these estimated models and the local analytic models (see Fig. 1).

[2] If necessary, a discrete-time subspace-based algorithm associated with a d2c technique can be used to get consistent continuous-time black-box estimates.

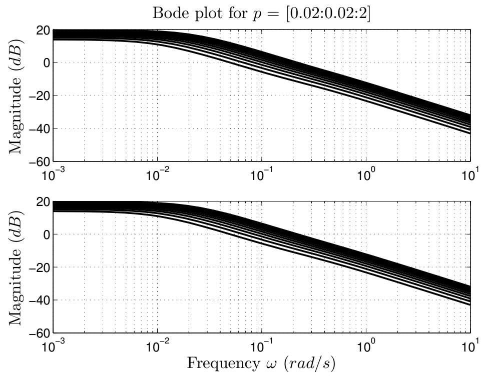
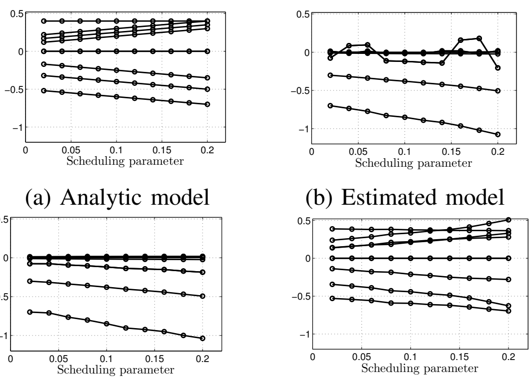


Fig. 1: Bode plots of the analytic (top) and the estimated black-box (bottom) local models.



(a) Analytic model

(b) Estimated model

(c) Estimated model after sign-change

(d) Restructured model

Fig. 2: Evolution of the entries of the A matrix w.r.t. p.

A. Results with the black-box model-based approach

With the black-box model-based approach, the local LTI models are balanced by using the standard `balreal` MATLAB function. Fig. 2b and Fig. 2c show the evolution of the A matrix coefficients w.r.t. p before and after the sign change treatment. More precisely, Fig. 2c shows that, with this specific balancing step, the evolution of the local matrices parameters is smooth w.r.t. p and the magnitude of the parameters is mild which ensures, in a way, a good conditioning of the local state-space forms. Notice here that the nature of the parameter evolution of the other matrices are similar.

This smooth evolution of the parameters (Fig. 2c) can be captured with an affine LPV state-space form through a least-squares regression. Once this interpolation step is performed, the interpolated LPV model must be validated. This validation is carried out by exciting both the system and the model equivalently so that the whole range of p is visited, then comparing the outputs of both representations. The input signal is now chosen equal to $u(t) = 5+5\sin(t/40+3\pi/2)$. As far as p is concerned, $p(t) = 0.1 + 0.08\sin(t/\alpha + \pi/2)$. According to the value of α or, more qualitatively, according to the variation speed of p, different conclusions can be drawn. First, for $\alpha = 400$, *i.e.*, the interpolated model is able to capture the dynamic behavior of the system to identify, as shown in Fig. 3 as well as the associated fit measurements, the Best Fit Percentage (BFT) and the Variance Accounted For (VAF) [20], [3], which are equal to 90.6% and 99.12% respectively. On the contrary, when the variation of p is a lot faster, *i.e.*, when $\alpha = 50$, then the fit measurements drop drastically to $BFT = 69.87\%$ and $VAF = 90.92\%$ (see also Fig. 4).

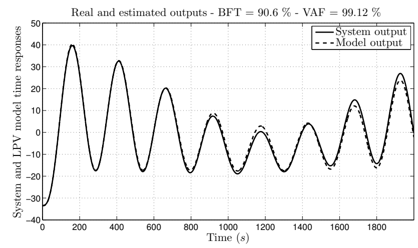


Fig. 3: Comparison of the system and the black-box interpolated LPV model time responses for $\alpha = 400$.

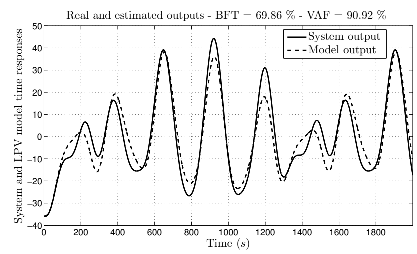


Fig. 4: Comparison of the system and the black-box interpolated LPV model time responses for $\alpha = 50$.

B. Results with the gray-box model-based approach

In order to improve the previous results and to give access to a better interpolated LPV model, the gray-box model-based technique described beforehand is used. The starting point of this method is the set of local balanced LTI state-space representations used by the black-box model-based approach. From these local fully-parameterized LTI state-space forms, N local re-structured state-space representations are extracted by following the steps of the null-space-based algorithm suggested in [26] and recalled in Section IV. More precisely, for each working point, a state-space representation (through a parameter vector $\boldsymbol{\theta}_i$, $i \in [1, N]$) satisfying

$$\boldsymbol{A}(\boldsymbol{\theta}_i) = \begin{bmatrix} \vartheta_{i_1} & \vartheta_{i_2} & 0 \\ \vartheta_{i_3} & \vartheta_{i_4} & \vartheta_{i_5} \\ 0 & \vartheta_{i_6} & -\vartheta_{i_5} \end{bmatrix} \ \boldsymbol{B}(\boldsymbol{\theta}_i) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \ \boldsymbol{C}^\top(\boldsymbol{\theta}_i) = \begin{bmatrix} \vartheta_{i_7} \\ \vartheta_{i_7} \\ \vartheta_{i_7} \end{bmatrix}$$

is estimated. Unfortunately, because there are 7 parameters $\{\vartheta_{i_j}\}_{j=1}^{7}$ to estimate, *i.e.*, a quantity larger than[3] $n_x(n_u + n_y) = 6$, the structure involving $\{\vartheta_{i_j}\}_{j=1}^{7}$ is not identifiable. This problem can be circumvented by

1) estimating the parameters of this unidentifiable model,
2) determining the unique similarity transformation $\mathbf{S}$ such that (i) $\mathbf{S}\boldsymbol{\mathcal{A}}(\boldsymbol{\tau}^*)\mathbf{S}^{-1}$ satisfies the structure of

[3]It is well-known that an identifiable structure can have $n_x(n_u + n_y)$ unknown parameters at most [20].

$\boldsymbol{A}(\boldsymbol{\theta})$ and (ii) the constraint $\theta_2+\theta_4+\theta_6 = 0$ is verified (as the system to identify does).

Such a procedure gives access to local structured LTI state-space matrices, whose parameters evolution of the A matrix is in Fig. 2d. A comparison of Fig. 2a and Fig. 2d illustrates the performance of this gray-box model-based approach. These good results are confirmed with Fig. 5 where the system and the interpolated gray-box LPV model time responses are compared for $\alpha = 50$.

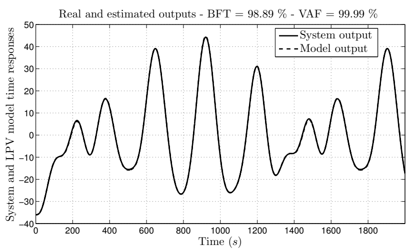


Fig. 5: Comparison of the system and the gray-box interpolated LPV model time responses for $\alpha = 50$.

VI. CONCLUSION

Under specific practical cases, *e.g.*, when the variation of the scheduling variables are fast w.r.t. to the dynamics of the system, the local approach may lead to unreliable LPV models when no prior information of the system is available. In this paper, this idea is illustrated through the study of affine LPV state-space representations. For instance, despite its numerical robustness, the technique suggested in [1] may give access to interpolated LPV models unable to capture the dynamics of the system as illustrated in Section V of this paper. In order to bypass these difficulties and to use the local approach efficiently, the introduction of prior information such as the structure of the LPV model seems to be a good solution. More precisely, combining a technique able to restructure local fully-parameterized state-space forms with the basic idea of the local approach can be seen as an efficient and promising solution to ensure the coherence of the coordinate bases used for each local model.

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