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Nonlinear Learning-based Adaptive Control for Electromagnetic Actuators

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Abstract— **We present in this paper our preliminary results on the problem of learning-based adaptive trajectory tracking control for electromagnetic actuators. First, we develop a nominal nonlinear backstepping controller that stabilizes the tracking errors asymptotically and globally. Second, we robustify the nominal controller using a model-free learning technique, namely, multiparameter extremum seeking, to estimate the uncertain model parameters. In this sense we are proposing to solve an adaptive control problem with model-free learning-based algorithms. We show the performance of the proposed controller on a numerical example.**

I. INTRODUCTION

Electromagnetic actuators have been utilized for a variety of practical applications, e.g. valves of combustion engines, artificial hearts, etc. This work deals with a particular control problem of nonlinear electromagnetic actuator, namely, the so-called 'soft landing' problem. The soft landing requires accurate control of the moving element of the actuator between two desired positions. The main objective, is to attain small contact velocity, which in turn ensures a low-noise, low-component-wear operation of the actuator. Furthermore, this 'soft-landing' requirement has to be guaranteed over long period of time during which the actuator's components may age. Due to these practical constraints we have developed a robust control algorithm that 1) aims for a zero impact velocity, and 2) adapts to the actuator's aging parts via a learning-based adaptive algorithm. We present here the results of this study.

Many papers have been dedicated to the soft-landing problem for electromagnetic actuators, e.g. [1], [2], [3], [4], [5], [6], [7], [8]. Some linear controllers have been proposed in [1], [4], [8]. The results based on linear control theory use linear approximations of the actuator dynamics and thus are usually valid only in the vicinity of linearization points. To control the system over a larger operating state space, the controller has to be based on more complex nonlinear models of the actuators. Thus, in this paper we have decided to take into account the nonlinear dynamics of the system and design a fully nonlinear controller. Different nonlinear controllers have been used in [2], [3], [5], [7], [9], [10]. In [2], the authors studied the problem of electromagnetic valve actuator control in an internal combustion engine. The solution proposed by the author is based on iteratively solving a constrained nonlinear optimal problem using Nelder-Mead algorithm. The optimal solution defines a feedforward control signal. The robustness of this approach to system aging has neither been proven nor tested, and there are no feedback terms to robustify the feedforward control. In [5], the authors proposed a nonlinear controller to solve the problem of armature stabilization for an electromechanical valve actuator. The authors proved a global asymptotic stability result using Sontag's nonlinear controller. However, this approach does not solve the problem of armature trajectory tracking and does not consider robustness of the controller with respect to system's uncertainties and changes in parameters over time. In [10], the authors designed a backstepping based controller for electromagnetic actuators position regulation. However, robustness w.r.t. uncertainties in parameters of the system are not considered in this paper. In [7], a nonlinear sliding mode approach is used to solve the problem of trajectory tracking for an electromagnetic valve actuator. The authors used a nonlinear model to design the sliding mode control. The reported results show good tracking performances, however, this sliding mode controller does not ensure robustness with respect to model uncertainties. In [3], the authors used a single parameter extremum seeking learning method to solve the problem of soft landing for an electromechanical valve actuator. The authors first designed a nonlinear controller based on a nonlinear model of the actuator and then used an extremum seeking algorithm to tune one gain of the controller. Although, the learning algorithm was not directly tailored to ensure robustness of the controller to model uncertainties or parameters drift over time, one could argue that this robustness is intrinsic due the iterative nature of the learning process. However, in this controller design only one gain of the control is tuned online, which limits the robustness capabilities of the controller with respect to model uncertainties or system's aging.

In this work we use a nonlinear model of the electromagnetic actuator to design a nonlinear backstepping controller that is proven to ensure trajectory tracking for the nominal system, which assumes that there are no uncertain or drift of parameters. Subsequently, this controller is robustified by a *multiparameter* extremum seeking (MES) algorithm that is used to *identify online the model's uncertain parameters*, this includes tracking over time any slow drifts of these parameters. Notice that contrary to [3], we are using a MES approach to learn *a vector of the model parameters*, and not the gain of the controller. In this sense, we are proposing a new *learning-based adaptive control*. One noticeable feature of such a learning algorithm is that there is no a priori constraint on the uncertainty structure, e.g. the assumption

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used in classical adaptive control, of linearity w.r.t. the uncertain parameters is not necessary.

This paper is organized as follows: We first present in Section II some notations and preliminaries. In Section III, we recall the nonlinear model of electromagnetic actuators. Then, in Section IV, we report the main result of this work, namely the learning-based adaptive nonlinear controller. Numerical validation of the proposed controller is given in Section V. Finally, concluding remarks are stated in Section VI.

II. PRELIMINARIES

Throughout the paper we will use $\|.\|$ to denote the Euclidean norm; i.e., for $x \in \mathbb{R}^n$ we have $\|x\| = \sqrt{x^T x}$. Also, we will use the notations $diag\{m_1, ..., m_n\}$ for $n \times n$ diagonal matrix, and $\dot{(.)}$ for the short notation of time derivative. We denote by C^k functions that are k times differentiable.

Definition 1 ($\mathcal{K}$ function [11]): A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.

Definition 2 ($\mathcal{KL}$ function [11]): A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

III. SYSTEM MODELLING

Following [9], we consider the nonlinear electromagnetic actuator model

$$\begin{aligned} & m\frac{d^2x}{dt^2} = k(x_0 - x) - \eta\frac{dx}{dt} - \frac{ai^2}{2(b+x)^2} + f_d \\ & u = Ri + \frac{a}{b+x}\frac{di}{dt} - \frac{ai}{(b+x)^2}\frac{dx}{dt},\ 0 \le x \le x_f, \end{aligned} \tag{1}$$

where, x represents the armature position physically constrained between the initial position of the armature 0, and the maximal position of the armature x_f, $\frac{dx}{dt}$ represents the armature velocity, m is the armature mass, k the spring constant, x_0 the initial spring length, η the damping coefficient (assumed to be constant), $\frac{ai^2}{2(b+x)^2}$ represents the electromagnetic force (EMF) generated by the coil, a, b being constant parameters of the coil, f_d a constant term modelling disturbance forces, e.g. static friction, R the resistance of the coil, $L = \frac{a}{b+x}$ the coil inductance (assumed to be armature-position dependent), $\frac{ai}{(b+x)^2}\frac{dx}{dt}$ represents the back EMF. Finally, i denotes the coil current, $\frac{di}{dt}$ its time derivative and u represents the control voltage applied to the coil. In this model we do not consider the saturation region of the flux linkage in the magnetic field generated by the coil, since we assume a current and armature motion ranges within the linear region of the flux.

Based on this well known nonlinear model of the electromagnetic actuator we develop in the next section a backstepping nonlinear control and we extend it to an adaptive version based on a MES algorithm.

IV. LEARNING-BASED ADAPTIVE NONLINEAR CONTROL

A. Nominal Backstepping Controller

In this section we first apply the backstepping approach to the model (1), assuming that all the coefficients of the model are known. We then prove the stability of the equilibrium point for the closed-loop dynamics. Subsequently, we extend this result to its adaptive version by using a learning technique based on extremum seeking theory to learn the uncertain coefficient of the model.

Consider the dynamical system (1). Defining the state vector $\mathbf{z} := [z_1 \ \ z_2 \ \ z_3]^T = [x \ \ \dot{x} \ \ i]^T$, the objective of the control is to make the variables (z_1, z_2) track a sufficiently smooth (at least C^2) time-varying position and velocity trajectories $z_1^{ref}(t),\ z_2^{ref}(t) = \frac{dz_1^{ref}(t)}{dt}$ that satisfy the following constraints: $z_1^{ref}(t_0) = z_{1_{int}},\ z_1^{ref}(t_f) = z_{1_f},\ \dot{z}_1^{ref}(t_0) = \dot{z}_1^{ref}(t_f) = 0, \ddot{z}_1^{ref}(t_0) = \ddot{z}_1^{ref}(t_f) = 0$, where t_0 is the starting time of the trajectory, t_f is the ending time, $z_{1_{int}}$ is the initial position and z_{1_f} is the final position.

To start, let us first write the system (1), in the following way:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{k}{m}(x_0 - z_1) - \frac{\eta}{m}z_2 - \frac{a}{2m(b+z_1)^2}z_3^2 + \frac{f_d}{m} \\ \dot{z}_3 &= -\frac{R}{\frac{a}{b+z_1}}z_3 + \frac{z_3}{b+z_1}z_2 + \frac{u}{\frac{a}{b+z_1}}. \end{aligned} \tag{2}$$

Also, consider the following control input:

$$\begin{aligned} u = \frac{a}{b+z_1}\Big(& \frac{R(b+z_1)z_3}{a} - \frac{z_2 z_3}{(b+z_1)} + \frac{1}{2z_3}\big(\dot{\tilde{u}} + \frac{a(z_2 - z_2^{ref})}{2m(b+z_1)^2} \\ & - c_2(z_3^2 - \tilde{u})\big)\Big) \end{aligned} \tag{3}$$

with

$$\begin{aligned} \tilde{u} = \frac{2m(b+z_1)^2}{a}\Big(& \frac{k}{m}(x_0 - z_1) - \frac{\eta}{m}z_2 - \dot{z}_2^{ref} \\ & + c_3(z_1 - z_1^{ref}) + c_1(z_2 - z_2^{ref}) + \frac{f_d}{m}\Big). \end{aligned} \tag{4}$$

We provide the stability analysis for the nominal system (1) with the control input given by (3) and (4) in the next lemma.

Lemma 1: Consider the closed-loop dynamics given by (2), (3) and (4). Then, there exist positive gains c_1, c_2, and c_3 such that $(z_1(t), z_2(t))$ are uniformly bounded and satisfy $\lim_{t\to\infty}(z_1(t), z_2(t)) = (z_1^{ref}(t), z_2^{ref}(t))$, for any initial condition $(z_1(t_0), z_2(t_0), z_3(t_0))^T$.

Proof: Consider the mechanical subsystem that consists of only the first two equations, and define the virtual control input $\tilde{u} := z_3^2$:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{k}{m}(x_0 - z_1) - \frac{\eta}{m}z_2 - \frac{a}{2m(b+z_1)^2}\tilde{u} + \frac{f_d}{m}. \end{aligned} \tag{5}$$

Consider the Lyapunov function $V_{sub} = \frac{c_3}{2}(z_1 - z_1^{ref})^2 + \frac{1}{2}(z_2 - z_2^{ref})^2$, where $c_3 > 0$ is a design parameter. Taking the derivative of V_{sub} along the trajectories of (5), we obtain

$$\begin{aligned} & \dot{V}_{sub} = c_3(z_1 - z_1^{ref})(\dot{z}_1 - \dot{z}_1^{ref}) \\ & + (z_2 - z_2^{ref})(\dot{z}_2 - \dot{z}_2^{ref}) \\ & = (z_2 - z_2^{ref})\big(c_3(z_1 - z_1^{ref}) + \frac{k}{m}(x_0 - z_1) \\ & - \frac{\eta}{m}z_2 - \dot{z}_2^{ref} + \frac{f_d}{m}\big) - (z_2 - z_2^{ref})\left(\frac{a}{2m(b+z_1)^2}\tilde{u}\right) \end{aligned}$$

We want to design the virtual input so that $\dot{V}_{sub} = -c_1(z_2 - z_2^{ref})^2$, with $c_1 > 0$. In order to do so, we design the virtual input as the follows:

$$\tilde{u} = \frac{2m(b+z_1)^2}{a}\Big(\frac{k}{m}(x_0 - z_1) - \frac{\eta}{m}z_2 - \dot{z}_2^{ref} + \frac{f_d}{m} + c_3(z_1 - z_1^{ref}) + c_1(z_2 - z_2^{ref})\Big). \tag{6}$$

Next, we define the augmented Lyapunov function $V_{aug} = V_{sub} + \frac{e^2}{2}$. Taking the derivative of V_{aug} along the trajectories of the full system, we have

$$\begin{aligned}
\dot{V}_{aug} &= c_3(z_1 - z_1^{ref})(\dot{z}_1 - \dot{z}_1^{ref}) + (z_2 - z_2^{ref})(\dot{z}_2 - \dot{z}_2^{ref}) \\
&\quad + e(2z_3\dot{z}_3 - \dot{\tilde{u}}) \\
&= (z_2 - z_2^{ref})\Big(c_3(z_1 - z_1^{ref}) + \frac{k}{m}(x_0 - z_1) - \frac{\eta}{m}z_2 + \frac{f_d}{m} - \frac{az_3^2}{2m(b+z_1)^2}\Big) - (z_2 - z_2^{ref})(\dot{z}_2^{ref}) \\
&\quad + e(2z_3(\frac{(b+z_1)u}{a} - \frac{R(b+z_1)z_3}{a} + \frac{z_3 z_2}{(b+z_1)}) - \dot{\tilde{u}}) \\
&= e\Big(\frac{-a(z_2 - z_2^{ref})}{2m(b+z_1)^2} + 2z_3(\frac{b+z_1}{a}u - \frac{Rz_3}{a}(b+z_1) + \frac{z_2 z_3}{(b+z_1)}) - \dot{\tilde{u}}\Big) \\
&\quad - c_1(z_2 - z_2^{ref})^2
\end{aligned}$$

If we denote $T = \Big(\frac{-a(z_2 - z_2^{ref})}{2m(b+z_1)^2} + 2z_3(\frac{b+z_1}{a}u - \frac{Rz_3}{a}(b + z_1) + \frac{z_2 z_3}{(b+z_1)}) - \dot{\tilde{u}}\Big)$, we want to design the control input u such that $T = -c_2 e$, which in turn will give $\dot{V}_{aug} = -c_1(z_2 - z_2^{ref})^2 - c_2 e^2 < 0$ for all $z_2 \neq z_2^{ref}$, $e \neq 0$. The control input given by (3) and (4) will achieve this. The derivative of the virtual input term $\tilde{u}$ can be explicitly stated as

$$\begin{aligned}
\dot{\tilde{u}} &= \frac{4m(b+z_1)z_2}{a}\left(\frac{k}{m}(x_0 - z_1) - \frac{\eta z_2}{m} + c_1(z_2 - z_2^{ref}) + \frac{f_d}{m}\right) \\
&+ \frac{2m(b+z_1)^2}{a}\left(c_1\left(\frac{f_d}{m} + \frac{k(x_0 - z_1)}{m} - \frac{\eta z_2}{m} - \frac{az_3^2}{2m(b+z_1)^2} - \dot{z}_2{}^{ref}\right)\right) \\
&+ \frac{2m(b+z_1)^2}{a}\left(\frac{-kz_2}{m} - \frac{\eta}{m}(\frac{k}{m}(x_0 - z_1) - \frac{\eta z_2}{m} - \frac{az_3^2}{2m(b+z_1)^2} + \frac{f_d}{m})\right) \\
&+ \frac{4m(b+z_1)z_2}{a}\left(c_3(z_1 - z_1^{ref}) - \dot{z}_2{}^{ref}\right) \\
&+ \frac{2m(b+z_1)^2}{a}\left(c_3(z_2 - z_2{}^{ref}) - \ddot{z}_2{}^{ref}\right).
\end{aligned} \tag{7}$$

Utilizing the control input $u(t)$ given by (3), (4) and (7), we have

$$\dot{V}_{aug} = -c_1(z_2 - z_2^{ref})^2 - c_2 e^2 := W(\mathbf{z}) \leq 0 \tag{8}$$

where $W(\cdot)$ is positive semi-definite. This implies, by the virtue of LaSalle-Yoshizawa Theorem [11], that all the solutions of (2) are globally uniformly bounded and satisfy $\lim_{t\to\infty} W(\mathbf{z}(t)) = 0$. This means that $\lim_{t\to\infty} z_2 = z_2^{ref}$ and $\lim_{t\to\infty} e = 0$ starting from any initial condition $z_2(t_0)$ and $i(t_0)$. Hence, we obtain the following zero dynamics for the subsystem given in (5):

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= \frac{k}{m}(x_0 - z_1) - \frac{\eta}{m}z_2 - \frac{a}{2m(b+z_1)^2}\tilde{u} + \frac{f_d}{m}.
\end{aligned} \tag{9}$$

Substituting $\tilde{u}$ from (4) into (9), we obtain

$$\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= \dot{z}_2^{ref} - c_3(z_1 - z_1^{ref}) - c_1(z_2 - z_2^{ref}).
\end{aligned} \tag{10}$$

Writing the second equation in (10) in terms of z_1 and z_1^{ref} only, and introducing $e_{z_1} := z_1 - z_1^{ref}$, we obtain

$$\ddot{e}_{z_1} + c_1\dot{e}_{z_1} + c_3 e_{z_1} = 0 \tag{11}$$

It can be seen that if c_3 and c_1 are selected such that

$$-c_1 \pm \sqrt{c_1^2 - 4c_3} < 0 \tag{12}$$

the roots of the characteristic equation of (11) would be negative, which in turn would imply $\lim_{t\to\infty} z_1 = z_1^{ref}$ starting from any initial condition $z_1(t_0)$. In the next section we discuss how multiparameter extremum seeking scheme is utilized along with backstepping controller to render the backstepping robust to uncertainties in the system parameters.

B. Robustification of the backstepping Controller

The controller (3), (4) ensures asymptotic tracking of the desired trajectories for the nominal system (2), however, in real applications the model is never known perfectly. For instance the damping coefficient η can be hard to estimate and might drift over time due to the wear and tear of the system. Similarly, the spring constant k can vary very slowly over time. For these reasons, we need to improve, i.e. robustify, the controller (3), (4), with respect to the unknown parameters k, η. The main idea of this work is that instead of using classical adaptive techniques to estimate these parameters e.g. [12], we use model-free learning techniques to do so, e.g. MES. One noticeable advantage is less constraints on the uncertainties' structure, e.g. the assumption of linearity in the parameters is not necessary anymore, since the model-free learning approaches are not limited to a particular class of uncertainties. Furthermore, these type of learning algorithms alow for the estimation of a set of uncertain parameters simultaneously, which can be hard to achieve with classical adaptive algorithms, e.g. gradient descent-based estimation filters [12]. A full comparison between classical adaptive control techniques and the idea that we proposing here is ongoing, and will not be reported here due to space limitations, but will appear in a longer version of this work. To use the MES learning algorithm, we first define the cost function to be minimized as

$$\begin{aligned}
Q(z(\beta)) &= C_1(z_1(t_f) - z_1^{ref}(t_f))^2 \\
&+ C_2(z_2(t_f) - z_2^{ref}(t_f))^2
\end{aligned} \tag{13}$$

where $C_1,\ C_2 > 0$, and $\beta = (\partial\hat{k},\ \partial\hat{\eta})'$ represents the variations of the learned parameters $(\hat{k}, \hat{\eta})$ defined such that

$$\begin{aligned}
\hat{k} &= k_{nominal} + \partial\hat{k} \\
\hat{\eta} &= \eta_{nominal} + \partial\hat{\eta}
\end{aligned} \tag{14}$$

where $k_{nominal},\ \eta_{nominal}$ are the nominal initial values of the uncertain parameters, i.e. the best know values from which to start learning the actual values.

Folowing multi-parametric extremum seeking theory [13], [14], the variations of the estimated parameters are defined as

$$\begin{aligned}
\dot{x}_{\hat{k}} &= a_{\hat{k}} sin(\omega_1 t + \tfrac{\pi}{2})Q(z(\beta)) \\
\partial\hat{k}(t) &= x_{\hat{k}}(t) + a_{\hat{k}} sin(\omega_1 t + \tfrac{\pi}{2}) \\
\dot{x}_{\hat{\eta}} &= a_{\hat{\eta}} sin(\omega_2 t + \tfrac{\pi}{2})Q(z(\beta)) \\
\partial\hat{\eta}(t) &= x_{\hat{\eta}}(t) + a_{\hat{\eta}} sin(\omega_2 t + \tfrac{\pi}{2})
\end{aligned} \tag{15}$$

where $a_{\hat{k}},\ a_{\hat{\eta}}$, are positive tuning parameters, and

$$\omega_1 \neq \omega_2. \tag{16}$$

1) Stability discussion: Under the constraints (16) and quadratic approximation of the cost function, the convergence of the previous learning algorithm has been proven, e.g. [13], [14]. Proving the stability of the combined controller (3), (4) and the learning (14) and (15), is much more challenging. A rigorous convergence proof of the whole controller is under development, and will be presented in a long version of this work. However, we want to discuss here the intuition behind the proof. Indeed, we showed in Lemma 1 that the nominal controller (3), (4), achieves asymptotic tracking of the desired trajectory if applied to the nominal model (2). Now the question is how does it behave if the parameters k, η

Parameter	Value
m	$0.27\ [kg]$
R	$6\ [\Omega]$
η	$7.53\ [kg/sec]$
x_0	$8\ [mm]$
k	$158\ [N/mm]$
a	$14.96 \times 10^{-6}\ [Nm^2/A^2]$
b	$4 \times 10^{-5}\ [m]$

TABLE I

NUMERICAL VALUES OF THE MECHANICAL PARAMETERS

are uncertain, i.e. their real value differ from their assumed value. In the targeted application, i.e. electromagnetic actuator, the drift of these parameters happens over a long period of time, and thus we can expect the nominal controller to still maintain some kind of bounded input/ bounded output stability for small errors of the uncertain parameters, which means that the tracking error bound will be correlated to the error in the parameters. Since we are using the MES learning algorithm to track these slow changes of the parameters, we can expect a bounded tracking error over time, with a decreasing error-bound correlated to a decreasing parameter estimation error over the learning iterations. Again, this discussion does not pretend to be a proof of stability of the whole controller, it is only a practical explanation of the stable behavior that we observed during the application of this scheme to our example, shown on the numerical results presented in the next section, however, a more rigorous proof will be presented in our future reports.

V. SIMULATIONS

We show here the behavior of the proposed approach on the example of electromagnetic actuator presented in [10], where the model (1) is used with the numerical values of Table I. The desired trajectory has been selected as the $5th$ order polynomial $x^{ref}(t) = \sum_{i=0}^{5} a_i(t/t_f)^i$, where the a_is have been computed to satisfy the boundary constraints $x^{ref}(0) = 0.2, x^{ref}(t_f) = x_f, \dot{x}^{ref}(0) = \dot{x}^{ref}(t_f) = 0, \ddot{x}^{ref}(0) = \ddot{x}^{ref}(t_f) = 0$, with $t_f = 0.5\ sec$, $x_f = 0.7\ mm$. In these simulations we neglect the effect of f_d, since this force is usually very small comparatively to the spring force. We introduced initial errors both on the position and the velocity $z_1(0) = 0.01\ mm$, $z_2(0) = 0.1\ mm/sec$. Due to paper length limitations, we will not report here the numerical results of the nominal case, i.e. controller (3) and (4), applied to the nominal model (2) without any parametric uncertainties. The convergence from any states initial conditions is obvious from Lemma 1. We rather report here the more challenging cases of uncertain model, and learning control. We consider that the parameters $k,\ \eta$ have an initial estimation error of $\delta k = -10\ N/mm$, $\delta \eta = -1\ kg/sec$. We test the robustness of the whole controller, i.e. the backstepping (3) and (4), merged with the learning algorithm (14) and (15), with the following parameters: $c_1 = 100$, $c_2 = 5000$, $c_3 = 100$, $C_1 = C_2 = 100$, $k_{nominal} = 158\ N/mm$, $\eta_{nominal} = 7.53\ kg/sec$, $a_{\hat{k}} = a_{\hat{\eta}} = 1$, $\omega_1 = 7.5\ rad/sec$, $\omega_2 = 7.4\ rad/sec$. We decided to stop the simulations, when the controller has driven the cost function under the threshold 10^{-4}. Figure 1, shows the cost function over time. It is clear that the learning algorithm is converging and drives the cost function to less that 10^{-4} after about 800 iterations. The number of iterations might seem high, but we underline here that we are imposing a very small tracking error target (cost target of 10^{-4}). Furthermore, we have deliberately imposed high uncertainties values on k and η, to test the controller in challenging cases. In any real application the drift of this parameters happens very slowly over time and the learning algorithm will track it in realtime, and will need much less iterations to track the actual values of the parameters. We report on Figures 2, 3 the behavior of the learned parameters $\partial\hat{k},\ \partial\hat{\eta}$ in equation (14). Both parameters converge to the desired 'actual' value of uncertainties -10 and -1, respectively. Note that, since we choose a relatively high excursion values of the dither signals, $a_{\hat{k}} = a_{\hat{\eta}} = 1$, the learning converges to a neighborhood of desired values with an oscillation of the same amplitude as the dither signal. This can be easily tuned by using smaller amplitudes $a_{\hat{k}},\ a_{\hat{\eta}}$, but will lead to a slower convergence. Another way is to stop the learning when the cost function is less than the desired threshold and resume the learning when the cost function rises above the threshold if the system parameters start drifting again. In our simulations we decided to stop the learning when the cost function values reached the threshold, in this case the final learned parameters values were $\partial\hat{k} = -9.1924\ N/mm,\ \partial\hat{\eta} = -0.8417\ kg/sec$. We report on Figures 4, 5 the armature position and velocity trajectories, before and after the learning, vs. the desired trajectories. We see clearly that without the learning algorithm, the tracking is lost, whereas, when the learning is used the tracking of the desired trajectories is recovered. Now we show that the proposed learning-based adaptive control is not limited by the structure of the uncertainties, i.e. the classical assumption of linearity w.r.t. the uncertain parameters is not needed here. To do so, we choose as an example to learn the parameter b appearing as nonlinear terms in the model (1). We assume an uncertainty of $\delta b = 0.01\ mm$ which is 25% of the nominal value, no uncertainties are considered for k and η. We use the same learning algorithm (15) for only one variable $\partial\hat{b}$, with $a_{\hat{b}} = 10^{-3}$ and $\omega_{\hat{b}} = 7.5\ rad/sec$. The same cost function is used with $C_1 = C_2 = 100$, and the same gains are used for the backstepping controller. We show on Figures 6, 7, 8 and 9, the obtained results. It is clear that the uncertain parameter converges towards a neighborhood of its actual value $0.01\ mm$ (Figure 7), keeping in mind again that we purposely choose a large uncertain value of 25% to show the behavior of the algorithm on a challenging case. The convergence of the cost function is shown on Figure 6, here again we stopped the learning after the cost function reached the threshold 10^{-4}. The tracking performance, with and without learning, are shown on Figures 8 and 9. The correction of the tracking errors achieved with the adaptive learning-based algorithm is clearly depicted.

VI. CONCLUSION

We have studied in this paper the problem of adaptive control for electromagnetic actuators. We have developed a nominal nonlinear trajectory tracking controller based on backstepping approach. We have proven the global stability of this nominal controller. We have complemented the nominal controller with a model-free learning algorithm, i.e. MES, to estimate on-line a vector of uncertain parameters in the actuator model. In this sense we have solved an adaptive control problem using a model-free learning algorithm. We have shown the performance of the combined backstepping and MES on a numerical example.

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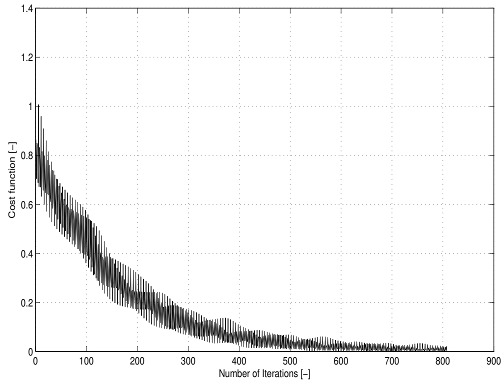

Fig. 1. Cost function vs. learning iterations

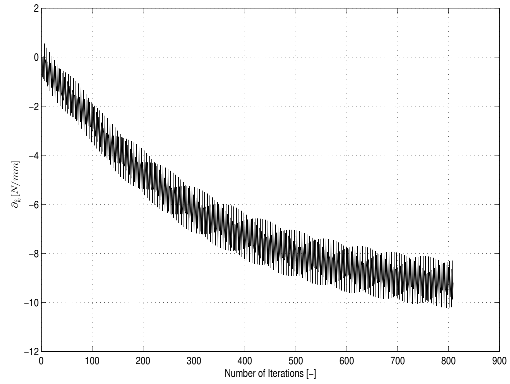

Fig. 2. $\partial\hat{k}$ vs. learning iterations

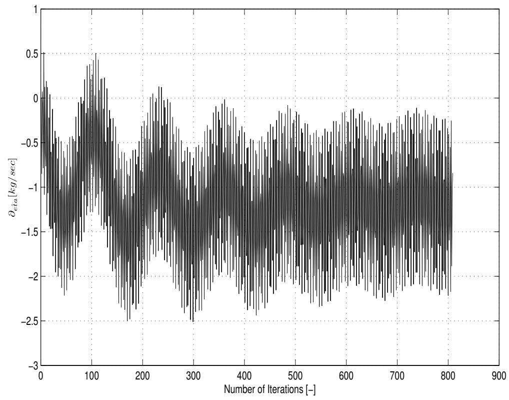

Fig. 3. $\partial\hat{\eta}$ vs. learning iterations

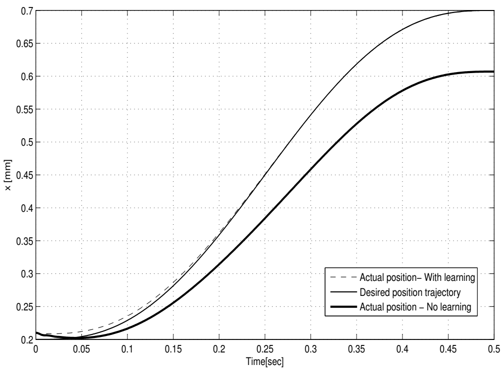

Fig. 4. Obtained armature position vs. reference trajectory

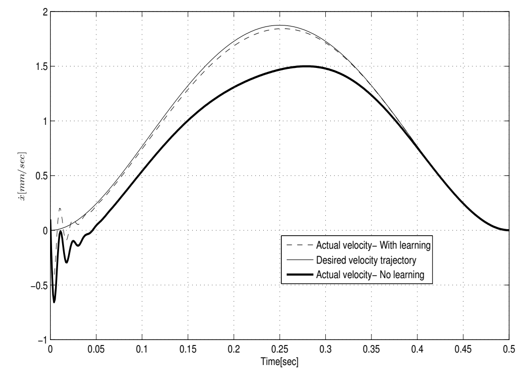

Fig. 5. Obtained armature velocity vs. reference trajectory

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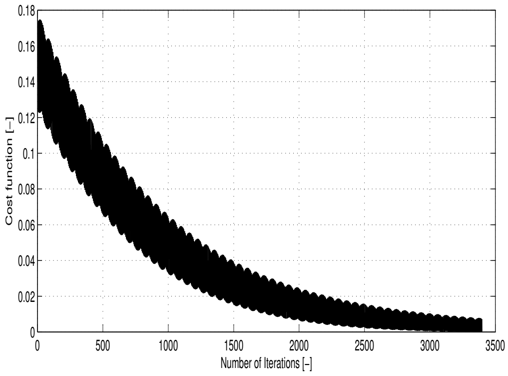

Fig. 6. Cost function vs. learning iterations

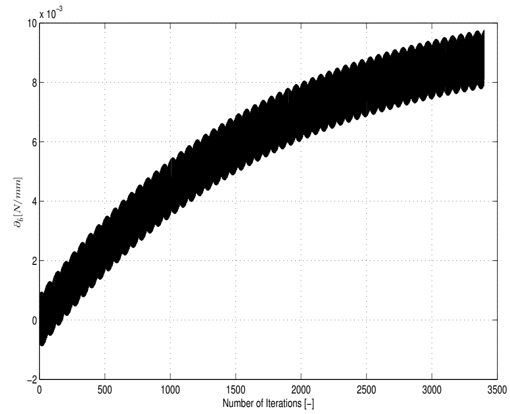

Fig. 7. $\partial\hat{b}$ vs. learning iterations

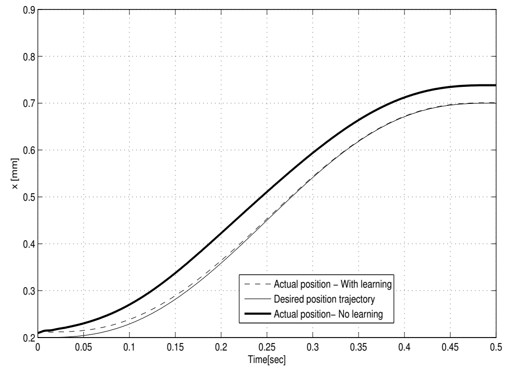

Fig. 8. Obtained armature position vs. reference trajectory

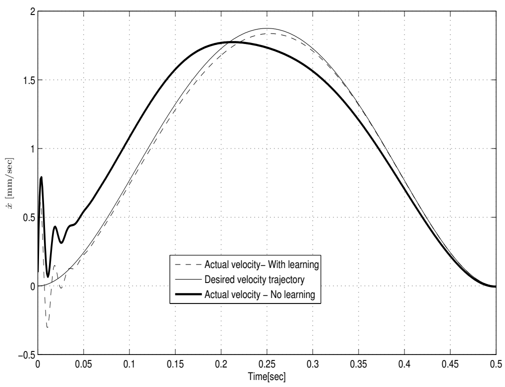

Fig. 9. Obtained armature velocity vs. reference trajectory