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Direct and indirect adaptive regulation strategies for rejection of time varying narrow band disturbances applied to a benchmark problem

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Abstract— **The paper will compare the performances which can be obtained using a direct adaptive regulation scheme based on the Youla-Kučera (YK) parametrization of the controller and using an adaptive finite impulse response (FIR) filter for implementing the internal model of the disturbance with a new indirect adaptive regulation scheme. The main features of this new scheme are: (i) the use of adaptive Band-stop Filters (BSFs) tuned at the frequencies of the disturbance instead of the Internal Model Principle (IMP) and (ii) a procedure for direct identification of frequencies contained in the disturbance. The use of adaptive BSFs allows to introduce the desired attenuation of the disturbance (instead of total rejection) and allows to get a much better shaping of the output sensitivity function (to meet the specification for the tolerated amplification outside the frequencies of the disturbance). The two approaches are comparatively evaluated on the benchmark simulator and on the benchmark active vibration control system.**

I. Introduction

Direct adaptive regulation schemes using a Youla-Kučera (YK) parametrization of the controller and the Internal Model Principle (IMP) have been successfully used in a number of application ([5], [4]). It is a simple scheme and offers excellent adaptation transients. This approach has been considered to be applied to the benchmark. For a single or two narrow band disturbances, the design of the central stabilizing controller does not rise difficult problems. However for 3 narrow band disturbances (level 3 of the benchmark) the design of the central stabilizing controller is much more challenging in order to obtain a flat "water bed" effect on the Bode integral of the output sensitivity function (in order to meet the specifications on maximum tolerated amplification in the frequency domain). The IMP introduces too much attenuation with respect to the specifications of the benchmark and contributes to accentuate the "water bed" effect. In order to achieve a certain level of attenuation of narrow band disturbances, a very efficient solution is to use Band-stop Filters (BSFs) for shaping the output sensitivity functions. Basically, the numerator of the filter is implemented in the controller and the denominator is considered as part of the desired closed poles to be achieved (for details see [7]). The use of this approach allows to easily obtain the desired characteristics of the output sensitivity functions (since the water bed effect is distributed over the entire range of frequencies). No direct adaptive procedure for adapting these filters in a controller is available. Therefore an indirect adaptive procedure has been developed based on the estimation of the frequencies of the narrow band disturbances followed by the computation of the controller using BSFs for disturbance attenuation. Reduction of the complexity of the computations has been achieved by considering an appropriate YK parametrization of the controller. Previous approaches for indirect adaptive regulation were still based on the use of the IMP and the identification of a model of the disturbance was enough for implementing the procedure ([4]). For the use of adaptive BSF what is required is the frequency of the disturbance. Therefore a procedure for the direct estimation of the frequencies of the disturbance has been implemented.

The paper will present the two approaches for solving the benchmark problem, enhancing the design aspects and comparing the obtained results. Some conclusions upon when and how these schemes have to be used will be provided.

The paper is organized as follows. Section II presents the general plant and controller structure in the context of the YK parametrization. The direct adaptive approach is reviewed in Section III. The indirect adaptive control method is presented in Section IV. Section V discusses briefly the design of the central controller and comparative experimental results for the two methodologies are given in Section VI. Concluding remarks are presented in Section VII.

II. Plant Representation and Controller Structure

The structure of the LTI discrete time model of the plant, also called *secondary path*, used for controller design is

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} = \frac{z^{-d-1}B^*(z^{-1})}{A(z^{-1})}, \quad (1)$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A}, \quad (2)$$

$$B(z^{-1}) = b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} = z^{-1}B^*, \quad (3)$$

$$B^* = b_1 + \cdots + b_{n_B} z^{-n_B+1}, \quad (4)$$

and d is the plant pure time delay in number of sampling periods[1].

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[1]The complex variable z^{-1} will be used to characterize the system's behaviour in the frequency domain and the delay operator q^{-1} will be used for the time domain analysis.

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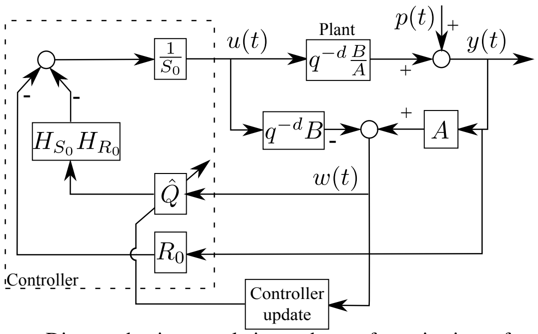

Fig. 1. Direct adaptive regulation scheme for rejection of unknown disturbances

The output of the plant $y(t)$ and the input $u(t)$ in closed loop with the central controller may be written as (for the equivalent scheme in Fig. 1 consider $\hat{Q}=0$):

$$y(t)=\frac{q^{-d}B(q^{-1})}{A(q^{-1})}\cdot u(t)+p(t), \tag{5}$$

$$S_0(q^{-1})\cdot u(t)=-R_0(q^{-1})\cdot y(t). \tag{6}$$

In (5), $p(t)$ is the effect of the disturbances on the measured output[2] and $R_0(z^{-1})$, $S_0(z^{-1})$ are polynomials in z^{-1} having the following expressions[3]:

$$S_0=1+s_1^0 z^{-1}+\ldots+s_{n_S}^0 z^{-n_S}=S_0'(z^{-1})\cdot H_{S_0}(z^{-1}), \tag{7}$$

$$R_0=r_0^0+r_1^0 z^{-1}+\ldots+r_{n_R}^0 z^{-n_R}=R_0'(z^{-1})\cdot H_{R_0}(z^{-1}), \tag{8}$$

where $H_{S_0}(q^{-1})$ and $H_{R_0}(q^{-1})$ represent pre-specified parts of the controller (used for example to incorporate the internal model of a disturbance or to open the loop at certain frequencies) and $S_0'(q^{-1})$ and $R_0'(q^{-1})$ are computed.

We define the output sensitivity function (the transfer function between the disturbance $p(t)$ and the output of the system $y(t)$) as

$$S_{yp}(z^{-1})=\frac{A(z^{-1})S_0(z^{-1})}{P_0(z^{-1})} \tag{9}$$

and the input sensitivity function (the transfer function between the disturbance $p(t)$ and the control input $u(t)$) as

$$S_{up}(z^{-1})=-\frac{A(z^{-1})R_0(z^{-1})}{P_0(z^{-1})}, \tag{10}$$

where

$$P_0(z^{-1})=A(z^{-1})S_0(z^{-1})+z^{-d}B(z^{-1})R_0(z^{-1}), \tag{11}$$

the characteristic polynomial, specifies the desired closed loop poles of the system[4] (see also [7]). It is important to remark that one should only reject disturbances located in frequency regions where the plant model has enough gain. This can be seen by looking at eq. (9) and noticing that perfect rejection at a certain frequency, ω_0, is obtained iff $S_0(e^{-j\omega_0})=0$. On the other hand, from eq. (10) one can see that this has a bad effect on the control input if the gain of the secondary path is too small at ω_0.

In this paper, the Youla-Kučera parametrization ([1], [12]) is used. Supposing a generalized infinite impulse response (IIR) representation of the adaptive Q filter

$$Q(z^{-1})=\frac{B_Q(z^{-1})}{A_Q(z^{-1})}, \tag{12}$$

the controller's polynomials are:

$$R=R_0A_Q+AB_QH_{S_0}H_{R_0}, \tag{13}$$

$$S=S_0A_Q-z^{-d}BB_QH_{S_0}H_{R_0}. \tag{14}$$

where R_0 and S_0 define the central controller which verifies the desired specifications in the absence of the disturbance. The characteristic polynomial of the closed loop becomes

$$P=A_Q\left(AS_0+z^{-d}BR_0\right). \tag{15}$$

III. Direct Adaptive Regulation for Disturbance Rejection

This section presents the direct adaptive control scheme ([5], [4]) that will be used for the benchmark problem. A key aspect of this methodology is the use of the IMP. It is supposed that $p(t)$ is a deterministic disturbance given by

$$p(t)=\frac{N_p(q^{-1})}{D_p(q^{-1})}\cdot\delta(t), \tag{16}$$

where $\delta(t)$ is a Dirac impulse and N_p, D_p are coprime polynomials of degrees n_{N_p} and n_{D_p}, respectively[5]. In the case of stationary narrow-band disturbances, the roots of $D_p(z^{-1})$ are on the unit circle.

Internal Model Principle: The effect of the disturbance given in (16) upon the output

$$y(t)=\frac{A(q^{-1})S(q^{-1})}{P(q^{-1})}\cdot\frac{N_p(q^{-1})}{D_p(q^{-1})}\cdot\delta(t), \tag{17}$$

where $D_p(z^{-1})$ is a polynomial with roots on the unit circle and $P(z^{-1})$ is an asymptotically stable polynomial, converges asymptotically towards zero iff the polynomial $S(z^{-1})$ in the RS controller has the form (based on eq. (7))

$$S(z^{-1})=D_p(z^{-1})H_{S_0}(z^{-1})S'(z^{-1}). \tag{18}$$

Thus, the pre-specified part of $S(z^{-1})$ should be chosen as $H_S(z^{-1})=D_p(z^{-1})H_{S_0}(z^{-1})$ and the controller can be computed in an indirect adaptive scheme by solving the Bezout equation

$$P=AD_pH_{S_0}S'+z^{-d}BH_{R_0}R', \tag{19}$$

where P, D_p, A, B, H_{R_0}, H_{S_0} and d are given[6].

For the purpose of direct adaptive regulation, $Q(z^{-1})$ is considered to be a finite impulse response (FIR) filter ($A_Q(z^{-1})=1$ and $Q(z^{-1})=B_Q(z^{-1})$)

$$Q(z^{-1})=q_0+q_1z^{-1}+\ldots+q_{n_Q}z^{-n_Q}. \tag{20}$$

[2]The disturbance passes through a socalled *primary path* which is not represented in this figure, and $p(t)$ is its output.

[3]The argument (z^{-1}) will be omitted in some of the following equations to make them more compact.

[4]It is assumed that a reliable model identification is achieved and therefore the estimated model is assumed to be equal to the true model.

[5]Throughout the paper, n_X denotes the degree of the polynomial X.

[6]Of course, it is assumed that D_p and B do not have common factors.

To compute $Q(z^{-1})$ in order that the polynomial $S(z^{-1})$ given by (14) incorporates the internal model of the disturbance (18), one has to solve the diophantine equation

$$S'D_p + z^{-d}BH_{R_0}Q = S'_0, \tag{21}$$

where D_p, d, B, S'_0, and H_{R_0} are known and S' and Q are unknown. Eq. (21) has a unique solution for S' and Q with: $n_{S'_0} \le n_{D_p} + n_B + d + n_{H_{R_0}} - 1$, $n_{S'} = n_B + d + n_{H_{R_0}} - 1$, $n_Q = n_{D_p} - 1$. One sees that the order n_Q of the polynomial Q depends upon the structure of the disturbance model. The use of the Youla-Kučera parametrization, with Q given in (20), is interesting in this case because it allows to maintain the closed loop poles as given by the central controller but at the same time introduce the parameters of the internal model into the controller. To build the parametric adaptation algorithm (PAA), one has to find first an *error equation* (see also [12], [5], [4]). Using the Q-parametrization, the output of the system in the presence of a disturbance can be expressed as

$$y(t) = \frac{S_0 - q^{-d}BH_{S_0}H_{R_0}Q}{P} \cdot w(t), \tag{22}$$

where $w(t)$ is given by (see also Fig. 1)

$$w(t) = \frac{AN_p}{D_p} \cdot \delta(t) = A \cdot y(t) - q^{-d} \cdot B \cdot u(t). \tag{23}$$

Taking into consideration that the adaptation of Q is done in order to obtain an output $y(t)$ which tends asymptotically to zero, one can define $\varepsilon^0(t+1)$ as the value of $y(t+1)$ obtained with $\hat{Q}(t,q^{-1})$ (the estimate of Q at time t, written also $\hat{Q}(t)$)

$$\varepsilon^0(t+1) = \frac{S_0}{P} \cdot w(t+1) - \hat{Q}(t)\frac{q^{-d}B^*H_{S_0}H_{R_0}}{P} \cdot w(t). \tag{24}$$

Similarly, the *a posteriori* error becomes (using $\hat{Q}(t+1)$) as

$$\varepsilon(t+1) = \frac{S_0}{P} \cdot w(t+1) - \hat{Q}(t+1)\frac{q^{-d}B^*H_{S_0}H_{R_0}}{P} \cdot w(t). \tag{25}$$

Replacing S_0 in the last equation by (7) and (21), one obtains

$$\varepsilon(t+1) = [Q - \hat{Q}(t+1)] \cdot \frac{q^{-d}B^*H_{S_0}H_{R_0}}{P} \cdot w(t) + v(t+1), \tag{26}$$

where $v(t) = \frac{S'D_pH_{S_0}}{P} \cdot w(t) = \frac{S'H_{S_0}AN_p}{P} \cdot \delta(t)$ is a signal which tends asymptotically towards zero.

Define the estimated polynomial $\hat{Q}(t,q^{-1}) = \hat{q}_0(t) + \hat{q}_1(t)q^{-1} + \ldots + \hat{q}_{n_Q}(t)q^{-n_Q}$ and the associated estimated parameter vector $\hat{\theta}(t) = [\hat{q}_0(t)\ \hat{q}_1(t)\ \ldots\ \hat{q}_{n_Q}(t)]^T$. Define the fixed parameter vector corresponding to the optimal value of the polynomial Q as: $\theta = [q_0\ q_1\ \ldots\ q_{n_Q}]^T$.

Denote

$$w_2(t) = \frac{q^{-d}B^*H_{S_0}H_{R_0}}{P} \cdot w(t) \tag{27}$$

and define the following observation vector

$$\phi^T(t) = [w_2(t)\ w_2(t-1)\ \ldots\ w_2(t-n_Q)]. \tag{28}$$

Eq. (26) becomes

$$\varepsilon(t+1) = [\theta^T - \hat{\theta}^T(t+1)] \cdot \phi(t) + v(t+1). \tag{29}$$

One can remark that $\varepsilon(t+1)$ corresponds to an adaptation error ([6]).

From eq. (24), one obtains the *a priori* adaptation error

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t), \tag{30}$$

with

$$w_1(t+1) = \frac{S_0(q^{-1})}{P(q^{-1})} \cdot w(t+1), \tag{31}$$

$$w(t+1) = A(q^{-1}) \cdot y(t+1) - q^{-d}B^*(q^{-1}) \cdot u(t), \tag{32}$$

where $B(q^{-1})u(t+1) = B^*(q^{-1})u(t)$.

The *a posteriori* adaptation error is obtained from (25)

$$\varepsilon(t+1) = w_1(t+1) - \hat{\theta}^T(t+1)\phi(t). \tag{33}$$

For the estimation of the parameters of $\hat{Q}(t,q^{-1})$ the following PAA is used ([6]):

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\phi(t)\varepsilon(t+1), \tag{34}$$

$$\varepsilon(t+1) = \frac{\varepsilon^0(t+1)}{1 + \phi^T(t)F(t)\phi(t)}, \tag{35}$$

$$\varepsilon^0(t+1) = w_1(t+1) - \hat{\theta}^T(t)\phi(t), \tag{36}$$

$$F(t+1) = \frac{1}{\lambda_1(t)}\left[F(t) - \frac{F(t)\phi(t)\phi^T(t)F(t)}{\frac{\lambda_1(t)}{\lambda_2(t)} + \phi^T(t)F(t)\phi(t)}\right], \tag{37}$$

$$1 \ge \lambda_1(t) > 0,\ 0 \le \lambda_2(t) < 2, \tag{38}$$

where $\lambda_1(t)$, $\lambda_2(t)$ allow to obtain various profiles for the evolution of the adaption gain $F(t)$ (for details see [6], [7]).

IV. Indirect Adaptive Regulation Based on BSFs for Disturbance Attenuation

The purpose of this method is to allow the possibility of choosing the desired attenuation and bandwidth of attenuation for each of the estimated narrow-band disturbances. This is the main advantage with respect to classical internal model methods which in the case of several narrow-band disturbances, as a consequence of complete cancellation of the disturbances, may lead to unacceptable values of the modulus of the output sensitivity function outside the attenuation regions.

In this section, a technique of output sensitivity function shaping for narrow-band disturbance compensation is presented. The design uses BSFs to shape the output sensitivity function. Following [7], [10], there exist digital filters[7] $\frac{H_{S_i}}{P_{F_i}}$, which will assure the desired attenuation of a narrow-band disturbance ($i \in \{1,\ldots,n\}$ and $n = \frac{n_{D_p}}{2}$ from the previous section).

The structure of the BSFs is

$$\frac{S_{BSF_i}(z^{-1})}{P_{BSF_i}(z^{-1})} = \frac{1 + \beta_1^i z^{-1} + \beta_2^i z^{-2}}{1 + \alpha_1^i z^{-1} + \alpha_2^i z^{-2}}, \tag{39}$$

resulting from the discretization of a continuous filter (see also [10], [7])

$$F_i(s) = \frac{s^2 + 2\zeta_{n_i}\omega_i s + \omega_i^2}{s^2 + 2\zeta_{d_i}\omega_i s + \omega_i^2} \tag{40}$$

[7]The numerators of these filters will be implemented in the controller while the denominators will define additional closed loop poles.

using the bilinear transformation. This filter introduces an attenuation of $M_i = -20 \cdot \log_{10}\left(\frac{\zeta_{n_i}}{\zeta_{d_i}}\right)$ at the frequency ω_i. Positive values of M_i denote attenuations ($\zeta_{n_i} < \zeta_{d_i}$) and negative values denote amplifications ($\zeta_{n_i} > \zeta_{d_i}$)[8].

Under the hypothesis that the plant model parameters are constant and that an accurate identification experiment can be run, a reliable estimate $\hat{p}(t)$ of the disturbance signal can be obtain by using the disturbance observer

$$\hat{p}(t+1) = y(t+1) - q^{-d}\frac{B^*(q^{-1})}{A(q^{-1})}u(t). \tag{41}$$

The signal $\hat{p}(t)$ can then be used to estimate the spike frequencies ($\hat{\omega}_i$) with adaptive notch filters (ANF) described by IIR filters of the type $\frac{A^f(q^{-1})}{A^f(\rho q^{-1})}$, where $A^f(q^{-1})$ has a mirror symmetric form and $0 < \rho < 1$ is a scalar usually chosen close to 1 (see also for more details [9], [8]).

Remark: The design parameters for each BSF are the desired attenuation (M_i), the central frequency of the filter ($\hat{\omega}_i$) and the damping of the denominator (ζ_{d_i}). The denominator damping is used to adjust the frequency bandwidth of the BSF. For very small values of the frequency bandwidth the influence of the filters on frequencies other than those defined by $\hat{\omega}_i$ is negligible. Therefore, the number of BSFs and subsequently that of the narrow-band disturbances that can be compensated can be as large as necessary[9].

For n narrow-band disturbances, n BSFs will be used

$$H_{BSF}(z^{-1}) = \frac{S_{BSF}(z^{-1})}{P_{BSF}(z^{-1})} = \frac{\prod_{i=1}^{n} S_{BSF_i}(z^{-1})}{\prod_{i=1}^{n} P_{BSF_i}(z^{-1})}. \tag{42}$$

$S(z^{-1})$ and $R(z^{-1})$ are obtained as solutions of the Bezout equation $P(z^{-1}) = A(z^{-1})S(z^{-1}) + z^{-d}B(z^{-1})R(z^{-1})$, where $R(z^{-1}) = H_R(z^{-1})R'(z^{-1})$, $S(z^{-1}) = H_S(z^{-1})S'(z^{-1})$, and $P(z^{-1})$ is given by $P(z^{-1}) = P_0(z^{-1})P_{BSF}(z^{-1})$.

In the last equation, P_{BSF} is the combined denominator of all the BSFs, (42), and P_0 was defined in (11). The fixed part of the controller denominator H_S is in turn factorized into

$$H_S(z^{-1}) = S_{BSF}(z^{-1})H_{S_0}(z^{-1}), \tag{43}$$

where S_{BSF} is the combined numerator of the BSF, (42), and H_{S_0} has been introduced in (7), while the fixed part of R remains as given by the central controller, $H_R = H_{R_0}$. It is easy to see that the output sensitivity function becomes

$$S_{yp}(z^{-1}) = \frac{A(z^{-1})S'(z^{-1})H_{S_0}(z^{-1})S_{BSF}(z^{-1})}{P_0(z^{-1})P_{BSF}(z^{-1})} \tag{44}$$

and the shaping effect of the BSFs upon the sensitivity functions is obvious.

The unknowns S' and R' are solutions of

$$\begin{aligned} P(z^{-1}) = & P_0(z^{-1})P_{BSF}(z^{-1}) = A(z^{-1})H_S(z^{-1})S'(z^{-1}) + \\ & + z^{-d}B(z^{-1})H_{R_0}(z^{-1})R'(z^{-1}) \end{aligned} \tag{45}$$

[8]For frequencies bellow $0.17f_s$ (f_s is the sampling frequency) the design can be done with a very good precision directly in discrete time ([7]).

[9]Of course, there is a compromise between the attenuation imposed and the number of narrow-band disturbances.

and can be computed by putting (45) into matrix form (see also [7]). The size of the matrix equation that needs to be solved is given by

$$n_{Bez} = n_A + n_B + d + n_{H_{S_0}} + n_{H_{R_0}} + 2\cdot n - 1, \tag{46}$$

where n_A, n_B, and d are respectively the order of the plant's model denominator, numerator, and delay (given in (2) and (3)), $n_{H_{S_0}}$ and $n_{H_{R_0}}$ are the orders of $H_{S_0}(z^{-1})$ and $H_{R_0}(z^{-1})$ respectively and n is the number of narrow-band disturbances. Eq. (45) has an unique minimal degree solution for S' and R', if $n_P \le n_{Bez}$, where n_P is the order of the pre-specified characteristic polynomial $P(q^{-1})$. Also, it can be seen from (45) and (43) that the minimal orders of S' and R' will be:

$$n_{S'} = n_B + d + n_{H_{R_0}} - 1, \quad n_{R'} = n_A + n_{H_{S_0}} + 2\cdot n - 1. \tag{47}$$

Note that for real time applications, the diophantine equation (45) has to be solved either at each sampling time (adaptive operation) or each time when a change in the narrow-band disturbances' frequencies occurs (self-tuning operation).

The computational complexity related to the Bezout equation (45) is significant. In this section, we show how the computation load of the algorithm can be reduced by the use of the Youla-Kučera parametrization.

The BSFs (42) should be computed based on the estimated frequencies of the multiple narrow-band signal. The objective is to implement the design method described above using a Youla-Kučera parametrization ([11]) around a central controller defined in (7), (8), and (11).

An IIR Youla-Kučera parametrization (see (12)) can offer the desired characteristics for disturbance rejection maintaining also the fixed parts of the nominal controller ($H_{R_0}(z^{-1})$ and $H_{S_0}(z^{-1})$) and is subsequently used. For this purpose, the controller polynomials are factorized as in eqs. (13) and (14), where $A_Q(z^{-1})$ will be chosen as the cumulated denominator of the BSFs, $P_{BSF}(z^{-1})$. On the other hand, $B_Q(z^{-1})$ is computed so that it allows to introduce the BSFs' numerators into the fixed part of $S(z^{-1})$, as in (43). Taking into account (14), this is equivalent to finding $B_Q(z^{-1})$ from the Bezout equation

$$S'_0 P_{BSF} = S_{BSF}S' + q^{-d}BH_{R_0}B_Q, \tag{48}$$

where the common term $H_{S_0}(z^{-1})$ has been eliminated.

In the last equation, the left side of the equal sign is known and on its right side only $S'(z^{-1})$ and $B_Q(z^{-1})$ are unknown. This is also a Bezout equation which can be solved by finding the solution to a matrix equation of dimension $n_{Bez_{YK}} = n_B + d + n_{H_{R_0}} + 2\cdot n - 1$. As it can be observed, the size of the new Bezout equation is reduced in comparison to (46) by $n_A + n_{H_{S_1}}$. For systems with large dimensions, this has a significant influence on the computation time (in Section VI, $n_A = 23$, $n_B = 26$, $n \in \{2,3\}$, $n_{H_{R_0}} = 2$, $n_{H_{S_0}} = 0$, and $d = 0$). The nominal controller, being an unique and minimal degree solution to a Bezout equation, satisfies $n_{S'_0} = n_B + d + n_{H_{R_0}} - 1$. By adding $2\cdot n$ in both sides of the last equation, one obtains $n_{S'_0}+2\cdot n=2\cdot n+n_B+d+n_{H_{R_0}}-1$ which means that the solution of the simplified Bezout equation (48) is unique and of minimal degree. Furthermore, the order of the B_Q FIR filter is equal to $2\cdot n-1$.

V. CENTRAL CONTROLLER DESIGN

A key element for both designs is the central controller, which is presented in eqs. (13) and (14). Its main objective is that of assuring closed loop system's robustness for the entire disturbances frequency range, which is not trivial considering the control specifications of the benchmark and the frequency characteristics of the system. As such, for direct adaptive regulation using IMP a controller was designed for each level of the benchmark and the complexity of the resulting controller had to be increased with the number of narrow band disturbances which had to be attenuated. In the case of the BSF design, a single central controller has been used for the three levels of the benchmark as closed loop robustness could be satisfied easier.

For all central controllers, the following steps of design are common. First, to preserve the robustness and dynamics in open loop, all the poles of the system are conserved (stable system). Then, fixed parts are introduced in the central controller's numerator polynomial, $R_0(z^{-1})$, for opening the loop at $0f_s$ and $0.5f_s$. No fixed parts were considered for $S_0(z^{-1})$.

Specifically for the design of the central controller for the IMP scheme, it was found that the introduction of some fixed auxiliary resonant low damped poles can minimise the influence of the IMP adaptive controller outside the attenuation frequencies improving robsutness. Near to the frequency region of interest, 50 to 95 Hz, the system has two pairs of low damped complex zeros, one at 45.6 and the other at 98.5 Hz. To maintain robustness when disturbances are located close to the limits of this region, auxiliary complex poles had to be introduced. Their damping factors had to be chosen in a way that the effect of the rejection method is not eliminated, just attenuated.

The design of the central controller for indirect adaptive regulation using BSF is easier. The attenuation level is specified directly in the design of the BSFs which are of IIR type, thus introducing poles in the closed loop. These poles act as the additional poles that had to be introduced for each central controller in the IMP based scheme.

VI. EXPERIMENTAL RESULTS

The specifications of the benchmark (see [3], [2]) consider three levels in terms of the number of narrow band disturbances to be attenuated. For each level, there are three types of trials for which performance specifications have to be achieved. The first series of trials deal with global attenuation (GA in dB), disturbance attenuation (DA in dB), maximum amplification (MA in dB) outside the attenuation frequencies (these quantities are evaluated once the adaptation has settled), transient duration (TD in sec), maximum value (MV in Volts) during transient, a measure of the integral of the square truncated two norm of the error during transient (N^2T), and after settling of the adaptation (N^2R). The second series of trials consider the evaluation of performances for step changes in frequencies and the third consider the evaluation of performances when the disturbances are chirp signals.

The two adaptive schemes, called subsequently DIMP (direct adaptive regulation using IMP) and IBSF (indirect adaptive regulation using BSFs), have been evaluated in both simulation and real time. For lack of space, only experimental results for the third level are presented comparatively in this section. All other results can be found on the website dedicated to the benchmark [2].

A. Level 3 Results

The results for the Simple Step Test are given in Fig. 2. The benchmark specifications are almost fulfilled by both algorithms with respect to the disturbance attenuation and maximum amplification (only slightly better results for IBSF). Global attenuation requirements are fully satisfied. Figs. 3 and 4 show the Power Spectral Densities (PSD) esti-

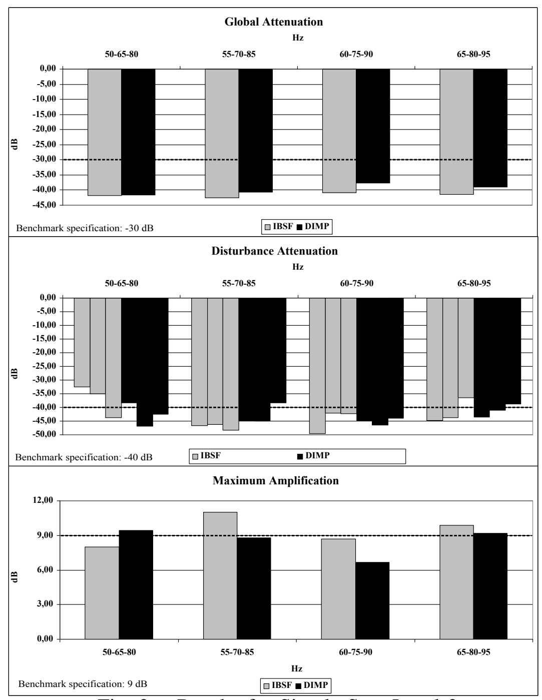

Fig. 2. Results for Simple Step Level 3

mates obtained using IBSF and DIMP for the case of three narrow band disturbances with frequencies of $[60,75,90]$ Hz, respectively. The behaviour of IBSF is slightly better than DIMP's.

Table I summarises the results for the Step Changes in Frequency Test. It can be seen that both algorithms satisfy the benchmark specifications on MV (≤ 0.1 V).

Figs. 5 and 6 show the time responses for both approaches for a single step test (upper plots), for a step changes test using the first disturbance sequence of Table I (middle plots), and for a chirp test (lower plots). It can be observed that the IBSF algorithm is better than DIMP with respect to the Step Changes in Frequency and Chirp Tests.

The basic specification for transient performance is the requirement that the transient duration when a disturbance is

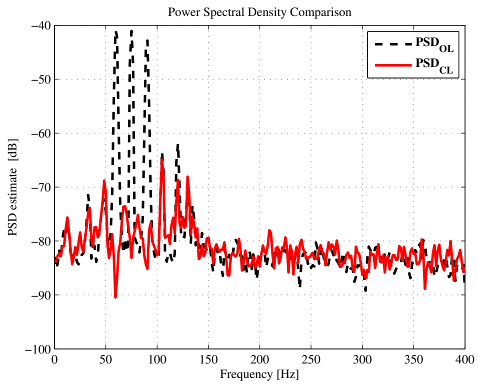

Fig. 3. PSD of the result for IBSF at $[60,75,90]$ Hz.

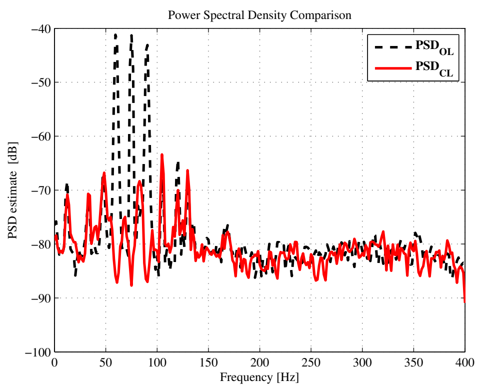

Fig. 4. PSD of the result for DIMP at $[60,75,90]$ Hz

TABLE I

RESULTS - STEP CHANGES LEVEL 3

Sequence 1	IBSF		DIMP	
	N^2T	MV [mV]	N^2T	MV [mV]
$[55,70,85] \to [60,75,90]$ Hz	97.33	65.06	235.74	62.10
$[60,75,90] \to [55,70,85]$ Hz	86.38	64.84	208.75	55.69
$[55,70,85] \to [50,65,80]$ Hz	103.87	62.61	242.60	59.61
$[50,65,80] \to [55,70,85]$ Hz	115.15	68.74	235.63	76.77
Sequence 2	N^2T	MV [mV]	N^2T	MV [mV]
$[60,75,90] \to [65,80,95]$ Hz	96.45	57.76	275.33	64.50
$[65,80,95] \to [60,75,90]$ Hz	100.68	61.44	225.24	56.83
$[60,75,90] \to [55,70,85]$ Hz	85.00	61.44	196.53	51.93
$[55,70,85] \to [60,75,90]$ Hz	96.50	63.57	183.17	54.69

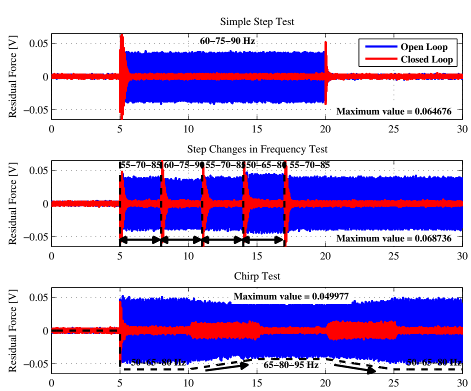

Fig. 5. Time response results for Level 3 - IBSF.

applied, be smaller than 2 sec. Details of the measurement procedure can be found in [2]. A benchmark satisfaction index (BSI) has been introduced to evaluate the transient behaviour of the algorithms. For IBSF, one obtains a BSI_{Trans}

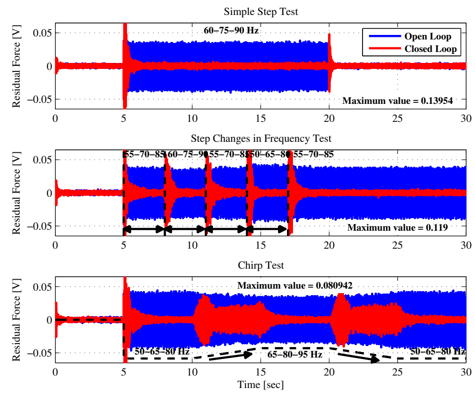

Fig. 6. Time response results for Level 3 - DIMP.

of 100%, while for the DIMP it is of 99.50%. The difference points here in the advantage of the IBSF algorithm.

VII. CONCLUDING REMARKS

Two adaptive algorithms for rejection of multiple narrow band disturbances have been presented in this paper and evaluated experimentally. From a comparison of the results of these two methods, it can be concluded that the indirect approach gives, overall, slightly better results than the direct approach, given the possibility of easily adjusting the BSFs in order to improve closed loop robustness. However, the computational complexity is in favour of the direct approach which is far less demanding.

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