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Identification of errors–in–variables models as a quadratic eigenvalue problem

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Abstract— **The paper proposes a new approach for identifying linear dynamic errors–in–variables (EIV) models, whose input and output are affected by additive white noise. The method is based on a nonlinear system of equations consisting of part of the compensated normal equations and of a set of high order Yule–Walker equations. This system allows mapping the EIV identification problem into a quadratic eigenvalue problem that, in turn, can be mapped into a linear generalized eigenvalue problem. The system parameters are thus estimated without requiring the use of iterative identification algorithms. The effectiveness of the method has been tested by means of Monte Carlo simulations and compared with those of other EIV identification methods.**

I. INTRODUCTION

Identification of linear dynamic systems from noise–corrupted output measurements is a classical problem in system identification, for which many different solutions are available [1], [2]. On the other hand, the identification of the so called errors–in–variables models, that is the identification of linear dynamic systems in presence of input and output measurement noises, is recognized as a more difficult problem. This class of models is present in several engineering applications, like time series analysis, audio and image processing, biomedicine, econometrics, chemical engineering, etc. [3], [4].

Several solutions for this problem are nowadays available in the literature [5]. One of the simplest and oldest method is the instrumental variable (IV) method with all its variants. Most IV approaches use delayed inputs and outputs as instruments, leading thus to a set of high order Yule–Walker (HOYW) equations [6]–[8]. The method is computationally very efficient and can be easily used under fairly general assumptions on the noise. However, the obtained parameter estimates are often characterized by a poor accuracy since they are based on the estimation of high–lag auto and cross correlations. Possible improvements can be achieved by combining IV methods with the subspace fitting approach [9] but the resulting estimators involve a highly nonlinear optimization problem

Many recently proposed methods belong to the class of bias–compensated least–squares (BCLS) methods or can be interpreted as BCLS methods. Among these approaches it is worth recalling the bias–eliminating least squares (BELS) [10]–[12], the extended compensated least squares [13], [14] and the dynamic Frisch scheme [15]–[18]. All these methods are characterized, in general, by a good estimation accuracy and a modest computational cost. The relations between the above mentioned methods have been analyzed in [19] whereas in [20] it is shown how these methods can be put into a general framework, resulting into a generalized instrumental variable estimator.

The BCLS methods rely on the so–called compensated normal equations whose unknowns are the plant parameters and the noise variances [5]. Since the number of such equations coincides with the number of system parameters, more equations are needed for solving the EIV identification problem (at least two more equations when both the input and output noises are white processes). One way to obtain the additional equations consists in considering a set of HOYW equations [12], [16], [21]–[23].

The approach proposed in this paper lies, in a certain sense, between IV and BCLS methods. In fact, it exploits part of the compensated normal equations and the HOYW equations. The method is based on a nonlinear system of equations whose unkowns are the system parameters and one of the additive noise variances, the input noise variance or the output noise one, depending on which subset of the compensated normal equations is considered. The obtained set of equations allows mapping the EIV identification problem into a quadratic eigenvalue problem that, in turn, can be mapped into a linear generalized eigenvalue problem. The system parameters are thus estimated without requiring the use of iterative identification algorithms. Several Monte Carlo simulations show that the performance of the proposed approach lies between those of the Frisch scheme and the classic IV estimator for what concerns both estimation accuracy and computational efficiency.

The contents of the paper are organized as follows. Section II defines the EIV identification problem. Section III describes the high–order Yule–Walker equations which are exploited by the proposed method. In Section IV, starting from a subset of the compensated normal equations and the HOYW equations, it is shown how the EIV identification problem can be mapped into a quadratic eigenvalue problem involving the input, or the output, noise variance. The obtained quadratic eigenvalue problem is thus solved by mapping it into a generalized eigenvalue problem. The proposed method is tested and compared with other EIV identification approaches by means of Monte Carlo simulations whose results are discussed in Section V. Finally, some concluding remarks are given in Section VI.

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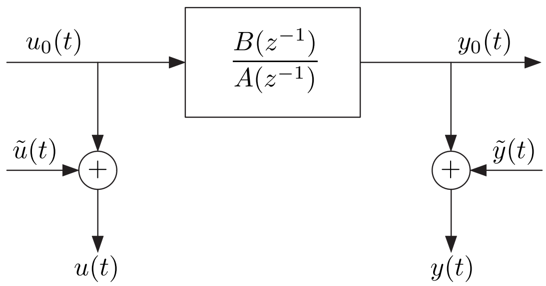

Fig. 1. Errors–in–variables model.

II. PROBLEM STATEMENT

Consider the linear, discrete–time and time–invariant errors–in–variables model described by the equations (see Fig. 1)

$$A(z^{-1})\, y_0(t) = B(z^{-1})\, u_0(t) \tag{1}$$

$$u(t) = u_0(t) + \tilde{u}(t) \tag{2}$$

$$y(t) = y_0(t) + \tilde{y}(t), \tag{3}$$

where $u_0(t), y_0(t)$ denote the noise–free input and output, $u(t), y(t)$ are the available observations affected by the additive noise $\tilde{u}(t), \tilde{y}(t)$ while $A(z^{-1}), B(z^{-1})$ are the following polynomials in the backward shift operator z^{-1}

$$A(z^{-1}) = 1 + a_1\, z^{-1} + \cdots + a_n\, z^{-n} \tag{4}$$

$$B(z^{-1}) = b_0 + b_1\, z^{-1} + \cdots + b_n\, z^{-n}. \tag{5}$$

The following assumptions are introduced.

A1. The dynamic system (1) is asymptotically stable, i.e. $A(z)$ has all zeros outside the unite circle.

A2. All system modes are observable and controllable, i.e. $A(z^{-1})$ and $B(z^{-1})$ do not share any common factor.

A3. The order n of the system is assumed as *a priori* known.

A4. The noise–free input $u_0(t)$ is a zero–mean ergodic process that is persistently exciting of sufficiently high order.

A5. $\tilde{u}(t)$ and $\tilde{y}(t)$ are zero–mean white processes with unknown variances $\tilde{\sigma}_u^2$ and $\tilde{\sigma}_y^2$ respectively. These processes are mutually uncorrelated and uncorrelated with the noise–free input $u_0(t)$.

By introducing the vectors

$$\varphi(t) = [-y(t)\, \ldots\, -y(t-n)\, u(t)\, \ldots\, u(t-n)]^T \tag{6}$$

$$\varphi_0(t) = [-y_0(t)\, \ldots\, -y_0(t-n)\, u_0(t)\, \ldots\, u_0(t-n)]^T \tag{7}$$

$$\tilde{\varphi}(t) = [-\tilde{y}(t)\, \ldots\, -\tilde{y}(t-n)\, \tilde{u}(t)\, \ldots\, \tilde{u}(t-n)]^T \tag{8}$$

and the parameter vector

$$\bar{\theta}_0 = \left[\, 1\, a_1\, \cdots\, a_n\; b_0\, \cdots\, b_n \,\right]^T = \left[\, 1\, \theta_0^T \,\right]^T \tag{9}$$

it is possible to rewrite model (1)–(3) in the form

$$\varphi_0^T(t)\, \bar{\theta}_0 = 0 \tag{10}$$

$$\varphi(t) = \varphi_0(t) + \tilde{\varphi}(t), \tag{11}$$

that will be used in the sequel. The problem under investigation is the following.

Problem 1: Determine an estimate of the system parameters a_k $(k = 1, \ldots, n)$, b_k $(k = 0, \ldots, n)$ of $A(z^{-1}), B(z^{-1})$ starting from the input–output observations $u(1), y(1), u(2), y(2), \ldots, u(N), y(N)$.

III. YULE–WALKER EQUATIONS IN EIV IDENTIFICATION

Define the following covariance matrices

$$\Sigma = E[\varphi(t)\, \varphi^T(t)] \tag{12}$$

$$\Sigma_0 = E[\varphi_0(t)\, \varphi_0^T(t)] \tag{13}$$

$$\tilde{\Sigma} = E[\tilde{\varphi}(t)\, \tilde{\varphi}^T(t)]. \tag{14}$$

From (10), (11) and Assumption A5 it follows that

$$\Sigma = \Sigma_0 + \tilde{\Sigma} \tag{15}$$

$$\Sigma_0\, \bar{\theta}_0 = 0 \tag{16}$$

$$\tilde{\Sigma} = \begin{bmatrix} \tilde{\sigma}_y^2\, I_{n+1} & 0 \\ 0 & \tilde{\sigma}_u^2\, I_{n+1} \end{bmatrix}, \tag{17}$$

therefore

$$\left(\Sigma - \tilde{\Sigma}\right) \bar{\theta}_0 = 0. \tag{18}$$

This relation cannot be directly used to solve the EIV identification problem since both the parameter vector θ_0 and the noise variances $\tilde{\sigma}_u^2, \tilde{\sigma}_y^2$ are unknown. The number of equations is thus $2n + 2$ while that of unknowns is $2n + 3$. At least one additional equation must be introduced.

To this end, define the $p \times 1$ vectors

$$\varphi_{u_0}^p(t) = [\, u_0(t-n-1)\, \ldots\, u_0(t-n-p)\,]^T \tag{19}$$

$$\varphi_u^p(t) = [\, u(t-n-1)\, \ldots\, u(t-n-p)\,]^T \tag{20}$$

$$\varphi_{\tilde{u}}^p(t) = [\, \tilde{u}(t-n-1)\, \ldots\, \tilde{u}(t-n-p)\,]^T, \tag{21}$$

and the $p \times (2n + 2)$ cross–covariance matrix

$$\Sigma^p = E\,[\varphi_u^p(t)\, \varphi^T(t)]. \tag{22}$$

Equations (2), (11) and Assumption A5 lead to

$$\begin{aligned} \Sigma^p &= E\,[(\varphi_{u_0}^p(t) + \varphi_{\tilde{u}}^p(t))\, (\varphi_0(t) + \tilde{\varphi}(t))^T] \\ &= E\,[\varphi_{u_0}^p(t)\, \varphi_0^T(t)], \end{aligned} \tag{23}$$

so that from (10)

$$\Sigma^p\, \bar{\theta}_0 = 0. \tag{24}$$

Relation (24) represents a set of high order Yule–Walker (HOYW) equations that could be directly used to obtain an estimate of the parameter vector θ_0 if $p \geq 2n + 1$. This approach can also be viewed as an instrumental variable (IV) method that uses delayed inputs as instruments [6], [8]. The HOYW equations can also be solved by using a total least squares approach [7], [8]. The main advantages of IV methods are the computational efficiency and the applicability under fairly general noise conditions. On the other hand, the obtained estimation accuracy is often poor [5]. To improve the estimation accuracy, the set of HOYW equations can be used together with the normal compensated equations (18). This approach has been followed in various bias–compensation based schemes [12], [16], [21]–[23].

The approach proposed in this paper exploits the set of HOYW equations (24) and part of the compensated normal equations (18). As it will be shown in the next section, this allows mapping the EIV identification problem into a quadratic eigenvalue problem that, in turn, can be solved as a generalized eigenvalue problem.

IV. EIV IDENTIFICATION AS A QUADRATIC EIGENVALUE PROBLEM

By introducing the following partition of the covariance matrix of the noisy data

$$\Sigma = \begin{bmatrix} \Sigma_{yy} & \Sigma_{yu} \\ \Sigma_{yu}^T & \Sigma_{uu} \end{bmatrix}, \tag{25}$$

relation (18) can be splitted as

$$\begin{bmatrix} \Sigma_{yy} - \tilde{\sigma}_y^2 \, I_{n+1} & \Sigma_{yu} \end{bmatrix} \bar{\theta}_0 = 0 \tag{26}$$

$$\begin{bmatrix} \Sigma_{yu}^T & \Sigma_{uu} - \tilde{\sigma}_u^2 \, I_{n+1} \end{bmatrix} \bar{\theta}_0 = 0. \tag{27}$$

Each of the above relations consists in a set of $n+1$ equations in $2n+2$ unknowns. By choosing $p \geq n+1$, equation (24) can be combined with (26) or (27) in order to map the EIV identification problem into a quadratic eigenvalue problem. To this end, consider, for example, (27) and (24) to obtain the following nonlinear systems of $n+1+p$ $(p \geq n+1)$ equations

$$\begin{bmatrix} \Sigma_{yu}^T & \Sigma_{uu} - \tilde{\sigma}_u^2 \, I_{n+1} \\ & \Sigma^p \end{bmatrix} \bar{\theta}_0 = 0, \tag{28}$$

where both $\tilde{\sigma}_u^2$ and the entries of θ_0 are unknown. This set of equations can be rewritten as

$$\left(S - \tilde{\sigma}_u^2 \, J \right) \bar{\theta}_0 = 0, \tag{29}$$

where

$$S = \begin{bmatrix} \Sigma_{yu}^T & \Sigma_{uu} \\ & \Sigma^p \end{bmatrix} \tag{30}$$

$$J = \begin{bmatrix} 0_{(n+1)\times(n+1)} & I_{n+1} \\ & 0_{p\times(2n+2)} \end{bmatrix}. \tag{31}$$

Multiplying both sides of (29) by $\left(S - \tilde{\sigma}_u^2 \, J \right)^T$ leads to the equation

$$\left(A_2 \, \tilde{\sigma}_u^4 + A_1 \, \tilde{\sigma}_u^2 + A_0 \right) \bar{\theta}_0 = 0, \tag{32}$$

where

$$A_0 = S^T S \tag{33}$$

$$A_1 = -\left(S^T J + J^T S \right) \tag{34}$$

$$A_2 = J^T J. \tag{35}$$

The coefficients of θ_0 can thus be estimated by solving the following quadratic eigenvalue problem (QEP)

$$\left(A_2 \, \lambda^2 + A_1 \, \lambda + A_0 \right) v = 0. \tag{36}$$

The set of $4n+4$ eigenvalues solving (36) are real or appear in complex conjugate pairs and can also be infinite [24]. If the system is identifiable, the only real eigenvalue (with multiplicity two) that solves (36) is $\lambda = \tilde{\sigma}_u^2$. It is thus possible to conclude that the solution of the EIV identification problem is the eigenvector associated with the only real eigenvalue that solves (36).

The QEP (36) can be solved in several ways [24]. Many of these approaches are based on a linearization that allows mapping the QEP problem into a linear generalized eigenvalue problem (GEP). The easiest way to do this consists in rewrite equation (36) as

$$A_2 \, v' \, \lambda + A_1 \, v \, \lambda + A_0 \, v = 0, \tag{37}$$

where $v' = \lambda \, v$, that leads to the $4n+4$–dimensional linear GEP [24]

$$(P - \lambda \, Q) \; \eta = 0, \tag{38}$$

where

$$P = \begin{bmatrix} A_0 & 0 \\ 0 & I_{2n+2} \end{bmatrix} \tag{39}$$

$$Q = \begin{bmatrix} -A_1 & -A_2 \\ I_{2n+2} & 0 \end{bmatrix} \tag{40}$$

$$\eta = \begin{bmatrix} v^T & v'^T \end{bmatrix}^T. \tag{41}$$

The only real eigenvalue solving (38) is $\tilde{\sigma}_u^2$ and the first $2n+2$ entries of the corresponding eigenvector η are, after a normalization of the first entry to 1, the entries of $\bar{\theta}_0$:

$$\eta_0 = \frac{\eta}{\eta(1)} = \begin{bmatrix} \bar{\theta}_0^T & \tilde{\sigma}_u^2 \bar{\theta}_0^T \end{bmatrix}^T. \tag{42}$$

Since only a finite number N of data is available, Σ_{yu}, Σ_{uu} and Σ^p will be replaced by sample estimates and all the eigenvalues solving (38) will exhibit, in general, a small imaginary part. A criterion leading to good results consists in choosing the eigenvalue having the smallest modulus [25], [26]. The whole identification procedure can be summarized as follows.

Algorithm 1.

1) Compute, on the basis of the available observations, the sample estimates

$$\hat{\Sigma} = \frac{1}{N-n} \sum_{t=n+1}^{N} \varphi(t) \, \varphi^T(t) \tag{43}$$

$$\hat{\Sigma}^p = \frac{1}{N-n-p} \sum_{t=n+p+1}^{N} \varphi_u^p(t) \, \varphi^T(t), \tag{44}$$

and partition $\hat{\Sigma}$ as in (25).

2) Construct the matrices $\hat{S}$ and J as in (30)–(31) and compute the matrices $\hat{A}_0, \hat{A}_1, \hat{A}_2$ (33)–(35).

3) Construct the matrices $\hat{P}$ and $\hat{Q}$ as in (39)–(40).

4) Solve the generalized eigenvalue problem

$$\left(\hat{P} - \lambda \, \hat{Q} \right) \eta = 0, \tag{45}$$

and select the generalized eigenvalue having minimum modulus. Let $\hat{\eta}$ be the corresponding eigenvector.

5) Divide $\hat{\eta}$ by its first entry $\hat{\eta}(1)$ to obtain an estimate of $\bar{\theta}_0$ (see (42) and (9))

$$\hat{\eta}_0 = \begin{bmatrix} \hat{\bar{\theta}}_0^T \; \# \end{bmatrix}^T = \begin{bmatrix} 1 \; \hat{\theta}_0^T \; \# \end{bmatrix}^T, \tag{46}$$

where $\#$ denotes the last $2n+2$ entries of $\hat{\eta}/\hat{\eta}(1)$, which are not of interest.

TABLE I

TRUE AND ESTIMATED VALUES OF PARAMETERS AND NRMSE FOR THE QEP–BASED METHODS (32) AND (51), THE FRISCH APPROACH [16], THE CLASSIC INSTRUMENTAL VARIABLE ESTIMATOR AND THE EXTENDED FRISCH METHOD [22]. A MONTE CARLO SIMULATION OF 100 RUNS HAS BEEN PERFORMED WITH $N = 500$. THE NOISELESS INPUT IS AN ARMA PROCESS.

	a_1	a_2	b_0	b_1	b_2	NRMSE
true	-0.5	0.3	1.5	-0.9	-0.45	–
QEP (32)	-0.5065 ± 0.0449	0.2903 ± 0.0334	1.3096 ± 0.0365	-0.7646 ± 0.0828	-0.4270 ± 0.0727	0.1411
QEP (51)	-0.5169 ± 0.0531	0.2924 ± 0.0369	1.7879 ± 0.0520	-1.1809 ± 0.1232	-0.3730 ± 0.1021	0.2358
Frisch	-0.4973 ± 0.0552	0.2970 ± 0.0374	1.5084 ± 0.0720	-0.9052 ± 0.1387	-0.4495 ± 0.0995	0.1034
IV	-0.4892 ± 0.0876	0.2887 ± 0.0377	1.4110 ± 0.2816	-0.8005 ± 0.3284	-0.4746 ± 0.3132	0.2934
ExtFri	-0.4955 ± 0.0474	0.2912 ± 0.0342	1.4331 ± 0.1754	-0.8440 ± 0.1461	-0.4492 ± 0.0844	0.1391

Remark 1: As an alternative, it is possible to start from (26) and (24) and to obtain the following nonlinear systems of $n+1+p$ $(p \geq n+1)$ equations

$$\begin{bmatrix} \Sigma_{yy} - \tilde{\sigma}_y^2 \, I_{n+1} & \Sigma_{yu} \\ \Sigma^p & \end{bmatrix} \bar{\theta}_0 = 0, \tag{47}$$

with $\tilde{\sigma}_y^2$ and θ_0 as unknowns, that can be rewritten as

$$\left(S - \tilde{\sigma}_y^2 \, J\right) \bar{\theta}_0 = 0, \tag{48}$$

where

$$S = \begin{bmatrix} \Sigma_{yy} & \Sigma_{yu} \\ \Sigma^p & \end{bmatrix} \tag{49}$$

$$J = \begin{bmatrix} I_{n+1} & 0_{(n+1)\times(n+1)} \\ 0_{p\times(2n+2)} & \end{bmatrix}. \tag{50}$$

It is thus possible to define the quadratic eigenvalue problem

$$\left(A_2 \, \tilde{\sigma}_y^4 + A_1 \, \tilde{\sigma}_y^2 + A_0\right) \bar{\theta}_0 = 0, \tag{51}$$

whose solution leads to an estimate of θ_0. In this case, the only real eigenvalue (with multiplicity two) that solves (36) is $\lambda = \tilde{\sigma}_y^2$.

Remark 2: Different sets of HOYW equations can be obtained by using the instrument vectors

$$\varphi_y^p(t) = [\, y(t-n-1) \, \ldots \, y(t-n-p) \,]^T \tag{52}$$

$$\varphi_{uy}^p(t) = [\, y(t-n-1) \, \ldots \, y(t-n-p_y) \quad u(t-n-1) \, \ldots \, u(t-n-p_u) \,]^T, \tag{53}$$

with $p \geq n+1$ in (52) and $p_y + p_u \geq n+1$ in (53). It is easy to show that both choices satisfy (24).

Remark 3: The set of nonlinear equations (28) leading to the quadratic eigenvalue problem (32) is exploited also in [22] with reference to the identification of EIV models with colored output noise. Nevertheless, the solution proposed in [22] is different since it is based on a two–step estimation algorithm where the first step consists in a one–dimensional optimization problem involving the input noise variance.

V. NUMERICAL SIMULATIONS

In this section the effectiveness of the proposed identification method is tested by means of numerical simulations. The system to be identified is described by the polynomials

$$A(z^{-1}) = 1 - 0.5\, z^{-1} + 0.3\, z^{-2},$$
$$B(z^{-1}) = 1.5 - 0.9\, z^{-1} - 0.45\, z^{-2}.$$

Both the quadratic eigenvalue problems (32) and (51) will be considered. Their peformance will be compared with those of the classic instrumental variable (IV) estimator which uses delayed inputs as instruments [6], [8], the Frisch scheme approach decribed in [16] and the extended Frisch method (ExtFri) proposed in [22] (second algorithm), which is based on the set of equations (28). Note that all the above mentioned methods make use of a set of high order Yule–Walker equations. The number of HOYW equations p has been set to $p = 8$ for QEP and ExtFri, $p = 5$ for Frisch and $p = 11$ for IV so that each method relies on the same number of equations (11 in this case).

In the first example the noise–free input $u_0(t)$ is a first–order ARMA process given by

$$u_0(t) = \frac{1 - 0.3\, z^{-1}}{1 - 0.7\, z^{-1}} e(t),$$

where $e(t)$ is a gaussian white noise sequence with unit variance. The additive noise variances are $\tilde{\sigma}_u^2 = 0.12$ and $\tilde{\sigma}_y^2 = 0.34$, corresponding to a signal–to–noise ratio (SNR) of about 10 dB on both the input and output sides. The number of available data is $N = 500$. A Monte Carlo simulation of 100 independent runs has been carried out. The obtained results are summarized in Table I that reports the true values of the parameters, the means of their estimates and the normalized root mean square error given by

$$\text{NRMSE} = \frac{1}{\|\theta_0\|} \sqrt{\frac{1}{100} \sum_{i=1}^{100} \|\hat{\theta}^i - \theta_0\|^2},$$

where $\hat{\theta}^i$ denotes the estimate of θ_0 obtained in the i–th trial of the Monte Carlo simulation.

In the second example the noise–free input is a pseudo random binary sequence of unit variance and length $N = 500$. A Monte Carlo simulation of 100 independent runs has been performed by adding to the noise–free sequences different gaussian white noise realizations with variances $\tilde{\sigma}_u^2 = 0.1$ and $\tilde{\sigma}_y^2 = 0.35$, leading to signal to noise ratios

TABLE II

TRUE AND ESTIMATED VALUES OF PARAMETERS AND NRMSE FOR THE QEP–BASED METHODS (32) AND (51), THE FRISCH APPROACH [16], THE CLASSIC INSTRUMENTAL VARIABLE ESTIMATOR AND THE EXTENDED FRISCH METHOD [22]. A MONTE CARLO SIMULATION OF 100 RUNS HAS BEEN PERFORMED WITH $N = 500$. THE NOISELESS INPUT IS A PSEUDO RANDOM BINARY SEQUENCE.

	a_1	a_2	b_0	b_1	b_2	NRMSE
true	-0.5	0.3	1.5	-0.9	-0.45	–
QEP (32)	-0.5009 ± 0.0412	0.3018 ± 0.0356	1.3674 ± 0.0351	-0.8132 ± 0.0666	-0.4161 ± 0.0687	0.1047
QEP (51)	-0.4865 ± 0.0447	0.2945 ± 0.0381	1.7664 ± 0.0453	-0.9791 ± 0.0918	-0.4051 ± 0.0881	0.1674
Frisch	-0.4966 ± 0.0441	0.2988 ± 0.0351	1.5024 ± 0.0462	-0.8872 ± 0.0798	-0.4629 ± 0.0753	0.0698
IV	-0.4993 ± 0.0481	0.3062 ± 0.0402	1.2077 ± 0.5468	-0.6654 ± 0.4340	-0.5505 ± 0.5096	0.4982
ExtFri	-0.4935 ± 0.0428	0.2995 ± 0.0364	1.4453 ± 0.1373	-0.8478 ± 0.0977	-0.4458 ± 0.0907	0.1119

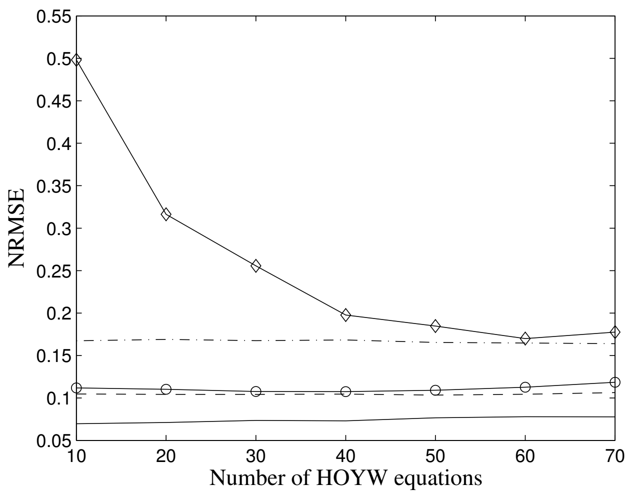

Fig. 2. NRMSE versus p: Frisch (solid), QEP (32) (dashed), QEP (51) (dashed–dotted), ExtFri (o), IV ($\diamond$). Monte Carlo simulations of 100 independent runs have been performed, $N = 500$.

on the input and output sides of about 10 dB. The obtained results are summarized in Table II.

The performance of the considered algorithms with respect to the number of HOYW equations has been evaluated as well. In the same scenario of the second example, the algorithms under investigation have been implemented by varying the number of HOYW equations from $p = 10$ to $p = 70$. For each value of p a Monte Carlo simulation of 100 independent runs has been carried out. The results are shown in Fig. 2, that reports the NRMSE versus the number p of HOYW equations.

Other Monte Carlo simulations have been performed in addition to those described above. On the basis of the obtained results, the following remarks can be made.

- The accuracy of the parameter estimates given by QEP approaches lies between those of IV and Frisch. This is not surprising since QEP relies on a subset of the compensated normal equations while the Frisch scheme exploits all such equations. On the contrary, the IV estimator relies only on HOYW equations.
- The results obtained with ExtFri and QEP (32) are quite similar. However, QEP (32) is computationally more efficient than ExtFri (see Remark 3). In addition, the results confirm what has already been noted in [22], namely, when both the input and output noise are white, the Frisch scheme leads to a better estimation accuracy than that of ExtFri.
- The QEP (32) gives better results than those of the QEP (51). This remark has been confirmed by other Monte Carlo simulations whose results have not been reported. The use of the input noise variance $\tilde{\sigma}_u^2$ as unknown seems thus the best choice.
- It has been observed that both versions of QEP algorithms are $2 \div 3$ times slower than IV. On the other hand, the IV estimator is one order of magnitude faster than Frisch (see also [5]). Therefore, the QEP approach lies between Frisch and IV also for what concern the computational load.
- The estimation of the input noise variance obtained with QEP (32) as well as that of the output noise variance obtained with (51) are quite poor and have not been reported. With respect to this, it is worth remembering that the solutions of the quadratic eigenvalue problems are always complex numbers and it has been observed that the only feasible solution is the one with smallest modulus.
- The use of large numbers of HOYW equations is not recommended since it does not lead to any improvement and increases the computational burden. This property holds also for the Frisch scheme. On the contrary, the accuracy of the IV estimator increases with p until a certain value (approximately 60 in the considered case). Nevertheless, the IV performance is always inferior to that of QEP approach.

VI. CONCLUSIONS

This paper proposes a new approach for identifying errors–in–variables models that lies between the classic instrumental variable method and the bias–compensated least squares methods with respect to estimation accuracy and computational load. It exploits part of the compensated normal equations and a set of high order Yule–Walker equations.

The solution of the EIV identification problem is obtained by solving a generalized eigenvalue problem, hence the approach is computationally simple. Monte Carlo simulations show that the obtained estimation accuracy lies between those of IV and Frisch, as expected.

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